Mathematics 1

Guide

NOVA SCOTIA
NOUVELLE-ÉCOSSE

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Mathematics 1

Prepared by the Department of Education and Early Childhood Development

This is the most recent version of the current curriculum materials as used by teachers in Nova Scotia.

Mathematics 1

Draft
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Mathematics

Prepared by the Department of Education and Early Childhood Development

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Contents

Introduction

Background and Rationale

Mathematics curriculum is shaped by a vision that fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in society. It is essential that the mathematics curriculum reflect current research in mathematics instruction. To achieve this goal, Western and Northern Canadian Protocol's (WNCP) *The Common Curriculum Framework for K–9 Mathematics* (2006) has been adopted as the basis for the new mathematics curriculum in Nova Scotia.

The Common Curriculum Framework (WNCP 2006) was developed by the seven ministries of education (Alberta, British Columbia, Manitoba, Northwest Territories, Nunavut, Saskatchewan, and Yukon Territory) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators, and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and performance indicators agreed upon by the seven jurisdictions. The outcomes and performance indicators have been adapted for Nova Scotia. This document is based on both national and international research by the WNCP and the National Council of Teachers of Mathematics (NCTM).

There is an emphasis in the Nova Scotia curriculum on particular key concepts at each grade that will result in greater depth of understanding and, ultimately, stronger student achievement. There is also a greater emphasis on number sense and operations concepts in the early grades to ensure students develop a solid foundation in numeracy.

Purpose

This document provides sets of outcomes and performance indicators to be used as a mandated common base for defining mathematics curriculum expectations. This common base should result in consistent student outcomes in mathematics within the province of Nova Scotia. It should also enable easier transfer for students moving within the province or from any jurisdiction that has adopted the WNCP framework. This document is intended to clearly communicate to all education partners across the province the high expectations for students' mathematical learning.

Program Design and Components

Assessment

Ongoing assessment for learning is essential to effective teaching and learning. Research has shown that assessment for learning (formative assessment) practices produce significant and often substantial learning gains, close achievement gaps, and build students' ability to learn new skills (Black & Wiliam 1998; OECD 2006). Student involvement in assessment promotes learning. Timely and effective teacher feedback and student self-assessment allow students to reflect on and articulate their understanding of mathematical concepts and ideas.

Assessment in the classroom includes
- providing clear goals, targets, and learning outcomes
- using exemplars, rubrics, and models to help clarify outcomes and identify important features of the work
- monitoring progress towards outcomes and providing feedback as necessary
- encouraging self-assessment
- fostering a classroom environment where conversations about learning take place, where students can check their thinking and performance and develop a deeper understanding of their learning

(Davies 2000)

Assessment for learning practices act as the scaffolding for learning, which only then can be measured through assessment of learning (summative assessment). Assessment of learning tracks student progress, informs instructional programming, and aids in decision making. Both forms of assessment are necessary to guide teaching, stimulate learning, and produce achievement gains.

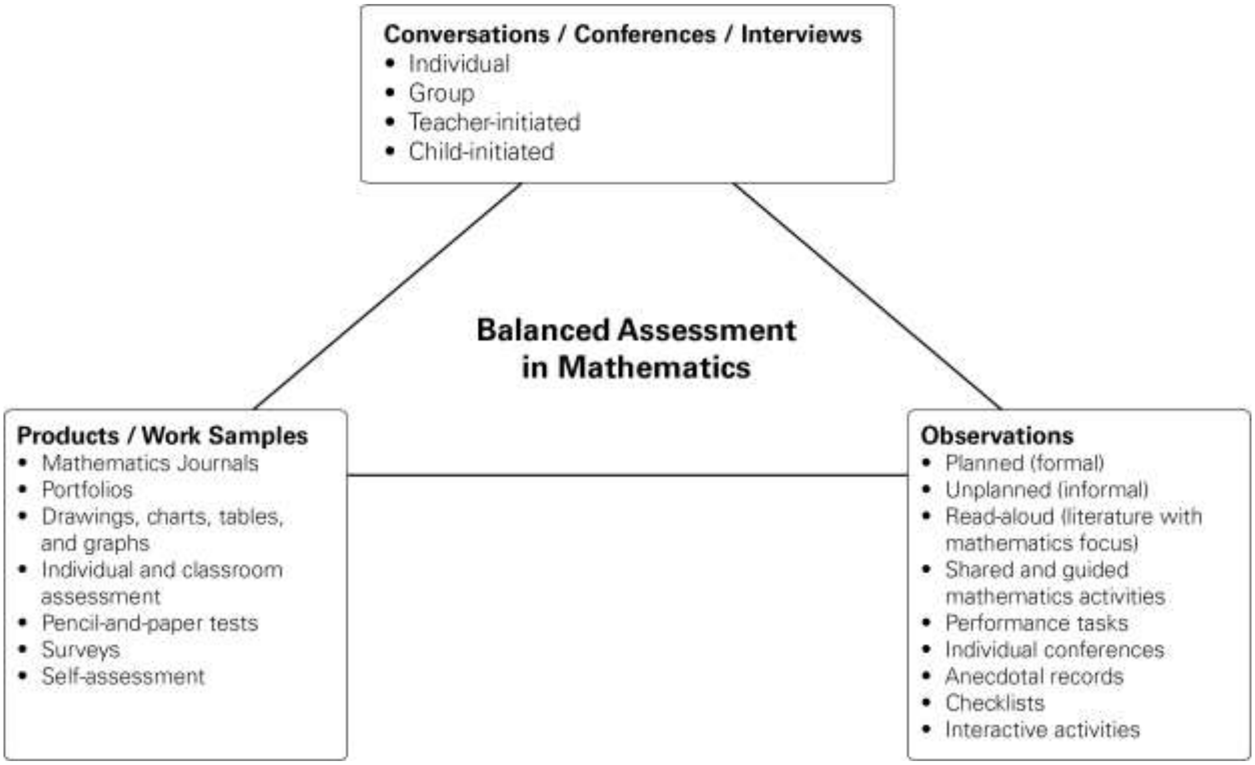
Assessment of student learning should
- align with curriculum outcomes
- clearly define criteria for success
- make explicit the expectations for students' performance
- use a wide variety of assessment strategies and tools
- yield useful information to inform instruction

Cognitive Levels

Cognitive levels of questions in mathematics are defined as:
- Knowledge questions may require students to recall or recognize information, names, definitions, or steps in a procedure.
- Application/comprehension questions may require students to make connections, represent a situation in more than one way (translating between representations), or solve contextual problems.
- Analysis questions may require students to go beyond comprehension and application to higher order thinking skills, such as generalizations and non-routine problem solving.

Classroom assessment and instruction should incorporate tasks with varied levels of cognitive demand. The recommended percentages for questions used in classroom-based assessments are:
  - Knowledge 20–30%
  - Application / Comprehension 50–60%
  - Analysis 10–20%

Assessment evidence may be gathered through conversation and interviews, observations, and products.



Time to Learn for Mathematics

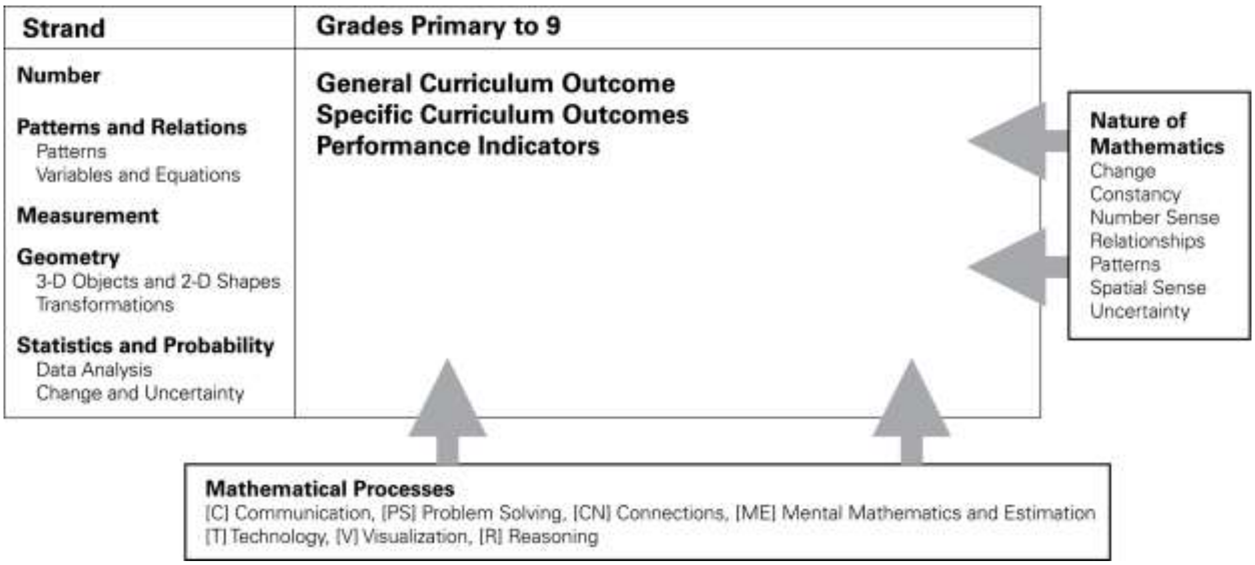
The *Plan for Instruction Revised Time to Learn Strategy: Grades Primary*–3 (Nova Scotia Department of Education 2015) and *Revised Time to Learn Strategy: Grades 4*–6 (Nova Scotia Department of Education 2016) requires a block of time for mathematics instruction each day. In order to support a constructivist approach to teaching through problem solving, 90 minutes of mathematics instruction is required daily in grades primary–6 . It is recommended that this instruction be provided in an uninterrupted block of time.

Time to Learn guidelines can be found at
www.ednet.ns.ca/files/ps-policies/semestering.pdf
The Time to Learn Plan for Instruction document for Grade P-3 can be found at:
https://www.ednet.ns.ca/files/curriculum/Time2Learn-Rev-June25-2015.pdf

Outcomes

Conceptual Framework for Mathematics Primary–9

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



(Adapted with permission from Western and Northern Canadian Protocol, *The Common Curriculum Framework for K–9 Mathematics*, p. 5.

Structure of the Mathematics Curriculum

Strands

The learning outcomes in the Nova Scotia Framework are organized into five strands across grades primary to 9.

- Number (N)
- Patterns and Relations (PR)
- Measurement (M)
- Geometry (G)
- Statistics and Probability (SP)

General Curriculum Outcomes (GCO)

Some strands are further subdivided into sub-strands. There is one general outcome (GCO) per sub-strand. GCOs are overarching statements about what students are expected to learn in each strand/sub-strand. The general curriculum outcome for each strand/sub-strand is the same throughout the grades.

Number (N)
GCO: Students will be expected to demonstrate number sense.

Patterns and Relations (PR)
Patterns
GCO: Students will be expected to use patterns to describe the world and solve problems.

Variables and Equations
GCO: Students will be expected to represent algebraic expressions in multiple ways.

Measurement (M)
GCO: Students will be expected to use direct and indirect measure to solve problems.

Geometry (G)
3-D Objects and 2-D Shapes
GCO: Students will be expected to describe the characteristics of 3-D objects and 2-D shapes and analyze the relationships among them.

Transformations
GCO: Students will be expected to describe and analyze position and motion of objects and shapes.

Statistics and Probability (SP)
Data Analysis
GCO: Students will be expected to collect, display, and analyze data to solve problems.

Chance and Uncertainty
GCO: Students will be expected to use experimental or theoretical probabilities to represent and solve problems involving uncertainty.
The Mathematics 1 curriculum contains outcomes for Number (N), Patterns and Relations (PR), Measurement (M), and Geometry (G).

Statistics and Probability (SP) outcomes are not part of the Mathematics 1 curriculum. Data analysis outcomes are introduced in Mathematics 2. Chance and uncertainty outcomes are introduced in Mathematics 5.

Specific Curriculum Outcomes (SCOs) and Performance Indicators

Specific curriculum outcomes (SCOs) are statements that identify the specific conceptual understanding, related skills, and knowledge students are expected to attain by the end of a given grade.

Performance indicators are statements that identify specific expectations of the depth, breadth, and expectations for the outcome. Teachers use these statements to determine whether students have achieved the corresponding specific curriculum outcome.

Process Standards Key

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

NUMBER (N)

N01 Students will be expected to say the number sequence by

- 1s, forward and backward between any two given numbers, 0 to 100
- 2s to 20, forward starting at 0
- 5s to 100, forward starting at 0, using a hundred chart or a number line
- 10s to 100, forward starting at 0, using a hundred chart or a number line [C, CN, V, ME]

Performance Indicators

N01.01 Recite forward by 1s the number sequence between two given numbers, 0 to 100.
N01.02 Recite backward by 1s the number sequence between two given numbers, 0 to 100.
N01.03 Record a given numeral, 0 to 100, presented orally.
N01.04 Read a given presented numeral, 0 to 100.
N01.05 Skip count by 2s to 20 starting at 0.
N01.06 Skip count by 5s to 100 starting at 0, using a hundred chart or a number line.
N01.07 Skip count forward by 10s to 100 starting at 0, using a hundred chart or a number line.
N01.08 Identify and correct errors and omissions in a given number sequence.

N02 Students will be expected to recognize, at a glance, and name the quantity represented by familiar arrangements of one to ten objects or dots. [C, CN, ME, V]

Performance Indicators

N02.01 Look briefly at a given familiar arrangement of objects or dots, and identify the number represented without counting.
N02.02 Identify the number represented by a given arrangement of counters or dots on a ten-frame.

N03 Students will be expected to demonstrate an understanding of counting to 20 by

- indicating that the last number said identifies "how many"
- showing that any set has only one count
- using the counting-on strategy [C, CN, ME, R, V]

Performance Indicators

N03.01 Answer the question, How many are in the set? using the last number counted in a given set.
N03.02 Identify and correct counting errors in a given counting sequence.
N03.03 Show that the count of the number of objects in a given set does not change regardless of the order in which the objects are counted.
N03.04 Record the number of objects in a set using the numeral symbol.
N03.05 Determine the total number of objects in a given set, starting from a known quantity and counting on.

N04 Students will be expected to represent and partition numbers to 20. [C, CN, V]

Performance Indicators

N04.01 Represent a given number up to 20 using a concrete materials, such as ten-frames, linking cubes, and created materials.
N04.02 Represent a given number up to 20 pictorially.
N04.03 Find examples of a given number in the environment.
N04.04 Place given numerals on a number line with benchmarks 0, 5, 10, 15, and 20.
N04.05 Partition any given quantity up to 20 into two parts and identify the number of objects in each part and in the whole.
N04.06 Model a given number using two different objects.

N05 Students will be expected to compare sets containing up to 20 objects to solve problems using referents and one-to-one correspondence. [C, CN, ME, PS, R, V]

Performance Indicators

N05.01 Construct a set that has more, fewer, or as many objects as a given set, up to 20 objects.

N05.02 Construct several sets of different objects that have the same given number of objects in the set.

N05.03 Compare two given sets using one-to-one correspondence and describe them using comparative words, such as *more*, *fewer,* or *as many*.

N05.04 Compare a set to a given referent using comparative language.

N05.05 Solve, using pictures and words, given story problems that involve the comparison of two quantities.

N06 Students will be expected to estimate quantities to 20 by using referents. [C, ME, PS, R, V]

Performance Indicators

N06.01 Estimate a given quantity by comparing it to a given referent (known quantity).

N06.02 Select between two or more possible estimates for a given quantity and explain the choice.

N07 Students will be expected to demonstrate an understanding of conservation of number for up to 20 objects. [C, R, V]

Performance Indicators

N07.01 Explain why for a given number of counters, no matter how they are arranged, the total number of counters does not change.

N07.02 Group a set of given counters in more than one way.

N07.03 Explain why for a given number of counters, no matter how they are grouped, the total number of counters does not change.

N08 Students will be expected to identify the number, up to 20, that is one more, two more, one less, and two less than a given number. [C, CN, ME, R, V]

Performance Indicators

N08.01 Name the number, up to 20, that is one more, two more, one less, or two less than a given number, up to 20.

N08.02 Represent a number, up to 20, on ten-frames that is one more, two more, one less, or two less than a given number.

N09 Students will be expected to demonstrate an understanding of the addition of two one-digit numbers and the corresponding subtraction, concretely, pictorially, and symbolically, in join, separate, equalize/compare, and part-part-whole situations. [C, CN, ME, PS, R, V]

Performance Indicators

N09.01 Act out story problems that are presented orally or through shared reading.

N09.02 Model story problems with manipulatives or pictures, find and share solutions using counting strategies, and record number sentences that represent how they thought about the problems.

N09.03 Create story problems that connect to student experiences.

N09.04 Create story problems for given number sentences.

N10 Students will be expected to use and describe strategies to determine sums and differences using manipulatives and visual aids. Strategies include

- counting on or counting back
- one more or one less
- making ten
- doubles
- near doubles [C, CN, ME, PS, R, V]

Performance Indicators

N10.01 Use and describe a personal strategy to determine a sum.
N10.02 Use and describe a personal strategy to determine a difference.
N10.03 Use and describe how two different strategies can be used to determine a sum or difference.

PATTERNS AND RELATIONS (PR)

PR01 Students will be expected to demonstrate an understanding of repeating patterns (two to four elements) by identifying, describing, reproducing, extending, and creating patterns using manipulatives, diagrams, sounds, and actions. [C, PS, R, V]

Performance Indicators

PR01.01 Describe a given repeating pattern containing two to four elements in its core.
PR01.02 Identify errors in a given repeating pattern.
PR01.03 Identify the missing element(s) in a given repeating pattern.
PR01.04 Create and describe a repeating pattern using a variety of manipulatives, musical instruments, and actions.
PR01.05 Reproduce and extend a given repeating pattern using manipulatives, diagrams, sounds, and actions.
PR01.06 Identify and describe a repeating pattern in the environment (e.g., classroom, outdoors) using everyday language.
PR01.07 Identify repeating events (e.g., days of the week, birthdays, seasons).
PR01.08 Identify the core of a repeating pattern.

PR02 Students will be expected to translate repeating patterns from one representation to another. [C, R, V]

Performance Indicators

PR02.01 Represent a given repeating pattern using another mode(e.g., actions to sound; colour to shape; ABC, ABC, ABC to blue, yellow, green, blue, yellow, green, blue, yellow, green,…)
PR02.02 Describe a given repeating pattern using a letter code(e.g., ABC, ABC, ABC ….)

PR03 Students will be expected to describe equality as a balance and inequality as an imbalance, concretely and pictorially (0 to 20). [C, CN, R, V]

Performance Indicators

PR03.01 Construct two equal sets using the same objects (same shape and mass) and demonstrate their equality of number using a balance scale.
PR03.02 Construct two unequal sets using the same objects (same shape and mass) and demonstrate their inequality of number using a balance scale.
PR03.03 Determine if two given concrete sets are equal or unequal and explain the process used.

PR04 Students will be expected to record equalities using the equal symbol. [C, CN, PS, V]

Performance Indicators

PR04.01 Represent a given pictorial or concrete equality in symbolic form.
PR04.02 Represent a given equality using manipulatives or pictures.
PR04.03 Provide examples of equalities where the given sum or difference is on either the left or right side of the equal symbol (=).
PR04.04 Record different representations of the same quantity (0 to 20) as equalities.

Measurement (M)

M01 Students will be expected to demonstrate an understanding of measurement as a process of comparing by

- identifying attributes that can be compared
- ordering objects
- making statements of comparison
- filling, covering, or matching [C, CN, PS, R, V]

Performance Indicators

M01.01 Identify common attributes, such as length, mass, volume, capacity, and area that could be used to compare a given set of two objects.
M01.02 Compare and order two given objects and identify the attributes used to compare.
M01.03 Predict which object in a set is longest/shortest, determine by matching and explain the reasoning.
M01.04 Predict which object in a set is heaviest/lightest, determine by comparing and explain the reasoning.
M01.05 Predict which object in a set is largest/smallest, determine by comparing and explain the reasoning.
M01.06 Predict which object in a set holds the most/least, determine by filling and explain the reasoning.
M01.07 Predict which figure in a set has the greatest/least area, determine by covering and explain the reasoning.

Geometry (G)

G01 Students will be expected to sort 3-D objects and 2-D shapes using one attribute and explain the sorting rule. [C, CN, R, V]

Performance Indicators

G01.01 Sort a given set of familiar 3-D objects or 2-D shapes using a given sorting rule.
G01.02 Sort a given set of familiar 3-D objects using one attribute determined by the student, and explain the sorting rule.
G01.03 Sort a given set of 2-D shapes using a one attribute determined by the student and explain the sorting rule.
G01.04 Determine the difference between two given pre-sorted sets of familiar 3-D objects or 2-D shapes and explain a possible sorting rule used to sort them.

G02 Students will be expected to replicate composite 2-D shapes and 3-D objects. [CN, PS, V]

Performance Indicators

G02.01 Select 2-D shapes from a given set of 2-D shapes to reproduce a given composite 2-D shape.
G02.02 Select 3-D objects from a given set of 3-D objects to reproduce a given composite 3-D object.

G02.03 Predict and select the 2-D shapes used to produce a composite 2-D shape and verify by deconstructing the composite shape.
G02.04 Predict and select the 3-D objects used to produce a composite 3-D object and verify by deconstructing the composite object.

G03 Students will be expected to identify 2-D shapes in 3-D objects. [C, CN, V]

Performance Indicators
G03.01 Identify the shape of the faces of a 3-D object.
G03.02 Identify 3-D objects in the environment that have faces that are a given 2-D shape.

Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics.

Students are expected to
- communicate in order to learn and express their understanding of mathematics (Communication [C])
- develop and apply new mathematical knowledge through problem solving (Problem Solving [PS])
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines (Connections [CN])
- demonstrate fluency with mental mathematics and estimation (Mental Mathematics and Estimation [ME])
- select and use technologies as tools for learning and solving problems (Technology [T])
- develop visualization skills to assist in processing information, making connections, and solving problems (Visualization [V])
- develop mathematical reasoning (Reasoning [R])

The Nova Scotia curriculum incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning. The key to these process standards is presented in a box, as shown below, with each specific curriculum outcome within the strands.

Process Standards Key

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing, and modifying ideas, knowledge, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics.

Students also need to communicate their learning using mathematical terminology. Communication can help students make connections between and among the different representational modes—contextual, concrete, pictorial, linguistic/verbal, and symbolic—of mathematical ideas. Students must communicate *daily* about their mathematics learning. This enables them to reflect, to validate, and to clarify their thinking and provides teachers with insight into students' interpretations of mathematical meanings and ideas.

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, How would you ...? or How could you ...? the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement, perseverance, and collaboration.

Problem solving is also a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive, mathematical risk takers.

When students are exposed to a wide variety of problems in all areas of mathematics, they explore various methods for solving and verifying problems. In addition, they are challenged to find multiple solutions for problems and to create their own problem.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to one another or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

"Because the learner is constantly searching for connections on many levels, educators need to *orchestrate the experiences* from which learners extract understanding. … *Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching*." (Caine and Caine 1991, 5).

Mathematics should be viewed as an integrated whole rather than as the study of separate strands or units. Connections must also be made between and among the different representational modes—contextual, concrete, pictorial, linguistic/verbal, and symbolic. The process of making connections, in turn, facilitates learning. Concepts and skills should also be connected to everyday situations and other curricular areas. For example, when developing literacy skills students learn to make text-to-world, text-to-text, and text-to-self connections. Students can also make connections to make mathematics come alive through math-to-world, math-to-math, and math-to-self connections.

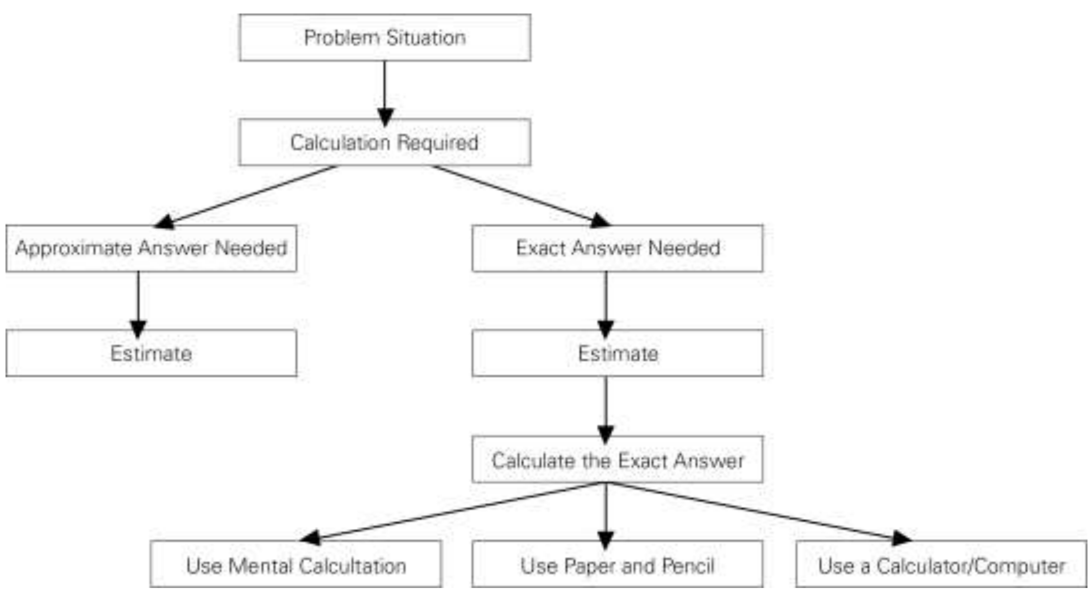
Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy, and flexibility. "Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental math." (National Council of Teachers of Mathematics, May 2005).

Students proficient with mental mathematics "become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers, and are more able to use multiple approaches to problem solving." (Rubenstein 2001) Mental mathematics "provides a cornerstone for all estimation processes, offering a variety of alternative algorithms and nonstandard techniques for finding answers." (Hope 1988, v)

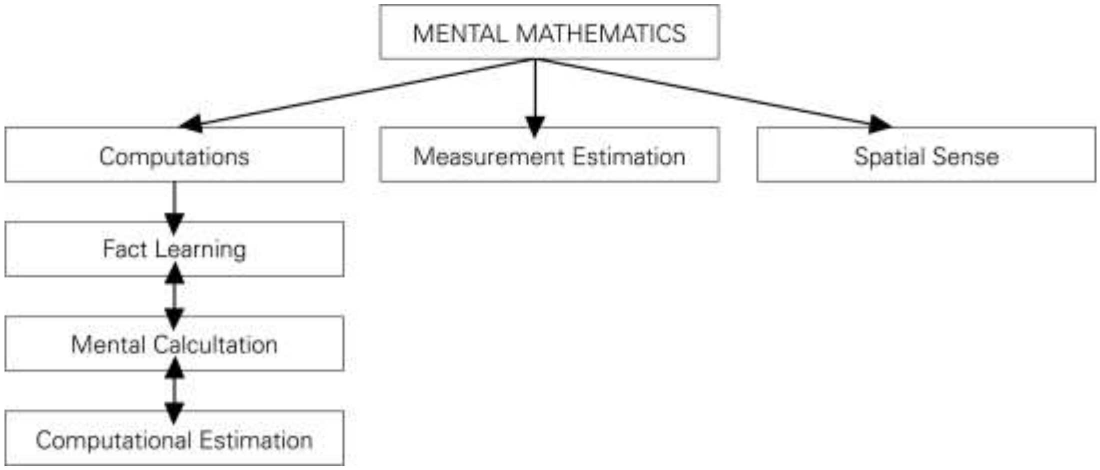
Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and what strategy to use when estimating. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision-making process as illustrated on the following page.



The skill of estimation requires a sound knowledge of mental mathematics. Both are necessary to many everyday experiences, and students should be provided with frequent opportunities to practise these skills.

For the purpose of this document and to provide some uniformity in communication, it is important that some mental mathematics terms are defined. Nova Scotia uses the term mental mathematics to encompass the whole range of mental processing of information in all strands of the curriculum. This mental mathematics is broken into three categories in this document: computations, measurement estimation, and spatial sense. Computations are further broken down into fact learning, mental calculations, and computational estimation.



Fact learning refers to the acquisition of the 100 number facts related to the single digits 0 to 9 for each of the four operations, addition, subtraction, multiplication, and division. When students know these facts, they can quickly retrieve them from memory (usually in three- to five- seconds). Ideally, through practice over time, students will achieve automaticity; that is, they will have instant recall without using strategies. *Mental calculation* refers to getting exact answers by using strategies to do the calculations in one's head, while *computational estimation* refers to getting approximate answers by using strategies to do calculations in one's head.

While each category in computations has been defined separately, this does not suggest that the three categories are entirely separable. Initially, students develop and use strategies to get quick recall of the facts. These strategies and the facts themselves are the foundations for the development of other mental calculation strategies. When the facts are automatic, students are no longer employing strategies to retrieve them from memory. In turn, the facts and mental calculation strategies are the foundations for computational estimation strategies. Actually, attempts at computational estimation are often thwarted by the lack of knowledge of the related facts and mental calculation strategies.

Measurement estimation is the process of using internal and external visual (or tactile) information to get approximate measures, or to make comparisons of measures, without the use of measurement instruments.

Spatial sense is an intuition about shapes and their relationships, and an ability to manipulate shapes in one's mind. It includes being comfortable with geometric descriptions of shapes and positions.

In Mathematics 1, students are expected to continue to develop some important concepts about number and pre-operation skills that will support the study of mental mathematics in later grades. These concepts include

- counting
  - rote counting - saying the number names in order
  - meaningful counting – determining quantity
- representing numbers
- spatial relationships and subitizing (recognize at a glance a quantity of objects without counting)
- comparing numbers
  - more than
  - the same as
  - fewer than
  - one more/one less
  - two more/two less

- benchmarks and referents - anchors to 10
- partitioning numbers (part-part-whole)

Students also begin to use strategies to determine sums and differences, though they are not expected to quickly recall addition or subtraction facts.

Throughout the year, students should be working toward developing these very important concepts using dot cards, dot cubes, dot plates, five-frames, ten-frames, and number cubes.

Technology [T]

Technology can be effectively used to contribute to and support the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Technology can be used to

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties
- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense

The use of calculators is recommended to enhance problem solving, to encourage discovery of number patterns, and to reinforce conceptual development and numerical relationships. They do not, however, replace the development of number concepts and skills. Carefully chosen computer software can provide interesting problem-solving situations and applications.
Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in grades primary–6 to enrich learning, it is expected that students will achieve all outcomes without the use of technology, unless the use of technology is specified in the outcome.

The Integration of Information and Communication Technology Within the Curriculum, (Revision 2014) (P–6) can be found online in several locations including: http://lrt.ednet.ns.ca.

Visualization [V]

Visualization “involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world.” (Armstrong 1999). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers. These mental images are needed to develop concepts and understand procedures. Images and explanations help students clarify their understanding of mathematical ideas in all strands.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure, when to estimate and to know several estimation strategies. (Shaw and Cliatt 1989)

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Mathematics reasoning involves informal thinking, conjecturing, and validating—these help students understand that mathematics makes sense. Students are encouraged to justify, in a variety of ways, their solutions, thinking processes, and hypotheses. In fact, good reasoning is as important as finding correct answers.

Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that define the nature of mathematics, and these are woven throughout this document. These components include change, constancy, number sense, relationships, patterns, spatial sense, and uncertainty.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as

- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen 1990, 184).

Constancy

Different aspects of constancy are described by the terms **stability**, **conservation**, **equilibrium**, **steady state**, and **symmetry** (AAAS–Benchmarks 1993, 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems. Examples of constancy include the following:

- The area of a rectangular region is the same regardless of the methods used to determine the solution.
- The sum of the interior angles of any triangle is 180°.
- The theoretical probability of flipping a coin and getting heads is 0.5.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (British Columbia Ministry of Education 2000, 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences and use benchmarks and referents. This results in students who are computationally fluent, flexible with numbers, and have intuition about numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, and concepts. The search for possible relationships involves the collection and analysis of data, and describing relationships visually, symbolically, orally, or in written form.

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all strands, and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with an understanding of their environment. Patterns may be represented in concrete, visual, or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create, and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking, which is foundational for working with more abstract mathematics in higher grades.

Spatial Sense

Spatial sense involves visualization, mental imagery, and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to reason and interpret among and between 3-D and 2-D representations and identify relationships to mathematical strands. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes. Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions. For example,

- knowing the dimensions of an object enables students to communicate about the object and create representations
- the volume of a rectangular solid can be calculated from given dimensions
- doubling the length of the side of a square increases the area by a factor of four

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Curriculum Document Format

This guide presents the mathematics curriculum so that a teacher may readily view the scope of the outcomes that students are expected to meet during that year. Teachers are encouraged, however, to examine what comes before and what follows after, to better understand how students' learning at a particular grade level is part of a bigger picture of concept and skill development.

The order of presentation in no way assumes or prescribes a preferred order of presentation in the classroom, but simply lays out the specific curriculum outcomes in relation to the overarching general curriculum outcomes (GCOs).

The footer of the document shows the name of the course, and the strand name is presented in the header. When a specific curriculum outcome (SCO) is introduced, it is followed by the mathematical processes and performance indicators for that outcome. A scope and sequence is then provided, which relates the SCO to previous and next grade SCOs. Also, for each SCO, there is background information, assessment strategies, suggested instructional strategies, suggested models and manipulatives, mathematical language, and a section for resources and notes. For each section, the guiding questions should be used to help with unit and lesson preparation.

SCO

Mathematical Processes
[C] Communication **[PS]** Problem Solving **[CN]** Connections
[ME] Mental Mathematics and Estimation
[T] Technology **[V]** Visualization **[R]** Reasoning

Performance Indicators

Describes observable indicators of whether students have achieved the specific outcome.

Scope and Sequence

Previous grade or course SCOs	Current grade SCO	Following grade or course SCOs

Background

Describes the “big ideas” to be learned and how they relate to work in previous grade and work in subsequent courses.

Performance Indicator Background

Contains further elaborations for the performance indicators.

Assessment, Teaching, and Learning

Assessment Strategies

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Sample tasks that can be used to determine students’ prior knowledge.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Some suggestions for specific activities and questions that can be used for both instruction and assessment

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Correlations to related resources.

Planning for Instruction

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcome and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Suggested strategies for planning daily lessons.

SUGGESTED LEARNING TASKS

Suggestions for general approaches and strategies suggested for teaching this outcome.

Guiding Questions
- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

SUGGESTED MODELS AND MANIPULATIVES

MATHEMATICAL LANGUAGE

Teacher and student mathematical language associated with the respective outcome.

Resources/Notes

Contexts for Learning and Teaching

Beliefs about Students and Mathematics Learning

“Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.” (National Council of Teachers of Mathematics 2000, 20).

The Nova Scotia mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning that have grown out of research and practice. These beliefs include the following:

- Mathematics learning is an active and constructive process.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.
- Learners are individuals who bring a wide range of prior knowledge and experiences and who learn via various styles and at different rates.
- Learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking and that nurtures positive attitudes and sustained effort.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Children develop a variety of mathematical ideas before they enter school. They make sense of their environment through observations and interactions at home and in the community. Mathematics learning is embedded in everyday activities, such as playing, reading, storytelling, and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Curiosity about mathematics is fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks, and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do and need to construct their own meaning of mathematics. This meaning is best constructed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with and translating through a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial, contextual, and symbolic representations of mathematics.

The learning environment should value and respect all students’ experiences and ways of thinking, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must realize that it is acceptable to solve problems in different ways and that solutions may vary.

Goals for Mathematics Education

The main goals of mathematics education are to prepare students to

- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- make connections between mathematics and its applications
- become mathematically literate adults, using mathematics to contribute to society

Students who have met these goals will

- gain understanding and appreciation of the contributions of mathematics as a science, a philosophy, and an art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity about mathematics and situations involving mathematics

Opportunities for Success

A positive attitude has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for student success help develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations, and engage in reflective practices.

To experience success, students must be taught to set achievable goals or assess their progress as they work toward these goals. Striving toward success and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessing of personal goals.

Engaging All Learners This now matches the background in the 4-6 documents.

“No matter how engagement is defined or which dimension is considered, research confirms this truism of education: *The more engaged you are, the more you will learn.*” (Hume 2011, 6)

Student engagement is at the core of learning. Engagement in learning occurs when students are provided with opportunities to become more invested in their learning. This is critical for teachers to take into account when planning and implementing instruction. Effective instruction engages, embraces, and supports all learners through a range of learning experiences that are both age and developmentally appropriate.

This curriculum is designed to provide learning opportunities that are equitable, accessible, and inclusive of the many facets of diversity represented in today’s classrooms. When teachers know their students as individual learners and as individual people, their students are more likely to be motivated to learn, persist in challenging situations, and apply reflective practices.

Supportive Learning Environments

A supportive and positive learning environment has a profound effect on students' learning. Students need to feel physically, socially, emotionally, and culturally safe in order to take risks with their learning. In classrooms where students feel a sense of belonging, see their teachers' passion for learning and teaching, are encouraged to actively participate, and are challenged appropriately, they are more likely to be successful.

Teachers recognize that not all students progress at the same pace nor are they equally positioned in terms of their prior knowledge of particular concepts, skills, and learning outcomes. Teachers are able to create more equitable access to learning when

- instruction and assessment are flexible and offer multiple means of representation
- students have options to engage in learning through multiple ways
- students can express their knowledge, skills, and understanding in multiple ways

(Hall, Meyer, and Rose 2012)

In a supportive learning environment, teachers plan learning experiences that support *each* student's ability to achieve curriculum outcomes. Teachers use a variety of effective instructional approaches that help students to succeed, such as

- providing a range of learning opportunities that build on individual strengths and prior knowledge
- providing all students with equitable access to appropriate learning strategies, resources, and technology
- involving students in the creation of criteria for assessment and evaluation
- engaging and challenging students through inquiry-based practices
- verbalizing their own thinking to model comprehension strategies and new learning
- balancing individual, small-group, and whole-class learning experiences
- scaffolding instruction and assignments as needed and giving frequent and meaningful descriptive feedback throughout the learning process
- integrating "blended learning" opportunities by including an online environment that extends learning beyond the physical classroom
- encouraging students to take time and to persevere, when appropriate, in order to achieve a particular learning outcome

Multiple Ways of Learning

"Advances in neuroscience and education research over the past 40 years have reshaped our understanding of the learning brain. One of the clearest and most important revelations stemming from brain research is that there is no such thing as a 'regular student.'" (Hall, Meyer, and Rose 2012, 2) Teachers who know their students well are aware of students' individual learning differences and use this understanding to inform instruction and assessment decisions.

The ways in which students make sense of and demonstrate learning vary widely. Individual students tend to have a natural inclination toward one or a few learning styles. Teachers are often able to detect learning strengths and styles through observation and through conversation with students. Teachers can also get a sense of learning styles through an awareness of students' personal interests and talents. Instruction and assessment practices that are designed to account for multiple learning styles create greater opportunities for all students to succeed.

While multiple learning styles are addressed in the classroom, the three most commonly identified are:

- auditory (such as listening to teacher-modelled think-aloud strategies or participating in peer discussion)
- kinesthetic (such as examining artifacts or problem-solving using tools or manipulatives)
- visual (such as reading print and visual texts or viewing video clips)

For additional information, refer to *Frames of Mind: The Theory of Multiple Intelligences* (Gardner 2007) and *How to Differentiate Instruction in Mixed-Ability Classrooms* (Tomlinson 2001).

A Gender-Inclusive Curriculum and Classroom

It is important that the curriculum and classroom climate respect the experiences and values of all students and that learning resources and instructional practices are not gender-biased. Teachers promote gender equity and inclusion in their classrooms when they

- articulate equally high expectations for all students
- provide equal opportunity for input and response from all students
- model gender-fair language, inclusive practices, and respectful listening in their interactions with students
- identify and openly address societal biases with respect to gender and sexual identity

Valuing Diversity: Teaching with Cultural Proficiency

"Instruction that is embedded in socially meaningful contexts, and tasks that are meaningful and relevant to the lives of students, will engage students in high-level problem-solving and reasoning and enhance students' engagement (Frankenstein 1995; Gutstein 2003; Ladson-Billings 1997; Tate 1995)." (Herzig 2005)

Teachers appreciate that students have diverse life and cultural experiences and that individual students bring different prior knowledge to their learning. Teachers can build upon their knowledge of their students as individuals, value their prior experiences, and respond by using a variety of culturally-proficient instruction and assessment practices in order to make learning more engaging, relevant, and accessible for all students. For additional information, refer to *Racial Equity Policy* (Nova Scotia Department of Education 2002) and *Racial Equity / Cultural Proficiency Framework* (Nova Scotia Department of Education 2011).

Students with Language, Communication, and Learning Challenges

Today's classrooms include students who have diverse language backgrounds, abilities, levels of development, and learning challenges. By observing and interacting with students and by conversing with students and/or their families, teachers gain deeper insights into the student as a learner. Teachers can use this awareness to identify and respond to areas where students may need additional support to achieve their learning goals. For students who are experiencing difficulties, it is important that teachers distinguish between those students for whom curriculum content is challenging and those for whom language-based factors are at the root of apparent academic difficulties. Students who are learning English as an additional language may require individual support, particularly in language-based subject areas, while they become more proficient in their English language skills. Teachers understand that many students who appear to be disengaged may be experiencing difficult life or family circumstances, mental health challenges, or low self-esteem, resulting in a loss of confidence that affects their engagement in learning. A caring, supportive teacher demonstrates belief in the students' abilities to learn and uses the students' strengths to create small successes that help nurture engagement in learning and provide a sense of hope.

STUDENTS WHO DEMONSTRATE EXCEPTIONAL TALENTS AND GIFTEDNESS

Modern conceptions of giftedness recognize diversity, multiple forms of giftedness, and inclusivity. Some talents are easily observable in the classroom because they are already well developed and students have opportunities to express them in the curricular and extracurricular activities commonly offered in schools. Other talents only develop if students are exposed to many and various domains and hands-on experiences. Twenty-first century learning supports the thinking that most students are more engaged when learning activities are problem-centred, inquiry-based, and open-ended. Talented and gifted students usually thrive when such learning activities are present. Learning experiences may be enriched by offering a range of activities and resources that require increased cognitive demand and higher-level thinking with different degrees of complexity and abstraction. Teachers can provide further challenges and enhance learning by adjusting the pace of instruction and the breadth and depth of concepts being explored. For additional information, refer to *Gifted Education and Talent Development* (Nova Scotia Department of Education 2010).

Connections across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the students' understanding of mathematical concepts and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating mathematics in health education, literacy, music, physical education, science, social studies, and visual arts.

Number (N)

GCO: Students will be expected to demonstrate number sense.

SCO N01 Students will be expected to say the number sequence by ▪ 1s, forward and backward between any two given numbers, 0 to 100 ▪ 2s to 20, forward starting at 0 ▪ 5s to 100, forward starting at 0, using a hundred chart or a number line ▪ 10s to 100, forward starting at 0, using a hundred chart or a number line [C, CN, V, ME]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N01.01 Recite forward by 1s the number sequence between two given numbers, 0 to 100.
N01.02 Recite backward by 1s the number sequence between two given numbers, 0 to 100.
N01.03 Record a given numeral, 0 to 100, presented orally.
N01.04 Read a given presented numeral, 0 to 100.
N01.05 Skip count by 2s to 20 starting at 0.
N01.06 Skip count by 5s to 100 starting at 0, using a hundred chart or a number line.
N01.07 Skip count forward by 10s to 100 starting at 0, using a hundred chart or a number line.
N01.08 Identify and correct errors and omissions in a given number sequence.

Scope and Sequence

Mathematics Primary	**Mathematics 1**	**Mathematics 2**
N01 Students will be expected to say the number sequence by ▪ 1s, from 1 to 20 ▪ 1s, starting anywhere from 1 to 10 and from 10 to 1	**N01** Students will be expected to say the number sequence by ▪ 1s, forward and backward between any two given numbers, 0 to 100 ▪ 2s to 20, forward starting at 0 ▪ 5s to 100, forward starting at 0 using a hundred chart or a number line ▪ 10s to 100, forward starting at 0, using a hundred chart or a number line	**N01** Students will be expected to say the number sequence by ▪ 1s, forward and backward, from any point to 200 ▪ 2s, forward and backward, starting from any point to 100 ▪ 5s and 10s, forward and backward, using starting points that are multiples of 5 and 10 respectively to 100 ▪ 10s, starting from any point to 100

Background

Meaningful counting involves an understanding of the six principles of counting. These principles are addressed in various outcomes (N01, N03, and N07). The principle in bold face is the one addressed in this outcome.

1. One number is said for each item in the group and is counted once and only once. (one-to-one correspondence)
2. Counting begins with the number 1, and there is a set number sequence. (stable order)
3. The quantity in the set is the last number said. (cardinality)
4. The starting point and order of counting the objects does not affect the quantity. (order irrelevance)
5. The arrangement or types of objects does not affect the count. (conservation)

6. It does not matter what is being counted, the resulting count will always be the same. (abstraction)

Stable Order Principle: In everyday use there are two meanings of “to count.” One meaning is to say the numbers in order (outcome N01 addresses this meaning) which addresses the stable order principle. As early as two years of age, children can repeat words such as one, two, and three; however, children most likely do not understand the quantity represented by those numbers. Understanding quantity is the second meaning of “to count” (outcomes N03 and N07 address this second meaning). In Mathematics Primary, students learned the names of the numbers in the counting sequence to 20 and focused on meaningful counting for collections up to 10. In Mathematics 1, students will extend the number names in the counting sequence to 100 and will develop meaningful counting for collections up to 20.

Learning the number names is an important prerequisite for counting items in a set. “Before there can be any meaningful counting, students must be able to recite the sequence beginning 1, 2, 3, 4, 5, etc.” (Small 2009, 84) There is a difference between being able to recite the number words (one, two, three...) and understanding how counting is used to describe a set. The counting sequence itself is a rote procedure; however, “The *meaning* attached to counting is the key conceptual idea on which all other number concepts are developed.” (Van de Walle and Lovin 2006, 39).

Being able to immediately state the number before or after any given number up to 100 is an important pre-operation skill. The emphasis here being able to state the number with no hesitation. This ability is essential for students to use later to identify the number that is *one more and one less* than a given number in outcome N08. It will also allow students to reach the counting-on stage necessary for success with outcome N03. Once students are able to give automatic responses to numbers from 1 to 9, this should be extended to the teen numbers, and then numbers in the 20s and beyond using an awareness of the pattern in the sequence of whole numbers.

Students should learn how to count on and count back from a given number without starting at 0. In order to do this, students must understand the concepts of more and less as well as the counting sequence. Students’ counting experiences should lead to a mental mathematics stage in which students can, without concrete materials or pictures, count on by 1s from any given number (0 to 100) and count back by 1s from any given number (0 to 100). Students counting experiences should also enable them to skip count by 2 to 20 without concrete materials or pictures. Tasks that involve skipping counting by 5s to 100 and 10s to 100 starting from 0 should involve the use of a number line or hundred chart.

(Note: In Mathematics 1, students are expected only to recite number names to 100 and are not expected to develop quantity sense for these numbers. Quantity sense in Mathematics 1 focuses on numbers to 20.)

Performance Indicator Background

N01.01 and N01.02 Most students use the auditory patterns that exist in our number system to learn these counting sequences from 20 to 99, including the sequence of decades (20, 30, 40, …, 90) and the 1 to 9 sequence within each decade (20, 21, 22, 23, …, 29, 30, 31, 32, 33, …, 39). Students should be very comfortable counting forward up to 100 before counting backward. Initial backward counting experiences should build on their experience counting back from 10 in Mathematics Primary, by counting back from a decade, such as 50, 49, 48, …, 41, 40.

Students should experience a wide variety of situations throughout the school year that require counting so they become familiar with counting patterns. Counting should be integrated into work in other subjects. Students can reinforce saying the number sequence forward and backward through various opportunities throughout the day, such as lining up for recess.

N01.03 and N01.04 In order to use a hundred chart, or number line, in support of developing number sequences to 100, students will need to read and record numerals. Students should be able to say “twenty-seven” when shown the numeral “27.” When a number is presented orally, students should be able to identify and record the numeral. For example, say the number “twenty-seven”. Ask students to identify where the number “twenty-seven” is on the hundred chart and to write the numeral “27.”

While students are reading, identifying, and recording numerals to 100, they are doing so through patterning, not through place value knowledge. They will view the two digits in the numerals as a way to “spell” the number. For example, fifty-seven is spelled with a 5 and a 7 to produce 57.

Because it is important that students develop an efficient means of recording numerals, numeral writing should be taught. Allow the students to experiment freely on lined and unlined paper, whiteboards, chart paper, and other mediums. Observe students as they write their numerals, both when copying from a model and when forming them from memory. Students should be encouraged to start at the top when writing numerals. Be very careful not to assume that because a student has learned to write the numerals they are learning anything about the quantities they represent.

In these early years, it is not unusual for many students to reverse some of the digits as they write them. These students could be helped by getting them to compare the digits they write with ones you write, and by reminding them how to start making the digit. This practice should always be done in context. A number reversal is when a student records a digit in reverse “”. The number 14 is still considered to be written correctly if there is a digit reversal; if it is written as 1. The ones digit is reversed but the numeral still correctly represents the number 14 because the place value of tens and ones is correctly represented. It is mathematically incorrect when the number *fourteen* is written symbolically as 41 or 1. This is not a reversal of a digit. This is a mathematical error because this numeral does not represent 14. The “4” is recorded in the tens place and the “1” is in the ones place. Thus, it is an error in recording the numeral because it represents a place value error. Teachers need to investigate the source of such an error. It may be caused by issues of directionality of print or it may be a misunderstanding of the recording of two-digit numbers. For example, a student may think because we say “fourteen” and we hear the “four” first, we also record the numeral 4 first.

N01.05, N01.06, and N01.07 Hundred charts and number lines are essential tools to support skip counting by 2s to 20 and by 5s and 10s to 100. For example, when skip counting by 5s, the student may place a counter on every fifth number, starting at 5, reading the number as each counter is placed on the numeral, and subsequently saying the sequence of numbers hidden by the counters. It is important that students realize that they read the hundred chart left to right and top to bottom. Students will have opportunities to apply these skip-counting sequences to count sets of objects to 20 in outcomes N03 and N07.

N01.08 After a variety of experiences saying the number sequences, students should be able to identify errors in counting sequences. For example, students should be able to say which number is omitted or incorrect in the sequence 31, 32, 34, 33, 35, … or 5, 10, 15, 20, 30, 35, … and they should be able to correct sequences that have been presented incorrectly.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to start at 1 and count forward to 20.
- Ask students to start at 10 and count backward.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to say the number sequence forward by 1s starting at a given number less than 100.
- Ask students to say the number sequence backwards by 1s starting at a given number less than 100.
- Ask students to begin saying the number sequence by 1s at 13 and stop at 25.
- Ask students to begin saying the number sequence by 1s at 68 and stop at 49.
-
- Ask students to skip count by 2s (or 5s or 10s) starting at 0.
- Begin reciting a number sequence from a starting point other than 1, for example 67, but omit some numbers. Ask students to tell you the numbers you omitted.
- Ask students to count by 2s (5s or 10s) as you clap. Have students tell you or record the final number when you finish clapping.
- Recite a number sequence with an error or a missing number. Ask students to correct the sequence.
- Have students "count-off" by 1s, 2s, 5s, and 10s. Observe whether students are able to follow the number sequence.
- Give each student a card with a numeral on it. Invite students put themselves in order using the number cards. The cards can be used to show counting by 1s, 2s, 5s, and 10s.
- Ask students, If you count by twos, starting at 0, will you say 17? Why or why not?
- Provide a hundred chart. Tell the students, I counted from 10 to 50 and only said 5 numbers. What numbers do you think I said?
- Tell students, I said, "10, 15, 20, 25," when I was skip counting. By what number was I skip counting?

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)
- Mathematics 1 Checkpoint 3, pp. 40–41

Planning for Instruction

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of my students be met?

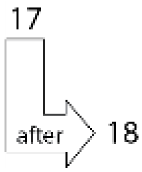
CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Use songs, poems, finger plays, and children's literature to support increasing and decreasing counting sequences. Sing songs and to read poems that involve reciting number sequences backward and forward and skip counting.
- Use a hundred chart or hundred mat. Both are excellent tools to explore counting patterns. For example, when skip counting by 2s, students might put a counter on every other number, reading the number as the counter is placed on it. This can be extended to counting by 5s and 10s.
- Have students use a walk-on number line (on the floor) to explore counting and skip counting.

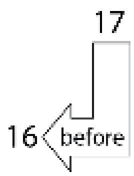
SUGGESTED LEARNING TASKS

- Engage students in echo counting. The teacher recites a number sequence and the students echo the teacher.
- Engage students in orchestra counting. Put students into groups. The first group begins reciting a number sequence, for example the sequence forward by 1s starting at 1. When the teacher points to another group, they continue from the last number stated.
- .
- Invite students to use calculators to count. For example, as some students place cookies into a bag and count aloud, others may repeatedly add one on calculators to keep track of the count electronically.
- Ask students to count items that occur naturally in 2s (e.g., shoes, hands, eyes) or 5s (fingers or toes on one hand) or 10s (the fingers on two hands).
- Begin saying a number sequence. Ask students to continue up to a particular number less than 100. For example, say "33, 34, 35, …" This activity can be repeated by reversing the number sequence. For example, "42, 41, 40, …"
- Say a number and the class or individual student responds with the number that comes after or before the stated number.
- Say two numbers, for example 56 and 58. The class or individual student responds with the number that comes between (after 56 and before 58) those two stated numbers.

- Draw a number machine on the board. It requires an input hopper and an output chute. Explain to students how the machine works; for example, This is a magic counting machine. It takes in a number up here and sends out the number that comes *after* that number down here. Place a number at the top of the machine. Ask students to say the number or hold up the numeral card that will be sent out the bottom of the machine.
  I put in my number.
  

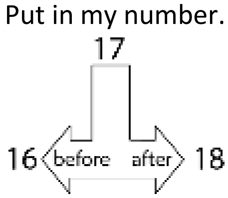
  The number that comes after 17 is 18.

  Change the machine so that it shows the number that comes before the given number.

  Put in a number.
  

  The number that comes before 17 is 16.

  Change the machine so that it shows both the numbers that come before and after the given number.

  Put in my number.
  

  The number that come before 17 is 16. The number that comes after 17 is 18.
  These activities should begin with the numbers to 20 and be extended to other decades as students develop proficiency with saying the number sequence forward and backward between any two given numbers, 0 to 100.
- Ask students to use a hundred chart or number line to count by 5s, starting at 0 and ending at 45. Record their responses on the whiteboard. Then ask, Do you see a pattern in the numbers that have been said? Describe it.
- Place number cards 1 to 100 in a container. Have a student select two cards. The students begin counting with the first number selected and stop when they get to the second number drawn. This may require students to either count forward or backward by 1s.
- Use a horizontal number line or a large floor chart with numbers 1 to 100. Ask the class to skip count by 2s (or 5s or 10s) and ask a student volunteer to step on each number as it is said. Discuss the movements made by the student volunteer.
- Ask students to use the repeat function on the calculator to skip count to a target number. For example, If you start at 0 and want to end on 40, by which number(s) could you skip count? What if you started at a different point? What if you wanted to end at a different point?
- Ask the student to count aloud to 50 by 5s while using the “constant function” on the calculator. (Enter “5,” press “+ 5,” press “=.” Each time the “=“ is pressed, another 5 is added: 10, 15, 20, 25, …)

- Have students work with a partner to create a number sequence with a missing number. Have students exchange their sequences and identify the missing number in the sequence they've received.
- Ask students how many ways they can count to 30. Have students talk about their findings.

SUGGESTED MODELS AND MANIPULATIVES

- calculators
- counters
- hundred chart
- hundred mat
- linking cubes
- walk-on number line

MATHEMATICAL LANGUAGE

Teacher	**Student** (oral language)
▪ counting numbers: one to one hundred ▪ count on, count back ▪ forward, backward ▪ hundred chart, number lines, hundred mat ▪ number sequence ▪ number, numeral ▪ position words: between, , next, after, before ▪ recite, ▪ skip count,	▪ counting numbers: one to one hundred ▪ count on, count back ▪ forward, backward ▪ hundred chart, number lines, hundred mat, ▪ number sequence ▪ number, numeral ▪ position words: between, , next, after, before ▪ skip count,

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 83–87
- *Making Math Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 139–144
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 37–41

Videos

- *An Introduction to Using Number Lines* (13:04 min.) (ORIGO Education 2010)
- *Analyzing Patterns (Skip Counting) on a Hundred Board* (27:16 min.) (ORIGO Education 2010)
- *Powerful Models to Help Struggling Students: Number Lines* (17:37 min.) (ORIGO Education 2010)
- *Teaching Number: 0 to 9* (14:47 min.) (ORIGO Education 2010)
- *Teaching Number: Counting* (10:49 min.) (ORIGO Education 2010)

Notes

SCO N02 Students will be expected to recognize, at a glance, and name the quantity represented by familiar arrangements of one to ten objects or dots. [C, CN, ME, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N02.01 Look briefly at a given familiar arrangement of objects or dots, and identify the number represented without counting.
N02.02 Identify the number represented by a given arrangement of counters or dots on a ten-frame.

Scope and Sequence

Mathematics Primary	**Mathematics 1**	**Mathematics 2**
N02 Students will be expected to recognize, at a glance, and name the quantity represented by familiar arrangements of one to five objects or dots.	**N02** Students will be expected to recognize, at a glance, and name the quantity represented by familiar arrangements of one to 10 objects or dots.	—

Background

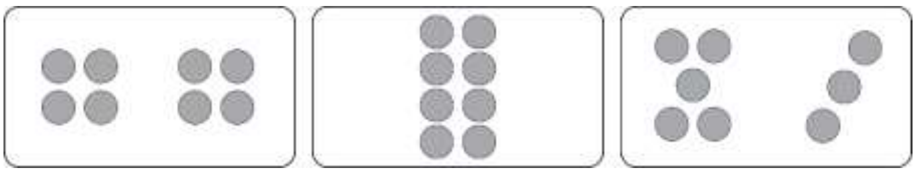
This outcome addresses **subitizing**. Subitizing is the ability to recognize, at a glance without counting, a quantity of objects (cardinality), for example, the ability to immediately recognize the number of dots on a standard dot cube when it is rolled or to immediately recognize a number on a five- or ten-frame when it is presented briefly. Subitizing small arrangements of objects helps students with counting on and composing and decomposing (partitioning) numbers. Students in Mathematics 1 should come to realize that subitizing and counting are two ways of determining how many are in a set of objects, and not think that counting is the only legitimate way to find how many. Students should recognize that there are many ways to arrange a set of objects, and that some arrangements are easier to subitize than others.

In Mathematics Primary, students subitized and named familiar arrangements of 1 to 5 dots and objects and identified the number represented by a given dot arrangement on a five-frame. In Mathematics 1, most students should be able to instantly recognize up to sets of 5, and through experience they can extend this to recognition of sets of up to 10. Initially, for some arrangements, some students may count the objects or dots, but by the end of Mathematics 1, they must be able to recognize at a glance without counting, configurations or spatial arrangements for numbers of items up to 10. Some students may subitize two parts of an arrangement and combine them. Having students share their subitizing strategies will benefit all students.

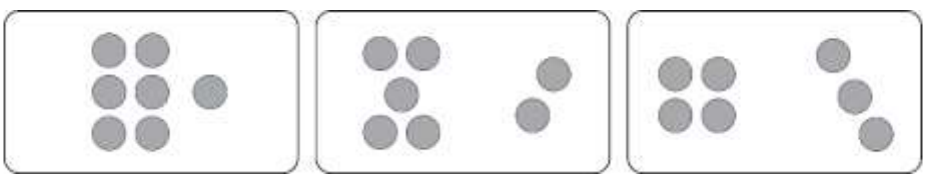
Performance Indicator Background

N02.01 Students must develop the ability to state the quantity when briefly presented with familiar arrangements of **up to 10** dots. They should **be given** just enough time to subitize and not do one-to-one counting. Start with arrangements up to 5 dots and gradually increase the number of dots until students can subitize arrangements of up to 10 dots.

To avoid the misconception that a specific quantity may only be represented if it is arranged in a certain way, it is very important to vary the arrangement of the objects, dots, or pictures. When asking students to identify the number of fingers use different combinations of fingers so that they do not believe that there is only one way to represent the number. For example, the number 6 can be represented with 5 fingers on one hand and 1 on the other, 2 fingers on one hand and 4 on the other, and 3 fingers on each hand. Similarly, each of the following three arrangements shows the quantity of 8.



Provide opportunities for students to discover and discuss which configurations are easier to recognize. For example, ask students to show 7 in several ways, and then decide which configuration(s) is (are) easiest to identify and explain why they prefer this arrangement. Possible configurations of 7 may include

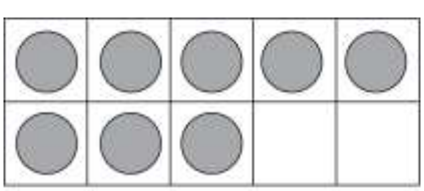


Set recognition should be reinforced through flash math activities where students are presented with a number configuration for a few seconds and are asked to identify the number after it is hidden. Sets of number configurations on a Smartboard, overhead, cards, and on paper plates would be good sources of materials for these reinforcements. Playing dominoes and games with standard dot cubes, also support students in developing set recognition.

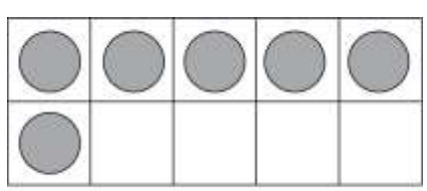
N02.02 Students should also have experiences with modelling numbers on a ten-frame to develop visual representations of numbers to 10. **A ten-frame is a rectangle with 2 rows of 5 boxes each large enough to hold a counter.** Using five-frames, and then ten-frames, helps to develop part-part-whole thinking.

When using a ten-frame, students should know that
- only one counter is permitted in each box of the ten-frame
- counters are typically placed from left to right in the top row until it is full and then from left to right in the bottom row

Students may think of numbers between 5 and 10 as “5 and some more.” For example, **8 is 5 and 3 more**



8 is 5 and 3 more

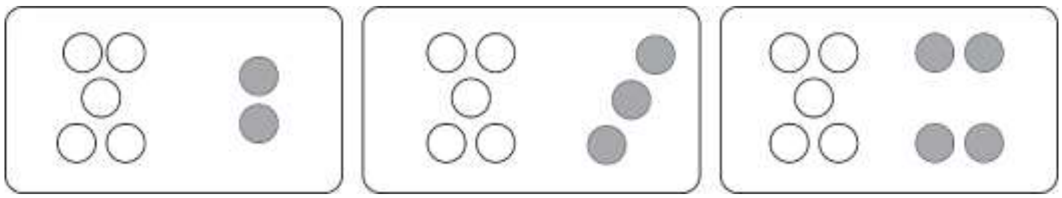


6 is 1 more than 5

When subitizing, it is important to use the familiar (typical) arrangement; however, when students are creating their own representations of a number other arrangements are acceptable.

Students' work with ten frames should eventually lead to a mental mathematics stage where they can visualize the standard ten-frame representations for numbers and answer questions from their visual memories.

Students should also understand that patterns for the numbers 6 to 10 can be made up of two patterns for smaller numbers. For example,
7 may be visualized as 5 and 2   8 may be visualized as 5 and 3   9 may be visualized as 5 and 4



Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge.

- Flash five-frame representations of numbers 1 to 5 for students in random order. Ask students to indicate the number shown on each five-frame by selecting the appropriate numeral card from a set of cards.

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Prepare a set of ten-frame cards. Show a card for one to three seconds depending upon the number shown. For example, a ten-frame containing 1 or 2 dots may be shown for a second, while a ten-frame containing 8 dots might be shown for two or three seconds. Ask students to respond by writing the numeral on their individual white board or paper that represents the ten-frame card shown. Ask students to describe what they saw (e.g., I saw 5 and 2 more, so I saw 7.).
- Use prepared dot arrangements for numbers 0 to 10. Hold up a dot plate for one to three seconds. Say, How many? How did you see it? Students might say, I saw 6; I saw 3 on one side and 3 on the other. Observe how quickly students can recognize the number of dots without counting.

- Ask students to draw an arrangement of counters that makes it easy to recognize 6.
- Ask students to arrange 8 counters (or another number) in a way that will make it easy to tell that there are 8.
- Ask students to make a sketch showing how they "see" 9 (or another number).
- Provide students with a set of counters. Flash a ten-frame card with dots for approximately three seconds. Have the students take the number of counters they think they would need to cover the dots displayed on the ten-frame. After students have made their sets of counters, place the card in front of one student who should then place counters on the dots, while the other students count and check. Ask the student to explain how they identified the number represented on the ten-frame. Repeat this activity using other ten-frame cards with different representations of numbers to 10.
- Play "rabbit ears." Students place their right hand beside their right ear and their left hand beside their ear. Ask them to use their fingers to show the number that you say. For example, if you say 7, a student may hold up 4 fingers on one hand and 3 fingers on the other hand. This allows you to see who is counting by ones and who is able to subitize.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Mathematics 1 Checkpoint 2, Task 2, pp. 32–33 (Line Masters 2.5 and 2.6)
- Mathematics 2 Checkpoint 2, Task 2, pp. 34–35 (Line Masters 2.7 and 2.8)

Planning for Instruction

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of my students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

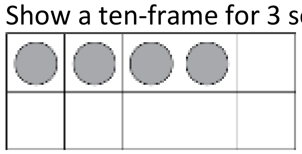
Consider the following strategies when planning daily lessons.

- Initially, focus subitizing activities on arrangements of numbers from 1 to 5 and gradually increase to 10.

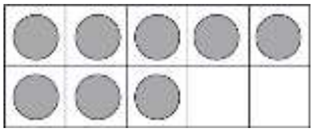
- Use “dot cards” and other models with easily recognizable configurations of numbers frequently so that students increase their familiarity with them. The level of difficulty can be adjusted by the arrangements used.
- After students become proficient in subitizing familiar arrangements, incorporate less familiar arrangements to promote greater automaticity in recognizing quantities. Less familiar arrangements could make use of two easily recognized arrangements, for example, 6 could be represented as 5 and 1.
- Have a set of cards or objects in your pocket. At any time during the day, show students one of the cards or group of objects and ask them to tell you how many.

Suggested Learning Tasks

- Ask students to quickly sort dot card arrangements into groups that display the same number. Students should not count the dots but should use subitizing to determine the sort.
- Arrange counters on an overhead projector or interactive whiteboard. Show the counters to the students for a few seconds, but not long enough for them to count the counters. Ask, What number was represented? (Repeat several times, using different configurations of the same number.) Ask, Which configuration was easiest to recognize? Why?
- Hold up a dot card for a few seconds. Ask, How many dots? What did you see first (all of the dots or a smaller group)? Include dot cards with both familiar and unfamiliar arrangements. Consider also using cards with two colours of dots to represent the two parts of the whole.
- State a number or hold up a numeral card and ask students to quickly find the corresponding dot card or ten-frame from a collection on their desks.
- Play a “concentration game.” Each pair of students will need two sets of dot cards, or ten-frame cards, or both that show the same numbers. They place 20 dot cards face down in a 5 x 4 array. Students take turns turning over two cards at a time trying to find a match. If the student finds a match, the student keeps the pair. If the student does not make a match, the cards are turned back over. Play continues until all pairs of dot cards or ten-frame cards are found.
- Students play in pairs. Players place their sets of ten-frame or dot cards face down in a pile in front of them. On a signal, both players turn over the top card and identify the number shown. The player who names the number first keeps the card.
- Play a favourite board game with your students using dot cards or dot cubes instead of a number cube.
- Provide each student with about 10 counters and a piece of construction paper as a mat. Hold up a dot card, dot plate, or ten-frame for about 3 seconds. Ask, How many dots did you see? How did you see them? Make the pattern you saw using the counters on the mat. Tell the number of counters that you saw. Spend some time discussing the configuration of the pattern and how many dots. Do this with a few new patterns each day.
- Give each student a set of numeral cards (1-10). Hold up a dot card or ten-frame and ask students to hold up the corresponding numeral card.
- Ask students to visualize 7 on a ten-frame. Ask, How many dots are in the first row? How many are in the second row? How many more dots are needed to make 10? What number would you have if you added one more dot? What number would you have if you removed two dots?
- Show a ten-frame for 3 seconds. For example,

  Ask, What number did you see? How many dots would be added to make 5? How many more to make 10? What number would be shown if you removed two dots?
- Display a ten-frame for a few seconds. For example,



Ask students to explain what they saw. A student might say, I saw 8 dots - 5 in the full top row and 3 in the bottom row. Another student might say, I saw 8 dots because I saw there are 2 empty cells. I need two more dots to make 10.

- Show three dot plates that display familiar arrangements of dots (two of the plates should display the same amount but have different arrangements, and the third should display a different amount). Ask students to identify, without counting, which plate shows the different amount.
- Using individual ten-frames, ask students to show you a number that you announce to the class. Walk around and see if they have the correct number of corresponding dots (counters).
- Display a ten-frame, and after three seconds cover it. Ask students to place counters on an empty ten-frame to copy what they saw. Repeat with different quantities.

SUGGESTED MODELS AND MANIPULATIVES

- collections of counters or objects
- dominoes
- dot cards
- five-frames and ten-frames
- number cubes
- playing cards

MATHEMATICAL LANGUAGE

Teacher	**Student** (oral language)
▪ dots, dot cards, familiar dot arrangements ▪ five-frame or ten-frame, counters ▪ quantity of objects or dots ▪ sets	▪ dots ▪ five-frame or ten-frame, counters

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 89–90
- *Making Math Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 145–148
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 43–44

Videos

- *Developing Sight Recognition of Quantity* (Subitizing) (12:03 min.) (ORIGO Education 2010)
- *Using a Hands-on Approach to Represent Numbers to 10* (13:06 min.) (ORIGO Education 2010)
- *Using Structured Patterns to Develop Number Combinations* (18:11 min.) (ORIGO Education 2010)

Notes

SCO N03 Students will be expected to demonstrate an understanding of counting to 20 by ▪ indicating that the last number said identifies "how many" ▪ showing that any set has only one count ▪ using the counting-on strategy [C, CN, ME, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N03.01 Answer the question, How many are in the set? using the last number counted in a given set.
N03.02 Identify and correct counting errors in a given counting sequence.
N03.03 Show that the count of the number of objects in a given set does not change regardless of the order in which the objects are counted.
N03.04 Record the number of objects in a set using the numeral symbol.
N03.05 Determine the total number of objects in a given set, starting from a known quantity and counting on.

Scope and Sequence

Mathematics Primary	**Mathematics 1**	**Mathematics 2**
N03 Students will be expected to relate a numeral, 1 to 10, to its respective quantity. **N06** Students will be expected to demonstrate an understanding of counting to 10.	**N03** Students will be expected to demonstrate an understanding of counting to 20 by ▪ indicating that the last number said identifies "how many" ▪ showing that any set has only one count ▪ using the counting-on strategy	**N01** Students will be expected to say the number sequence by ▪ 1s, forward and backward starting from any point to 200 ▪ 2s, forward and backward starting from any point to 100 ▪ 5s to 100, forward and backward, using starting points that are multiples of 5 and 10 respectively, to 100 ▪ 10s, starting from any point, to 100

Background

Meaningful counting involves an understanding of the six principles of counting. These principles are addressed in various outcomes (N01 and N07). The principles in bold face are the ones addressed in this outcome.

1. One number is said for each item in the group and is counted once and only once. (one-to-one correspondence)
2. Counting begins with the number 1, and there is a set number sequence. (stable order)
3. The quantity in the set is the last number said. (cardinality)
4. The starting point and order of counting the objects does not affect the quantity. (order irrelevance)
5. The arrangement or types of objects does not affect the count. (conservation)
6. It does not matter what is being counted, the resulting count will always be the same. (abstraction)

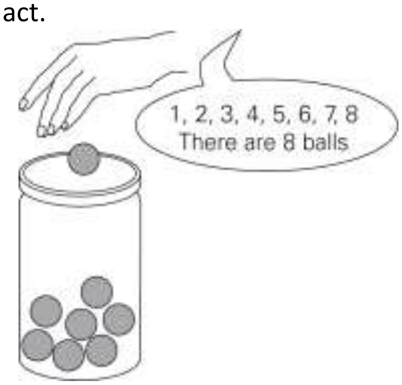
In Mathematics Primary, students were expected to successfully count sets up to 10 objects demonstrating the one-to-one correspondence, the stable order, and the cardinality principles, and they explored the other principles. In Mathematics 1, the development of counting continues to sets up to 20 objects with the expectation of understanding all six principles. The conservation principle has its own outcome (N07) and should be addressed in conjunction with this outcome (N03).

Meaningful counting is the foundation on which all other number concepts are developed. For this reason, instruction on counting must emphasize quantity, numerals, and oral and written forms of numbers. It is also necessary to assess each student individually in order to determine their understanding of number, not only in the oral and written expression of numbers, but also in counting abilities and quantity sense. Interviews are an important method to assess meaningful counting.

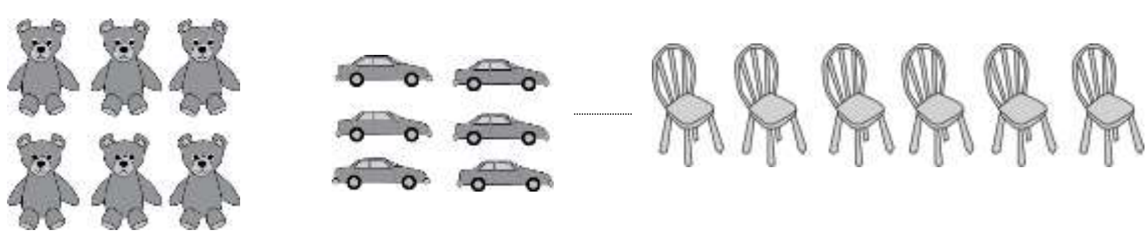
Before students count sets of a certain size, they should be fluent with the number sequence involved, stable order principle (addressed in outcome N01), so they can concentrate on the counting process rather than trying to recall the number names.

One-to-One Correspondence: When students are counting, watch to see if they are touching objects while they count them and saying the appropriate numbers when they touch them. Encourage students to touch or move the objects as they count as this deepens their awareness of the one-to-one principle of counting.

Cardinality: Some students may successfully perform a counting act of 8 objects, but when asked to show 8, pick up the last counter, thinking that the counter they touched when they said “eight” is 8, not the set of 8 counters. Help students develop this principle by engaging them in activities where objects are placed in a container as they are counted and directing students’ attention to the number of objects in the container after the counting act.



Abstraction Principle: Students need to come to understand that the size of the objects does not influence the number that represents the quantity in a set; that is, the objects themselves do not have to be uniform in size and shape, and two sets may both represent the same number even if the objects in one set are larger than those in the other. For example, students can count 6 bears, 6 toy cars, and 6 chairs. The three sets are equivalent because they all represent 6.



Performance Indicator Background

N03.01 Watch students as they count to see if they are touching objects (one-to-one correspondence) and saying the appropriate number when they touch them. Listen to how the students are counting and verify that they start at 1 and say the number words in the correct sequence (stable order). Verify that students realize the last number said is the quantity of objects they have counted (cardinality).

N03.02 Students should be able to identify errors in a counting process. Errors may include counting objects more than once, missing an object, or making a mistake in the number sequence.

N03.03 The goal in Mathematics 1 is for students to see that the order in which they count objects does not change the quantity. Have them count a set of objects from the right. Then ask them how many they think they will get if they start counting from the left or from the middle. Their response should indicate that the count will be the same and there is no need to recount.

N03.04 Students should be able to record numerals presented orally (**outcome** N01) and those representing the quantity of objects in a set. Opportunities should begin by reviewing counting and recording numbers to 10 before moving to sets of up to 20 objects.

N03.05 Counting on **is a** fundamental prerequisite for addition and subtraction and its importance should not be underestimated. Counting on requires an ability to say the number sequence, starting at any number (**outcome** N01). It also requires an understanding that numbers are inclusive; for example, counting on from 6 implies knowing that 1, 2, 3, 4, and 5 are included in the number 6 and there is no reason to recount those numbers (1, 2, 3, 4, 5). Counting on is an alternative to “counting all”.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students’ prior knowledge.

- Place a set of up to 10 counters in a five-frame or ten-frame and ask students to count the objects from left to right. Ask students to predict how many they think they would get if they started counting at the right or in the middle.
- Ask students to select a numeral card (1 to 10) and to count out that number of counters. After the students count out the set, spread the counters out and ask students to tell you how many they think they have now.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Provide students with a number of objects. Ask them to count them and to tell you how many objects are in the set. After they have counted them once, rearrange the objects and ask them to tell you (without recounting) how many objects are there now. Observe students to determine their understanding of each of the principles underlying meaningful counting. Note the way in which students count:
  - Do they touch each object as they count?
  - Do they set items aside as they count them?
  - Do they show confidence in their count or feel the need to check?
  - Do they check their counting in the same order as the first count or in a different order?
- Ask students to count a number (1 to 20) of items in a photo. Observe how they count. Ask them to record, using numeral symbols, the number of items they counted.
- Ask the student to count out 16 blocks/counters onto the table. Rearrange them by moving them around the table and then display them in two parts to display a "16" combination (e.g., 9 in one group, 7 in the other). Ask the student how many you have altogether. Repeat using different combinations of 16. Observe the student's method of determining how many.
- Show two groups of objects. Hide one under a piece of paper labelled with its amount. Leave the other group showing. Ask the student how many objects there are in all.
- Ask the student to count out 6 counters. Once they have 6 counters, ask them to show you a total of 14 counters. Students should count on from 6 rather than recount starting at 1.
- Show students a number sequence with an error or a missing number. Ask students to correct the sequence.
- Place a large number line on the floor, positioning a student on the 8 and facing the higher numbers. Ask, Where would you be if you moved 4 spaces forward? (Ask additional questions, such as, Where would you be if you moved 2 spaces back?)
- Show students a set of counters arranged in a row. Count the counters from left to right with the students. Ask, If we start counting from the other end of the row, how many counters will we have?
- Show students a set of counters, linking cubes, or countable objects. Ask students to tell the number of counters in the set. While students watch, rearrange the counters (do not add or remove any counters). Ask, How many counters are in my set now?

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)
- Mathematics 1 Checkpoint 1, pp. 23–24
- Mathematics 1 Checkpoint 2, Task 1, pp. 32–35 (Line Masters 2.2, 2.3, and 2.4)
- Mathematics 1 Checkpoint 15, pp. 141–142
- Mathematics 2 Checkpoint 1, Tasks 1 and 3, pp. 25–26

Planning for Instruction

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of my students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Observe how the students count. Students who are successful counters have strategies to keep track of their count, such as touching and moving each object as it is counted.
- Encourage students to count items in natural situations that arise in the classroom.
- Play a variety of games that require counting. For example,
  - bowling (counting both the pins knocked down and the pins left standing)
  - board games (counting the number of spaces to be moved based on a spin)
  - tossing bean bags (counting how many land in the target box)

SUGGESTED LEARNING TASKS

- Request that students draw pictures of their favourite toys, and then ask them to count the number of toys in their pictures.
- Allow students to count the number of napkins, cups, plates, etc., that are on the table or are needed for snack time or for a special party.
- Tell students, I am thinking of something in the classroom of which there are exactly 5. What do you think it could be?
- Place 5 counters under a cup and tell the students that there are 5 underneath the cup. Show 3 more beside the cup. Ask, How many counters are there altogether? Students should count-on to determine how many are there.
- Use a walk-on number line. Invite a student to roll two number cubes. That student chooses the value on one of the cubes to stand on that number on the number line and then moves along the number line by counting on the amount shown on the other number cube.
- Have students work in pairs. Place numeral cards face down. The first student draws a card making sure not to show it to their partner and reads the number shown. The partner records the numeral and makes a set of that size. The first student checks the written numeral and the set that was created. They then change roles.
- Provide students with a bag of counters. Tell them they must find out how many there are in total without counting by ones. Invite students to illustrate or demonstrate to the class how they counted and discuss which way of counting was the most efficient.
- Count some items with the students. Cover the starting quantity of items. Have the students
- count on as more items are added. (up to 20).
- Ask students to make a set of 6 counters and a set of 3 counters. Ask students to cover the set of 6 counters with their hands, and then point to each counter in the set of 3 as they as, 7, 8, 9.

Count out two sets of objects, for example a set of 6 and a set of 5. Hide one of sets of objects under a piece of paper. Label the paper to indicate the number of objects that are hidden. Place the second set beside the paper. Ask students to tell how many objects there are in total. Students should count on from the number shown. In the example, students would start counting at 6, and count on saying, 7, 8, 9, 10, 11.

SUGGESTED MODELS AND MANIPULATIVES

- coins, calculators
- collections of objects
- dot cards
- five-frames and ten-frames
- hundred chart
- number cubes
- number lines
- open number lines
- walk-on number lines

MATHEMATICAL LANGUAGE

Teacher	**Student** (oral language)
▪ counting on ▪ left, right, middle ▪ numeral ▪ order, arrangements ▪ quantity of objects	counting on ▪ left, right, middle ▪ numeral ▪ quantity of objects

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 83–95
- *Making Math Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 139–152
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006) pp. 39–47, 54–58

Videos

- *Teaching Number: Counting* (10:49 min.) (ORIGO Education 2010)
- *Using a Hands-on Approach to Represent Numbers to 10* (13:06 min.) (ORIGO Education 2010)

Notes

SCO N04 Students will be expected to represent and partition numbers to 20. [C, CN, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N04.01 Represent a given number up to 20 using a concrete materials, such as ten-frames, linking cubes, and created materials.
N04.02 Represent a given number up to 20 pictorially.
N04.03 Find examples of a given number in the environment.
N04.04 Place given numerals on a number line with benchmarks 0, 5, 10, 15, and 20.
N04.05 Partition any given quantity up to 20 into two parts and identify the number of objects in each part and in the whole.
N04.06 Model a given number using two different objects.

Scope and Sequence

Mathematics Primary	Mathematics 1	Mathematics 2
N04 Students will be expected to represent and partition numbers 2 to 10 in two parts, concretely and pictorially.	**N04** Students will be expected to represent and partition numbers to 20.	**N04** Students will be expected to represent and partition numbers to 100.

Background

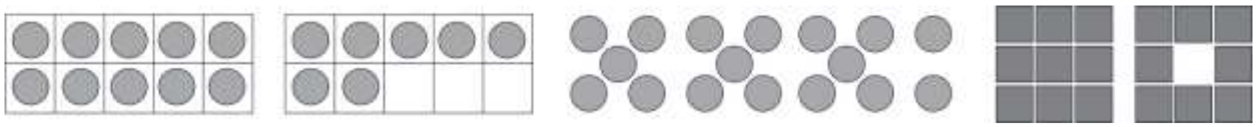
In order to fully develop deep meaning of and flexibility with numbers to 20, students need to experience multiple representations of those numbers. They should be given opportunities to model numbers concretely with a variety of materials, draw a variety of pictures, and identify a variety of real-world uses. **Note:** Verbal and symbolic representations are specifically addressed in outcome N01; however, these representations should be used in conjunction with this outcome.

The ability to recognize that any number can be partitioned into two or more smaller numbers is an essential understanding in number development. Understanding the relationship between/among the parts and the whole (part-part-whole) is essential to developing a sense of numbers and an understanding of mathematical operations. It is important to understand that the action of partitioning a set of objects does not affect the count. Students should be able to see small groups (parts) as part of a larger group (the whole). They should also be able to recognize how many objects are in each of the small groups (the parts) and how many objects are in the larger group (the whole) . Partitioning numbers helps students develop part-part-whole thinking. (This is also seen in outcome N02.)

In Mathematics Primary, students had experiences representing and describing numbers to 10, in one or two parts, concretely and pictorially. They have also been ordering quantities using objects, five-frames, ten-frames, or dot cards as well as relating a numeral, 1 to 10, to its respective quantity.

Performance Indicator Background

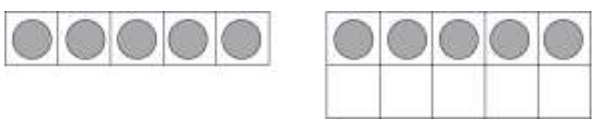
N04.01 and N04.02 Students should be able to show and sketch models of quantities to 20 in a variety of ways. For example, a student might show 17 in ten-frames, or with three groups of 5 counters and 2 more counters, or with a set of 9 squares and a set of 8 squares as seen below.



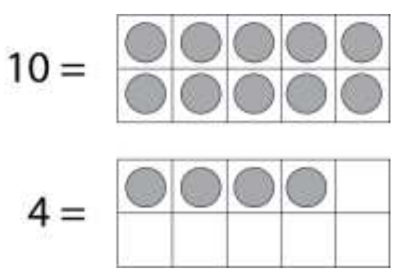
Students may also use created materials to represent a number. For example, to represent 17 they may use a bundle of 10 sticks and 7 single sticks or 3 sticks with 5 beans glued on each and 2 loose beans.

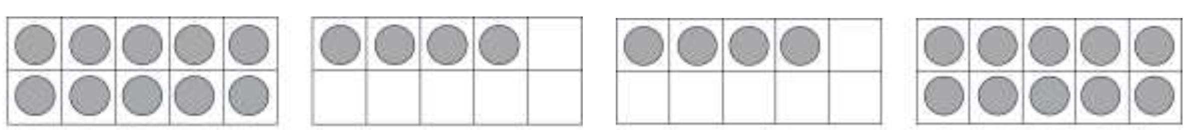
Ten plays an important role in our number system and since five and five make ten, it is important for students to develop and understand the relationships for the numbers 1 to 10 to the important anchors of 5 and 10.

Students will use five-frames and ten-frames to help develop and learn about these anchors of 5 and 10.



Students in Mathematics 1 are expected to develop deep understanding of the “teen” numbers; that is the numbers between 10 and 20. The focus of the work done with these numbers should be on their relationship with 10. Ten-frames are useful tools for exploring this relationship. Students should learn that the numbers between 10 and 20 can be made with *ten* (a full ten-frame) *and some more*. For example, 14 is ten and four more or 14 is four and ten more. Students need to experience teen numbers as *ten and some more* to be convinced they do not need to count on from 10 or to count all in order to determine the quantity. This is an important pre-place value concept.





10 and 4 more is the same as 14

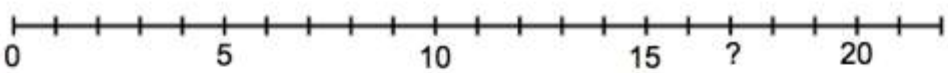
14 is the same as 4 and 10 more

Mathematics 1 is the first time number lines are used to represent numbers. Number lines provide a visual representation of the sequence of numbers and their relative positions to one another. Initial exposure to number lines should include kinesthetic experiences, such as concrete or walk-on number lines that display all numerals, followed by number lines on paper that also display all numerals, and finally to more abstract number lines that display only some numerals. Number lines in Mathematics 1 should always contain tick marks for each number, whether or not the numeral is displayed.

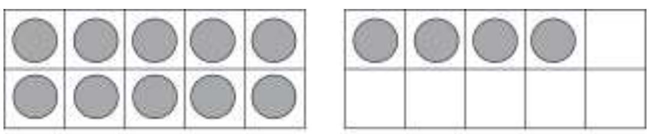
Note: Students will need to be introduced to the concept of zero and its application to a number line. On a number line that starts with zero, a number is represented by the position that is that many spaces from the starting point; for example, 4 is four spaces from the starting point (zero).

N04.03 Students should be able to identify where the quantities of objects, up to 20, may be found outside of mathematics class, such as eggs in a carton and number of crayons in a box. They should also be able to identify where numerals can be found that do not represent a quantity, such as numbers on sports jerseys or house numbers.

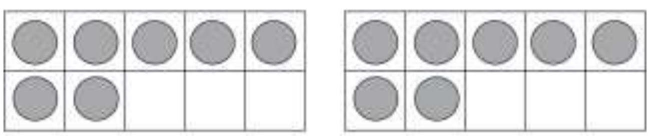
N04.04 Students need a lot of experience using benchmarks to place numbers from 1 to 20 on a number line. For example, students should be able to say what number on the number line is above the "?".



N04.05 Students should be given opportunities to partition numbers to 20 in a variety of ways. For example, they can use ten-frames to show different ways to model 14.

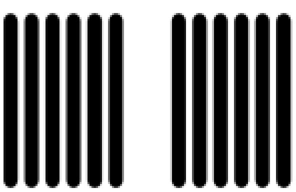


14 can be a set of 10 and a set of 4.

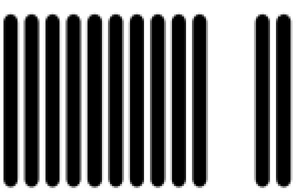


14 can also be a set of 7 and another set of 7.

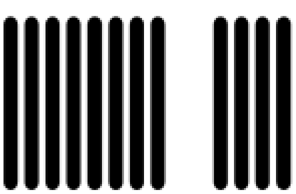
Students may use objects, such as stir sticks, toothpicks, or counters, to model partitioning in a real context**,** such as the number of boys and the number of girls in a group of 12 students. Students may show three possible ways to partition the group of 12 students using sticks as shown below.



12 can be 6 and 6.



12 can be 10 and 2.



12 can be 8 and 4.

They should be able to relate the arrangements they have created to the context; for example, the 12 students could be 6 boys and 6 girls, 10 boys and 2 girls, or 8 boys and 4 girls. They should be able to explain that no matter how they partition the quantity, there are always 12 students in the group.

N04.06 Students need to understand that the size of the objects does not influence the number that represents the quantity in a set; that is, the objects themselves do not have to be uniform in size and shape, and two sets may both represent the same number even if the objects in one set are larger than those in the other. This is the abstraction principle. For example, they should see that 4 apples, 4 bicycles, or a set of 4 unrelated objects are all sets of 4. To develop the "4-ness" of a set, students should represent a set of 4 with a large variety of different objects, both concretely and pictorially.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge.

- Present the following problem: I have 10 cubes in this bag. Some are blue and some are red. How many cubes might be blue and how many might be red? Can you tell another possible combination of blue and red cubes that might be in the bag?

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to show the number 15 in as many different ways as they can using manipulatives. This may also be done pictorially. The parts and the whole should be identified and recorded using words and numbers.
- Tell students, In my bowl, I have apples and bananas. There are 14 pieces of fruit altogether. How many apples are there? Draw a picture of the fruit. Are there other possibilities? The parts and the whole should be identified and recorded using words and numbers. Students should relate the arrangements they have created to the context of the problem.
- Tell students, I was counting objects in our classroom. I counted exactly 18 of the same thing. What could I have been counting? Tell me why. What are some things I could not have been counting? Why could they not be the objects I was counting?
- Shake and spill a handful of two different colours of counters (20 or less). Ask students to record (with pictures) how many of each colour there are and how many counters there are altogether.

They should also name the quantity in each part by recording the respective numeral beside or below each part of their drawing.

- Provide students with a given number (20 or less) and ask them to make two-colour bars, using linking cubes to represent that number. Ask students to tell you about the number of cubes, describing each part and the whole. Ask them to find as many different two-colour bars (part-part-whole) as they can to represent their given number.
- Present students with a number line showing benchmarks 0, 5, 10, 15, and 20. Give them a number (1–20) and ask them to place the number on the number line and to explain how they know where it goes.
- Hold out a bar of linking cubes, a dot strip, a two-column strip, or a dot plate showing 6 or fewer. Say, "I wish I had six." The students should respond with the part that is needed to make 6. Counting on can be used to check. The game can focus on a single whole, or the "I wish I had …" number can change each time.
- Give the students a picture card and ask them to model or tell a number story about a part-part-whole relationship.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Mathematics 1 Checkpoint 5, pp. 57–58 (Line Master 5.2)
- Mathematics 2 Checkpoint 5, pp. 59–60 (Line Master 5.2)

Planning for Instruction

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of my students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

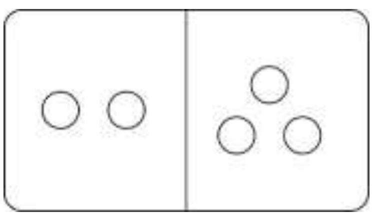
- Provide students with many opportunities to represent numbers in part-part-whole relationships using a variety of physical materials or pictorial representations and to explain how many are in each part of a whole. Students should also label each part and the whole with numerals.

- Provide students with counting activities in which sets of 11 to 19 items are counted. Students will develop number sense and will recognize that certain partitions, such as a group of 10 and 7 more, make it easier to determine the size of the set. This is an important pre-place value concept.

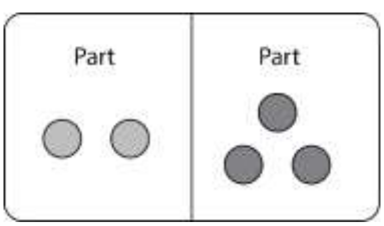
SUGGESTED LEARNING TASKS

- Tell students, There are 16 squirrels in a park that has one big tree and one small tree. One day, all the squirrels were in the trees. How many squirrels could be in the big tree and in the small tree? Draw two trees on the board and have construction paper squirrels to place in the trees. Change the position of the squirrels as students offer alternative answers.
- Show students a set of 15 counters. Ask students to count how many are in the set. Place the 15 counters on the part-part-whole mats to show two parts, for example 12 and 3. Ask, How many objects are there? How do you know? Observe whether students must recount all of the objects or if they know that partitioning the whole (the objects) has not changed the quantity. Repeat with a different number of objects.
- Ask students to count out 10 counters and place them on one side of a part-part-whole mat. Ask them to place 5 counters on the other side. Together, count all the counters by ones to determine the total number of counters. Then, identify each part and the whole. Say, 10 and 5 more is 15 and 15 is 10 and 5 more. Turn the mat around and say, 5 and 10 more is 15 and 15 is 5 and 10 more. Repeat with other numbers (11 to 20) without changing the 10 side of the mat.
- Ask students to find objects in the classroom that represent numbers from 1 to 20 (e.g., 12: there are 12 windows in the classroom).
- Ask students to explore multiple ways to partition a number. For example, students could be asked to create books in which each page shows a different way to represent a given number as two parts, using pictures.
- Keep a number line in the classroom showing benchmark numbers (0, 5, 10, 15, and 20). Prepare number cards from 0 to 20. Have one student each day select a number card and place it on the number line. Ask each student to explain their thinking about the number and its placement in relation to the benchmark numbers.
- Say, If one part is 4 what is the other part needed to make 5? The students should use linking cubes to determine the answer and respond by saying how many more are needed to make 5. Repeat for other partitions of 5. Repeat for other numbers (to 20).
- Display a number between 5 and 10 on a ten-frame. Students respond, Five and … make …. using the appropriate numbers. For example, if you say, 8, the students respond, five and three make eight or five and three are the same as eight.
- Show students a number represented on a ten-frame. Ask students to make statements about the number represented. For example, if shown a ten-frame representing 9, students might say, nine is the same as 5 and 4. Nine is one less than ten. Nine is eight and one more. Eight is one less than nine.
- The teacher calls out a number between 10 and 20. Students represent the number using double ten-frames and then respond, Ten and ____ more make _____ using the appropriate numbers. For example, if the teacher says, Eighteen, the students respond, ten and eight more make eighteen or ten and eight are the same as eighteen or eighteen is ten and eight more.
- Show students a double ten-frame. The students respond by stating the relationship to ten. For example, show a full ten-frame and a ten-frame with three dots. The students would respond, Ten and three more is thirteen, or I see thirteen because I see three more than ten.
- Show a full ten-frame. Say, I wish I had 17. Students respond by holding up the other part needed to represent 17, and they say, You need 7 more because 10 and 7 is 17. Repeat with other numbers between 10 and 20.

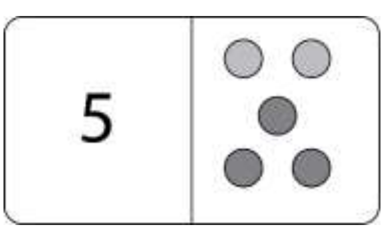
- Provide each student with a part-part-whole mat and counters. Say a number between 2 and 10 and ask students to place counters on their mats in two parts to match that number. Ask for different configurations for each number you say.
- Hold up dot cards with dots arranged in two parts. Ask student to name the parts and the whole. For example, if you hold up a dot card with 2 dots and 4 dots, the students say, One part is 2. The other part is 4. The whole is 6.
- Invite students to close their eyes. Tell them to picture in their heads what a number looks like. For example, ask, How do you see 4? After students have mentally pictured the number, invite them to use counters to create a representation of their mental image. Ask them to use language to describe the way they saw 4. For example, a student might say, I see 4 as 2 and 2.
- Hold up a five-frame. Ask students to respond by stating the relationship between the display number and five. For example, show a five-frame with three dots. The students respond, Three is two fewer than five. Add the two missing dots to the five-frame. Students respond, Three and two make 5. This can be repeated with ten-frames later in the year.
- Play, *I wish I had* … For example, Hold up a ten-frame with 4 dots. Say, I wish I had seven. The students place counters on their ten-frame to represent 4 and then count on with counters of a different colour to represent 7. Students then respond with the part that is needed to make 7. For example, If one part is 4, the other part is 3. The whole is 7.
- Show a part-part-whole mat on which you have displayed two parts.

  Say, if there are 2 counters on one part and three counters on the other part, how many counters are there in the whole?
  Alternately, one side of the card could show

  and students can be invited to call out the "whole" before you turn the card over to show

- Make sets of dot cards or ten-frame cards with various arrangements to represent numbers to 20. Each pair of students is given two decks of cards. One deck of cards has numerals from 1 to 20. The other deck of cards is a set of dot cards or a set of ten-frame cards. The set of dot cards or ten-frame cards is placed face up on the table. Students take turns selecting a numeral card and then finding a dot card or ten-frame card (or all the dot cards or ten-frame cards) that represent that number.

- Show four dot cards. (Three of the cards should represent the same number; one should represent a different number). Ask students to identify the dot card that isn't the same as the others. These tasks should also be done with ten-frames.

Suggested Models and Manipulatives

- counters
- dominoes
- linking cubes
- part-part-whole mats
- various number lines

Mathematical Language

Teacher	**Student** (oral language)
▪ numeral ▪ partition numbers/quantities ▪ partition ▪ part-part-whole relationships ▪ parts or whole ▪ represent ▪ ten-frames, number lines	▪ numeral ▪ partition ▪ parts or whole ▪ part-part-whole ▪ represent ▪ ten-frames, number lines

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 83–95
- *Making Math Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 139–152
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 47–58

Videos

- *Powerful Models to Help Struggling Students: Number Lines* (17:37 min.) (ORIGO Education 2010)
- *Teaching Number: Relative Position* (20:41 min.) (ORIGO Education 2010)
- *Teaching Place Value: Teen Numbers* (8:30 min.) (ORIGO Education 2010)
- *Using Structured Patterns to Develop Number Combinations* (18:11 min.) (ORIGO Education 2010)

Notes

SCO N05 Students will be expected to compare sets containing up to 20 objects to solve problems using referents and one-to-one correspondence. [C, CN, ME, PS, R, V]
[C] Communication **[PS]** Problem Solving **[CN]** Connections **[ME]** Mental Mathematics and Estimation **[T]** Technology **[V]** Visualization **[R]** Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N05.01 Construct a set that has *more*, *fewer*, or *as many* objects as a given set, up to 20 objects.
N05.02 Construct several sets of different objects that have the same given number of objects in the set.
N05.03 Compare two given sets using one-to-one correspondence and describe them using comparative words, such as *more*, *fewer*, or *as many*.
N05.04 Compare a set to a given referent using comparative language.
N05.05 Solve, using pictures and words, given story problems that involve the comparison of two quantities.

Scope and Sequence

Mathematics Primary	Mathematics 1	Mathematics 2
N05 Students will be expected to compare sets containing 1 to 10 objects using one-to-one correspondence.	**N05** Students will be expected to compare sets containing up to 20 objects to solve problems using referents and one-to-one correspondence.	**N05** Students will be expected to compare and order numbers up to 100.

Background

Comparing is something students do many times a day. Even before coming to school, they will have developed a sense of fairness when comparing sets of objects. In Mathematics Primary, students had experiences comparing quantities, 1 to 10, by constructing a set to show *more than*, *fewer than*, or *as many as* a given set and by direct comparison.

In Mathematics 1, these same comparisons will be expected for numbers to 20. Students will explore the concept of quantity as it relates to countable objects and to compare these quantities. Therefore, the word *fewer* is used; for example, there are fewer counters in this set than in that set. It is interesting to note that although the concept of fewer is logically equivalent to the concept of more, the word *fewer* proves to be more difficult for students than the word *more*.

The words *fewer* and *less* are often used incorrectly. The word *fewer* should be used when referring to countable quantities (e.g., there are fewer students in this class). The word *less* is used when referring to measures and non-countable items (e.g., there is less water in this bottle). However, when comparing numbers, the phrases, *less than* and *greater than*, should be used. Students will become more familiar with the term *less than* to compare numbers in later years. They will learn that 5 is less than 7 because there are 2 fewer items in a set of 5 than in a set of 7. When talking about sets that have the same number of objects encourage the use of the terms *the same number as*, *as many as*, and *equal*. In Mathematics 1, students will be introduced to referents (benchmarks) and will solve story problems involving comparison situations. Some students may know one number is more than another simply because it is said later in the counting sequence.

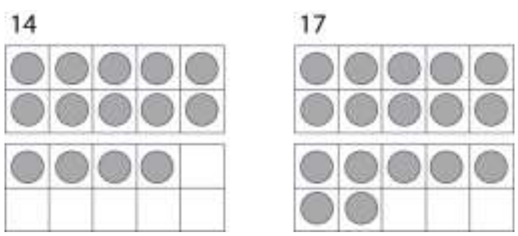
Performance Indicator Background

N05.01 and N05.02 Through planned activities, students should have opportunities to build sets of quantities that have *more*, *fewer*, or *as many* objects as a given set.

To reinforce the idea that the size of the objects does not affect the quantity, students may be shown a set of toothpicks and be asked to build a set of cubes that has fewer in the set. Similarly, students should be given opportunities to build sets that contain the same quantity using objects of various sizes.

N05.03 Students should be able to examine two sets that have been presented and be able to compare them by matching items in one set with those in the other to see if each item has a "partner" or if one set has leftover items. Through such one-to-one correspondence, students should be able to explain which set has more and which has fewer or explain why the two sets are the same.

N05.04 Students should be able to compare a set to a referent, using language such as "*fewer than*", "*more than*," or "the same as." Ten-frames can be very helpful in comparing quantities in general, but also can help students to use referents, such as benchmark numbers, to make comparisons. For example, a student may show that 14 is less than 17 because 14 fills **fewer** than 3 rows of the ten-frames (14 is less than 15) and 17 fills more than 3 rows (17 is more than 15).



Working with referents (benchmarks) is connected to outcome N08 where the relationships of *one more than*, *one less than*, *two more than*, and *two less than* are explored. Making connections of a quantity to benchmarks of 5 and 10 (and their multiples) using these relationships is critical. For example, students using 15 as a benchmark should know that 17 is 2 more than 15 or 14 is 1 less than 15. Modelling with ten-frames is a useful way to develop these benchmarks and can be used in conjunction with number lines.

N05.05 Students should be orally presented with story problem situations in which they need to compare quantities and use pictures and words to solve these problems. See the story problem chart and general discussion of **addition and subtraction** story problems in outcome N09. Students may also examine quantities of objects in stories they have been reading (or have been read by the teacher), using comparative language to discuss their observations.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Show students a set of 9 counters. Ask them to make a set that has as many counters as your set. Ask them to make a set that has fewer counters than yours. Ask them to make a set that has more counters than yours.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Identify a target number and give each student a set of three cards labelled "*more*," "*fewer*," and "*same*." Show students a double ten-frame that represents a number between 11 and 20. Ask students to compare the double ten-frame display with the target number and to hold up the appropriate card to show whether the double ten-frame has more, fewer, or the same number of counters as the target number. Later in the year, ask them to tell how many more or fewer are shown.
- Provide representations of two different sets each with 20 or fewer objects. Ask students which set has more and which has fewer (e.g., one set has 19 and the other has 12) objects and to explain how they know.
- Hold up a dot card that has between 11 and 20 dots. Give each student a set of three cards labelled "more," "fewer," and "same." Beside each card, ask students to make a collection of counters; one set that has more, one set that has fewer, and one set that has the same number of counters as the original card.
- Give students a tower of 14 cubes and another tower of 17 cubes. Ask, Which is closer to 10? and then ask, How do you know? Which is closer to 20? How do you know? You could also ask students to tell you which tower is closer to 15 and to explain their thinking. Allow students to use more linking cubes to assist in solving this problem.
- Give small groups of students interlocking cubes. Give a variety of directions that use the words *more*, *fewer*, and *the same as* and have students build towers. For example, Build a tower that is one more than 11; Build a tower that is two fewer than 9; Build a tower that is two more than 18. Challenge students by saying, "What number do you think is one fewer than 15? Let's build a tower and see."
- Present students with story problems that involve comparing two quantities and ask them to use manipulatives or pictures to solve the problems.
- Provide three sets of objects that are not equal and have students number them, in order, from least to greatest. Then, ask students to write comparative sentences about them.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)
- Mathematics 1 Checkpoint 4, pp. 49–50 (Line Masters 2.5 and 2.6)
- Mathematics 1 Checkpoint 8, pp. 83–84
- Mathematics 1 Checkpoint 15, pp. 141–142
- Mathematics 2 Checkpoint 4, Tasks 1 and 3, pp. 51–51 (Line Masters 2.5, 2.6, and 4.1)
- Mathematics 2 Checkpoint 1, Task 2, pp. 25–26

Planning for Instruction

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of my students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Encourage students to work in pairs or small groups to facilitate discussion and share thinking.
- Provide many opportunities for students to pose or answer questions such as,
  - Who has fewer? Who has more?
  - Are there more boys or girls here today?
  - Are there the same number of chairs as there are students?
- Provide students with many opportunities to represent numbers concretely.
- Allow students to make purposeful links between concrete, pictorial, and symbolic representations of numbers.
- Expect students to verbally explain their answers about numbers.
- Use real-world contexts for comparison problems whenever possible.

SUGGESTED LEARNING TASKS

- Place 15 counters on a paper plate and ask students to make sets that have more, fewer, or the same number of counters as the given set of counters.
- Display two double ten-frames, each representing a different number. Ask students, Which double ten-frame has more counters? How many more? Which double ten-frame has fewer? How many fewer?
- Have students work in pairs to play "Dot Challenge." Provide a deck of dot cards (1 to 20 dots each) for each student. Each student turns up a card; the student whose card has more dots gets both cards. This can also be played so that the student whose card has fewer dots captures both cards.
- Present students with story problems that involve comparing two quantities and ask them to use manipulatives or pictures to solve them. For example, Mary had 6 cookies. Chantelle had 8 more cookies than Mary. How many cookies did Chantelle have? or Fred had 6 marbles. Yousef had 10 marbles. How many fewer marbles did Fred have than Yousef had?

- Students work in pairs. Place a set of dot cards (1–20) face down. One student turns over the top card, and identifies the number shown. The second student turns over a word card (*more*, *fewer*, *the same as*) to determine whether they will make a set that has more, fewer, or the same as the one shown on the original card.
- Hold up a ten-frame and the students respond by stating the relationship to ten. For example, show the ten-frame with three dots. The students would respond, Three is seven fewer than ten. (These will always be fewer than statements except when ten is held up-ten is the same as ten)
- Show students a number represented on a ten-frame. Ask students to make statements about the number represented. For example, if shown a ten-frame representing 8, students might say, Eight is three more than five. Eight is two fewer than ten. Eight is one more than seven. Eight is one fewer than nine

SUGGESTED MODELS AND MANIPULATIVES

- counters
- dot cards
- linking cubes
- ten-frames
- various number lines

MATHEMATICAL LANGUAGE

Teacher	**Student** (oral language)
▪ compare sets ▪ more than, greater than, fewer than, as many as, or the same as (equal), less than ▪ one-to-one ▪ size of objects ▪ ten-frames, number lines	▪ compare sets ▪ more than, greater than, fewer than, as many as, or the same as (equal) ▪ size of objects ▪ ten-frames, number lines

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 87–89
- *Making Math Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 144–145
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 37–38, 54–58, 69

Notes

SCO N06 Students will be expected to estimate quantities to 20 by using referents. [C, ME, PS, R, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N06.01 Estimate a given quantity by comparing it to a given referent (known quantity).
N06.02 Select between two or more possible estimates for a given quantity and explain the choice.

Scope and Sequence

Mathematics Primary	Mathematics 1	Mathematics 2
—	**N06** Students will be expected to estimate quantities to 20 by using referents.	**N06** Students will be expected to estimate quantities to 100 by using referents.

Background

The ability to estimate quantities, a key reasoning skill in mathematics, should develop with regular practice over the course of the year. Estimation helps develop useful benchmarks for thinking about numbers. It is important for students to understand what makes a good estimate. All counting activities can be modified to include estimation. Students should estimate how many are in a set prior to finding the actual count. Prepare daily estimation tasks by presenting between 10 and 20 objects and having students record their estimates. Be sure to have students share their strategies for arriving at their estimates. Throughout the day, count the objects and compare the estimates to the actual number.

Students should understand the difference between guessing and estimating; that is, estimating involves the use of strategies to find a carefully considered approximation rather than a number “pulled out of the air.” The two words should never be combined as “guesstimation” because it further confuses the terms. Many students are reluctant to give an estimate because they want to be “right.” They need to be convinced that there can be a number of different estimates that are acceptable or “right” for a given collection of objects, so long as each estimate is reasonable.

After estimates are made, there should be discussion of reasonableness and accuracy of those estimates. Some students also find it easier to commit to an estimate if initial estimation activities involve selecting from two or more stated estimates. Estimation skills will improve over time with many activities and refinement.

Performance Indicator Background

N06.01 To develop estimation skills, students should be provided with collections of objects and asked to estimate the quantities in these collections. Referents such as 5, 10, 15, and 20 are useful benchmarks to facilitate the development of quantity estimation skills. For example, students could be shown labelled sets of 5 counters and 10 counters, and an unknown set of counters they are to estimate. The labelled sets provide them with referents to which they can visually compare the unknown set in order to arrive at an estimate. After they state their estimates, they should then find the quantity by counting and reflect on whether their estimates were *more* ***than***, *fewer* ***than***, or *the same as* the actual quantity and which estimates were closest. Through many repetitions of this process, students will refine their estimation skills. Eventually, activities should just involve estimating without checking by counting.

Through other planned activities, students should be presented with opportunities to estimate. For example, display between 10 and 20 cubes. After 5 seconds, cover the display. Ask the students to write down their estimates. Reveal the cubes and move 5 cubes off to the side. Tell students, “Here are 5 cubes. Do you want to change your estimate? If you do, will you change it to be more or less than your first estimate?” Ask students to explain their decisions.

N06.02 Students should **select** what they think is the best estimate among given estimates. For example, students could be shown a set of buttons and asked to decide if 5, 10, or 15 is the best estimate of the number of buttons they were shown. For a smaller collection, students may be asked whether it is closer to 5 or 10. Keep in mind that six-year-old students should be able to subitize random sets of 1 to 6 objects; therefore, sets of more than 6 objects should be used for estimation. For larger collections, they may be asked whether the collection is closer to 10 or 20. Include situations in which sets have the same number of items but differ in the amount of physical space they occupy.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Tasks such as the following could be used to determine students’ prior knowledge.

- Place a set of 10 or fewer counters on the table. Ask students, About how many counters are on the table? Ask students to explain their thinking. Note whether the estimate is reasonable.

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Fill a container with cubes. The container should hold almost 20 cubes. Show it to the student and then ask, How many cubes do you think are in the container? Have the student count the cubes. Then ask, Was your estimate too small, too large, or just right? How do you know?
- Provide three different sets of objects such as cubes, colour tiles, or unifix cubes. Ask students to take a handful of one of the objects and estimate the number of objects they have. They should

then count to check. Ask, Was your estimate too small, too large, or just right? How do you know? Have students repeat the task with the other objects.
- Place between 10 and 20 counters on the overhead projector. Turn the projector on for five seconds and then turn it off. Ask students to write down their estimate. Turn the projector on and move 5 counters (referents) off to the side but still on the screen. Tell students, "Here are 5 counters. Do you want to adjust your estimate? If you want to adjust it, will it be more or less than your first estimate? Ask students to explain their decision.
- Show students a set of objects and ask them to choose between the two estimates you've provided. Ask them to explain their choice.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)
- Mathematics 1 Checkpoint 2, Task 3, pp. 32–33 (Line Masters 2.7 and 2.8)

Planning for Instruction

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of my students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

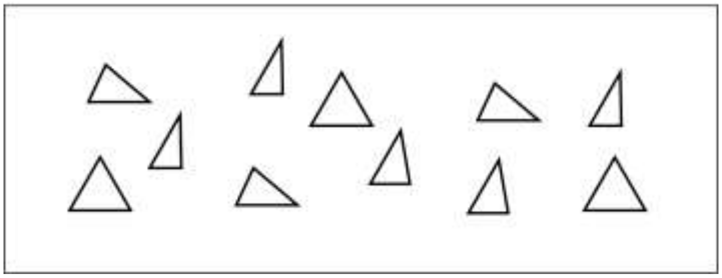
Consider the following strategies when planning daily lessons.

- Expect students to verbally explain their estimates and how they arrived at them.
- Provide students with a variety of estimation activities. For small collections of objects or pictures ask, Is it closer to 5 or 10? For large collections, ask, Is it closer to 10 or 20?
- Develop an understanding of the concept of "about" as it relates to estimation activities. Use varied language, such as
  - More or less than ___? Are there more or fewer than 15 counters on the overhead projector?
  - Closer to ___ or to ___? Do I have closer to 10 cubes or closer to 15 cubes in the clear glass?
  - Fewer than ___, between ___ and ___, or more than ___?

Suggested Learning Tasks

- Find or create pictures of sets of objects. Show a picture and suggest two possible estimates. Ask students to select one of the two possible estimates and to justify their choice. For example,

Estimate the number of triangles is it 9 or 15?



- Provide students with a set of 16 objects without telling them the number of objects in the set. Tell them that three students estimated the number of objects and their estimates were 4, 13, and 20. Ask them to discuss each estimate and to decide if the estimate is too large, too small, or just right and to explain their thinking.
- Use three identical transparent containers. Fill one container. Use the other containers as referents. For example, put 16 blocks in the first container. Put 5 blocks in the second container and 10 in the third container. Have students use the referents to estimate the number of blocks in the full container. Ask students to explain their thinking as they use the referents to estimate.
- Provide students with a bag of cubes. Ask them to reach in and grab about 5, 10, or 15 cubes. Ask the students to count the cubes they took from the bag and to determine if they took too many, too few, or just the right number of cubes.
- Have students gather groups of classroom objects (between 5 and 20). Take digital pictures of the sets, and have students identify two possible estimates for the set. The pages can be compiled into a class estimation book.

Suggested Models and Manipulatives

- cubes
- counters
- objects of various sizes

Mathematical Language

Teacher	**Student** (oral language)
▪ about	▪ about
▪ between	▪ between
▪ closer to	▪ closer to
▪ estimate quantities	▪ estimate
▪ fewer than	▪ fewer than
▪ more than	▪ more than
▪ referents, benchmarks	▪

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 144–145
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 58–60

Notes

SCO N07 Students will be expected to demonstrate an understanding of conservation of number for up to 20 objects. [C, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N07.01 Explain why for a given number of counters, no matter how they are arranged, the total number of counters does not change.
N07.02 Group a set of given counters in more than one way.
N07.03 Explain why for a given number of counters, no matter how they are grouped, the total number of counters does not change.

Scope and Sequence

Mathematics Primary	Mathematics 1	Mathematics 2
N06 Students will be expected to demonstrate an understanding of counting to 10.	**N07** Students will be expected to demonstrate an understanding of conservation of number for up to 20 objects.	—

Background

Meaningful counting involves an understanding of the six principles of counting. These principles are addressed in various outcomes (N01, N03, and N07). The principle in bold face is the one addressed in this outcome.

1. One number is said for each item in the group and is counted once and only once. (one-to-one correspondence)
2. Counting begins with the number 1, and there is a set number sequence. (stable order)
3. The quantity in the set is the last number said. (cardinality)
4. The starting point and order of counting the objects does not affect the quantity. (order irrelevance)
5. **The arrangement or types of objects does not affect the count. (conservation)**
6. It does not matter what is being counted, the resulting count will always be the same. (abstraction)

Of the six counting principles, the conservation principle is the only one addressed by its own outcome. This is done to highlight its importance in number development. however, this outcome should be addressed in conjunction with outcome N03. While it was explored in Mathematics Primary, it is often the last principle to be attained by students, so it is an essential part of counting expectations in Mathematics 1. Trusting that you will get the same count no matter where you start, or in which order, or how the objects are arranged is a very significant development in students' thinking. Students must learn to trust the count by the end of Mathematics 1. Many students in Mathematics 1 also believe they may get a different quantity if they count in a different way. It is critical that by the end of grade one, students are convinced they will get the same count whether they count by 1s, 2s, 5s, or 10s.
Conservation: Conservation of number is the understanding that the number of objects remains the same when they are rearranged spatially. When students do not demonstrate conservation of number,

they may believe that the number of objects increases or decreases when they are arranged differently, for example if the objects are moved farther apart, counted in a different order, or are grouped in different ways.

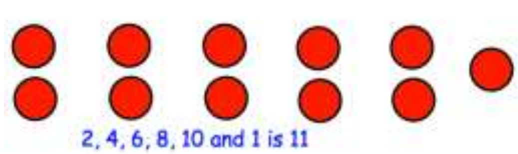
Performance Indicator Background

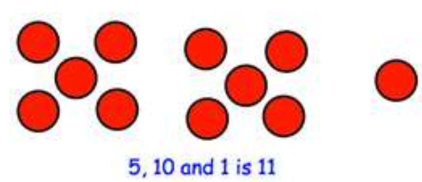
N07.01 Students explore the conservation principle through counting activities. For example, have students count a set of objects that are close together, then push the objects out so they cover a larger area, and ask students to predict how many there are now. (They may be unsure or predict a different number than the one they just counted. These responses indicate that conservation is not yet part of their understanding.) Have them recount the objects and focus their attention on the fact they got the same count both times. It may take many similar experiences before students are convinced they will get the same count regardless of how the objects are arranged. When students have internalized this understanding, they will likely think it is a silly question **if you** ask them again**, How** many **are** there**?** when you just rearrange**d** the objects after the initial count. They may explain, "It will be the same because you didn't add any more or take any away!"

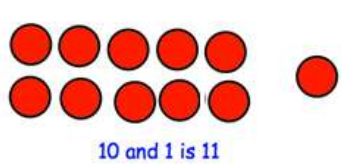
N07.02 and N07.03 Once students have demonstrated a competency in one-to-one counting, they should be encouraged to develop flexible thinking about numbers. Students need to explore ways in which previously learned number concepts are connected to the breaking apart of numbers (part-**part-**whole thinking). This development will lead the students toward increased understanding of the relative size of numbers and will enable students to construct meaning regarding the composing and decomposing of numbers. Therefore, indicators N07.02 and N07.03 should be addressed in conjunction with **outcome** N04.

Students will learn to trust that a quantity will not change if that quantity is thought of as being made up of parts. Students will also know that the quantity remains the same if some from one of those parts is moved to the other part. Students will need many opportunities to model this concretely.

Students should be encouraged to discuss what they see and talk about the ways they have partitioned the quantity. They should be able to explain how the quantity stays the same regardless of the groupings they have created. Skip-counting strategies can be used to determine the quantity of any collection. For example, encourage students to arrange counters in pairs and skip count by 2s, or organize objects into rows of 5 and skip count by 5s. Planned activities might include giving students a collection of buttons, counters, or cubes and asking them to see if they can organize them into groups and singles. A student might take 11 counters and organize them in some of the following ways. These groupings show how a student could count a set of 11 by 2s, 5s, or 10s with singles. This reinforces conservation concepts and partitioning ideas. **Students must understand that the different arrangements and the different methods for counting the set does not change the quantity; there are still 11 counters.**







Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with a set of 10 objects. Ask them to count the objects. Rearrange the objects while students watch. Ask students to tell you how many objects are there now. Note whether students recount the objects or whether they know that the number of objects has not changed.

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Observe and note the way in which students count.
  - Do they show confidence in their count or feel the need to check?
  - Do they correctly predict the count if the objects they have just counted have been rearranged?
  - Do they correctly predict the count if the objects they have just counted have been grouped to make counting easier (by 2s, 5s)?
- Provide a selection of buttons, linking cubes, or bread tags to represent any number less than 20. Ask students to count them and tell the number. Then, ask them to rearrange the objects to make counting easier and to predict the number of objects that are there before they count. Note whether students understand that the count has not changed even though the arrangement has.
- Give the students a set of counters and ask them to group the counters in different ways. They should record their work with drawings of their various groupings. Ask them to explain why the number of counters is the same though the groupings are not.
- Explain why it is possible to have a number, such as 13, described using two or more parts in more than one way.
- Ask a student to model a number in two or more parts. Ask students to represent that number in as many different ways they can. Ask them to explain why the arrangements are different but the number is the same.

Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Responding to Assessment

Numeracy Nets K–2 (Bauman 2011)
- Mathematics 1 Checkpoint 5, pp. 57–58 (Line Master 5.2)
- Mathematics 2 Checkpoint 5, pp. 59–60 (Line Master 5.2)

Planning for Instruction

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of my students be met?

Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

- Provide students with one type of material, such as linking cubes. Explore how many different combinations for a particular number can be made using different arrangements or groupings of the cubes. Direct students' attention to the fact that the arrangements and groupings do not change the total number of cubes they have.

Suggested Learning Tasks

- Observe and note the way in which students count.
  - Do they show confidence in their count or feel the need to check?
  - Do they correctly predict the count if the objects they have just counted have been rearranged?
  - Do they correctly predict the count if the objects they have just counted have been grouped to make counting easier (by 2s, 5s)?
- Ask students to bring in a set of between 2 and 20 of their favourite things. Alternatively, you could provide pairs of students with sets of between 2 and 20 objects. Ask students to count their favourite things/objects and to record the number. Then ask students to place their favourite things/objects in various locations (on a desk, on a large table, in a cup, on a window ledge, etc.) and in various arrangements (close together, spread out, in a row, in a circle) around the classroom. After placing them in each new location and/or new arrangement, ask students to count the objects to confirm that the number of objects has remained the same. If a digital camera is available, students could take pictures of each of their locations/arrangements and could use the pictures to create "Conservation" books.
- Give each pair of students a set of 2–20 objects. Ask them to count their objects and to record the number of objects. Then ask them to work together to find different ways to group the objects (by 2s, by 5s, with groups and singles). After each new grouping, ask students to count their objects again to confirm that the number of objects remains the same. Students can record with drawings

the various groupings they discover. When they have completed the task, ask students to explain why the number of objects is the same though the groupings are not.

- Provide each student with a set of paper plates or index cards and some dots. Have each student select a number (20 or less). Students will then make a set of dot cards/plates to represent the number chosen. The dot cards/plates should show various arrangements or groupings.
- Request up to 20 student volunteers. Play a game of "Simon Says" and count the group after each of Simon's directions and direct students' attention to the fact that the size of the group has not changed though Simon's directions have arranged them in different ways. For example, Simon says, "Stand in a small circle," or "Stand in a line at the front of the room," or "Spread out in the hallway (or the gym)," or "Stand in pairs around the room."

SUGGESTED MODELS AND MANIPULATIVES

- counters
- dot cards
- double ten-frames
- linking cubes
- ten-frames

MATHEMATICAL LANGUAGE

Teacher	**Student** (oral language)
▪ different groups ▪ groups ▪ same number ▪ singles	▪ groups ▪ same number ▪ singles

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8 (Small 2009),* pp. 83–86; 95
- *Making Math Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 141–142, 151
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 47–52; 54–55

Notes

SCO N08 Students will be expected to identify the number, up to 20, that is one more, two more, one less, and two less than a given number. [C, CN, ME, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N08.01 Name the number, up to 20, that is one more, two more, one less, or two less than a given number.

N08.02 Represent a number, up to 20, on ten-frames that is one more, one less, two more, or two less than a given number.

Scope and Sequence

Mathematics Primary	Mathematics 1	Mathematics 2
N01 Students will be expected to say the number sequence by ▪ 1s, from 1 to 20 ▪ 1s, starting anywhere from 1 to 10 and from 10 to 1	**N08** Students will be expected to identify the number, up to 20, that is one more, two more, one less, and two less than a given number.	**N08** Students will be expected to demonstrate and explain the effect of adding zero to or subtracting zero from any number. **N10** Students will be expected to apply mental mathematics strategies to quickly recall basic addition facts to 18 and determine related subtraction facts.

Background

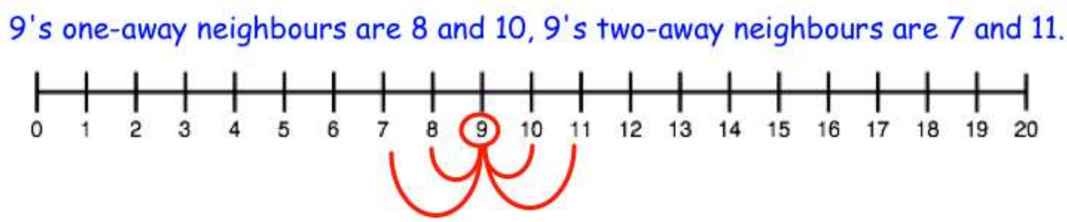
Developing an understanding of the relationship between numbers is an important aspect of number sense. Students do not necessarily reflect on the connection between two numbers when they are counting; therefore, they should have experiences that direct their attention to these relationships. When talking about numbers, such as 6 and 8, students need to say that 8 is two more than 6, and 6 is two less than 8. Similarly, the relationship between numbers that differ by 1 can be explored. Identifying these relationships should build on students' experiences with counting on and counting back allowing them to deepen their understanding of numbers. By the end of the year, students in Mathematics 1 should be able to name the number that is one more, one less, two more and two less than a given number to 20.

It is interesting to note that although the concept of fewer is logically equivalent to the concept of more, the word *fewer* proves to be more difficult for students than the word *more*. The words *fewer* and *less* are often used incorrectly. The word fewer should be used when referring to countable quantities; for example, there are fewer students in this class than the class next door. The word less is used when referring to measures and non-countable items; for example, there is less water in this bottle. However, when comparing numbers, the phrases, *less than* and *greater than*, should be used.

Performance Indicator Background

N08.01 Student's initial exploration of numbers that are one more, one less, two more, and two less than a given number should be done concretely using sets of objects. Have students use counters to create a set equal in number to a given set. Ask them to change their set to equal a number that is one more, one less, two more, or two less than their current set and to orally name the number using the appropriate language, such as 15 is two more than 13. When naming the number that is one more, two more, one less, or two less encourage students to count on or count back rather than beginning the count from one.

Students should also use a number line to explore which numbers come before and after a given number. Students should know the numbers that are one more, one less, two more, and two less than a given number. It might be helpful for students to take steps on a number line that has been placed on the floor, walking one or two more (forward) and walking one or two less (backward). The number line can help students think about a whole number's neighbours—the one-away and two-away neighbours.



The work students do in Mathematics 1 with one more, one less, two more, and two less should lead to a mental mathematics stage where students are presented with a number and asked for the number that is one more, one less, two more, or two less than this number. Reinforcements could use number configurations on paper plates, cards, and overhead acetates; digit cards; and ten frame cards.

N08.02 Have students show a given quantity on a ten-frame or a double ten-frame for numbers from ten to twenty. Ask them to add or remove counters to build and name the number that is one more, two more, one less, or two less. As students' understanding of this concept develops, they can be given tasks where they are shown a quantity in a ten-frame or double ten-frame and asked to visualize and say what number is one more, two more, one less, or two less **than that quantity**.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge.

- Show students a ten-frame representing the number 6. Ask students to identify the number shown. Then, ask them to select another ten-frame from the group on the table that has more (or fewer) than the ten-frame showing 6.

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Give students dot cards and some counters. Ask them to create sets that are one more than, one less than, two more than, and two less than the given dot card and to name the sets that are created.
- Call out a number up to 20. Ask students to select a ten-frame card that shows the number that is one more, one less, two more or two less than the number you called out.
- Display ten-frames to show numbers up to 20. After showing the ten-frame(s), ask students to write the number that is one more, one less, two more, or two less than the number shown.
- Create two groups of students, for example 7 boys and 6 girls. Ask, What must be changed to make the number of girls equal to the number of boys? What must be changed to make the number of boys equal to the number of girls?
- Ask students to sort a collection of buttons by various criteria into two sets so that the sets are one more or one less than each other. Compare the size of the sets.
- Ask students to create a set equal in number to or the same as a given set. Then, ask students to change their set to equal a number that is one or two more (less) than their current set. For example, change your set of 8 counters to show a set that is two more than 8. Name the new set that you have created.
- Place 3 red counters and 3 blue counters in one group and 3 blue and 2 red counters in another group, as follows: Set 1: RRR BBB Set 2: BBB RR. Ask, How do you know that there are more blue counters than red counters in the second group? How many more?
- Name a number up to 20. Ask students to name the number that is one more, one less, two more, or two less than that number. Repeat for other numbers.

Follow-up on Assessment

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Responding to Assessment

Numeracy Nets K–2 (Bauman 2011)
- Mathematics Primary Checkpoint 6, pp. 64–65
- Mathematics 1 Checkpoint 1, Task 1, pp. 23–24
- Mathematics 1 Checkpoint 4, Task 2, pp. 49–50 (Line Masters 2.5 and 2.6)

Planning for Instruction

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of my students be met?

Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

- Ensure that students' initial exploration of numbers that are one more than, one less than, two more than, and two less than is done concretely using sets of objects.
- Give students frequent opportunities to transfer their thinking from one representation to another. For example, represent the numbers 6 and 7 with linking cubes and then display the same numbers with counters on ten-frames.
- Ensure students are able to create a set that is one more and one less than a given set and two more and two less than a given set.

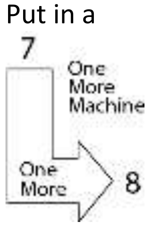
Suggested Learning Tasks

- Show students a set of up to 20 counters on an overhead projector or interactive whiteboard. Have the students count them. Have students close their eyes while you change the amount of counters by one or two. Ask the students to open their eyes and tell you how the group of counters has changed.
- Invite students to play "One More Than Dot Cover."
  - Create cards using symbols and dot patterns for numbers up to 8.
    Rules:
    - Take turns rolling a number cube.
    - Cover any one square on the card that is one more (or two more, one less, or two less) than the top number on the number cube.
    - The player who first covers three in a row is the winner.
    - This game could be extended to numbers to 20 using two number cubes or by using dice with more sides and changing the student cards.
- Students work in pairs. Give each pair of students a set of numeral cards marked 1 to 20 and a set of word cards with the words *one more*, *one less*, *two less*, and *two more* written on them. The first student takes a number card and makes that number on a ten-frame. The other student takes a word card and makes the new number on a ten-frame. Students compare the ten-frames to verify that the two numbers are correct. They then record their work using words. For example, if the first student drew a "16" and the second student drew the *two more* "two more" word card, the students would record "18 is two more than 16."
- Invite students to solve story problems involving situations of one more, one less, two more, and two less. For example, If I have 20 teeth and I lose a tooth, how many teeth will I have? If I have 18

teeth and one more tooth grows in, how many teeth will I have? I had 20 teeth, but I lost my 2 front teeth. How many teeth do I have now?

- Invite students to create their own story problems that involve one more, one less, two more, and two less situations.
- Provide students with a set of numeral cards from 1 to 19 and have them turn them up in random order on their desktops. Explain that a number will be stated and they should quickly find the number card for the number that comes after the stated number. When directed they should hold up their cards.
- Provide each pair of students with a calculator. Direct them to press +, 1, and =. Partner A chooses a number from 2 to 19, puts it in the calculator. Partner B quickly states the number that comes after the chosen number while Partner A presses = to check. (Remind students not to press clear; rather, just enter the next number.) After 5 numbers, partners change roles.
- Hold up a dot card or a ten-frame and ask students to construct a set of counters that is one more than the set shown. This should later be done for one less than the set, then two more than the set, and finally two less than the set.
- Show students a dot card or ten-frame. Ask students to say the number or to hold up a numeral card that is one more than the set shown. This should later be done for one less than the set, then two more than the set, and finally two less than the set.
- Draw a number machine on the board. It requires an input hopper and an output chute. Explain to students how the machine works; for example, This is a magic machine. It takes in a number up here and sends out the number that is one more down here.
- Place a number at the top of the machine. Ask students to tell the number or hold up the numeral card that will be sent out the bottom of the machine.
  The number machine can be used for one less, two more, and two less.

  Put in a



- A set of dot cards is placed face up on the table. Each pair of students is given two decks of cards. One deck of cards has numerals from 1 to 20. The other deck of cards is a set of word cards, *one more*, *two more*, *one less*, and *two less*. The students take turns selecting both a word card and a numeral card. The student must then find a dot card that represents that combination. For example, if a student drew a 4 and two more card, he/she would look for a dot card that had 6 dots. The student would say, I found 6 because 6 is two more than 4.
- Roll a large number cube. Ask students to quickly write on their individual white boards the number that is one less (one more, two more, or two less) than the number they will see on the large number cube.
- Ask students to quickly think of the number that is two less (two more, one less, or two more) than each ten-frame configuration that is shown. When directed students should put up the number of fingers “rabbit ears” that represents their answer.

SUGGESTED MODELS AND MANIPULATIVES

- dot cards
- five-frames
- number cubes
- Rekenrek
- ten-frames

MATHEMATICAL LANGUAGE

Teacher	**Student** (oral language)
▪ count on ▪ count back ▪ one more, two more, one less, two less, ▪ before, after ▪ ten-frames, double ten-frames, number lines	▪ count on ▪ count back ▪ one more, one less, two more, two less, ▪ before, after ▪ ten-frames, double ten-frames, number lines

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 87–89
- *Making Math Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 144–145
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 37–38, 44–45, 54–55

Videos

- *Using a Hands-on Approach to Develop Mental Strategies for Addition* (11:04 min.) (ORIGO Education 2010)
- *Using a Hands-on Approach to Develop Mental Strategies for Subtraction* (6:45 min.) (ORIGO Education 2010)

Notes

SCO N09 Students will be expected to demonstrate an understanding of the addition of two one-digit numbers and the corresponding subtraction, concretely, pictorially, and symbolically, in join, separate, equalize/compare, and part-part-whole situations. [C, CN, ME, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N09.01 Act out story problems that are presented orally or through shared reading.
N09.02 Model story problems with manipulatives or pictures, find and share solutions using counting strategies, and record number sentences that represent how they thought about the problems.
N09.03 Create story problems that connect to student experiences.
N09.04 Create story problems for given number sentences.

Scope and Sequence

Mathematics Primary	Mathematics 1	Mathematics 2
—	**N09** Students will be expected to demonstrate an understanding of the addition of two one-digit numbers and the corresponding subtraction, concretely, pictorially, and symbolically, in join, separate, equalize/compare, and part-part-whole situations.	**N09** Students will be expected to demonstrate an understanding of addition (limited to 1- and 2-digit numerals) with answers to 100 and the corresponding subtraction by ▪ using personal strategies for adding and subtracting with and without the support of manipulatives ▪ creating and solving problems that involve addition and subtraction ▪ explaining and demonstrating that the order in which numbers are added does not affect the sum ▪ explaining and demonstrating that the order in which numbers are subtracted matters when finding a difference

Background

As with many early concepts, the development of the meaning of addition and subtraction cannot be rushed. It is desirable to explore adding and subtracting situations in meaningful contexts. Students should have extensive investigative experiences in which they use a variety of concrete materials and pictures to model and compare contexts, before recording the number sentence that represents their thinking. It is important that problems be personalized, but students also need experience interpreting how addition and subtraction are portrayed in print.

When working with addition and subtraction, it is important to include examples of active and static situations. Active situations involve the physical joining of sets (join problems) or the physical separating of sets (separate problems.) Static situations (part-part-whole and comparison) involve the implied joining or separating of sets that are not physically joined to form a whole or separated to form parts.

Examples of active situations are

-
- I had 8 pencils. I gave 4 of them to my friend. How many do I have left?
- I had 4 pencils. My teacher gave me 3 more. How many do I have now?

Examples of static situations (part-part-whole and comparison are

- There are 4 cars parked on one side of the road. There are 3 cars parked on the other side of the road. Altogether, how many cars are parked on the road?)
- There are 7 red and green cars parked on the road. Four of them are red. How many cars are green? [In this situation, the whole remains the same, nothing is added or taken away, we are looking to find the two parts that make up the whole]).

Addition and subtraction problems can be categorized based on the kinds of relationships they represent. It is important that all of the following structures of problems be presented and that these are developed from students' experiences. These structures include

ACTIVE			**STATIC**	
Join			**Part-Part-Whole**	**Compare**
Result Unknown	**Change Unknown**	**Start Unknown**	**Whole Unknown**	**Difference Unknown**
Pat has 8 marbles. Her brother gives her 4. How many does she have now? $8 + 4 = ?$	Pat has 8 marbles. Her brother gives her some more marbles. Then, Pat has 12 marbles. How many marbles did Pat's brother give her? $8 + ? = 12$ or $12 - 8 = ?$ or $8 + ? = 12$	Pat has some marbles. Her brother gave her 4 and now she has 12. How many did she have to start? $? + 4 = 12$ or $12 - 4 = ?$	Pat has 8 blue marbles and 4 green marbles. How many does she have in all? $8 + 4 = ?$	Pat has 8 blue marbles and 4 green marbles. How many more blue marbles than green marbles does she have? $8 - 4 = ?$ or $4 + ? = 8$
Separate			**Part-Part-Whole**	**Compare**
Result Unknown	**Change Unknown**	**Start Unknown**	**Part Unknown**	**Smaller or Larger Unknown**
Pat has 12 marbles. She gives her brother 4 of them. How many does she have left? $12 - 4 = ?$	Pat has 12 marbles. She gives her brother some. Now she has 8. How many marbles did she give to her brother? $12 - ? = 8$ or $12 - 8 =$ or $8 + ? = 12$?	Pat had some marbles. She gave her brother 4 of them. Now, she has 8 marbles. How many marbles did she have to start? $? - 4 = 8$ or $8 + 4 = ?$	Pat has 12 marbles. Eight are blue and the rest are green. How many are green? $8 + ? = 12$ or $12 - 8 = ?$ or $12 = 8+ ?$	Pat has 8 blue marbles and some green marbles. She has 4 more blue marbles than green ones. How many green marbles does she have? $8 - 4 = ?$ or $? + 4 = 8$

Initial work with addition and subtraction may focus on join and separate types of problems because students associate the actions in these problems with the operations. Eventually, however, students should experience all story problem structures. **Note:** Compare problems are also addressed as part of outcome N05.

More information on each of these structures can be found in *Teaching Student-Centered Mathematics* by Van de Walle and Lovin (2006).

Symbols for addition, subtraction, and equal should be introduced slowly and should only be used as a way to record number sentences that represent the contexts that are modelled. (See outcome PR04.) Model the symbolic recording when students share solutions orally. This modelling can be done well before expecting students to begin recording number sentences. When working with the symbols for addition and subtraction, the sign (–) is called a minus and the sign (+) is called a plus. Students should be exposed to a variety of language phrases that correspond to these signs, depending upon the contexts. For addition phrases, such as 4 + 2, these could include

- 4 add 2
- 4 and 2
- 4 plus 2
- 2 more than 4
- the sum of 4 and 2
- the total of 4 and 2

For subtraction phrases, such as 8 – 2, these could include

- 8 subtract 2
- 8 take away 2
- 8 minus 2
- 2 less than 8
- the difference between 8 and 2

Care should be taken with the equal sign (=). The equal sign means *is the same as* or *is equivalent to*. However, most students come to think of it as a symbol that tells you that the answer is coming up. A good idea is to often use the phrase *is the same as* in place of, or in conjunction with, equals as you read number sentences with students.

Students need to be able to explain how they got their answers demonstrating their use of addition and subtraction. By observing students at work, we can assess their understanding of how they solve addition and subtraction problems.

Performance Indicator Background

N09.01 Students will be more captivated and engaged if initially they act out story situations in which they can play the central characters. Present an addition or subtraction story for each structure. Have students act out the story situation to find the solution. For example, five children are sitting on the story mat. Two children left the mat to go back to their seats. How many children stayed on the mat? At this point students are not yet associating these situations with addition and subtraction; rather, they are using counting strategies to act out and find solutions.

N09.02 After acting out story situations, students should model the situations using concrete materials and pictures. They might use story boards or mats to represent the story situations. Students should spend a considerable amount of time modelling and talking about the problems they have solved and should be encouraged to share using language that reflects what they have done, such as 2 and 3 more is equal to 5. The focus is to get students comfortable and competent with all addition and subtraction story structures including joining, separating, part-part-whole, and comparing problems. In all, eleven story structures are emphasized and students develop connections and the relationship between addition and subtraction.

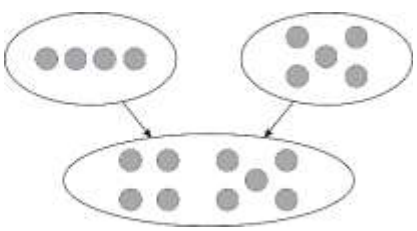
Students should use set models concretely before recording their actions with pictures. The pictures students draw should describe what they did with the concrete models. It is important to note that students may draw a variety of pictures representing what they did with concrete materials. Any student generated drawing should be accepted provided the drawing is complete, logical, efficient, accurate, and mathematically correct, and represents the context and key elements of the problem. For example, when drawing pictures to represent join problems, students must incorporate the key elements of those problems:

- The sets being joined
- The result of the joining
- The action of joining

Teachers should monitor students' pictures to ensure that the key elements are present in those student drawings. One or more samples might be offered to a student who is struggling to record their thinking pictorially.

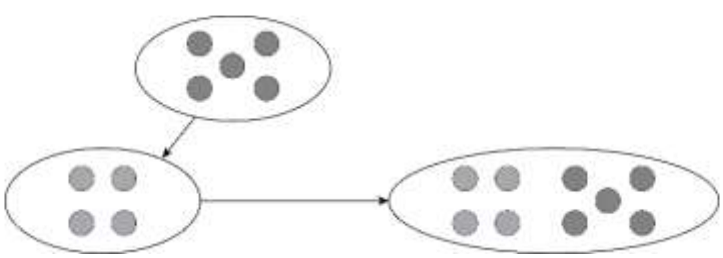
When students are solving join or separate story problems and using pictures to represent their thinking, they need to show a representation of the action in the question, not just a symbolic answer or a picture without action. Arrows may be used to represent the joining or separating of the sets. Part-part-whole mats are appropriate for representing part-part-whole story problems, and comparison problems may be represented by showing one-to-one matching of the sets.

To represent the joining problem, Maya had 4 pencils. She got 5 more pencils at the store. How many pencils did Maya have then? students might draw,



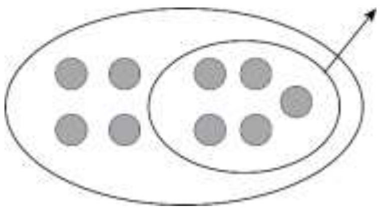
They would record the number sentence symbolically as 4 + 5 = 9.

Students might also draw,

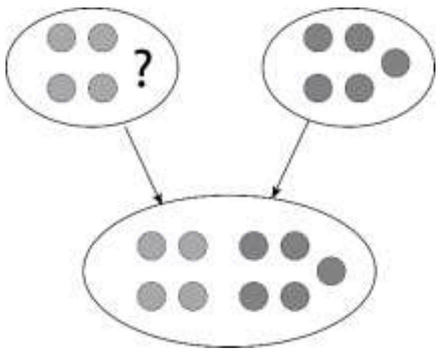


They would record the number sentence symbolically as 4 + 5 = 9.

To represent the separating problem, Maya had some pencils. She got 5 more pencils at the store. Then, she had 9 pencils. How many pencils did Maya have to start? students might draw,

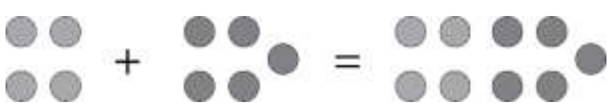


Students might also draw,

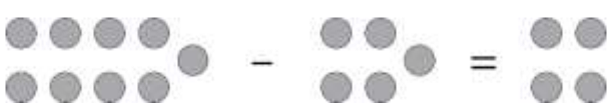


Many students draw pictures using counters, but they use the addition sign (+) or the subtraction sign (-) and the equal sign (=) to indicate the action of addition or subtraction. This should be avoided. The symbols are just that, “symbols”, they do not represent “the action” of either operation. The addition sign (+), the subtraction sign (-), and the equal sign are symbolic conventions. They should only be used with other symbols.

A picture that uses both pictures and symbols should not be accepted as correct. Symbols should only be used with other symbols. For example, neither of these pictures should be accepted as correct.



4 + 5 = 9

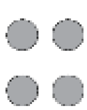


9 - 5 = 4

A picture that does not represent the action in the story problem and only represents the sum or difference should also not be accepted as correct.

For example, neither of these pictures should be accepted as correct.


4 + 5 = 9


9 - 5 = 4

Students will need support when they first start recording these situations and solutions with number sentences that represent how they thought about the problems. For example, consider the story problem: Pat has 8 marbles but she would like to have 12. How many more does she need to get? Some students may count out 8 marbles and add on until they reach 12, counting the 4 that they added on. These students would likely write 8 + 4 = 12 to represent how they thought about the problem. Other students may count out 12 marbles, remove the 8 they knew they had, and count the 4. These students would likely write 12 – 8 = 4 to represent how they thought about the problem. Either sentence would be acceptable, but students should be able to explain the number sentence they have written in relation to the situation they are modelling.

N09.03 and N09.04 Students first create story situations using prompts, such as storyboards. Many different problems can be created using the same storyboards. Students should share their story problems with others and record the corresponding number sentence for each of their problems. Students need experiences where they create story problems that will correspond to given number expressions, such as 3 + 7, and number sentences, such as 3 + 7 = 10. They should be asked to develop the script and then act out the story.

By applying their own experiences to the number expressions or sentences, they will create many different scripts. Students more naturally create join and separate (result unknown) types of problems, so they will need to be encouraged to create other types. One possible way to do this is to present three or more problems of one type, such as join (change unknown), discuss how the problems are alike and how they are different, and challenge students to create one like them.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Give students counters and ask them to act out the story problem you will tell them. Tell students a simple join story problem using numbers less than 5, such as "I have 3 flowers and my friend gave me 2 more flowers. How many flowers do I have now?" Observe as students solve the problem.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Model this problem for a pair of students: I had 5 pencils. I found some pencils. Now, I have 9 pencils. How many pencils did I find? Ask the pair to make up a similar problem, using objects of their choice, and to model and describe it.
- Ask students to tell an addition/subtraction story involving 8 and 5 while manipulating a model.
- Provide the students with a given number of counters. Ask them to add/remove 3 or another number of counters and tell how many are now there. Ask them to represent this symbolically.

- Tell the student that Jake had 9 pencils and lost 3, while Martha had 7 pencils and lost 2. Ask, Who has more pencils left? Explain how you know.
- Tell the student that you had 9 marbles but lost some. Now, you have 4 marbles left. Ask, How many marbles did I lose? Show how you know.
- Present students with a number sentence. Ask them to create a story problem that matches the number sentence.
- Present students with a pictorial representation of a story problem. Ask them to tell a story and record a number sentence that matches the picture.
- Provide students with manipulative materials and present them with each of the following story problems. Ask students to solve each problem and record the number sentence that represents how they thought about the problem
  - Chen has 8 cards. David has 4 more cards than Chen. How many cards does David have?
  - Brodie has 18 toys. Eight of his toys are cars and the rest are animals. How many animals does Brodie have?
  - Sophie had 12 apples. She gave some to her mother and now she has 8 apples. How many did she give to her mother?
  - Shona had 15 hairbands. Her dad gave her some more. Now she has 18 hairbands. How many hairbands did dad give her?
- Invite students to create their own word problems for the number family 7, 9, and 16 (numbers related by addition and subtraction). Ask them to write a word problem that uses these numbers in an addition situation and another word problem that uses these numbers in a subtraction situation.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)
- Mathematics 1 Checkpoint 6, pp. 66–67 (Line Master 6.1)
- Mathematics 2 Checkpoint 5, Task 1, pp. 59–60 (Line Master 5.2)
- Mathematics 2 Checkpoint 6, pp. 68–69 (Line Masters 6.1 and 6.2)

Planning for Instruction

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of my students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Personalize word problems for students. Encourage students to create a variety of meaningful problems based on situations with which they are familiar.
- Use daily classroom experiences to tell story problems that represent addition and subtraction.
- Manipulate concrete materials to represent the context as you or students relate a solution to a word problem. Verbalize the process as you manipulate the model.
- Provide a wide variety of problem types and include all four problem structures (join, separate, part-part-whole and comparison).

SUGGESTED LEARNING TASKS

- Choose a book, or make up a story, that tells about an addition or a subtraction situation and ask a student to model the situation with counters as you read the book. Reread the story and use pictures and number sentences to record the actions. Ask students to make up other problems about the story.
- Model the telling of story problems. Ask students to orally tell story problems for other students to act out and answer. Story mats can be used for support.
- Use a story mat and some counters for each student. Present problem scenarios and ask students to act them out using manipulatives.
- Have students model addition and subtraction questions concretely using a Rekenrek.
- Pose story problems, such as Janet has 6 baseball cards. Mario gives her some hockey cards. She now has 13 sports cards. How many sports cards did Mario give her? Ask students to model the solution using concrete materials, draw a picture showing how they solved the problem, and record a number sentence (an equation) that describes how they thought about the problem. Observe how students solve the problem. Students should be encouraged to share strategies and to explain their thinking.
- Ask students to make a drawing to model this and other structures of problems: Robert had some baseball cards. He gave his brother 2 of the cards. Robert now has 8 cards. How many cards did Robert have to start?
- Ask students to think of a situation in which someone might add. Ask students to think of a situation when they might subtract.
- Provide storyboards for students to use with manipulatives to create, model, and solve story problems. Storyboards can be created by drawing a simple scene, such as a fence, an ocean, or a tree, on a half-sheet of 8 ½" × 11" paper. As well, a piece of black construction paper can be used to represent outer space or nighttime, sandpaper for a beach, and blue paper for the sky. Many different problems can be created using the same storyboards. Students should share their story problems with others and record the corresponding number sentence for each of their problems.
- Present students with a number sentence. Ask students to create a story problem that matches the number sentence.

SUGGESTED MODELS AND MANIPULATIVES

- counters
- dominoes
- linking cubes
- number cubes
- Rekenrek
- ten-frames

MATHEMATICAL LANGUAGE

Teacher	**Student** (oral language)
▪ add, subtract, addition, subtraction ▪ count on, count back, one more, one less, doubles, near doubles, make 10 ▪ equal, is the same as, is equivalent to ▪ join, separate, compare, part-part-whole ▪ minus, plus ▪ more than, less than ▪ number sentence, equation ▪ story problem, situation ▪ sum, difference, total, addend, minuend, subtrahend	▪ add, subtract, addition, subtraction ▪ count on, count back, one more, one less, doubles, near doubles, make 10 ▪ equal, is the same as, is equivalent to ▪ minus, plus ▪ more than, less than ▪ number sentence, equation ▪ story problem, situation ▪ sum, difference, total

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 103–110
- *Making Math Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 159–166
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 65–75

Videos

- *Using Language Stages to Develop Addition Concepts* (15:38 min.) (ORIGO Education 2010)
- *Using Language Stages to Develop Subtraction Concepts* (18:32 min.) (ORIGO Education 2010)
- *Using Static Problems to Relate Addition and Subtraction and Introduce Equality* (13:25 min.) (ORIGO Education 2010)
- *Using Static Problems to Relate Addition and Subtraction and Introduce Functions* (18:59 min.) (ORIGO Education 2010)

Notes

SCO N10 Students will be expected to use and describe strategies to determine sums and differences using manipulatives and visual aids. Strategies include
- counting on or counting back
- one more or one less
- making ten
- doubles
- near doubles

[C, CN, ME, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N10.01 Use and describe a personal strategy to determine a sum.
N10.02 Use and describe a personal strategy to determine a difference.
N10.03 Use and describe how two different strategies can be used to determine a sum or difference.

Scope and Sequence

Mathematics Primary	Mathematics 1	Mathematics 2
—	**N010** Students will be expected to use and describe strategies to determine sums and differences using manipulatives and visual aids. Strategies include ▪ counting on or counting back ▪ one more or one less ▪ making ten ▪ doubles ▪ near doubles	**N010** Students will be expected to apply mental mathematics strategies to quickly recall basic addition facts to 18 and determine related subtraction facts.

Background

Students' first strategy for addition and subtraction involved counting sets and recounting to find results. For example, for 6 + 7, students used materials to count out a set of 6 and a set of 7, and then counted both sets together starting at 1 until they reached 13. Similarly, for 12 – 4, students used materials to count out a set of 12, from this set of 12 they took away 4 one at a time, and then counted how many were left. This outcome addresses exploring alternative, more efficient strategies to this *count-all* strategy. Students will need many rich experiences exploring these strategies concretely and pictorially. Modelling specific examples of the strategies will be an important aspect of the development.

In Mathematics 1, the strategies are the focus of the learning experiences; students are <u>not</u> expected to recall facts. In Mathematics 2, students will be applying these strategies mentally without using materials, so this foundational work with models in Mathematics 1 is critical. It is important that opportunities are provided for student discussion and sharing of a wide variety of strategies, including their own.

In general, a strategy should be introduced in isolation from other strategies, a variety of different reinforcement activities should be provided until students are competent in their use of that strategy, the strategy should be assessed in a variety of ways, and then it should be combined with other previously learned strategies.

A. Introducing a Strategy

The approach to introducing a strategy is to give students an example of a computation to solve for which the intended strategy would be useful to see if any of the students already can apply that strategy. If so, the student(s) can explain the strategy to the class with the teacher's help. If not, the teacher could share the strategy.

The explanation of a strategy should include anything that will help students see the pattern and logic of the strategy. This includes the use of concrete materials, visuals, and/or contexts. The introduction of a strategy should include explicit modeling of the processes used to carry out the strategy, and explicit discussion of the situations for which the strategy is most appropriate and efficient. Discussion should also include situations for which the strategy would not be the most appropriate and efficient one. Most important is that the logic of the strategy should be well understood before it is reinforced.

B. Reinforcing a Strategy

Each strategy should be practiced in isolation until students can determine solutions in a reasonable time frame. Students must understand the logic of the strategy, recognize when it is appropriate, and explain the strategy. The amount of time spent on each strategy should be determined by the students' abilities and previous experiences.

Reinforcement activities for a strategy should be varied in type and should focus as much on the discussion of how students obtained their answers as on the answers themselves. The reinforcement activities should be structured to insure maximum participation. Student participation should be monitored and their progress assessed in a variety of ways to help determine the length of time that should be spent on a strategy.

After most of the students can model the strategy, teacher support is needed to help the students integrate it with other strategies they have developed. This can be done by providing tasks that include a mix of number expressions, for which this strategy and others would apply. Students should complete the tasks and discuss the strategy/strategies that could be used; or students should match the number expressions included in the task to a list of strategies and discuss the attributes of the number expressions that prompted them to make the matches.

C. Assessment

Assessment of strategies should take a variety of forms. Teachers should record any observations made during the reinforcements, ask the students for oral responses and explanations, and have them explain strategies. Individual interviews can provide many insights into a student's thinking, especially in situations where responses are not correct.

Assessments, regardless of their form, should shed light on students' abilities to determine sums and differences efficiently and accurately, to select appropriate strategies, and to explain their thinking.

Integration of Strategies

After students have achieved competency using one strategy, provide opportunities for them to integrate it with other strategies they have learned. The ultimate goal is for students to have a network of strategies that they can flexibly and efficiently apply whenever they need to determine a sum or difference. This integration can be aided in a variety of ways, some of which are described below.

Students should be presented with a variety of questions, some of which could be done just as efficiently by two or more different strategies and some of which are most efficiently done by one specific strategy. It is important to have a follow-up discussion of the strategies and the reasons for the selection of specific strategies.

Include written questions as part of the assessment of strategies. This could be completed as a journal entry, part of a portfolio, or other assessment for which students will get individual feedback. Ask students to explain how they could determine a sum or difference for a given question in one or more ways, to comment on a student response that has an error in thinking, or to generate sample questions that would be efficiently done by a specified strategy.

LANGUAGE

Students should hear a variety of language used in association with each operation, so they do not develop a single word-operation association. Through rich language usage, students will be able to quickly determine which operation and strategy they should employ. For example, when a student hears, “Six plus five”, “Six and five”, “The total of six and five”, “The sum of six and five”, or “Five more than six”, they should be able to quickly determine that they must add 6 and 5, and that an appropriate strategy to do this is the *near doubles* strategy.

CONTEXT

Students should be presented with a variety of contexts for each operation in some of the reinforcement activities, so they are able to transfer the use of operations and strategies to situations found in their daily lives. By using contexts such as measurement, money, and food, the numbers become more real to the students. Contexts also provide opportunities for students to recall and apply other common knowledge that should be well known. For example, when a student hears, “How many days in two weeks?” they should be able to determine that there are seven days in a week and that double seven is 14, so there are 14 days in two weeks.

Performance Indicator Background

N10.01 and N10.02 Through continued exposure to, discussion of, and practice applying a variety of strategies, students will adopt the strategies that make the most sense to them or with which they are most comfortable. These will be their personal strategies. The most important goal is that students are spontaneously using personal strategies, not count-all strategies, throughout their day. Appropriate strategies for determining sums and differences in Mathematics 1 are described below.

COUNTING ON

Students’ first strategy for addition involves counting sets and recounting to find results. For example, to determine the sum of 6 and 7, students use materials to count out a set of 6 and a set of 7, and then recount both sets together starting at 1 until they reach 13. This is a *count all strategy.* The *counting on strategy* is an alternative and more efficient strategy to use.

To determine the sum of 6 + 3 using the *counting on strategy*, make a set of 6 counters and a set of 3 counters. Cover the 6 counters with your hand as you say, “6,” Then point to each counter in the set of 3, as you say, “7, 8, 9.” Students must be able to say the number sequence forward from a given number before beginning the *count on strategy*.

Counting Back

Students' first strategy for subtraction involves counting sets and recounting to find results. For example, to determine 12 – 4, students count out a set of 12, and from this set take away a set of 4 one at a time, and then count to determine how many are left. This is a *count all strategy.* The *counting back strategy* is an alternative and more efficient strategy to this *count all strategy*.

To determine the difference of 8 – 2 using the *counting back strategy*, make a row of 8 counters, Remove 1 counter and place it below as you say, "7," Remove another counter and place it below as you say, "6." You stop because there are 2 counters below, and the 6 you said is how many counters were left in the row above. Students must be able to say the number sequence forward backward from a given number before beginning the count back strategy.

One More

If students have the ability to state the next number for any given number with no hesitation, they will be able to determine that adding 1 to a number, or adding a number to 1, will result in the next number in the counting sequence. To help students visualize the *one more strategy*, they could also use ten-frames for all the numbers from 1 to 9 and notice the one-dot difference between consecutive numbers.

To determine the sum of 1+1, 2+1, 3 + 1, 4 + 1, 5 + 1, 6 +1, 7 + 1, 8 + 1, and 9 + 1 using the *one more strategy*, build a set of cube towers for a series of consecutive numbers 4, 5, 6, and 7. Place the towers side by side, showing 4 + 1 by placing 1 cube on the 4 tower to see that it becomes a 5 tower. Similarly show that 5 + 1 becomes a 6 tower and 6 +1 becomes a 7 tower. Then generalize that adding 1 to a number results in the next number in the counting sequence.

One Less

Students should associate subtracting 1 as a call for the number that comes before it in the counting sequence. If students have the ability to state the number that comes before any given number with no hesitation, they will be able to determine that subtracting 1 from a number will result in the number that comes before it in the counting sequence.

To determine the difference of 10 – 1, 9 – 1, 8 – 1, 7 – 1, 6 – 1, 5 – 1, 4- 1, 3-1, 2-1 and 1 – 1 using the *one less strategy*, use the same set of towers as those used for the one more strategy. Remove 1 cube from the 10-tower to see that it becomes a 9-tower, and so on. Then generalize that subtracting 1 from a number results in the number that comes before it in the counting sequence.

Making Ten

This strategy involves taking 1 or 2 from the one addend to make the 9 (or 8) addend a 10, and then adding this 10 to what was left from the other addend. To help students visualize the *make ten strategy*, they should use double ten-frames to represent the addition of 9 and any addend less than 10.

To determine the sum of 9 + 5 using the *make ten strategy*, make 9 in one ten-frame with counters and 5 in another ten-frame. Move 1 counter from the 5 and put it with the 9, to make 10. (4 remains in the other ten-frame.), Say, "Nine plus five is the same as ten plus four, which is fourteen."

Doubles

The *doubles strategy* should be developed using associations with things with which students are familiar. That is, each double should be associated with a real-life context. Examples of associations for each of the doubles facts are shown in the table below. Students may identify other associations for the *doubles strategy*.

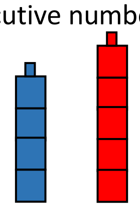
Double Strategy	Possible Association
1 + 1	Number of tires on two unicycles Number of eyes on one person
2 + 2	Number of tires on two bicycles Number of ears on two people
3 + 3	Number of tires on two tricycles Number of sides on two triangles Six pack of pop
4 + 4	Number of tires on two cars Number of sides on two squares Number of legs on a spider
5 + 5	Number of fingers on two hands Two rows of dots on a ten-frame
6 + 6	Dozen eggs in a carton
7 + 7	Number of days in two weeks
8 + 8	Number of crayons in two rows in a box Number of legs on two octopuses
9 + 9	Number of tires on an 18-wheeler truck

A set of cards with an expression on one side, for example, 9 + 9, and a picture of the associated context on the other side would be a good material to use to introduce and reinforce the *doubles strategy*. To determine the sum of 6 + 6 using the *doubles strategy*, display a picture of an egg carton, which is known to contain 12 eggs arranged in two rows of 6.

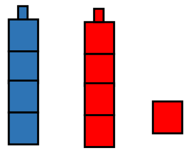
NEAR DOUBLES

The *near doubles strategy* is a combination of the *doubles strategy* and the *one more strategy*. It involves doubling the smaller number and adding one. (Some students might double the larger number and subtract 1.) Students need to be convinced that the larger number can be partitioned without changing the sum. For example, they need to see that for 4 + 5, the 5 can be partitioned into 4 and 1, and that the 4 can be combined with the other 4 before adding the 1.

To help students visualize the *near doubles strategy*, create linking cube towers for numbers 2 to 9. Place two consecutive number towers, such as the 4-tower and the 5-tower, beside each other.



Take one cube off the larger tower so student can see the double towers (4 and 4) and the 1.



In symbols this could be recorded as,

4 + 5 =
4 + (4 + 1) =
(4 + 4) + 1 =
8 + 1 = 9

To determine the sum of 6 + 7 using the *near doubles strategy*, build a 6-cube tower and a 7-cube tower. Place them side by side. Remove 1 cube from the tower of 7 to create two towers each with 6-cubes and a single cube., Say, Six plus seven is the same as six plus six plus one. “Six and six are twelve and one more is thirteen.

TEN FRAME VISUALIZATION STRATEGY

Students may be able to visualize many of the subtraction facts to 10, by visualizing the first number on a ten-frame (minuend) and removing the number of dots (subtrahend) to get the result. Begin with the facts that subtract from 5, so students will visualize the first row of a ten-frame filled and remove the necessary dots to see the result. For example, to determine the difference of 5 – 2, students would visualize the five dots in the first row of the ten-frame, remove 2 of the dots, and see 3 as a result. This could be followed by facts that subtract from 10, and finally other facts with minuends to 9. For 10 – 2, students would visualize the ten dots in the ten-frame, remove 2 of the dots, and see 8 as a result.

N10.03 To ensure that students develop more than one personal strategy, they should be expected to describe two ways to find a sum or difference. For example, if asked to find the sum 8 + 7, they may describe how it could be found by counting on **from 8**, or by near doubles **(7 + 7 and 1 more)**, or perhaps make 10 (**8 and 2 make 10; 10 and 5 more make 15).**

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students’ prior knowledge.

- Ask students to tell you the number that
  - comes after 8
  - comes before 6

- Ask students to show a set of 2 counters and a set of 8 counters. Ask them to tell you how many counters there are in all. Observe the strategy used to determine the sum. Students should count on from either 2 or 8, rather than counting all counters by 1s.

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to explain how they solve each of the following computations using the strategies studied.
  7 + 1   8–2   8 + 9   7–1   6 + 4   7 + 8   4 + 7   9 + 6

- Ask students to explain how they could solve each of the following computations using two different strategies.
  8 + 9   9 + 6
- Ask students how they could use 6 – 4 = 2 to determine the difference of 6 – 3. Students may use materials to model this.
- Have students work in pairs and answer the following question. If you did not know the answer to 9 + 6, what are some strategies you can use to get the answer? Encourage students to come up with more than one strategy to find the sum. Students discuss their ideas with their partner and then present their ideas to the class.

Follow-up on Assessment

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Responding to Assessment

Numeracy Nets K–2 (Bauman 2011)
- Mathematics 1 Checkpoint 6, pp. 66–67 (Line Master 6.1)
- Mathematics 2 Checkpoint 6, pp. 68–69 (Line Masters 6.1 and 6.2)

Planning for Instruction

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Long-term Planning
- Yearly plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of my students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide students with opportunities to develop their own strategies for determining a given sum or difference.
- Encourage students to use strategies for solving problems.
- Ask students to employ as many representations as possible for determining sums and differences, including physically acting out, drawing pictures, verbally explaining their ideas, using concrete materials and number lines, and writing number sentences.
- Encourage students to create their own word problems. They can write the problems or dictate them to a scribe.

SUGGESTED LEARNING TASKS

- Have students spill 10 two-coloured counters from a cup. Ask them to describe the addition and subtraction situations and the strategy they could use to determine the sum (or the difference) of the two numbers that are shown. For example, if 7 red and 3 yellow counters are spilled, students could state, “7 and 3 make 10; 3 and 7 make 10; 10 less 3 is 7; 10 less 7 is 3.”
- Have students show a variety of combinations of numbers that make 10 using ten-frames. Students can record their work with pictures, numbers, and words.
- Use picture cards of real-life contexts to illustrate doubles; for example, 5 + 5 could be illustrated with two hands; 6 + 6 could be illustrated with an egg carton showing a dozen eggs, etc.
- Prepare a set of addition and subtraction problem cards. Have students select a card and identify a strategy that could be used to find the answer. The cards can then be placed under that strategy heading.
- Show a ten-frame for a short time. Ask students to select the ten-frame from their set that is one more or one less, than the card you showed.
- Make missing part cards. Each card has a numeral for the whole and two dot sets with one set covered by a flap. Ask students how many are covered and write the number sentence.
- Ask students to build a linking cube train of 10 cubes with two colours in different ways and to record the parts and the whole using addition and subtraction equations
- Have a group of approximately 12 students stand in front of the room. Ask the class how many students are at the front. Divide the group into two smaller groups. Ask the class how many students are at the front now and how do they know. Explore the different ways to partition the larger group.
- Play “I Wish I Had” with students. The teacher shows a number of counters and says, “I have 6 counters. I wish I had 10.” Students respond with, “You need 4 more because 6 + 4 = 10.”
- Provide each student with a card on which a variety of number sentences are written. Each card should contain different number sentences. The teacher calls out a number, and each student covers any number sentence on their card that has the number as an answer. For example, if you call out 5, a student might cover 4 + 1, 6 – 1, or 10 – 5.
- Request students to work in pairs and by joining fingers, demonstrate the doubles from 1 to 10. This also offers opportunity to reinforce skip counting by 2s.

SUGGESTED MODELS AND MANIPULATIVES

- coins
- counters
- dominoes
- dot cards

- hundred chart
- linking cubes
- number lines
- ten-frames

MATHEMATICAL LANGUAGE

Teacher	**Student** (oral language)
▪ add, subtract, addition, subtraction ▪ counting on, counting back ▪ doubles, near doubles ▪ making ten ▪ one more, one less ▪ number sentence, equation ▪ strategy ▪ sum, difference	▪ add, subtract, addition, subtraction ▪ counting on, counting back ▪ doubles, near doubles ▪ making ten ▪ one more, one less ▪ number sentence, equationstrategy ▪ sum, difference

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 111–115
- *Making Math Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 166–170
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 40–41, 44–45, 47, 56

Videos

- *An Introduction to Teaching Addition Number Facts* (15:51 min.) (ORIGO Education 2010)
- *Teaching the Think-Addition Subtraction Fact Strategy* (13:41 min.) (ORIGO Education 2010)
- *Using a Hands-on Approach to Develop Mental Strategies for Addition* (11:04 min.) (ORIGO Education 2010)
- *Using a Hands-on Approach to Develop Mental Strategies for Subtraction* (6:45 min.) (ORIGO Education 2010)
- *Using Static Problems to Relate Addition and Subtraction and Introduce Equality* (13:25 min.) (ORIGO Education 2010)
- *Using Static Problems to Relate Addition and Subtraction and Introduce Functions* (18:59 min.) (ORIGO Education 2010)

Notes

Patterns and Relations (PR)

GCO: Students will be expected to use patterns to describe the world and solve problems.

GCO: Students will be expected to represent algebraic expressions in multiple ways.

SCO PR01 Students will be expected to demonstrate an understanding of repeating patterns (two to four elements) by identifying, describing, reproducing, extending, and creating patterns using manipulatives, diagrams, sounds, and actions. [C, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR01.01 Describe a given repeating pattern containing two to four elements in its core.
PR01.02 Identify errors in a given repeating pattern.
PR01.03 Identify the missing element(s) in a given repeating pattern.
PR01.04 Create and describe a repeating pattern using a variety of manipulatives, musical instruments, and actions.
PR01.05 Reproduce and extend a given repeating pattern using manipulatives, diagrams, sounds, and actions.
PR01.06 Identify and describe a repeating pattern in the environment (e.g., classroom, outdoors) using everyday language.
PR01.07 Identify repeating events (e.g., days of the week, seasons).
PR01.08 **Identify the core of a repeating pattern.**

Scope and Sequence

Mathematics Primary	Mathematics 1	Mathematics 2
PR01 Students will be expected to demonstrate an understanding of repeating patterns (two or three elements) by identifying, describing, reproducing, extending, and creating patterns using manipulatives, sounds, and actions.	**PR01** Students will be expected to demonstrate an understanding of repeating patterns (two to four elements) by identifying, describing, reproducing, extending, and creating patterns using manipulatives, diagrams, sounds, and actions. **PR02** Students will be expected to translate repeating patterns from one representation to another.	**PR01** Students will be expected to demonstrate an understanding of repeating patterns (three to five elements) by describing, extending, comparing, and creating patterns using manipulatives, diagrams, sounds, and actions.

Background

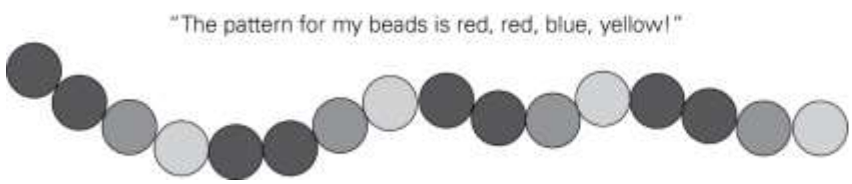
The foundation of all mathematics learning is investigating patterns and their representations. Patterns are explored in all the strands and are also developed through connections with other disciplines, such as science, social studies, English language arts, physical education, visual arts, and music. In Mathematics Primary, students sorted objects by attributes; copied, extended, described, and created a repeating pattern with a core of two or three elements; and identified the core of a repeating pattern. This outcome focuses on repeating patterns and ways of representing these patterns. Not only do students need to recognize a repeating pattern, but they must also be able to describe and extend a repeating pattern.

Repeating patterns have what mathematicians refer to as a “core” with a number of “elements.” The core of a pattern is the shortest set of elements that iterates (repeats). For example, the colour pattern red, yellow, green, red, yellow, green, red, yellow, green, …, has a core of three different elements that are repeated. The pattern red, red, yellow, yellow, red, red, yellow, yellow, red, red, yellow, yellow …, has a core of four elements even though this core has repeated elements. If students are expected to identify, describe, extend, or reproduce a pattern, it is important that the core of the pattern be repeated at least three times. Students should have many opportunities to work with materials that have one attribute difference, such as cubes, coloured tiles, two-colour counters, collections of small items of one kind (bread tags of different colours), before using materials that have more than one visible attribute, such as attribute blocks or sets of small plastic animals.

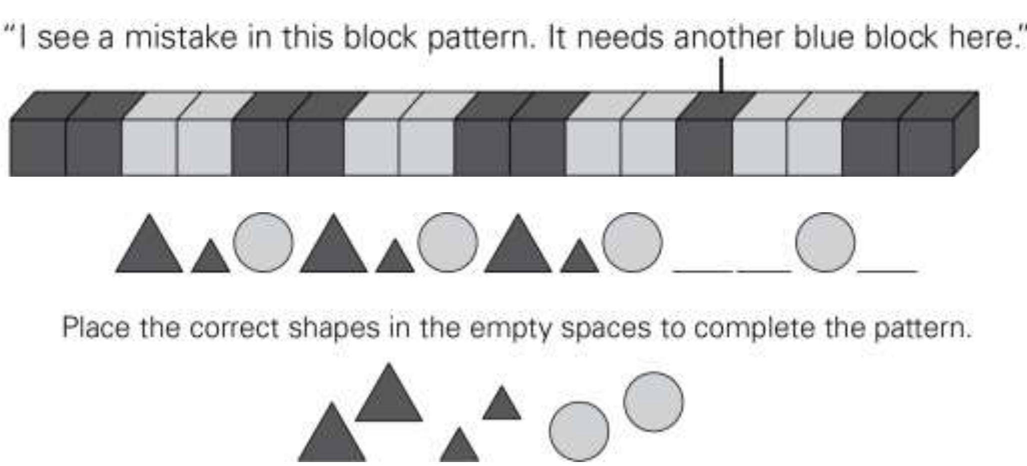
While the patterns being explored in Mathematics 1 are repeating patterns, you should be aware that some students may create or describe other types of patterns, such as growing (increasing) patterns. The detection of patterns should be an ongoing and natural process. Students need to experience both teacher-directed and independent patterning activities. Teacher-directed activities should encourage students to analyze a variety of visual, kinesthetic, and auditory patterns. Independent activities provide students with the opportunity to explore, reproduce, extend, and create patterns appropriate to their level of understanding.

Performance Indicator Background

PR01.01 Student descriptions of a repeating pattern should include the attribute and the core. Students should be given the opportunity to describe patterns orally as it helps them interpret the patterns they experience visually and solidify their understanding of the concept. It also allows students to learn from each other. Repeating the core of a pattern three times is more likely to lead students to the anticipated result; however, there may be alternative extensions unless the pattern rule is described.



PR01.02 and PR01.03 Once students are confident in identifying patterns, describing them, and extending them, provide repeating patterns containing two to four elements in which there are errors or missing elements. Ask students to identify the errors or omissions and correct the patterns. Most students may find the obvious errors; however, some students may find less typical errors and these should also be acknowledged.



PR01.04 Students should be encouraged to create and describe their own patterns. In all patterning activities, students should be given the opportunity to describe the patterns verbally. This not only helps them to interpret the specific patterns they experienced, but also to solidify their understanding of the concept of patterning in general. As well, it provides another opportunity for students to learn from each other. **Students should describe the elements and the core of their patterns and how the pattern repeats.**

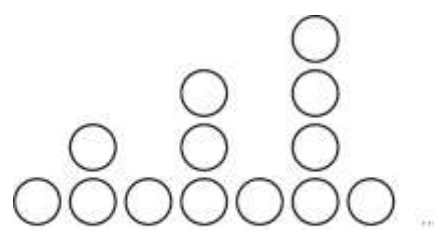
Give students opportunities to make patterns on an informal and independent basis. The choice of manipulatives can affect the difficulty of the task. Connecting cubes and colour tiles are the easiest manipulatives from which students can make patterns, as they have only one visible attribute. They can also create a song that has a pattern word, a sentence, or a beat.

PR01.05 When presenting a pattern for students to reproduce or extend, repeat the core three times (e.g., red, red, blue, red, red, blue, red, red, blue, ...). As students become more efficient reproducing and extending patterns, repeat the core three times and begin the fourth repetition (e.g., red, red, blue, red, red, blue, red, red, blue, red, …). Observe whether the student is able to continue the pattern from the last element given or repeats the entire core.

PR01.06 and PR01.07 By discussing patterns that occur naturally in students' everyday lives, patterning becomes more meaningful to them. Ask questions that alert students to patterns that occur naturally, such as in classroom routines, games played, songs sung, dance routines, books read, days of the week, and seasons of the year. Ask students to be on the alert for examples of patterns at home, on the playground, and at school. Students may be able to identify geometric and number patterns, such as tiles on the floor or ceiling, and numbers on doors down a corridor. **Opportunities to look for and create patterns should be integrated across all subjects.**

PR01.08 Repeating patterns include the continued extension of a sequence beyond what the student can actually see. The core of a repeating pattern is the shortest set of elements that repeats. A non-repeating pattern has no detectable core; therefore, the identification of a core determines if a pattern is repeating or non-repeating.

A Repeating Pattern



A Non-Repeating Pattern

It may be more difficult for students to identify patterns where the first and last elements of the core are the same, for example, ABA patterns.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge.

- Make an AB pattern with cubes. Ask the students to copy, extend, describe, and identify the core of the pattern.

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Show a pattern of cubes; for example, RGGRGGRGG (red, green, green, …), and ask students to tell you what the pattern is. Then show a different pattern of cubes; for example, YYBYYBYYB (yellow, yellow, blue, …). Have students identify the new pattern. Then, ask students to tell you how the patterns are different and how they are the same.
- Tell students, "Mary has 6 green triangles and 3 orange squares." Show students the pieces on the overhead. Ask, Can she make two different patterns? Ask students to draw two possible patterns that Mary could make and explain the patterns. Ask, What comes next in each pattern?
- Tell the students that you think there is a pattern to the days (Monday, Tuesday, ...) in a week. Ask students to explain the pattern.
- Have students look at a repeating visual pattern, or listen to a repeating sound pattern, that contains an error. Ask students to correct the error and explain their thinking
- Take students on a walk around the inside and outside of the school looking for patterns. Students can draw a pattern they found and can describe the pattern to a classmate.
- Show students a pattern and have them draw the missing part of the pattern.

Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Responding to Assessment

Numeracy Nets K–2 (Bauman 2011)

- Mathematics 1 Checkpoint 7, pp. 74–75

Planning for Instruction

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of my students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide students with many opportunities to represent patterns concretely. As students create repeating patterns, ask them questions such as
  - What comes next/before/after? How do you know?
  - Can you extend the pattern to the left? To the right?
  - Which part of the pattern repeats? What is the pattern core?
  - What are the elements in the core?
  - Can you make a new pattern with the same materials?
  - Can you make a new pattern with the same elements?
  - What else could you use to make the same pattern?
  - Are these patterns the same? How do you know?
  - How is the pattern the same as that pattern?
  - How is this pattern different from that pattern?
- Allow students to identify patterns in their daily lives. This can include repetitive songs and rhythmic chants that are based on repeating patterns. Songs and poems can be used to explore patterns.
- Provide a variety of examples of patterns to explore: rhythmic/sound patterns, action patterns, colour patterns, shape patterns, patterns of attributes, and patterns of size.
- Provide students with opportunities to create patterns and to identify the core of a pattern visually.
- Integrate patterns in physical education, music, visual arts, and other subject areas to provide a context.
- Expect students to describe and to explain their thinking about patterns.

SUGGESTED LEARNING TASKS

- Ask students to use pattern blocks or attribute blocks to construct a pattern with two to four elements in its core. Ask them to explain the pattern to another student and describe the core.
- Ask students to create clapping patterns, or to use stickers or coloured counters to make visual patterns.
- Show students a pattern with an error or missing element in the pattern. Ask students to identify and correct the error or add the missing element.
- Show students a pattern that you have begun. Ask them to continue the pattern in two different ways.

- Give students pattern blocks and ask them to create an ABB pattern. Then ask students, What would this pattern sound like? Continue asking for other patterns, such as ABC, AAB, or ABBC.

SUGGESTED MODELS AND MANIPULATIVES

- attribute blocks
- calendars
- dot cards
- geometric solids
- linking cubes
- pattern blocks
- stamps and ink pads
- stickers

MATHEMATICAL LANGUAGE

Teacher	**Student** (oral language)
▪ core ▪ element ▪ describe, reproduce, extend, create ▪ repeating pattern ▪ repeating	▪ core ▪ element ▪ describe, reproduce, extend, create ▪ repeating pattern ▪ repeating

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 568–569, 571–573
- *Making Math Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 605–611
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 276–280

Video

- *Using a Teaching Sequence for Repeating Patterns* (22:55 min.) (ORIGO Education 2010)

Notes

SCO PR02 Students will be expected to translate repeating patterns from one representation to another. [C, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR02.01 Represent a given repeating pattern using another mode(e.g., actions to sound; colour to shape; ABC, ABC, ABC to blue, yellow, green, blue, yellow, green, blue, yellow, green, …)

PR02.02 Describe a given repeating pattern using a letter code(e.g., ABC).

Scope and Sequence

Mathematics Primary	Mathematics 1	Mathematics 2
PR01 Students will be expected to demonstrate an understanding of repeating patterns (two or three elements) by identifying, reproducing, extending, and creating patterns using manipulatives, sounds, and actions.	**PR01** Students will be expected to demonstrate an understanding of repeating patterns (two to four elements) by describing, reproducing, extending, and creating patterns using manipulatives, diagrams, sounds, and actions. **PR02** Students will be expected to translate repeating patterns from one representation to another.	**PR01** Students will be expected to demonstrate an understanding of repeating patterns (three to five elements) by describing, extending, comparing, and creating patterns using manipulatives, diagrams, sounds and actions.

Background

Students should recognize many different forms of the same pattern structure. They need to see that patterns constructed with different materials, or in other ways, have the same pattern structure. For example, all these repeating patterns have the same AAB pattern structure:

- clap, clap, snap, clap, clap, snap, clap, clap, snap, ... (sound pattern)
- red, red, yellow, red, red, yellow, red, red, yellow, … (colour pattern)
- square, square, triangle, square, square, triangle, square, square, triangle, … (shape pattern)
- 4, 4, 1, 4, 4, 1, 4, 4, 1, … (number pattern)

Performance Indicator Background

PR02.01 When students are given a repeating pattern in one mode, they should be able to represent that pattern in other modes. For example, if presented the action pattern: up, down, up, down, up, down, …, some students may use cubes to make a colour pattern: red, blue, red, blue, red, blue, …; other students may use cut-out shapes to make a shape pattern: square, triangle, square, triangle, square, triangle, …; while still others may use musical instruments to make a sound pattern: drum beat, cymbal crash, drum beat, cymbal crash, drum beat, cymbal crash, …

PR02.02 Translating two or more modes of the same pattern structure to a common format, such as **A, A, B, A, A, B, A, A, B…** helps students see beyond the materials making up the pattern. In fact, repeating patterns are sometimes described using letter codes **to** help students name and compare patterns. For example, all four patterns in the background of t**his outcome** above are AAB patterns. Students should be provided with many experiences describing repeating patterns containing two to four elements such as AB, AAB, ABB, ABC, AABB, ABA, and other combinations. Patterns, such as ABA, can be more challenging for students to identify as they will see the A again and assume the pattern has started over.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Show students an ABC pattern with cubes. Ask students to describe the pattern. Ask them to continue the pattern.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Present students with a pattern made with pattern blocks. Ask them to describe the pattern using a letter code and to make a sound pattern that could be described with the same letter code.
- Show a pattern to students. Provide a variety of materials for students and ask them to represent that pattern in two different ways.
- Show students a collection of visual repeating patterns. Ask them to identify the patterns that match and to explain why they match.
- Clap out a pattern. Ask students to represent that same pattern using pattern blocks.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- No Checkpoint for this outcome.

Planning for Instruction

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of my students be met?

Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

- Model how patterns can be translated from one medium to another, using objects, pictures, sounds, actions, or letters.
- Have students create their own patterns and translate them to a different medium; for example, concrete to action, concrete to pictorial, or concrete to auditory; action to pictorial, action to concrete, or action to auditory; pictorial to concrete, pictorial to action, or pictorial to letters.
- Expect students to explain their thinking about patterns and to describe how the patterns are the same and how they are different.
- Provide students with opportunities to use letter codes to describe their patterns.

Suggested Learning Tasks

- Give students pattern blocks and ask them to create an ABB pattern. Then ask students, What would this pattern sound like? Continue using other patterns, such as ABC, AAB, or ABBC.
- Give students a variety of patterns. Ask students to translate these patterns into other representations, such as letters, actions, manipulatives, or sounds.
- Have students create audiotapes of clapping patterns and have them use stickers to translate the pattern to a visual pattern.
- Provide students with a selection of patterns such as ABB, ABC, AAB, or ABBC from which to choose. Ask students to select the pattern they wish to create and use materials to create it. When students have finished creating their patterns, invite them to look at the patterns that have been created by other students and to find all of the patterns that match the one they created.
- Prepare a set of pictorial patterns and their letter descriptions. Have students match the picture to the correct letter description.

Suggested Models and Manipulatives

- calendars
- dot cards
- geometric solids
- linking cubes
- pattern blocks
- Power Polygons
- stamps and ink pads
- stickers

MATHEMATICAL LANGUAGE

Teacher	**Student** (oral language)
▪ core ▪ element ▪ repeating pattern	▪ core ▪ element ▪ repeating pattern

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), p. 573
- *Making Math Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), p. 611
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 276–280

Video

- *Using a Teaching Sequence for Repeating Patterns* (22:55 min.) (ORIGO Education 2010)

Notes

SCO PR03 Students will be expected to describe equality as a balance and inequality as an imbalance, concretely and pictorially (0 to 20). [C, CN, R, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR03.01 Construct two equal sets using the same objects (same shape and mass) and demonstrate their equality of number using a balance scale.
PR03.02 Construct two unequal sets using the same objects (same shape and mass) and demonstrate their inequality of number using a balance scale.
PR03.03 Determine if two given concrete sets are equal or unequal and explain the process used.

Scope and Sequence

Mathematics Primary	Mathematics 1	Mathematics 2
—	**PR03** Students will be expected to describe equality as a balance and inequality as an imbalance, concretely and pictorially (0 to 20).	**PR03** Students will be expected to demonstrate and explain the meaning of equality and inequality by using manipulatives and diagrams (0 to 100).

Background

Balance activities form a basis for understanding equality. Working with balance scale problems involving concrete materials and pictures, helps students build foundations for further study in operations, algebra, and equation solving. Using concrete materials will help students see the equality or inequality relationship between the quantities on each pan of the balance scale. It is important that balances be reasonably calibrated and the items used be heavy enough that the differences in mass are visible.

Throughout balance activities, students should answer questions, such as

- What is the relationship between the two quantities if the scale is balanced? This provides opportunities to use the phrases *is the same as* and *is equal to*.
- What is the relationship between the quantities if the left balance pan is lower than the right pan? This provides opportunities to use the phrases *is more than* and *is not equal to*, and to discuss the *is fewer than* relationship between the right and left quantities on the pans.
- What is the relationship between the quantities if the left balance pan is higher than the right pan? This provides opportunities to use the phrases *is fewer than* and *is not equal to*, and to discuss the *is more than* relationship between the right and left quantities on the pans.

After students have had many concrete experiences interpreting the relationships between quantities on two pans of a balance scale, they could be asked to interpret a variety of balance scale pictures.

Performance Indicator Background

PR03.01 Using concrete materials, students can examine how a balance operates like the seesaw on the playground. Construct two equal sets using the same objects (same shape and mass) and demonstrate their *equality of number* using a balance scale. Place a set of 6 red cubes on the left pan of a balance scale and 4 yellow cubes on the right pan. Ask students to predict how many more cubes they would need to add, and on which pan, for the scale to balance. Have students place blue cubes, one at a time, onto the scale until it balances. Then have students count the number of cubes in each set, reinforcing the idea that both sides have the same number of cubes, saying, “Six red cubes are the same as four yellow cubes and two blue cubes” or “Six red cubes are equal to four yellow cubes and two blue cubes.”

PR03.02 When one set has more or fewer objects than the other, the sets are unequal. A balance scale can also be used to demonstrate these inequality relationships; for example, place 6 red cubes on the left pan of a balance scale and 4 yellow cubes on the right pan without actually showing students the quantities. Ask students what they observe. Ask them on which pan do they think there are more cubes and why they think so. Ask how this is like children on a seesaw. Have them count the quantities on both pans. Model how they could compare the quantities on the pans, saying, “The set of 6 red cubes is more than the set of 4 yellow cubes” and “The set of 6 red cubes is not the same as the set of 4 yellow cubes” or “The set of 6 red cubes is not equal to the set of 4 yellow cubes.”

When comparing sets many students may recognize that the set of 6 cubes is more than the set of 4 cubes but may not automatically realize that the set of 4 cubes is fewer than the set of 6 cubes. Both sides of the relationship need to be considered. Therefore, ask students what they could say about the set of 4 yellow cubes compared to the set of 6 red cubes, bringing out the relationship “is fewer than” and “is not the same as” or “is not equal to.”

PR03.03 Students should have experiences determining equality and inequality relationships through balance scales. For example, provide students with two sets of objects (all the same mass). Have them predict the relationship between the two sets, asking, Are these two sets equal, or unequal? Ask them to use a pan balance to check their predictions, explaining the process they use. Have them express the relationship between the two sets using appropriate language.

Students should also have experiences determining equality and inequality relationships through one-to-one correspondence, another form of balancing comparisons. For example, to show the relationship between the number of boys and the number of girls in a group, the boys and the girls could form two lines across from each other and make boy-girl partners. If there are leftover boys, discuss the relationship between the number of boys and the number of girls and between the number of girls and the number of boys.

One-to-one correspondence is also how students could show or verify equality and inequality relationships between pictures of two sets of objects. They could draw arrows from each object in one set to a corresponding object in the second set, and determine the relationship based on any leftover objects in either set. While the relationships interpreted from this arrow drawing may be obvious to adults, many students in grade one will require many experiences and discussion.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge.

- Give students a set of between 5 and 10 counters. Ask them to make a set that is the same.

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Working in partners, one student puts cubes in a paper bag and places the bag on a pan balance. The partner predicts the number of cubes in the bag. (The partner may change the prediction as the scale begins to balance). The partner then begins to add cubes to the other side of the balance to verify the prediction. Once the scale is balanced, ask, How many cubes do you think are in the bag? How do you know? The partners count and compare the number of cubes on both sides.
- Provide students with two bags of counters and ask them to determine if the sets are equal or unequal and to explain how they know.
- Show students a pan balance that has 5 cubes on the left side and 7 cubes on the right side. Ask if the two sets are equal or not equal. Ask them to predict how many more cubes need to be added to the left side to make the scale balance and to make the sets equal.
- Ask students to use cubes and a pan balance to prove that 8 is equal to 7 + 1.
- Ask students to use cubes and a pan balance to prove that 8 is not equal to 5 + 4.
- Provide students with two sets of cubes and ask them to tell if the two sets are equal or not equal using a pan balance.
- Ask students to create two sets of cubes that are equal. Ask them to use a pan balance to prove that they are equal. Then ask them to change the sets so that they are not equal. Ask them to use a pan balance to prove that they are not equal sets.

Follow-up on Assessment

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Responding to Assessment

Numeracy Nets K–2 (Bauman 2011)
- Mathematics 2 Checkpoint 8, pp. 85–86

Planning for Instruction

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of my students be met?

Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

- Provide students with a variety of experiences so they come to understand that the equal sign represents a relation, not an operation. Use of the words "the same as" for the equal sign will help them see the relation.
- Use balance activities to assist with the development of an understanding of equality. Use concrete materials so students can examine how a balance operates like the seesaw on the playground.
- Provide students with many opportunities to represent number sentences (equations) concretely and to use language to describe those number sentences (equations).
- Expect students to verbally explain their answers about equalities and inequalities.

Suggested Learning Tasks

- Use a pan balance. Explain how the scale works. When the scale is balanced the sets are equal. Have students use objects of equal weight, such as unifix cubes, to practise making sets that are equal and sets that are not equal. Students should be encouraged to use oral language to describe the sets and to explain why they are or are not equal. They may use words such as *more than*, *fewer than*, and *the same as*, *balances*, or *does not balance*.
- Ask students to work in pairs. Ask them to use linking cubes to make trains to show as many different ways as possible to make a number from 1 to 20. Ask them to use the pan balance to prove that all of the trains for a given number are equal.
- Show students a pan balance that has some cubes on the left side and a different number of cubes on the right side. Ask students to tell if the two sets are equal or not equal and to explain how they know.
- Ask students to create two sets of cubes that are equal. Ask them to use a pan balance to prove that they are equal. Then, ask them to change the sets so that they are not equal.
- Provide students with number sentences. Ask them to use pan balances and cubes to show whether the given number sentences (equations) are true or not. For example, 4 = 1 + 3; 3 + 2 = 5; 8 = 8; 6 + 1 = 5 + 2.
- Ask students to work in pairs. Provide each pair of students with a set of word cards that say, *Equal* and "*Not Equal*. The first student draws a word card but does not let the partner see the card. The

card tells whether the first student will create two sets of objects that are equal or not equal. For example, if the student drew a not equal card, that student might make a set of 5 and a set of 7. The second student looks at the sets of objects that have been created and determines whether the card said equal or not equal by stating "five is not equal to seven." The students verify the statement by placing the sets on pan balances.

SUGGESTED MODELS AND MANIPULATIVES

- dot cards
- linking cubes
- pan balance

MATHEMATICAL LANGUAGE

Teacher	**Student** (oral language)
▪ balance, imbalance ▪ compare ▪ equal sign ▪ equality/inequality ▪ left, right ▪ more than, fewer than ▪ not equal to/not same as ▪ number sentence/equation ▪ one-to-one ▪ quantity ▪ same as/equal/balances ▪	▪ balance/imbalance ▪ compare ▪ equal sign, ▪ left, right ▪ more than, fewer than, ▪ same as/equal/balance, ▪ not equal to/not same as/does not balancenumber sentence, equation ▪ quantity ▪ same as/equal/balances

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 585–586
- *Making Math Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 626–627
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 299–302

Notes

SCO PR04 Students will be expected to record equalities using the equal symbol. [C, CN, PS, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR04.01 Represent a given pictorial or concrete equality in symbolic form.
PR04.02 Represent a given equality using manipulatives or pictures.
PR04.03 Provide examples of equalities where the given sum or difference is on either the left or right side of the equal symbol (=).
PR04.04 Record different representations of the same quantity (0 to 20) as equalities.

Scope and Sequence

Mathematics Primary	Mathematics 1	Mathematics 2
—	**PR04** Students will be expected to record equalities using the equal symbol.	**PR04** Students will be expected to record equalities and inequalities, symbolically, using the equal symbol or the not equal symbol.

Background

This outcome could be addressed at the same time as the equality aspect of outcome PR03. Balance activities form a basis for understanding equality. Throughout balance activities, students should answer the question, What is the relationship between the two quantities if the scale is balanced? This provides opportunities to use *is the same as*, *balances*, and *is equal to* and also to use the equal sign (=) in the related number sentences (equations).

Students need to understand that the equal sign (=) represents a relationship, not an operation. It means *the same as*; that is, what is on one side of the equal sign is the same as what is on the other side. Often, when students only have experience using the equal sign in addition and subtraction, they mistakenly believe the sign (=) means *is the answer to* and, therefore, do not accept as possible number sentences (equations) such as 8 = 5 + 3, 6 = 6, and 4 + 5 = 2 + 7. However, through balance activities, students see that such number sentences (equations) express equality relationships between quantities on the two sides of a balance scale, or in a one-to-one correspondence. As such, these numbers sentences (equations) are not only possible but are necessary to express the relationships.

It is important for students to develop meaning of equality and a relational understanding of the equal sign. While you may be tempted to use an equal sign (=) in concrete and picture models, it is strongly recommended that you use, and encourage students to use, the equal sign (=) only with other symbols, such as in number sentences (equations) and open sentences. When you read a number sentence, it is recommended that you read the equal sign (=) to be *is the same as or balances*. Number sentences (equations) should emerge only as a way to record concrete or pictorial models of equality relationships. Students are not expected to be able to work with symbolic relationships without reference to quantities, or to write and interpret greater than and less than symbols.

Performance Indicator Background

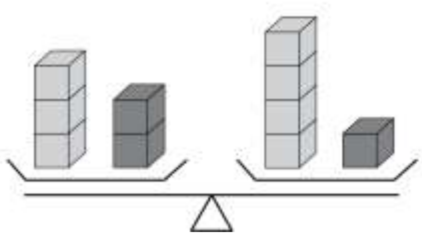
PR04.01 Initially, students should learn to write number sentences to express the equality between the quantities on the two pans of a balance scale. For example, if 2 red cubes and 5 blue cubes are on the left pan of the scale, and 7 yellow cubes are on the right pan, the students see the scale is balanced, say, 2 red cubes and 5 blue cubes is the same as 7 yellow cubes, and learn that 2 + 5 = 7 is how this is represented **with** symbols. **If 7 yellow cubes are on the left pan, and the 2 red cubes and 5 blue cubes are on the right pan, the students see the scale is balanced, say, 7 yellow cubes is the same as 2 red cubes and 5 blue cubes, and learn that 7 = 2 + 5 is how this is represented with symbols.**

PR04.02 After students are comfortable interpreting equalities on balance scales as number sentences(equations), they should be able to create balance scale representations for given number sentences (equations) and draw balance scale pictures to show these equalities. For example, given 8 = 7 + 1, they could set up a balance scale with, perhaps, 8 red cubes on the left pan, and 7 blue cubes and 1 green cube on the right pan. They could also draw a picture of this balance scale set up.

PR04.03 One possible activity to introduce students to writing equalities of sums on left and right sides of the equality sign involves a balance scale and cube towers. Have students use cubes of two colours, such as red and blue, to build different towers of 5 cubes. They could create these towers: 5 red cubes, 5 blue cubes, 4 red cubes and 1 blue cube, 3 red cubes and 2 blue cubes. Placing these towers, two at a time, on the pans of the balance scale and writing the related number sentences (equations) would provide opportunities to write a variety of number sentences, such as 5 = 4 + 1, 3 + 2 = 5, 4 + 1 = 3 + 2, and 5 = 5.

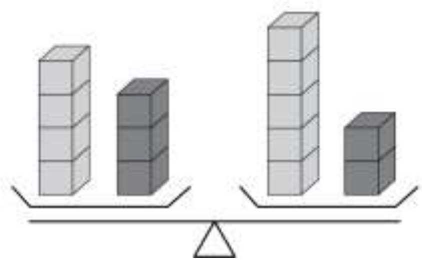
Three blue blocks and two red blocks balance four blue blocks and one red block. Three blue blocks and two red blocks are equal to four blue blocks and one red block.

3 + 2 = 4 + 1



Four blue blocks and three red blocks balance five blue blocks and two red blocks. Four blue blocks and three red blocks are equal to five blue blocks and two red blocks.

4 + 3 = 5 + 2



Students should also have experiences showing equality between two differences and between a sum and a difference. For example, have students choose a number from 4 to 7 and write their choice at the top of a white board. Suppose they choose 6. Below this number 6, draw a 2-column table, one column labelled “Two Quantities Added,” and the other column labelled “Two Quantities Subtracted.” Provide students with counters and ask them to suggest something to put in the first column that would have an answer of 6; for example, 5 + 1. Then ask for something to be put in the second column that would have an answer of 6; for example, 8 – 2. Continue until there are three or four entries in each column. Orally discuss the possible equalities that could be written using the number 6 and the entries in the table. Then ask the students to write at least five equalities.

PR04.04 This performance indicator is connected to the partitioning outcome N04, where students are concretely and pictorially representing a quantity in two parts. Here they start to symbolize this relationship. For example, ask students to place 11 two-colour counters in a row showing all one colour. Direct them to turn over one counter and write the equality. They would write 11 = 10 + 1 or 10 + 1 = 11.

Then direct them to turn over another counter and to write this equality. This can be continued until they have written a number of equalities.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

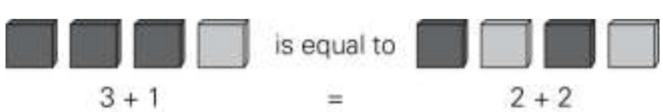
Tasks such as the following could be used to determine students' prior knowledge.

- Make a duplicate set of dot cards showing numbers to 10. Have students match the cards that are the same. Ask them to explain how they know the cards are the same.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to fill in the blank, (either a whole number or a combination of numbers showing an operation) to complete the number sentence (equation). Encourage students to explore these number sentences (equations) using materials, such as linking cubes of different colours and pan balances.
  _______ = 7 + 6
  4 + 2 = _______
  5 + 3 = ________
  _______ = 8 + 7
- Ask students to use materials and/or pictures to show an equality and then write the matching number sentence (equation). For example,



- Use pan balance pictures to show equalities. Ask students to write a number sentence (equation) to represent the equality. This can also be done for inequalities.
- Provide students with pictures of pan balances with number sentences (equations) underneath. Ask students to draw objects on the pan balance to match the number sentences (equations).

Follow-up on Assessment

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Responding to Assessment

Numeracy Nets K–2 (Bauman 2011)
- No Checkpoint for this outcome.

Planning for Instruction

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of my students be met?

Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.
- Provide students with a variety of experiences so they will understand that the equal sign represents a relation, not an operation. Use of the words "the same as" for the equal sign will help them see the relation. Avoid misuse of the equal sign, such as ⍰⍰⍰ = 3 or Alisa = 5.
- Have students create equations and describe the number sentences (equations) verbally.
- Provide students with many opportunities to represent number sentences (equations) concretely before recording them symbolically.
- Ensure that students learn to read number sentences (equations) from left to right and right to left.

Suggested Learning Tasks

- Ask the student to use Cuisenaire rods (or another suitable manipulative material such as linking cubes) to show the pattern for all of the number sentences (equations) for 8.
- Create dots cards representing number sentences, like flash cards. Ask students to record the number sentence symbolically. Ask students to represent the number sentence with linking cubes.
- Play a true and false game with students. Show number sentences (equations) and ask students to use linking cubes to represent each number sentence (equation). Then, ask them to hold up either a true or a false card in response to each number sentence (equation). Select one student to use linking cubes and a pan balance to prove that the statement is true or false. Number sentences (equations) could include
  $1 + 2 = 1 + 3$
  $9 = 7 + 2$
  $7 + 1 = 4 + 4$

4 + 3 = 7
6 + 4 = 5 + 5
8 = 8

- Ask students to work in pairs to show as many different ways as possible to make a number from 1 to 20 and to record them as equalities.

SUGGESTED MODELS AND MANIPULATIVES

- Cuisenaire rods
- dot cards
- linking cubes
- pan balance

MATHEMATICAL LANGUAGE

Teacher	**Student** (oral language)
▪ balance scale ▪ compare ▪ left, right ▪ make sets ▪ number expression ▪ number sentence, equation ▪ quantity ▪ same as, equality, equal sign, equals	▪ balance scale ▪ compare ▪ left, right ▪ make sets ▪ quantity ▪ same as, equal sign, equals

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 585–586
- *Making Math Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 624–625
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 299–302

Notes

Measurement

GCO: Students will be expected to use direct and indirect measure to solve problems.

SCO M01 Students will be expected to demonstrate an understanding of measurement as a process of comparing by ▪ identifying attributes that can be compared ▪ ordering objects ▪ making statements of comparison ▪ filling, covering, or matching [C, CN, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

M01.01 Identify common attributes, such as length, mass, volume, capacity, and area that could be used to compare a given set of two objects.
M01.02 Compare and order two given objects and identify the attributes used to compare.
M01.03 Predict which object in a set is longest/shortest, determine by matching and explain the reasoning.
M01.04 Predict which object in a set is heaviest/lightest, determine by comparing and explain the reasoning.
M01.05 Predict which object in a set is largest/smallest, determine by comparing and explain the reasoning.
M01.06 Predict which object in a set holds the most/least, determine by filling and explain the reasoning.
M01.07 Predict which figure in a set has the greatest/least area, determine by covering and explain the reasoning.

Scope and Sequence

Mathematics Primary	Mathematics 1	Mathematics 2
M01 Students will be expected to use direct comparison to compare two objects based on a single attribute, such as length, mass, volume, and capacity.	**M01** Students will be expected to demonstrate an understanding of measurement as a process of comparing by ▪ identifying attributes that can be compared ▪ ordering objects ▪ making statements of comparison ▪ filling, covering, or matching	**M03** Students will be expected to compare and order objects by length, height, distance around, and mass using non-standard units and make statements of comparison.

Background

Measurement involves identifying and comparing similar attributes of objects, and through measurement activities, students learn that the same object can have many measurable attributes. It is important that students explore measurement in their everyday contexts using direct comparison. For example, they might be asked, Which bean plant grew the tallest? Which ball has the greatest mass? Which box will hold the most treats? Which is the largest block in the set? Who has the largest piece of paper?

When exploring the different measurement attributes, have the students collect items from around the classroom. Students take turns predicting and checking their predictions. More than one student should do the same comparison. Observe whether there is agreement and how disagreements are resolved.

The first step in all these measurement activities should be for students to sort the objects for an attribute from smallest to largest, or vice versa, using their senses. That is, they would use sight to sort lengths, volumes, capacities, and areas; and use touch to sort masses. Then they would check their predictions by direct comparisons. Initially, they should be given only 3 objects to sort, moving on to more objects when they are ready. For the ones that will be sorted by sight, you should place the objects a good distance apart, bringing them together once the students have made their predictions. For mass, students should pick up each item with the same hand, to make their predictions and to compare the mass of those items.

Performance Indicator Background

M01.01 and M01.02 Students should learn that any object has multiple attributes that could be used to describe it. Similarly, any two objects can be compared in a variety of ways, depending upon the attributes they have in common. Once the comparisons of two or more attributes of objects have been developed, students should be presented with two objects and asked to describe how these objects could be compared. Then they should compare them in the ways they described, using predictions followed by direct comparisons to check those predictions.

M01.03 Organizing objects by length is usually the easiest attribute for students to do, so this would be the best attribute with which to start. Students should recognize that length describes the measurement of an object in one dimension. These linear measurements include measurements of the height, width, and length of an object, as well as the more abstract concepts of depth and distance that will be discussed in later grades. In Mathematics 1, their predictions of the lengths, widths, and heights of objects are checked by direct comparison. Direct comparison involves lining up items side by side, beginning at a common base **or starting point**. Students should be led to see why a common starting point is important.

Students should order objects from *longest* to *shortest, shortest* to *tallest, thickest* to *thinnest, and narrowest* to *widest*, learning the associated vocabulary through the direct comparison activities. Initially, the activities should involve objects for which the attribute being compared is the only variable. For example, if they are organizing a set of erasers from narrowest to widest, the erasers should initially be the same length. Later, students should be able to organize a set of erasers from narrowest to widest when the erasers are also of different lengths, or from longest to shortest when the erasers have different widths, and not be distracted by the other variable.

M01.04 Students should recognize that mass tells about the heaviness of an object. They should explore methods to compare and order masses. The most conceptual way for **students** to compare the masses of objects is to lift them in their hands. It is recommended that students use the same hand and lift one object after the other because there can be different sensations in their left and right arms. Their predictions can be checked by direct comparison using balance scales. Unlike other attribute development, you should have the objects be very different from one another, so students are not tempted to organize them by their visual characteristics. That is, you should have some objects that are smaller in size but with more mass than larger objects. However, initially the differences in mass should be quite pronounced so students can be successful. While developing measurement skills for mass, students should use terms such as *heaviest* and *lightest*.

M01.05 The volume of an object is the amount of space it occupies. Children naturally compare objects by what they refer to as their size, saying things such as, My toy wagon is bigger than yours. Your book is smaller than mine. That is the largest toy box in the classroom. Of course, they do not realize it is volume **to which** they are referring, but this can make an easy connection for them to the word *volume*. They should predict which objects around the classroom look *bigger/smaller* or *biggest/smallest* and then bring them close together to compare them directly to check their predictions. Initially, the differences in size should be obvious. While comparing objects based on volume, students should use language such as *takes up more space* or *takes up less space*.

MO1.06 Students should recognize that *capacity* tells how much something will hold. Identifying differences in capacities of containers is difficult for children; therefore, the differences among the capacities should be obvious so they can be successful. Initially, you should try to keep two of the dimensions of the containers constant. For example, 250 mL, 500 mL, and 1 L milk cartons have the same footprint, so their capacities will be directly related to the heights. These containers can be cut to different heights with their tops off so students can look into them to predict how much each object holds. Then the activities can involve containers of different shapes and varying dimensions.

Students should investigate strategies to compare the capacities of three or more containers. For this attribute, direct comparison involves filling one container and then pouring the contents into another to find which holds more. This method of comparing capacities will be more involved than comparing lengths. For example, suppose students have organized three containers from smallest to largest. They fill the first container with rice, pour it into what they believe to be the next largest, and if there is room for more rice in the second container, it will have to be filled with rice before it is poured into the third container. Obviously, it is important that the containers be organized by sight initially to provide a starting container, and that this will have to be revisited should the first, or subsequent, pourings prove to be incorrect. While developing measurement skills for capacity, students will use terms such as *holds more, holds less, holds the same, full,* and *empty*.

M01.07 In comparing *areas*, students are examining the amount of surface taken up by an object. For example, one place mat might cover more of the table top than another one, or one piece of paper covers more of the desk than another piece. Students should predict which objects cover more and use direct comparison to verify their prediction. Direct comparison involves placing one surface on top of another to see if they match or if one sticks out. While developing the concept of area, students should use terms such as *covers more/covers less, greatest/least area,* and *largest/smallest area*. As with capacities, for students' initial experiences, you should minimize the differences among the dimensions of surfaces and make the differences among the areas fairly large.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge.

- Set up comparing stations each with two different objects. Ask questions appropriate to the measurement focus for each station. For example,
  - Can you tell me which of these two objects is longer? shorter?
  - Can you tell me which of these two objects is heavier? lighter?
  - Can you tell me which of these two objects is bigger? smaller?
  - Can you tell me which of these two objects holds more? holds less?
- Observe how students decide upon their answers.

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Show two different objects that can be compared for mass. Ask students to predict which is heavier. Then, ask them to explain how they could compare them to determine which is heavier. Repeat for other measurement attributes.
- Have students prepare a set of ribbons for first, second, and third places in a race, so that the faster runner gets a longer ribbon.
- Provide students with two objects such as an eraser and a book. Ask,
  - Can you tell which of these two objects is longer?
  - Can you tell which of these two objects is heavier?
  - Can you tell me which of these two objects is bigger?
  - Can you tell which of these two objects takes up the most space?
  - Can you tell which of these two objects holds more?

After each question, have students explain their thinking. (Students should recognize that capacity is an attribute that cannot be used to measure these objects.) Use the questions as identified above, repeating the activity with other sets containing two objects.

- Show the students a coffee mug and a drinking glass. Ask them how they would find out which holds more.
- Give students sets of tangrams and have them compare the areas of the triangles in the sets.
- Provide students with "trains" of various lengths made from linking cubes. Ask them to order the trains from shortest to longest.
- Ask, What does "holds more" mean? Have the student explain their thinking.
- Give students a geometric shape, such as an attribute block or a pattern block, and have them draw another shape with a larger area. Have students explain how they know their drawn shape is larger.

Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Responding to Assessment

Numeracy Nets K–2 (Bauman 2011)
- Mathematics 1 Checkpoint 9, pp. 91–92
- Mathematics 1 Checkpoint 10, pp. 99–100

Planning for Instruction

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Long-term Planning
- Yearly plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of my students be met?

Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

- Have students participate in "dramas" in which someone measures incorrectly and the other students figure out what is wrong. For example, one student could line up pencils of different lengths to measure an item, or could use uniform units, but counts, "1, 2, 4, 5, ..."
- Use two objects of different sizes and ask students how they could compare the objects. For example, two glasses could be compared by height, mass, as well as capacity. Working with a variety of objects will allow many opportunities for students to make comparisons relating to measurement, using many attributes.
- Ask students to order objects from shortest to longest, shortest to tallest, lightest to heaviest, holds least to holds most, or covers least space to covers most space. Include situations in which students are dealing with an independent variable, such as objects that are not straight and objects that are also wide or thick.

Suggested Learning Tasks

- Provide students with a pan balance and two items. Ask them to predict and then determine which item is lighter. Observe students' strategies.
- Provide the students with containers of varying sizes and shapes. Ask students to select two containers, and predict and then determine, by filling with sand or water, which holds more. Students should choose two more containers and repeat the activity. Then, ask students to order the four containers based on how much they hold.
- Ask two students to perform standing long jumps. Encourage them to find a way to determine who jumped farther. Emphasize with the students the importance of a common starting point.
- Have the students make 3 play dough balls. Ask them to predict and then to determine, using a pan scale, which of the balls is the heaviest.
- Display a set of 5 objects of similar size and a 6th "target" object. Ask the students to sort them into groups with masses less than and greater than the "target" object.

- Provide opportunities for students to compare areas by superimposing objects, by comparing different sizes of the same shape, and by comparing different shapes. After comparing, ask, How do you know that the area of this shape is larger?
- Give each student a true and a false card. Make comparative statements and ask students to hold up either the true or false card in response. For example, The fish tank holds more than a juice box. My desk is longer than the white board. The white board eraser is shorter than this paper clip. I am heavier than my jacket. Ask student volunteers to explain their thinking.
- Show students five different objects one at a time. For each object, ask students if the length of the object is less than, greater than, or about the same as the length of a sheet of paper. After students record the estimation for each object, do a direct comparison of that object to a piece of paper. After students have estimated and compared the length of each object, ask them to place the objects in order from shortest to longest.

SUGGESTED MODELS AND MANIPULATIVES

- pan balance
- play dough
- various everyday objects

MATHEMATICAL LANGUAGE

Teacher	**Student** (oral language)
▪ area: covers more/covers less, greatest/least area, and largest/smallest area ▪ capacity: holds more/holds less, holds the same ▪ full, empty ▪ length, width, distance, height ▪ length: longest/shortest, shortest/tallest, widest/narrowest, thickest/thinnest, longer/shorter ▪ mass: heaviest/lightest, heavier/lighter ▪ order, compare, fill, cover, match ▪ volume: biggest/smallest, bigger/smaller	▪ area: covers more/covers less, greatest/least area, and largest/smallest area ▪ capacity: holds more/holds less, holds the same ▪ full, empty ▪ length, width, distance, height ▪ length: longest/shortest, shortest/tallest, widest/narrowest, thickest/thinnest, longer/shorter ▪ mass: heaviest/lightest, heavier/lighter ▪ order, compare, fill, cover, match ▪ volume: takes up more/less space, bigger/smaller

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 363–368, 370–375, 388–393, 415–419, 421–424, 430–433
- *Making Math Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 411–415, 418–419, 421–422, 429, 434–437, 451, 461–464, 467–470, 476–479, 481–483
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 223–242

Notes

Geometry

GCO: Students will be expected to describe the characteristics of 3-D objects and 2-D shapes and analyze the relationships among them.

SCO G01 Students will be expected to sort 3-D objects and 2-D shapes using one attribute and explain the sorting rule. [C, CN, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

G01.01 Sort a given set of familiar 3-D objects or 2-D shapes using a given sorting rule.
G01.02 Sort a given set of familiar 3-D objects using one attribute determined by the student, and explain the sorting rule.
G01.03 Sort a given set of 2-D shapes using one attribute determined by the student and explain the sorting rule.
G01.04 Determine the difference between two given pre-sorted sets of familiar 3-D objects or 2-D shapes and explain a possible sorting rule used to sort them.

Scope and Sequence

Mathematics Primary	Mathematics 1	Mathematics 2
G01 Students will be expected to sort 3-D objects using one attribute. **G02** Students will be expected to build and describe 3-D objects.	**G01** Students will be expected to sort 3-D objects and 2-D shapes using one attribute and explain the sorting rule.	**G02** Students will be expected to recognize, name, describe, compare, and build 3-D objects, including cubes and other prisms, spheres, cones, cylinders, and pyramids. **G03** Students will be expected to recognize, name, describe, compare, and build 2-D shapes, including triangles, squares, rectangles, and circles.

Background

Sorting objects into groups is a natural activity that needs to be fostered and extended to sorting three-dimensional (3-D) objects and two-dimensional (2-D) shapes because the study of objects and shapes is essential as students begin to describe, analyze, and understand the world in which they live. Activities selected in geometry should provide students with the opportunity to explore a variety of geometric shapes and objects. They need to see and feel, build and take apart, sort, identify their rule(s), and share their observations with their classmates. Sorting activities are based on students' visual discrimination abilities that should be further developed by discussing how objects or shapes are alike and how they are different, by hearing alternative sorting rules, and by being encouraged to explore alternative ways of sorting.

It is through such activities that students will become familiar with the geometric language of 2-D shapes and 3-D objects. All activities provide opportunities for students to become familiar with those names, and you should use correct names when talking about shapes. It is reasonable that students become familiar with the common names (2-D shapes: triangle, square, rectangle, and circle; 3-D objects: cylinder, sphere, cone, cube, prisms, and pyramids).

As students work with 3-D objects and 2-D shapes, they develop spatial sense. Spatial sense is an intuition about **objects and** shapes and their relationships and an ability to manipulate **objects and** shapes in one's mind. It includes being comfortable with geometric descriptions of **objects**, shapes, and positions.

There are seven abilities in spatial sense that need to be developed in the classroom:

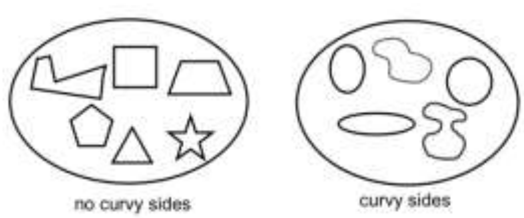
- **Eye-motor co-ordination**. This is the ability to co-ordinate vision with body movement. In the mathematics classroom, this is seen in children learning to write and draw using pencils, crayons, and markers; to cut with scissors; and to handle **3-D objects**.
- **Visual memory**. This is the ability to recall objects no longer in view.
- **Position-in-space perception**. This is the ability to determine the relationship between one object and another, and an object's relationship to the observer. This includes the development of associated language, such as over, under, beside, on top of; and the transformations of translations, reflections, and rotations that change an object's position.
- **Visual discrimination**. This is the ability to identify the similarities and differences between or among objects.
- **Figure-ground perception.** This is the ability to focus on a specific object within group of objects, while treating the rest of the objects as background.
- **Perceptual constancy**. This is the ability to recognize an object when it is seen from a different orientation, viewpoint, or from a different distance. his perception is connected to prior experiences to enable the brain to "see" what it expects to see when it interprets visual information it receives.
- **Perception of spatial relationships.** This is the ability to see the relationship between two or more objects.

These abilities develop naturally through many experiences in life; therefore, most students come to school with varying degrees of ability in each of these seven aspects. Classroom activities should be planned to allow students to further develop these perceptions. Most spatial sense activities involve many, if not most, of the spatial abilities, although often one of them is highlighted.

Activities should include a balance of students generating their own sorts and their sorting by assigned sorting rules. Throughout, students should learn to articulate their sorting rules, be challenged to create alternative sorting rules, and develop geometric language.

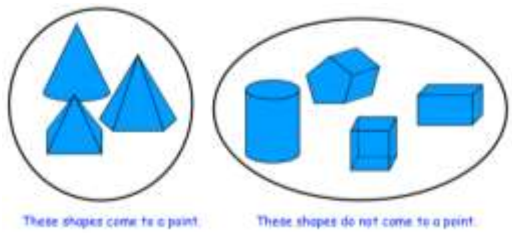
Performance Indicator Background

G01.01 Students should sort familiar objects, such as the objects in their pencil case. They should be able to sort these into two groups based on a given sorting rule, such as things that roll and things that do not roll. They should also have experiences sorting pictures of 2-D shapes. Shapes should not just be a set of normal geometric shapes but should include a variety of shapes. Students can be asked to sort these based on rules, such as has a curvy side and does not have a curvy side.



G01.02 Students' initial sorts of 3-D objects would likely be by type **of solid**, shape **of faces**, by size, and by ability to roll/slide. Through discussions of how two 3-D **objects** are alike and how they are different, through other focused questioning, and through sharing of sorting rules, students' sorting should expand to include some other attributes of 3-D **objects**:

- come up to a point
- number of edges
- number of corners/vertices
- number of faces



In open-ended sorting experiences, students may also sort based on other attributes of 3-D objects, such as

- the number of edges
- the number of vertices
- the number of faces
- Will it roll? Stack? Slide?

G01.03 Initially, students recognize **2-D** shapes by their overall appearance rather than by any of their attributes. For example, they would likely say they know something is a square because it looks like a square rather than through an awareness that it has four equal sides and all its angles are right angles. However, through sorting activities students begin to notice attributes of shapes. If students are asked to sort a given set of 2-D shapes, their initial sorts are likely to be by type of shape, by size, or by colour. However, their sorts should expand to involve the attributes of the number of sides and number of corners/vertices.

G01.04 Students need opportunities to recognize and discuss the sorting rule for two pre-sorted sets of familiar 2-D shapes. Sort 2-D shapes while students observe. Have them predict where each object would be placed, explaining the possible sorting rules used. Eventually they should be able to look at two sets and discuss what the difference is between the two sets of objects. Then they should use this difference to determine the rule. Pre-sorted sets should only contain one difference.

Note: The concept of 2-D shapes is very abstract because 2-D shapes do not have thickness. For practical purposes to study 2-D shapes, however, we commonly use pattern blocks, attribute blocks, and cut-out shapes that are actually 3-D objects (prisms and cylinders) that have thickness. These 3-D objects have very prominent 2-D faces that are the shapes to which we refer. For example, the triangle in the set of pattern blocks is actually a triangular prism with two prominent triangular faces. Even when you draw a figure, such as a square, on a piece of paper using a pencil, the square is actually the shape within the pencil segments, not including those pencil segments. It is not intended that you try to explain this to students in grade one unless you have an exceptionally advanced student who you think can understand and benefit from this knowledge. This concept will be clarified for students in later grades. You could, however, very consciously touch the prominent face of a pattern block when referring to its shape name as a subtle way to focus students on the actual shape being discussed.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge.

- Show students a pre-sorted set of 3-D objects from the environment, such as cans, boxes, or balls. Ask them to explain the sorting rule.
- Provide students with a set of 3-D objects. Ask them to sort them and to explain their sorting rule.

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Provide students with a collection of 2-D shapes or 3-D objects. Tell them a sorting rule. Ask them to sort the collection based on your sorting rule.
- Ask students to work in pairs. Have one student sort a collection of 2-D shapes or 3-D objects. The partner must describe the sorting rule.
- Provide students with a collection of 2-D shapes or 3-D objects. Ask them to sort the collection of 2-D shapes or 3-D objects and explain their sorting rule. Then, ask them to sort the collection again using a different sorting rule and to explain the new rule.
- Show students a set of 2-D shapes or 3-D objects that you have sorted. Ask students to identify your sorting rule.
- Show students a set of 2-D shapes or 3-D objects that you have sorted. Ask students to sort the shapes or objects in a different way and to explain their new sorting rule.
- Give students a set of 2-D shapes 3-D objects and ask them to sort the objects into two groups, telling you how they decided to sort them. Make sure the shapes or objects are clearly related in at least two ways so that the students have some obvious choices (e.g., objects with rounded parts and objects with straight edges only, or objects with triangular faces and objects with rectangular faces).
- Sort a set of 2-D shapes or 3-D objects into two groups leaving some objects out of the sort. Hold up one of the objects and ask students to identify the group to which it belongs. Have students explain their thinking and their sorting rule.
- Show students a set of 2-D shapes or 3-D objects that have been sorted. Ask them to describe the attributes of the objects in each group.
- Give pairs of students a set of 2-D shapes. One person should sort the shapes using a sort rule of their choice. Their partner should look at the sorted shapes and describe the sorting rule.

Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Mathematics 1 Checkpoint 12, Task 2, pp. 116–117
- Mathematics 1 Checkpoint 14, pp. 133–14 (Line Master 14.1)

Planning for Instruction

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of my students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ensure students have many opportunities to represent 2-D shapes and 3-D objects concretely.
- Provide an assortment of 2-D shapes cut from cardboard. Ask students to work in small groups to sort the shapes. Encourage discussion and have the groups share their sorting rules with their classmates.
- Encourage students to use oral language to describe how the 2-D shapes (or 3-D objects) are alike and how they are different.
- Allow students to identify 2-D shapes and 3-D objects in their daily lives. This should include common everyday objects.

SUGGESTED LEARNING TASKS

- Provide several different 2-D shapes or 3-D objects. Ask the student to sort them and to explain the sorting criteria. Ask the student to sort them again, using different criteria.
- Show students a set of shapes or objects that have been sorted but that contains an object or shape that does not belong. Ask students to remove the object or shape that does not belong and explain why.
- Provide students with a collection of 2-D shapes of various sizes. Ask students to sort the collection into two groups and name the sorting rule. Then, ask students to sort the collection in another way and to explain the new sorting rule. This task should also be done using 3-D objects.
- Have students sort a set of 2-D shapes or 3-D objects. Invite students to explain their sorting rule. Other questions could include
  - Explain why you put these shapes or objects together.

- According to your (the) sort, where would ___ belong?
- Which shape or object does not belong to this set?
- What other way could you sort these shapes or objects? Explain your sorting rule.

- Show students a set of 2-D shapes or 3-D objects that you have sorted. Ask students to describe your sorting rule.
- Invite students to sort a set of 3-D objects according to a specific attribute such as whether it rolls or doesn't roll, stacks or doesn't stack, its colour, its function, or the shape or number of faces.

SUGGESTED MODELS AND MANIPULATIVES

- attribute blocks
- geometric solids
- pattern blocks
- Polydrons
- tangrams

MATHEMATICAL LANGUAGE

Teacher	**Student** (oral language)
▪ 2-D shapes: triangle, square, rectangle, circle, trapezoid, hexagon, rhombus ▪ 3-D objects: cylinder, sphere, cone, cube, prism, pyramid ▪ attribute ▪ how are they alike/different ▪ roll, stack, slide ▪ sides, edges, corners/vertices, faces ▪ sorting groups of objects ▪ sorting rule	▪ 2-D shapes: triangle, square, rectangle, circle ▪ 3-D objects: cylinder, sphere, cone, cube, prism, pyramid ▪ how are they alike/different ▪ roll, stack, slide ▪ sides, corners, faces ▪ sorting groups of objects ▪ sorting rule

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 284–286, 289–290
- *Making Math Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 340–342, 344–346
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 186–195

Notes

GCO G02 Students will be expected to replicate composite 2-D shapes and 3-D objects. [CN, PS, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

G02.01 Select 2-D shapes from a given set of 2-D shapes to reproduce a given composite 2-D shape.
G02.02 Select 3-D objects from a given set of 3-D objects to reproduce a given composite 3-D object.
G02.03 Predict and select the 2-D shapes used to produce a composite 2-D shape and verify by deconstructing the composite shape.
G02.04 Predict and select the 3-D objects used to produce a composite 3-D object and verify by deconstructing the composite object.

Scope and Sequence

Mathematics Primary	Mathematics 1	Mathematics 2
G02 Students will be expected to build and describe 3-D objects.	**G02** Students will be expected to replicate composite 2-D shapes and 3-D objects.	**G02** Students will be expected to recognize, name, describe, compare, and build 3-D objects, including cubes and other prisms, spheres, cones, cylinders, and pyramids. **G03** Students will be expected to recognize, name, describe, compare, and build 2-D shapes, including triangles, squares, rectangles, and circles. **G04** Students will be expected to identify 2-D shapes as parts of 3-D objects in the environment.

Background

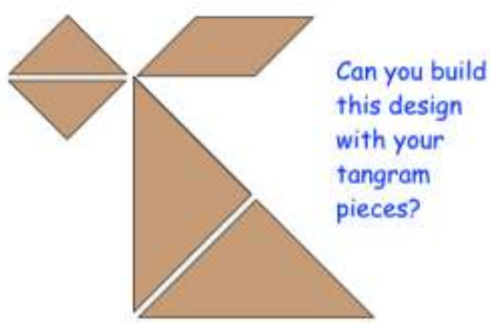
This outcome is promoting thinking in geometry that is like part-part-whole thinking in number. For example, determining the two tangram pieces that will solve a two-piece tangram puzzle is like determining two numbers that are parts of a given number. Experimentation and play with 2-D shapes and 3-D objects provide students with opportunities to explore the attributes of shapes or objects and how they can be put together and taken apart to make other shapes or objects. Blocks, tangram pieces, pattern blocks, interlocking cubes, and sets of 3-D solids are useful tools with which students can explore the relationships between and among various objects and shapes.

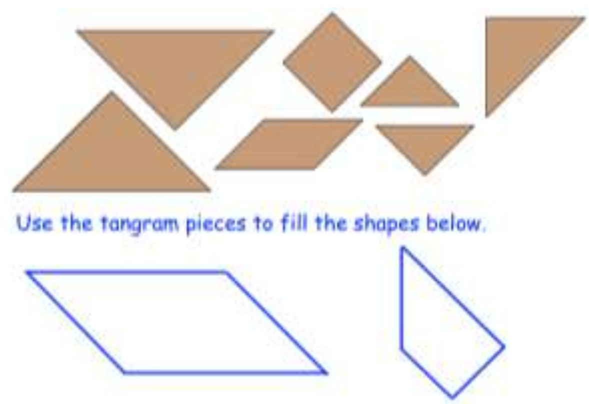
Creating composite shapes that are out of view and giving instructions to students on how to replicate them will provide opportunities for them to process and use the language of geometry and of position.

Performance Indicator Background

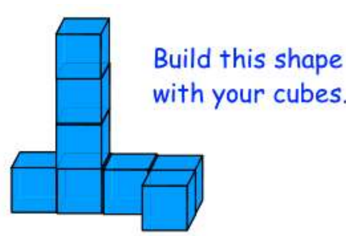
G02.01 and G02.03 Students should develop the ability to replicate composite 2-D shapes. This can begin by making a block design that matches a presented design involving 4 to 6 blocks. This will require students to select the blocks and arrange them in the same relative positions. Simply presenting a design created with a few blocks for a brief period of time, and then asking students to build that design, will extend their spatial abilities to include visual memory.

Students could create a block design using 2 blocks, trace around the design to create a 2-block puzzle, remove the blocks, and visualize where the 2 blocks would be placed if they returned them. After repeating this a few times, students could share their puzzles with other students to solve. This activity can be extended to puzzles involving more pieces depending upon individual student ability. Using tangram pieces in similar activities will provide students with experience with other shapes of varying sizes. It is through such replication activities that students become familiar with the attributes of various 2-D shapes. Students will use their knowledge of the properties of 2-D shapes to predict and select which shapes are necessary to produce a composite shape. To verify their predictions and selections, they will then deconstruct the original shape and compare the two sets.





G02.02 and G02.04 Students should develop the ability to replicate composite 3-D objects. For example, if students are presented with an object that has been constructed with 8 interlocking cubes, they are able to build the same shape; if they are presented with a display of various combinations of building blocks and 3-D objects, students are able to replicate this display. Such activities will use and develop students' visual discrimination and perception of spatial relationships abilities.



Through replication activities, students become more familiar with the attributes of 3-D objects. They will use these attributes, as well as relationships, to select a set of objects they predict are necessary to produce a composite object. To check their predictions, students should deconstruct the original object and compare the two sets. Any differences between their predictions and the actual set of objects should be discussed to help improve students' prediction abilities.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

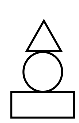
Tasks such as the following could be used to determine students' prior knowledge.

- Show students a 3-D object, such as a can or a small box. Ask them to describe the object to you. Listen to the language they use.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Give each student, or pair of students, a collection of pattern blocks. Say, "I am going to build a design with pattern blocks. I want you to use your pattern blocks to build a design just like mine." Observe their constructions.
- Provide students with a tangram puzzle outline and ask them to predict, and then select, the tangram pieces required to replicate the shape. Students may then replicate the shape to verify their predictions. Pattern blocks may also be used for this activity.
- Give each student, or pair of students, a collection of pattern blocks. Say to them, "I want you to build the following shape with your pattern blocks." Give oral directions and have students create your described shape. For example, Place a red trapezoid on your desk. On top of the trapezoid place a green triangle. On the left, place a blue rhombus and on the right, place another blue rhombus." Observe their constructions and have students share their thinking about the placement of each block.
- Show students a square that has been folded along the diagonal. Ask, What shape will this be when I unfold it?
- Provide students with a variety of 3-D solids. Show a composite 3-D object, such as a tower, and ask students to predict, and then select, the solids they need to replicate the object. Students should build the object using the solids they selected. They may then decompose the given object to verify their predictions.
- Show students a picture, such as the one below, for three to five seconds and ask them to draw it.



- Have students use a computer drawing program to make pictures using 2-D shapes. Then, have them challenge their classmates to replicate them.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)
- Mathematics Primary Checkpoint 13, pp. 122–123 (Line Master 13.1)
- Mathematics 1 Checkpoint 13, pp. 124–125 (Line Master 13.2)

Planning for Instruction

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of my students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ensure students have many opportunities to represent 2-D shapes and 3-D objects.
- Provide an assortment of 2-D shapes and/or 3-D objects. Ask students to work in small groups to describe the shapes and/or objects.
- Have students explore the properties of 3-D objects prior to identifying 3-D objects in the environment. Explorations may include tracing the faces of the solids or pressing the faces in play dough to investigate the attributes of the 3-D objects.
- Allow students to identify 2-D shapes and 3-D objects in their daily lives. This should include common everyday objects.

SUGGESTED LEARNING TASKS

- Ask students to cut a square, rectangle, or triangle into three parts. Have them exchange their pieces with a partner. Each partner should rearrange the pieces to make the original shape. (Activities such as these, in which students are required to assemble a figure from its parts, develops figure-ground perception skills.)
- Have students work in pairs with a geoboard to create a large square with a smaller square inside it.

- Display pictures of various 3-D objects, such as a rocket or sculpture. Ask students to identify which 3-D objects were used to build the object on display. Students can then build their own composite 3-D objects from individual 3-D objects, such as small cardboard boxes or modelling clay. Once the models are built, display the creations in class and ask students to identify the 3-D objects used to build the composite object.
- Ask students to create a design using circles, squares, rectangles and/or triangles. As students are working, ask them to describe their shapes.
- Work with prepared tangram puzzle outlines. Have students predict which shapes they will need to complete the picture and then work to complete the puzzle.
- Ask students to work with a partner. The first student makes a pattern block design with a specified number of pattern blocks. The second student selects the appropriate shapes and replicates the partner's design.
- Ask students to work with a partner. The first student makes an object with linking cubes. The second student selects the appropriate number of linking cubes and replicates the partner's design.
- Build a composite 3-D object. Have students select the appropriate 3-D objects and reproduce your object.
- Ask students to work in partners. Set up a screen between the students. Have one student create a composite 3-D object using linking cubes and then describe it to their partner. The partner uses the description to reproduce the composite object. They then lift the barrier to check to see if the objects are the same.
- Invite students to take a geometry walk around the school looking for 2-D shapes and 3-D objects.

SUGGESTED MODELS AND MANIPULATIVES

- attribute blocks
- geoboards
- geometric solids
- linking cubes
- pattern blocks
- Polydrons
- tangrams

MATHEMATICAL LANGUAGE

Teacher	**Student** (oral language)
▪ 2-D shapes: triangle, square, rectangle, circle, trapezoid, hexagon, rhombus ▪ 3-D objects: cylinder, sphere, cone, cube, prism, pyramid	▪ 2-D shapes: triangle, square, rectangle, circle, ▪ 3-D objects: cylinder, sphere, cone, cube, prism, pyramid

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 303–304, 309–314
- *Making Math Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 358–359, 365–368
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 196–200

Notes

SCO G03 Students will be expected to identify 2-D shapes in 3-D objects. [C, CN, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

G03.01 Identify the shape of the faces of a 3-D object.
G03.02 Identify 3-D objects in the environment that have faces that are a given 2-D shape.

Scope and Sequence

Mathematics Primary	Mathematics 1	Mathematics 2
	G03 Students will be expected to identify 2-D shapes in 3-D objects.	**G04** Students will be expected to identify 2-D shapes as part of 3-D objects in the environment.

Background

Initially, students recognize 3-D objects by their overall appearance, not by their properties. However, with directed activities, students examine 3-D objects to see how they are constructed and to become aware of their attributes. One thing students should notice is that many 3-D objects have faces that are 2-D shapes. For example, when they examine a typical cardboard box, they see that it has faces that are squares or rectangles. Typical classroom sets of 3-D solids include a cube, some prisms and pyramids, a cone, a cylinder, and a sphere. These solids have faces that are triangles, squares, rectangles, or circles—2-D shapes that are familiar to students.

Performance Indicator Background

G03.01 Students should have opportunities to find 3-D objects in their environment and identify the shape of their faces because these real-world associations are very important in the development of geometric concepts. With this foundation, they could also work with sets of 3-D solids and building blocks to explore the shapes of faces by tracing the different faces of the solids on paper or by pressing the faces in plasticine or sand.

G03.02 Once students have had experiences examining the faces of 3-D objects; they should be better able to concentrate on the component faces of a 3-D object. Prior to identifying 3-D objects in the environment with a face of a particular 2-D shape, students should have some preliminary experiences. For example, place four or five 3-D objects in front of students, ask them to identify an object that has a square face, and select a student to place a finger on the square face; repeat by naming other 2-D shapes. Label a mat with “All these objects have one face that is a triangle,” place a complete set of 3-D solids in view and ask students to select and place on the mat all the solids that meet this condition.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge.

- Place a collection of 3-D objects in front of students. Ask them to sort the objects and to describe their sorting rule.

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to examine a variety of containers (such as a yogurt container or cereal box). Ask, What shapes would be used to make this container? How do you know?
- Show students a triangle. Ask students to find three things in the classroom that make them think of that shape.
- Provide students with a set of paper 2-D shapes and have them circulate in the classroom or another environment, finding parts of 3-D objects that have that shape as one of its faces. Students may record their findings in their math journals.
- Provide students with a collection of 3-D objects. Ask them to find all of the objects that have circles for a face.
- Provide students with a collection of 3-D objects. Ask them to sort the objects by the shape of their faces. Ask them to explain their thinking as they sort the objects.

Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Responding to Assessment

Numeracy Nets K–2 (Bauman 2011)

- No Checkpoint for this outcome.

Planning for Instruction

Before introducing new material, consider ways to assess and build on students' knowledge and skills.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?

- What teaching strategies and resources should be used?
- How will the diverse learning needs of my students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ensure students have many opportunities to represent 2-D shapes and 3-D objects.
- Have students explore the properties of 3-D objects prior to identifying 3-D objects in the environment. Explorations may include tracing the faces of the solids or pressing the faces in play dough to investigate the attributes of the 2-D shapes and 3-D objects.
- Allow students to identify 2-D shapes and 3-D objects in their daily lives. This should include common everyday objects.

SUGGESTED LEARNING TASKS

- Have students examine a collection of objects found in their environment (e.g., cans, cereal boxes, ice cream cones, tissue boxes.) Ask them to identify the 2-D shapes found in each face of each 3-D object. Ask, What shape is the face? Do all of the faces have the same shape?
- Invite the students to hunt around the school to find various shapes (e.g., circles, squares, triangles, rectangles) in 3-D objects. If a digital camera is available, take pictures of the objects that are found. Have students share their findings and speculate on why certain shapes are more common than others.
- Ask students to find images of 3-D objects that have a face of a particular 2-D shape; for example, a face that is a square. Students can create books entitled "What's My Face?" using the digital images collected. Other students can read the books and identify the common faces.

SUGGESTED MODELS AND MANIPULATIVES

- attribute blocks
- geometric solids
- pattern blocks
- tangrams

MATHEMATICAL LANGUAGE

Teacher	**Student** (oral language)
▪ 2-D shapes: triangle, circle, square, rectangle, ▪ 3-D objects: cube, prism, pyramid, cone, cylinder, sphere ▪ faces	▪ 2-D shapes: triangle, circle, square, rectangle ▪ 3-D objects: cube, prism, pyramid, cone, cylinder, sphere ▪ faces

Resources/Notes

Print Resources

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 287–290
- *Making Math Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), pp. 343–346
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 216–217

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