Mathematics 2

Guide

NOVA SCOTIA
NOUVELLE-ÉCOSSE

2013

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Mathematics 2

Prepared by the Department of Education and Early Childhood Development

This is the most recent version of the current curriculum materials as used by teachers in Nova Scotia.

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NOVA SCOTIA

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Implementation Draft
May 2013

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Introduction

Background and Rationale

Mathematics curriculum is shaped by a vision that fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in society. It is essential that the mathematics curriculum reflect current research in mathematics instruction. To achieve this goal, Western and Northern Canadian Protocol's (WNCP) *The Common Curriculum Framework for K–9 Mathematics* (2006) has been adopted as the basis for the new mathematics curriculum in Nova Scotia.

The Common Curriculum Framework was developed by the seven ministries of education (Alberta, British Columbia, Manitoba, Northwest Territories, Nunavut, Saskatchewan, and Yukon Territory) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators, and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and performance indicators agreed upon by the seven jurisdictions. The outcomes and performance indicators have been adapted for Nova Scotia. This document is based on both national and international research by the WNCP and the National Council of Teachers of Mathematics (NCTM).

There is an emphasis in the Nova Scotia curriculum on particular key concepts at each grade that will result in greater depth of understanding and, ultimately, stronger student achievement. There is also a greater emphasis on number sense and operations concepts in the early grades to ensure students develop a solid foundation in numeracy.

Purpose

This document provides sets of outcomes and performance indicators to be used as a mandated common base for defining mathematics curriculum expectations. This common base should result in consistent student outcomes in mathematics within the province of Nova Scotia. It should also enable easier transfer for students moving within the province or from any jurisdiction that has adopted the WNCP framework. This document is intended to clearly communicate to all education partners across the province the high expectations for students' mathematical learning.

Program Design and Components

Assessment

Ongoing assessment for learning is essential to effective teaching and learning. Research has shown that assessment for learning (formative assessment) practices produce significant and often substantial learning gains, close achievement gaps, and build students' ability to learn new skills (Black & Wiliam 1998; OECD 2006). Student involvement in assessment promotes learning. Timely and effective teacher feedback and student self-assessment allow students to reflect on and articulate their understanding of mathematical concepts and ideas.

Assessment in the classroom includes

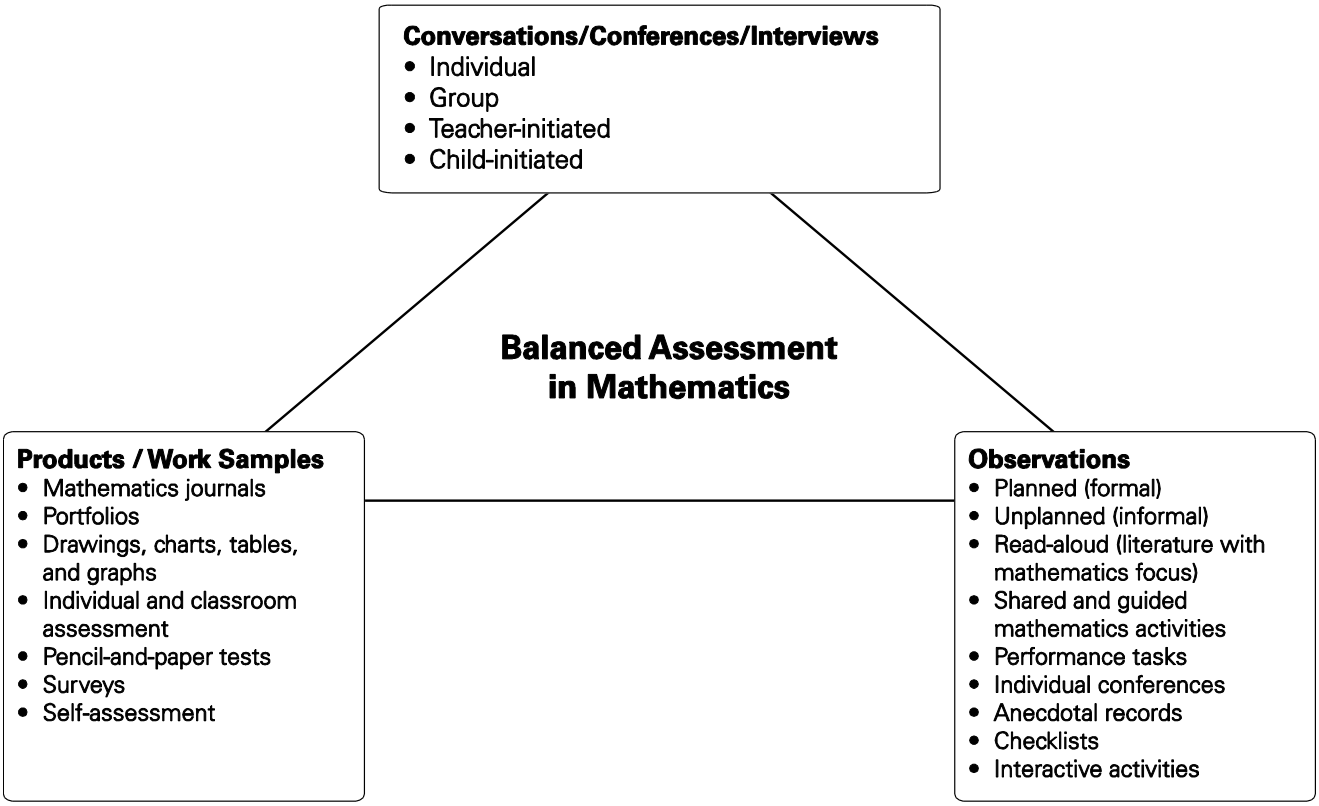
- providing clear goals, targets, and learning outcomes
- using exemplars, rubrics, and models to help clarify outcomes and identify important features of the work
- monitoring progress towards outcomes and providing feedback as necessary
- encouraging self-assessment
- fostering a classroom environment where conversations about learning take place, where students can check their thinking and performance and develop a deeper understanding of their learning

(Davies 2000)

Assessment for learning practices act as the scaffolding for learning, which only then can be measured through assessment of learning (summative assessment). Assessment of learning tracks student progress, informs instructional programming, and aids in decision making. Both forms of assessment are necessary to guide teaching, stimulate learning, and produce achievement gains.

Assessment of student learning should

- align with curriculum outcomes
- clearly define criteria for success
- make explicit the expectations for students' performance
- use a wide variety of assessment strategies and tools
- yield useful information to inform instruction



Time to Learn for Mathematics

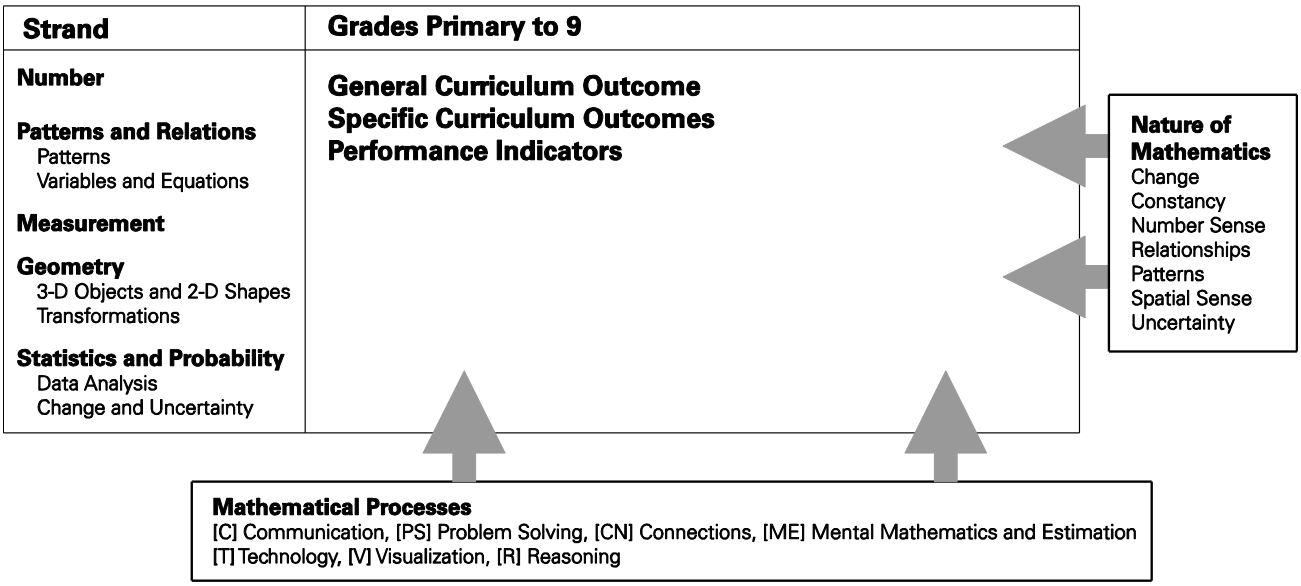
The Time to Learn Strategy *Guidelines for Instructional Time: Grades Primary*–6 includes time for mathematics instruction in the "Required Each Day" section. In order to support a constructivist approach to teaching through problem solving, it is highly recommended that the 45 minutes required daily in grades primary–2 and the 60 minutes required daily for grades 3–6 mathematics instruction be provided in an uninterrupted block of time.

Time to Learn guidelines can be found at
www.ednet.ns.ca/files/ps-policies/semestering.pdf
www.ednet.ns.ca/files/ps-policies/instructional_time_guidelines_p-6.pdf

Outcomes

Conceptual Framework for Mathematics Primary–9

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



(Adapted with permission from Western and Northern Canadian Protocol, *The Common Curriculum Framework for K–9 Mathematics*, p. 5. All rights reserved.)

Structure of the Mathematics Curriculum

Strands

The learning outcomes in the Nova Scotia Framework are organized into five strands across grades primary to 9.

- Number (N)
- Patterns and Relations (PR)
- Measurement (M)
- Geometry (G)
- Statistics and Probability (SP)

General Curriculum Outcomes (GCO)

Some strands are further subdivided into sub-strands. There is one general outcome (GCO) per sub-strand. GCOs are overarching statements about what students are expected to learn in each strand/sub-strand. The general curriculum outcome for each strand/sub-strand is the same throughout the grades.

NUMBER (N)

GCO: Students will be expected to demonstrate number sense.

PATTERNS AND RELATIONS (PR)

Patterns
GCO: Students will be expected to use patterns to describe the world and solve problems.

Variables and Equations
GCO: Students will be expected to represent algebraic expressions in multiple ways.

MEASUREMENT (M)

GCO: Students will be expected to use direct and indirect measure to solve problems.

GEOMETRY (G)

3-D Objects and 2-D Shapes
GCO: Students will be expected to describe the characteristics of 3-D objects and 2-D shapes and analyze the relationships among them.

Transformations
GCO: Students will be expected to describe and analyze position and motion of objects and shapes.

STATISTICS AND PROBABILITY (SP)

Data Analysis
GCO: Students will be expected to collect, display, and analyze data to solve problems.

Chance and Uncertainty
GCO: Students will be expected to use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Specific Curriculum Outcomes (SCOs) and Performance Indicators

Specific curriculum outcomes (SCOs) are statements that identify the specific conceptual understanding, related skills, and knowledge students are expected to attain by the end of a given grade.

Performance indicators are statements that identify specific expectations of the depth, breadth, and expectations for the outcome. Teachers use these statements to determine whether students have achieved the corresponding specific curriculum outcome.

Process Standards Key

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

NUMBER (N)

N01 Students will be expected to say the number sequence by

- 1s, forward and backward, starting from any point to 200
- 2s, forward and backward, starting from any point to 100
- 5s and 10s, forward and backward, using starting points that are multiples of 5 and 10 respectively to 100
- 10s, starting from any point, to 100 [C, CN, ME, R]

Performance Indicators

N01.01 Extend counting sequence (by 1s), forward and backward.
N01.02 Extend a given skip counting sequence (by 2s, 5s, or 10s) forward and backward.
N01.03 Skip count by 10s, given any number as a starting point.
N01.04 Identify and correct errors and omissions in a given skip counting sequence.
N01.05 Count a given sum of money with pennies, nickels, or dimes (to 100¢).
N01.06 Count quantity using groups of 2s, 5s, or 10s and counting on.

N02 Students will be expected to demonstrate if a number (up to 100) is even or odd. [C, CN, PS, R]

Performance Indicators

N02.01 Use concrete materials or pictorial representations to determine if a given number is even or odd.
N02.02 Identify even and odd numbers in a given sequence, such as on a hundred chart.
N02.03 Sort a given set of numbers as even numbers and odd numbers.

N03 Students will be expected to describe order or relative position using ordinal numbers (up to tenth). [C, CN, R]

Performance Indicators

N03.01 Indicate a position of a specific object in a sequence by using ordinal numbers up to tenth.
N03.02 Compare the ordinal position of a specific object in two different given sequences.

N04 Students will be expected to represent and partition numbers to 100. [C, CN, V]

Performance Indicators

N04.01 Represent a given number using concrete materials, such as ten-frames and base-ten materials.
N04.02 Represent a given number using coins (pennies, nickels, dimes, and quarters).
N04.03 Represent a given number using tallies.
N04.04 Represent a given number pictorially.
N04.05 Find examples of a given number in the environment.
N04.06 Represent a given number using expressions (e.g., 24 + 6, 15 + 15, 40 – 10)
N04.07 Read a number (0–100) given in symbolic or word form.
N04.08 Record in words a given number (0–20).
N04.09 Record, symbolically, any number (0–100).

N05 Students will be expected to compare and order numbers up to 100. [C, CN, R, V]

Performance Indicators

N05.01 Compare and order a given set of numbers in ascending or descending order and verify the result using a hundred chart, number line, ten-frames, or by making references to place value.
N05.02 Identify errors in a given ordered sequence.
N05.03 Identify missing numbers in a given hundred chart.
N05.04 Identify errors in a given hundred chart.

N06 Students will be expected to estimate quantities to 100 by using referents. [C, ME, PS, R]

Performance Indicators

N06.01 Estimate a given quantity by comparing it to a referent (known quantity).
N06.02 Estimate the number of groups of ten in a given quantity using 10 as a referent.
N06.03 Select between two possible estimates for a given quantity and explain the choice.

N07 Students will be expected to illustrate, concretely and pictorially, the meaning of place value for numerals to 100. [C, CN, R, V]

Performance Indicators

N07.01 Explain and show with counters the meaning of each digit for a given 2-digit numeral with both digits the same.
N07.02 Count the number of objects in a given set using groups of 10s and 1s, and record the result as a 2-digit numeral under the headings of 10s and 1s.
N07.03 Describe a given 2-digit numeral in at least two ways.
N07.04 Illustrate using ten-frames and diagrams that a given numeral consists of a certain number of groups of ten and a certain number of ones.
N07.05 Illustrate using proportional base-ten materials that a given numeral consists of a certain number of tens and a certain number of ones.
N07.06 Explain why the value of a digit depends on its placement within a numeral.
N07.07 Represent one unit if shown a pre-grouped model representing ten.

N08 Students will be expected to demonstrate and explain the effect of adding zero to or subtracting zero from any number. [C, R]

Performance Indicators

N08.01 Add zero to a given number and explain why the sum is the same as the addend.

N08.02 Subtract zero from a given number and explain why the difference is the same as the given number.

N09 Students will be expected to demonstrate an understanding of addition (limited to 1- and 2-digit numerals) with answers to 100 and the corresponding subtraction by

- using personal strategies for adding and subtracting with and without the support of manipulates
- creating and solving problems that involve addition and subtraction
- explaining and demonstrating that the order in which numbers are added does not affect the sum
- explaining and demonstrating that the order in which numbers are subtracted matters when finding a difference [C, CN, ME, PS, R, V]

Performance Indicators

N09.01 Solve a given story problem of any type by modelling it with materials or a diagram, and write a number sentence that represents the thinking in the solution.

N09.02 Solve a given story problem of any type by writing a number expression and combining the numbers to complete the number sentences.

N09.03 Match a number sentence to a given story problem.

N09.04 Create an addition or a subtraction number sentence and a story problem for a given solution.

N09.05 Model addition and subtraction using concrete materials or visual representations and record the process symbolically.

N09.06 Add a given set of numbers in two different ways and explain why the sum is the same.

N09.07 Recognize and create equivalent addition and subtraction number sentences.

N10 Students will be expected to apply mental mathematics strategies to quickly recall basic addition facts to 18 and determine related subtraction facts. [C, CN, ME, R, V]

Performance Indicators

N10.01 Explain the mental mathematics strategy that could be used to determine basic addition facts.

- Doubles Facts
- Plus One Facts
- One-Apart (Near Doubles) Facts
- Plus Two Facts
- Plus Zero Facts
- Make-10 Facts
- Two-Apart Facts
- Plus Three Facts

N10.02 Use and describe a personal strategy for determining a sum to 18.

N10.03 Quickly recall basic addition facts to 18 in a variety of contexts.

N10.04 Explain the think-addition strategy used to determine a basic subtraction fact.

N10.05 Use and describe a personal strategy for determining the subtraction facts.

PATTERNS AND RELATIONS (PR)

PR01 Students will be expected to demonstrate an understanding of repeating patterns (three to five elements) by describing, extending, comparing, and creating, patterns using manipulatives, diagrams, sounds, and actions. [C, CN, PS, R, V]

Performance Indicators

PR01.01 Identify the core of a given repeating pattern.
PR01.02 Describe and extend a given double attribute pattern.
PR01.03 Create a repeating non-numerical pattern and explain the rule.
PR01.04 Predict an element of a given repeating pattern using a variety of strategies and extend the pattern up to the tenth element to verify the prediction.
PR01.05 Translate a repeating pattern from one mode to another.
PR01.06 Compare two given repeating patterns, and describe how they are alike/different.

PR02 Students will be expected to demonstrate an understanding of increasing patterns by describing, extending, and creating numerical patterns (numbers to 100) and non-numerical patterns using manipulatives, diagrams, sounds, and actions. [C, CN, PS, R, V]

Performance Indicators

PR02.01 Identify and describe increasing patterns in a variety of given contexts.
PR02.02 Represent a given increasing pattern concretely and pictorially.
PR02.03 Identify errors in a given increasing pattern.
PR02.04 Explain the rule used to create a given increasing pattern.
PR02.05 Create an increasing pattern and explain the pattern rule.
PR02.06 Represent a given increasing pattern using another mode.
PR02.07 Solve a given problem using increasing patterns.
PR02.08 Identify and describe increasing patterns in the environment.
PR02.09 Determine missing terms in a given concrete, pictorial, or symbolic increasing pattern and explain the reasoning.

PR03 Students will be expected to demonstrate and explain the meaning of equality and inequality by using manipulatives and diagrams (0 to 100). [C, CN, R, V]

Performance Indicators

PR03.01 Determine whether two given quantities of the same object (same shape and mass) are equal by using a balance scale.
PR03.02 Construct and draw two unequal sets using the same object (same shape and mass) and explain the reasoning.
PR03.03 Demonstrate how to change two given sets, equal in number, to create inequality.
PR03.04 Choose from three or more given sets the one that does not have a quantity equal to the others and explain why.

PR04 Students will be expected to record equalities and inequalities symbolically, using the equal symbol or not equal symbol. [C, CN, R, V]

Performance Indicators

PR04.01 Determine whether two sides of a given number sentence are equal (=) or not equal (≠). Write the appropriate symbol and justify the answer.
PR04.02 Model equalities using a variety of concrete representations and record the equality.
PR04.03 Model inequalities using a variety of concrete representations and record the inequality.

MEASUREMENT (M)

M01 Students will be expected to demonstrate an understanding of the calendar and the relationships among days, weeks, months, and years. [C, CN, PS, R]

Performance Indicators

M01.01 Read a calendar.
M01.02 Name and order the days of the week and months of the year.
M01.03 Communicate the number of days in a week and the number of months in a year.
M01.04 Solve a given problem involving time which is limited to the number of days in a week and the number of months in a year.

M02 Students will be expected to relate the size of a unit of measure to the number of units (limited to non-standard units) used to measure length and mass. [C, CN, ME, R, V]

Performance Indicators

M02.01 Explain why one of two given non-standard units may be a better choice for measuring the length of an object.
M02.02 Explain why one of two given non-standard units may be a better choice for measuring the mass of an object.
M02.03 Select a non-standard unit for measuring the length or mass of an object and explain why it was chosen.
M02.04 Estimate the number of non-standard units needed for a given measurement task.
M02.05 Explain why the number of units of a measurement will vary depending upon the unit of measure used.

M03 Students will be expected to compare and order objects by length, height, distance around, and mass using non-standard units and make statements of comparison. [C, CN, ME, R, V]

Performance Indicators

M03.01 Estimate, measure, and record the length, height, distance around, or mass of a given object using non-standard units.
M03.02 Compare and order the measure of two or more objects in ascending or descending order and explain the method of ordering.

M04 Students will be expected to measure length to the nearest non-standard unit by using multiple copies of a unit and using a single copy of a unit (iteration process). [C, ME, R, V]

Performance Indicators

M04.01 Explain why overlapping or leaving gaps does not result in accurate measures.
M04.02 Count the number of non-standard units required to measure the length of a given object using a single copy or multiple copies of a unit.
M04.03 Estimate and measure a given object using multiple copies of a non-standard unit and using a single copy of the same unit many times, and explain the results.
M04.04 Estimate and measure, using non-standard units, a given length that is not a straight line.

M05 Students will be expected to demonstrate that changing the position of an object does not alter the measurements of its attributes. [C, R, V]

Performance Indicator

M05.01 Measure a given object, change the position, remeasure, and explain the results.

GEOMETRY (G)

G01 Students will be expected to sort 2-D shapes and 3-D objects using two attributes and explain the sorting rule. [C, CN, R, V]

Performance Indicators

G01.01 Determine the differences between two given presorted sets and explain the sorting rule.
G01.02 Identify and name two common attributes of items within a given sorted group.
G01.03 Sort a given set of 2-D shapes (regular and irregular) according to two attributes and explain the sorting rule.
G01.04 Sort a given set of 3-D objects according to two attributes and explain the sorting rule.

G02 Students will be expected to recognize, name, describe, compare, and build 3-D objects, including cubes and other prisms, spheres, cones, cylinders, and pyramids. [C, CN, R, V]

Performance Indicators

G02.01 Sort a given set of 3-D objects and explain the sorting rule.
G02.02 Identify common attributes of cubes and other prisms, spheres, cones, cylinders, and pyramids from given sets of the same 3-D objects.
G02.03 Identify and describe given 3-D objects with different dimensions.
G02.04 Identify and describe given 3-D objects with different positions.
G02.05 Create and describe a representation of a given 3-D object using materials such as modelling clay.
G02.06 Identify and name examples of cubes and other prisms, spheres, cones, cylinders, and pyramids found in the environment.

G03 Students will be expected to recognize, name, describe, compare and build 2-D shapes, including triangles, squares, rectangles, and circles. [C, CN, R, V]

Performance Indicators

G03.01 Sort a given set of 2-D shapes and explain the sorting rule.
G03.02 Identify common attributes of triangles, squares, rectangles, and circles from given sets of the same type of 2-D shapes.
G03.03 Identify given 2-D shapes with different dimensions.
G03.04 Identify given 2-D shapes with different positions.
G03.05 Identify and name examples of triangles, squares, rectangles, and circles found in the environment.
G03.06 Create a model to represent a given 2-D shape.
G03.07 Create a pictorial representation of a given 2-D shape.

G04 Students will be expected to identify 2-D shapes as part of 3-D objects in the environment. [C, CN, R, V]

Performance Indicators

G04.01 Compare and match a given 2-D shape, such as a triangle, square, rectangle, or circle, to the faces of 3-D objects in the environment.
G04.02 Name the 2-D faces of a given 3-D object.

STATISTICS AND PROBABILITY (SP)

SP01 Students will be expected to gather and record data about self and others to answer questions. [C, CN, PS, V]

Performance Indicators

SP01.01 Formulate a question that can be answered by gathering information about self and others.
SP01.02 Organize data as it is collected using concrete objects, tallies, checkmarks, charts, or lists.
SP01.03 Answer questions using collected data.

SP02 Students will be expected to construct and interpret concrete graphs and pictographs to solve problems. [C, CN, PS, R, V]

Performance Indicators

SP02.01 Determine the common attributes of concrete graphs by comparing a given set of concrete graphs.
SP02.02 Determine the common attributes of pictographs by comparing a given set of pictographs.
SP02.03 Answer questions pertaining to a given concrete graph or pictograph.
SP02.04 Create a concrete graph to display a given set of data and draw conclusions.
SP02.05 Create a pictograph to represent a given set of data using one-to-one correspondence.
SP02.06 Solve a given problem by constructing and interpreting a concrete graph or pictograph.

Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics.

Students are expected to

- communicate in order to learn and express their understanding of mathematics (Communication [C])
- develop and apply new mathematical knowledge through problem solving (Problem Solving [PS])
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines (Connections [CN])
- demonstrate fluency with mental mathematics and estimation (Mental Mathematics and Estimation [ME])
- select and use technologies as tools for learning and solving problems (Technology [T])
- develop visualization skills to assist in processing information, making connections, and solving problems (Visualization [V])
- develop mathematical reasoning (Reasoning [R])

The Nova Scotia curriculum incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning. The key to these process standards is presented in a box, as shown below, with each specific curriculum outcome within the strands.

Process Standards Key

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing, and modifying ideas, knowledge, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics.

Students also need to communicate their learning using mathematical terminology. Communication can help students make connections between and among the different representational modes—contextual, concrete, pictorial, linguistic/verbal, and symbolic—of mathematical ideas. Students must communicate *daily* about their mathematics learning. This enables them to reflect, to validate, and to clarify their thinking and provides teachers with insight into students' interpretations of mathematical meanings and ideas.

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, How would you ...? or How could you ...? the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement, perseverance, and collaboration.

Problem solving is also a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive, mathematical risk takers.

When students are exposed to a wide variety of problems in all areas of mathematics, they explore various methods for solving and verifying problems. In addition, they are challenged to find multiple solutions for problems and to create their own problem.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to one another or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

"Because the learner is constantly searching for connections on many levels, educators need to *orchestrate the experiences* from which learners extract understanding. … *Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching*." (Caine and Caine 1991, 5).

Mathematics should be viewed as an integrated whole rather than as the study of separate strands or units. Connections must also be made between and among the different representational modes—contextual, concrete, pictorial, linguistic/verbal, and symbolic. The process of making connections, in turn, facilitates learning. Concepts and skills should also be connected to everyday situations and other curricular areas. For example, when developing literacy skills students learn to make text-to-world, text-to-text, and text-to-self connections. Students can also make connections to make mathematics come alive through math-to-world, math-to-math, and math-to-self connections.

Mental Mathematics and Estimation [ME]

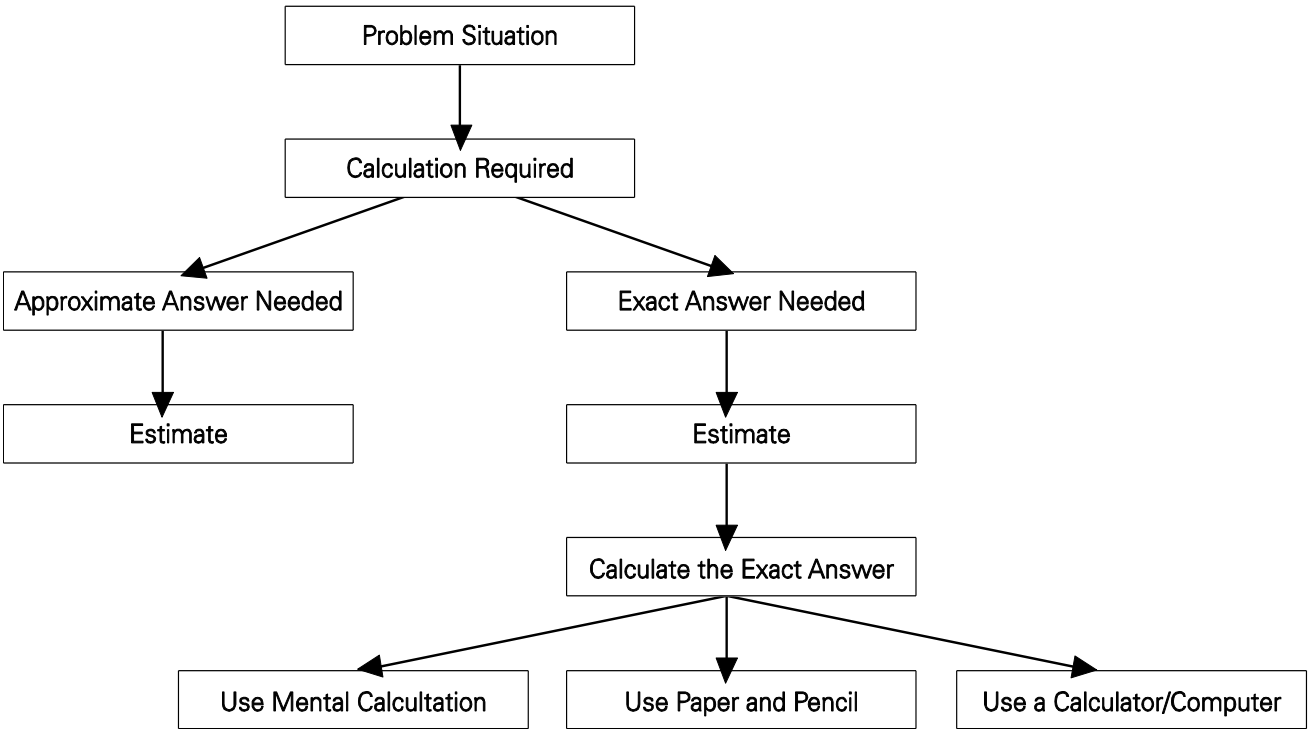
Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy, and flexibility. "Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental math." (National Council of Teachers of Mathematics, May 2005).

Students proficient with mental mathematics "become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers, and are more able to use multiple approaches to problem solving." (Rubenstein 2001) Mental mathematics "provides a cornerstone for all

estimation processes, offering a variety of alternative algorithms and nonstandard techniques for finding answers." (Hope 1988, v)

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and what strategy to use when estimating. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision-making process as illustrated below.



The skill of estimation requires a sound knowledge of mental mathematics. Both are necessary to many everyday experiences, and students should be provided with frequent opportunities to practise these skills.

Technology [T]

Technology can be effectively used to contribute to and support the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Techonology can be used to

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties

- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense

The use of calculators is recommended to enhance problem solving, to encourage discovery of number patterns, and to reinforce conceptual development and numerical relationships. They do not, however, replace the development of number concepts and skills. Carefully chosen computer software can provide interesting problem-solving situations and applications.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in grades primary to 3 to enrich learning, it is expected that students will achieve all outcomes without the use of technology.

Visualization [V]

Visualization "involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world." (Armstrong 1999). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers. These mental images are needed to develop concepts and understand procedures. Images and explanations help students clarify their understanding of mathematical ideas in all strands.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure, when to estimate and to know several estimation strategies. (Shaw and Cliatt 1989)

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Mathematics reasoning involves informal thinking, conjecturing, and validating—these help students understand that mathematics makes sense. Students are encouraged to justify, in a variety of ways, their solutions, thinking processes, and hypotheses. In fact, good reasoning is as important as finding correct answers.

Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that define the nature of mathematics, and these are woven throughout this document. These components include change, constancy, number sense, relationships, patterns, spatial sense, and uncertainty.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as

- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen 1990, 184).

Constancy

Different aspects of constancy are described by the terms **stability**, **conservation**, **equilibrium**, **steady state**, and **symmetry** (AAAS–Benchmarks 1993, 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems. Examples of constancy include the following:

- The area of a rectangular region is the same regardless of the methods used to determine the solution.
- The sum of the interior angles of any triangle is 180°.
- The theoretical probability of flipping a coin and getting heads is 0.5.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (British Columbia Ministry of Education 2000, 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences and use benchmarks and referents. This results in students who are computationally fluent, flexible with numbers, and have intuition about numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, and concepts. The search for possible relationships involves the collection and analysis of data, and describing relationships visually, symbolically, orally, or in written form.

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all strands, and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with an understanding of their environment. Patterns may be represented in concrete, visual, or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create, and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking, which is foundational for working with more abstract mathematics in higher grades.

Spatial Sense

Spatial sense involves visualization, mental imagery, and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to reason and interpret among and between 3-D and 2-D representations and identify relationships to mathematical strands. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes. Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions. For example,

- knowing the dimensions of an object enables students to communicate about the object and create representations
- the volume of a rectangular solid can be calculated from given dimensions
- doubling the length of the side of a square increases the area by a factor of four

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Curriculum Document Format

This guide presents the mathematics curriculum so that a teacher may readily view the scope of the outcomes that students are expected to achieve during that year. Teachers are encouraged, however, to examine what comes before and what follows after, to better understand how students' learning at a particular grade level is part of a bigger picture of concept and skill development.

The order of presentation in no way assumes or prescribes a preferred order of presentation in the classroom, but simply lays out the specific curriculum outcomes in relation to the overarching general curriculum outcomes (GCOs).

The footer of the document shows the name of the course, and the strand name is presented in the header. When a specific curriculum outcome (SCO) is introduced, it is followed by the mathematical processes and performance indicators for that outcome. A scope and sequence is then provided, which relates the SCO to previous and next grade SCOs. Also for each SCO, there is background information, assessment strategies, suggested instructional strategies, suggested models and manipulatives, mathematical language, and a section for resources and notes. For each section, the guiding questions should be used to help with unit and lesson preparation.

SCO

Mathematical Processes		
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections
[ME] Mental Mathematics and Estimation		
[T] Technology	**[V]** Visualization	**[R]** Reasoning

Performance Indicators

Describes observable indicators of whether students have achieved the specific outcome.

Scope and Sequence

Previous grade or course SCOs	Current grade SCO	Following grade or course SCOs

Background

Describes the "big ideas" to be learned and how they relate to work in previous grade and work in subsequent courses.

Additional Information

A reference to Appendix A, which contains further elaborations for the performance indictators.

Assessment, Teaching, and Learning

Assessment Strategies

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Sample tasks that can be used to determine students' prior knowledge.

Whole-Class/Group/Individual Assessment Tasks

Some suggestions for specific activities and questions that can be used for both instruction and assessment

Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Responding to Assessment

Correlations to related resources.

Planning for Instruction

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcome and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

Choosing Instructional Strategies

Suggested strategies for planning daily lessons.

Suggested Learning Tasks

Suggestions for general approaches and strategies suggested for teaching this outcome.

Guiding Questions

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Suggested Models and Manipulatives

Mathematical Language

Teacher and student mathematical language associated with the respective outcome.

Resources/Notes

Contexts for Learning and Teaching

Beliefs about Students and Mathematics Learning

"Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge." (National Council of Teachers of Mathematics 2000, 20).

The Nova Scotia mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning that have grown out of research and practice. These beliefs include the following:

- Mathematics learning is an active and constructive process.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.
- Learners are individuals who bring a wide range of prior knowledge and experiences and who learn via various styles and at different rates.
- Learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking and that nurtures positive attitudes and sustained effort.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Students develop a variety of mathematical ideas before they enter school. Children make sense of their environment through observations and interactions at home and in the community. Mathematics learning is embedded in everyday activities, such as playing, reading, storytelling, and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Curiosity about mathematics is fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks, and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do and need to construct their own meaning of mathematics. This meaning is best constructed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with and translating through a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial, contextual, and symbolic representations of mathematics.

The learning environment should value and respect all students' experiences and ways of thinking, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must realize that it is acceptable to solve problems in different ways and that solutions may vary.

Goals for Mathematics Education

The main goals of mathematics education are to prepare students to

- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- make connections between mathematics and its applications
- become mathematically literate adults, using mathematics to contribute to society

Students who have met these goals will

- gain understanding and appreciation of the contributions of mathematics as a science, a philosophy, and an art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity about mathematics and situations involving mathematics

Opportunities for Success

A positive attitude has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for student success help develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations, and engage in reflective practices.

To experience success, students must be taught to set achievable goals or assess their progress as they work toward these goals. Striving toward success and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessing of personal goals.

Engaging All Learners

*"*No matter how engagement is defined or which dimension is considered, research confirms this truism of education: *The more engaged you are, the more you will learn."* (Hume 2011, 6)

Student engagement is at the core of learning. This is critical for teachers to take into account when planning and implementing instruction. Effective instruction engages, embraces, and supports all learners through a range of learning experiences. This curriculum is designed to provide learning opportunities that reflect culturally proficient instructional and assessment practices and are equitable, accessible, and inclusive of the multiple facets of diversity represented in today's classrooms.

Engagement in learning occurs when students are provided with opportunities to become more invested in their learning. When teachers know their students as individual learners and as individual people, their students are more likely to be motivated to learn, participate in classroom activities, persist in challenging situations, and engage in reflective practices. Students often become more engaged when teachers demonstrate a genuine belief in each student's potential to learn.

SUPPORTIVE LEARNING ENVIRONMENTS

A supportive and positive learning environment has a profound effect on students' learning. In classrooms where students feel a sense of belonging, are encouraged to actively participate, are challenged without being frustrated, and feel safe and supported to take risks with their learning, students are more likely to experience success. It is realized that not all students will progress at the same pace or be equally positioned in terms of their prior knowledge of and skill with particular concepts and outcomes. Teachers provide all students with equitable access to learning by integrating a variety of instructional approaches and assessment activities that consider all learners and align with the following key principles:

- Instruction must be flexible and offer multiple means of representation.
- Students must have opportunities to express their knowledge and understanding in multiple ways.
- Teachers must provide options for students to engage in learning through multiple ways.

Teachers who know their students well become aware of individual learning differences and infuse this understanding into planned instructional and assessment decisions. They organize learning experiences to accommodate the many ways in which students learn, create meaning, and demonstrate their knowledge and understanding. Teachers use a variety of effective teaching approaches that may include

- providing all students with equitable access to appropriate learning strategies, resources, and technology
- offering a range of ways students can access their prior knowledge to connect with new concepts
- scaffolding instruction and assignments so that individual or groups of students are supported as needed throughout the process of learning
- verbalizing their thinking to model comprehension strategies and new learning
- balancing individual, small-group, and whole-class approaches to learning activities
- involving students in the co-creation of criteria for assessment and evaluation
- providing students with choice in how they demonstrate their understanding according to learning styles and preferences, building on individual strengths, and including a range of difficulty and challenge
- providing frequent and meaningful feedback to students throughout their learning experiences

LEARNING STYLES AND PREFERENCES

The ways in which students make sense of, receive, and process information, demonstrate learning, and interact with peers and their environment both indicate and shape learning preferences, which may vary widely from student to student. Learning preferences are influenced also by the learning context and purpose and by the type and form of information presented or requested. Most students tend to favour one learning style and may have greater success if instruction is designed to provide for multiple learning styles, thus creating more opportunities for all students to access learning. The three most commonly referenced learning styles are

- auditory (such as listening to teacher-presented lessons or discussing with peers)
- kinesthetic (such as using manipulatives or recording print or graphic/visual text)
- visual (such as interpreting information with text and graphics or viewing videos)

While students can be expected to work using all modalities, it is recognized that one or some of these modalities may be more natural to individual students than the others.

A Gender-Inclusive Curriculum

It is important that the curriculum respects the experiences and values of all students and that learning resources and instructional practices are not gender-biased. Teachers promote gender equity and inclusion in their classrooms when they

- articulate equally high expectations for all students
- provide equal opportunity for input and response from all students
- model gender-fair language and respectful listening in their interactions with students

Valuing Diversity: Teaching with Cultural Proficiency

Teachers understand that students represent diverse life and cultural experiences, with individual students bringing different prior knowledge to their learning. Therefore, teachers build upon their knowledge of their students as individuals and respond by using a variety of culturally-proficient instruction and assessment strategies. "Instruction that is embedded in socially meaningful contexts, and tasks that are meaningful and relevant to the lives of students, will engage students in high-level problem-solving and reasoning and enhance students' engagement (Frankenstein 1995; Gutstein 2003; Ladson-Billings 1997; Tate 1995)." (Herzig 2005)

Students with Language, Communication, and Learning Challenges

Today's classrooms include students who have diverse backgrounds, abilities, levels of development, and learning challenges. By observing and interacting with students as they work on assigned activities, teachers can identify areas where students may need additional support to achieve their learning goals. Teachers can then respond with a range of effective instructional strategies. Students who have English as an Additional Language (EAL) may require curriculum outcomes at different levels, or temporary individualized outcomes, particularly in language-based subject areas, while they become more proficient in their English language skills. For students who are experiencing difficulties, it is important that teachers distinguish between students for whom curriculum content is challenging and students for whom language-based issues are at the root of apparent academic difficulties.

Students who Demonstrate Gifted and Talented Behaviours

Some students are academically gifted and talented with specific skill sets or in specific subject areas. Most students who are gifted and talented thrive when challenged by problem-centred, inquiry-based learning and open-ended activities. Teachers may challenge students who are gifted and talented by adjusting the breadth, the depth, and/or the pace of instruction. Learning experiences may be enriched by providing greater choice among activities and offering a range of resources that require increased cognitive demand and higher-level thinking at different levels of complexity and abstraction. For additional information, refer to *Gifted Education and Talent Development* (Nova Scotia Department of Education 2010).

Connections across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the students' understanding of mathematical concepts and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating mathematics in health education, literacy, music, physical education, science, social studies, and visual arts.

Number

GCO: Students will be expected to demonstrate number sense.

Specific Curriculum Outcomes

Process Standards

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

N01 Students will be expected to say the number sequence by
- 1s, forward and backward, starting from any point to 200
- 2s, forward and backward, starting from any point to 100
- 5s and 10s, forward and backward, using starting points that are multiples of 5 and 10 respectively to 100
- 10s, starting from any point, to 100 [C, CN, ME, R]

N02 Students will be expected to demonstrate if a number (up to 100) is even or odd. [C, CN, PS, R]

N03 Students will be expected to describe order or relative position using ordinal numbers (up to tenth). [C, CN, R]

N04 Students will be expected to represent and partition numbers to 100. [C, CN, V]

N05 Students will be expected to compare and order numbers up to 100. [C, CN, R, V]

N06 Students will be expected to estimate quantities to 100 by using referents. [C, ME, PS, R]

N07 Students will be expected to illustrate, concretely and pictorially, the meaning of place value for numerals to 100. [C, CN, R, V]

N08 Students will be expected to demonstrate and explain the effect of adding zero to or subtracting zero from any number. [C, R]

N09 Students will be expected to demonstrate an understanding of addition (limited to 1- and 2-digit numerals) with answers to 100 and the corresponding subtraction by
- using personal strategies for adding and subtracting with and without the support of manipulates
- creating and solving problems that involve addition and subtraction
- explaining and demonstrating that the order in which numbers are added does not affect the sum
- explaining and demonstrating that the order in which numbers are subtracted matters when finding a difference [C, CN, ME, PS, R, V]

N10 Students will be expected to apply mental mathematics strategies to quickly recall basic addition facts to 18 and determine related subtraction facts. [C, CN, ME, R, V]

SCO N01 Students will be expected to say the number sequence by
- 1s, forward and backward, starting from any point to 200
- 2s, forward and backward, starting from any point to 100
- 5s and 10s, forward and backward, using starting points that are multiples of 5 and 10 respectively to 100
- 10s, starting from any point, to 100

[C, CN, ME, R]

[C] Communication **[PS]** Problem Solving **[CN]** Connections **[ME]** Mental Mathematics and Estimation
[T] Technology **[V]** Visualization **[R]** Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N01.01 Extend counting sequence (by 1s), forward and backward.
N01.02 Extend a given skip counting sequence (by 2s, 5s, or 10s) forward and backward.
N01.03 Skip count by 10s, given any number as a starting point.
N01.04 Identify and correct errors and omissions in a given skip counting sequence.
N01.05 Count a given sum of money with pennies, nickels, or dimes (to 100¢).
N01.06 Count quantity using groups of 2s, 5s, or 10s and counting on.

Scope and Sequence

Mathematics 1	Mathematics 2	Mathematics 3
N01 Students will be expected to say the number sequence by ▪ 1s, forward and backward between any two given numbers, 0 to 100 ▪ 2s to 20, forward starting at 0 ▪ 5s to 100, forward starting at 0, using a hundred chart or a number line ▪ 10s to 100, forward starting at 0, using a hundred chart or a number line	**N01** Students will be expected to say the number sequence by ▪ 1s, forward and backward, starting from any point to 200 ▪ 2s, forward and backward, starting from any point to 100 ▪ 5s and 10s, forward and backward, using starting points that are multiples of 5 and 10 respectively to 100 ▪ 10s, starting from any point, to 100.	**N01** Students will be expected to say the number sequence forward and backward by ▪ 1s through transitions to 1000 ▪ 2s, 5s, 10s, or 100s, using any starting point to 1000 ▪ 3s, using starting points that are multiples of 3 up to 100 ▪ 4s, using starting points that are multiples of 4 up to 100 ▪ 25s, using starting points that are multiples of 25 up to 200

Background

In Mathematics 2, students are continuing to develop an understanding of number and counting. In previous grades, students have had experience counting by 1s, forward and backward, between any two given numbers (0 to 100); 2s to 20, forward starting at 0; and 5s and 10s to 100, forward starting at 0. They also have demonstrated an understanding of counting by using the counting-on strategy and skip counting to count sets. Students will extend their experience with skip-counting patterns to 100 and will extend their knowledge of the number sequence by 1s to 200.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

Ask students to count by

- 1s, forward and backward, between any two given numbers (0 to 100)
- 2s, to 20, forward starting at 0
- 5s and 10s to 100, forward starting at 0, using a hundred chart or number line

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to start at
  - 92 and count forward by 1s (stop at 121)
  - 42 and count forward by 2s (stop at 60)
  - 13 and count by 2s (stop at 35)
  - 78 and count backward by 2s (stop at 58)
  - 30 and count by 10s (stop at 100)
  - 8 and count by 10s (stop at 58)
  - 100 and count backward by 10s (stop at 40)
  - 15 and count by 5s (stop at 60)
  - 85 and count backward by 5s (stop at 55)
- Ask students to count a collection of counters. Ask them to find the total number of counters and to explain their method of counting (e.g., by 2s, 5s, 10s).
- Ask students to count by 2s (5s or 10s) as you clap. Ask students to record the final number when you finish clapping.
- Show students a counting sequence with an error or missing number. Ask students to identify and correct the error.
- Provide students with a collection of pennies, nickels, or dimes and ask them to skip count to find the total.
- Provide students with a collection of different coins (pennies, nickels, and dimes) and ask them to count the coins to find the total.
- Tell students that you have 35 cents in your "piggy bank." Ask them to skip count to find the total as you add nickels or dimes to your piggy bank.

- Have students listen as you count, 5, 10, 15, 16, 17. Ask, What coins am I counting? Repeat for different coins.
- Tell students that you have some coins in your hand that total 44 cents. Ask them to record possible combinations of coins.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)
- Grade 2 Checkpoint 1, Task 4 (counting forward by 2s only), pp. 25–26
- Grade 2 Checkpoint 3 (counting forward by 1s only), pp. 42–43, (Line Master 3.1)

Numeracy Nets 3 (Bauman 2009)
- Grade 3 Checkpoint 2, pp. 23–24

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Use daily routines to reinforce number sense to 100.
- Use literature that involves skip counting.
- Use the repeat (constant) function (press 0, +, 2, =, =, =, …) on a calculator to skip count to a target number. For example, if you start at 0 and want to end at 40, by which number(s) could you skip count (2, 5, 10)? What if you started at a different point? What if you wanted to end at a different point?
- Use a variety of problems including open-ended (more than one solution) questions. For example, tell students that you have some coins that total 61 cents. Ask students to identify possible combinations of coins that would result in that total.

- Provide students with a variety of authentic tasks that require them to recognize when it is more efficient to skip count.
- Use number patterns, such as 65, _, 55, _, 45, _, … or 1, 3, _, 7, _, _, 13, … to encourage skip counting.

SUGGESTED LEARNING TASKS

- **Orchestra counting:** Put students into groups. The first group begins counting forward by 1s (or 2s, 5s, or 10s) from a given number. When the teacher points to another group, they continue the count.
- Use a horizontal number line, hundred chart, or a large floor chart with numbers 1 to 100. Ask the class to skip count by 2s (or 5s or 10s), and ask a student volunteer to step on each number as it is said. Discuss the movements made by the student volunteer.
- Provide students with a number of counters (60, for example). Ask students to separate the counters from the pile as they count them by groups. Have them discuss which method is the most efficient way of counting the counters.
- Ask students to reach into a bag and take a handful of counters. Then, ask them to spin a spinner labelled “by 2s, by 5s, by 10s.” Students group their counters by the number shown on the spinner and then count them to find the total.
- Have students create visuals to represent counting by 2s, 5s, and 10s. This could be done using a digital camera, drawings (by hand or computer), or prints. For example, skip counting by 5s could be represented by digital pictures of each student’s hand with the fingers spread apart, or counting by 2s could be represented by digital pictures of each student’s eyes, etc.
- Use ten-frames to model skip counting by 10s. Begin with a ten-frame representing a single-digit number, such as 3. Have students count by 10s, and add a full ten-frame to the display as they say the next number.
- Ask students to correct the following “counting by 2s” sequence: …, 82, 84, 86, 87, 88, 90, 92, …
- Ask students to work in pairs to create a number sequence with a missing number and then exchange their sequence with another pair of students. Each pair identifies the missing number in the sequence they received.
- Invite students to use calculators to count using the repeat (constant) function. For example, as a student places counters or coins in a bag and skip counts aloud, others may repeatedly add one on calculators to keep track of the count electronically. The calculator repeat function may also be used when skip counting by 2s, 5s, or 10s. Calculators may also be used with starting points other than 0 to model counting on.
- Give students 3 dimes, 2 nickels, and 6 pennies. Ask them to count the coins.
- Provide coins for the students. Ask, Can you use 7 of these coins to make 43 cents? Have students share their strategies for solving the problem.

SUGGESTED MODELS AND MANIPULATIVES

- calculators
- coins
- counters
- hundred chart
- number line
- ten-frames

MATHEMATICAL LANGUAGE

Teacher	Student
▪ coins: penny, nickel, dime ▪ forward, backward ▪ hundred chart, number line, calculator ▪ numbers: zero to two hundred ▪ position words: next, before, after, between ▪ sequence ▪ skip count	▪ coins: penny, nickel, dime ▪ forward, backward ▪ hundred chart, number line, calculator ▪ numbers: zero to two hundred ▪ position words: next, before, after, between ▪ skip count

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 86–87, 138–140
- *Making Math Meaningful to Canadian Students, K–8,* Second Edition (Small 2013), pp. 139, 142
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 39–41, 56–58, 123–127, 134–138, 150–152

Videos

- *An Introduction to Using Number Lines* (13:04 min.) (ORIGO Education 2010)
- *Analyzing Patterns (Skip Counting) on a Hundred Board* (27:16 min.) (ORIGO Education 2010)
- *Teaching Number: 0 to 9* (14:47 min.) (ORIGO Education 2010)
- *Teaching Number: Counting* (10:49 min.) (ORIGO Education 2010)

Notes

SCO N02 Students will be expected to demonstrate if a number (up to 100) is even or odd. [C, CN, PS, R]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N02.01 Use concrete materials or pictorial representations to determine if a given number is even or odd.
N02.02 Identify even and odd numbers in a given sequence, such as on a hundred chart.
N02.03 Sort a given set of numbers as even numbers and odd numbers.

Scope and Sequence

Mathematics 1	Mathematics 2	Mathematics 3
—	**N02** Students will be expected to demonstrate if a number (up to 100) is even or odd.	—

Background

Mathematicians use the term **even** to describe a quantity that can be shared equally into groups of two with no leftovers. The terms **odd** and **even** need to be carefully developed in the context of classifying numbers because they are also used in real-life situations with different meanings. Outside of mathematics, odd may mean strange, extraordinary, or unusual. Students may also have heard the term **even** used in relation to a scale being balanced. They may have also heard the term **evenly** being used in sharing situations (e.g., if 12 candies are shared evenly with 3 people, then they all have the same number of candies). It is important that the vocabulary be explored with students so that they have a clear understanding of the mathematical definitions.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

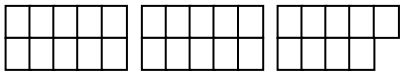
Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

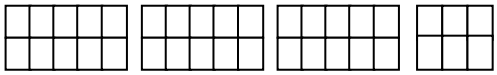
Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my Assessment Strategies with my teaching strategies?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to tell you if the following representations are even or odd and explain why.

  a. 

  b. 

- Ask students to sort the following numbers into even and odd by using linking cubes or ten-frames (or other materials): 11, 23, 30, 39, 40, 48.
- Ask students to use a hundred chart to explain if the following numbers are even or odd: 3, 18, 37, 55, 71.
- Ask students to fill in the missing numbers on a hundred chart. Ask, Are they even or odd? How do you know?
- Provide students with a hundred chart. Ask them to describe the odd and even patterns they see on the chart.
- Ask students to select any two numbers between 10 and 100. Ask them to tell whether each of the numbers they have selected is even or odd and to explain their thinking.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)
- No Checkpoint for this outcome.

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

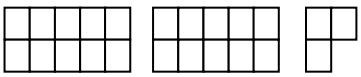
Consider the following strategies when planning daily lessons.

- Use concrete models to represent numbers to demonstrate their evenness or oddness.

7 (groups of 2 with one left over)

7 (2 equal groups with one left over)

- Use hundred charts and number lines to show even and odd number patterns.
- Use children's literature, such as *Even Stephen, Odd Todd* by K. Cristaldi (1996), to introduce the terms **odd** and **even** as they relate to number.
- Include discussions as part of the daily routine that relate to even and odd numbers (e.g., Do we have an even number of people here today?).
- Clarify misconceptions regarding numbers with both an even and odd digit (e.g., 23) by representing the number with ten-frames or square tiles grid pictures arranged as shown below.



SUGGESTED LEARNING TASKS

- Ask students to use sets of coloured tiles to determine whether a given number of tiles is even or odd.
- Show a set of counters on the overhead. Ask students to use ten-frames to determine whether the set of counters shown is odd or even by placing counters on their ten-frames. Ask students to explain their thinking.
- Work as a class to represent numbers with counters to determine their evenness or oddness. Then, ask students to colour the corresponding numeral on the hundred chart to begin creating a pattern (red for even, blue for odd). Ask students to continue discovering the even/odd pattern on the hundred chart.
- Ask students to sort the following numbers into even and odd: 9, 24, 30, 51, 67, 78, 86, and to explain their thinking.
- Ask students to explore the evenness or oddness of multiples of 10. Ask, Is 50 an even or an odd number? Explain how you know.
- Ask students to solve problems, such as Dave's class has 23 students. He is planning snacks for snack time and wants to know if he will need an even or odd number of snacks if he gives each student 1 snack? 2 snacks? 3 snacks?

SUGGESTED MODELS AND MANIPULATIVES

- colour tiles
- dot cards
- hundred chart
- linking cubes
- ten-frames

MATHEMATICAL LANGUAGE

Teacher	Student
▪ even, odd numbers ▪ groups of two, two equal groups ▪ hundred chart, number line ▪ numerals ▪ ones digit, tens digit ▪ quantity ▪ shared, left over	▪ even, odd numbers ▪ groups of two, two equal groups ▪ hundred chart, number line ▪ numerals ▪ quantity ▪ shared, left over

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), p. 149
- *Making Math Meaningful to Canadian Students, K–8,* Second Edition (Small 2013), p. 204
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 291–293

Notes

SCO N03 Students will be expected to describe order or relative position using ordinal numbers (up to tenth). [C, CN, R]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N03.01 Indicate a position of a specific object in a sequence by using ordinal numbers up to tenth.
N03.02 Compare the ordinal position of a specific object in two different given sequences.

Scope and Sequence

Mathematics 1	Mathematics 2	Mathematics 3
—	**N03** Students will be expected to describe order or relative position using ordinal numbers (up to tenth).	**PR01** Students will be expected to demonstrate an understanding of increasing patterns by describing, extending, comparing, and creating numerical (numbers to 1000) patterns and non-numerical patterns using manipulatives, diagrams, sounds, and actions.

Background

Ordinal numbers are used to describe the relative position of an object or event that have been ordered or ranked based on a criterion, such as size, importance, or chronology. Students are likely familiar with some ordinal numbers through everyday experiences, such as the second Tuesday, the third activity, and the first in line.

Students should discover important ideas about ordinal numbers including

- position matters — the ordinal number used to describe an object depends on the position of the object in relation to what has been deemed as being first
- For every cardinal number**,** there is an ordinal number. For example, counting a set of 9 objects, the last object touched will be the ninth object touched.
- There are two ways to record an ordinal number, in words and in symbolic form, such as third and 3rd.
- The position of first is not always fixed. It depends on the point of view. For example, the circle below can be described as the first from the left and the fifth from the right.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

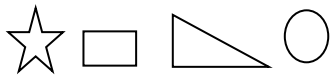
- Provide students with a triangle, a square, and a circle from the attribute blocks. Ask students to arrange them so that a triangle is first in line, a circle is second, and a square is last.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to name the position of the star in each row of shapes starting from the right. Then, ask them to name the position of the star in each row of shapes starting from the left.

a. 

b. 

- Ask students to make a "train" of 10 linking cubes in which the 3rd and 7th cars from the front of the train are different colours from the rest of the train. Ask, What position are these cars if you are standing at the other end of the train?
- Provide students with a calendar. Ask them to tell you the date of the third Thursday of the month.
- Provide students with a selection of attribute blocks and directions for ordering them in a sequence. For example, The first shape in the row is a triangle. The third shape from the right is a square. The fifth shape is a rectangle. The second shape is a circle. The fourth shape is a circle.
- Ask students to create a sequence of shapes using attribute blocks or pattern blocks and to record their sequence using pictures. Then, ask them to write a description of their sequence so that another student could recreate that sequence. Have students exchange descriptions with one another and attempt to use the written description he or she received to recreate the sequence.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)
- No Checkpoint for this outcome.

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Use calendars to provide a useful context for ordinal numbers.
- Have students observe position in line-ups. Who is third? If there are 6 people in front of you, where are you in the line? **Note:** Students who are seated can see these relationships more clearly than students in the line.
- Use literature, such as *Ten Little Rubber Ducks* by Eric Carle (2005) to explore ordinal numbers. Discuss the use of ordinal numbers in the literature.

SUGGESTED LEARNING TASKS

- Ask a child to place a set of objects in a line by following directions given orally or written on cards.
- Ask for ten student volunteers. Give each volunteer a card with an ordinal number (1st to 10th) on it. Ask students to place themselves in order from 1st to 10th.
- Ask students to create a sequence of shapes using attribute blocks or pattern blocks and to record the sequence using pictures. Then, ask them to write a description of their sequence so that another student could recreate that sequence. Have students exchange descriptions with one another and attempt to use the written description he or she received to recreate the sequence.
- Provide students with a set of pictures of sequences of objects and a set of ordinal descriptions of those same sequences. Ask students to match the pictures with the correct descriptions.
- Ask students to use counters to create a pattern in which the number of counters in the 4th position is fewer than the number of counters in the 3rd position.
- After reading literature that involves ordinal numbers, invite students to write and illustrate their own ordinal stories.
- Ask the student to use pattern blocks to make a row of five different shapes in which the first shape is a triangle and the third is a square. Ask, What would you have to do to make the square the fourth shape?

SUGGESTED MODELS AND MANIPULATIVES

- attribute blocks
- calendar
- pattern blocks

MATHEMATICAL LANGUAGE

Teacher	Student
▪ objects and events ▪ ordinal numbers: first to tenth ▪ position ▪ sequence	▪ objects and events ▪ first to tenth ▪ position

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 89, 95
- *Making Math Meaningful to Canadian Students, K–8,* Second Edition (Small 2013), pp. 145, 152

Videos

- *Teaching Number: Relative Position* (20:41 min.) (ORIGO Education 2010)

Notes

SCO N04 Students will be expected to represent and partition numbers to 100. [C, CN, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N04.01 Represent a given number using concrete materials, such as ten-frames and base-ten materials.
N04.02 Represent a given number using coins (pennies, nickels, dimes, and quarters).
N04.03 Represent a given number using tallies.
N04.04 Represent a given number pictorially.
N04.05 Find examples of a given number in the environment.
N04.06 Represent a given number using expressions (e.g., 24 + 6, 15 + 15, 40 – 10)
N04.07 Read a number (0–100) given in symbolic or word form.
N04.08 Record in words a given number (0–20).
N04.09 Record, symbolically, any number (0–100).

Scope and Sequence

Mathematics 1	**Mathematics 2**	**Mathematics 3**
N04 Students will be expected to represent and partition numbers to 20.	**N04** Students will be expected to represent and partition numbers to 100.	**N02** Students will be expected to represent and partition numbers to 1000.

Background

Numbers, like all mathematical concepts, can be represented in five ways—contextually, concretely, pictorially, symbolically, and verbally. Students need numerous experiences representing numbers to 100 in all of these ways, and translating between and among them.

In addition, to develop flexibility in working with number when performing operations, it is very important for students to understand that numbers can be broken down into parts, in many different ways. For example, 75 can be partitioned into 70 + 5, 50 + 25, or 60 + 12 + 3. It is important that students understand that these are three ways of writing 75 and not just three expressions that have an answer of 75.

While there are nine indicators, classroom tasks should provide opportunities to observe many or all of the indicators simultaneously. For example, a specific number such as 75 can be represented with pictures, coins, base-ten materials, tallies, ten-frames, words, parts, and contexts. According to John Van de Walle (2006), to conceptualize a number as being made up of two or more parts is the most important understanding that can be developed about number relationships.

In previous grades, students have had experiences with representing and describing numbers to 20 concretely, pictorially, and symbolically; reading number words to 20; determining compatible number pairs for 5, 10, and 20; and placing numerals on a number line with benchmarks 0, 5, 10, 15, and 20. Students have had no formal instruction in place value. This will be introduced later in this grade with outcome N07. In Mathematics 1, students demonstrated with concrete materials and with pictures, how

a number, up to 20, could be partitioned into two or more smaller groups. For example, they may have taken a number like 16 and grouped it into three groups of five and one single using five-frames, or one group of 10 using a ten-frame with six singles, which sets the stage for learning place value.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask children to show the number 15 in as many different ways as they can using manipulatives, words, pictures, and symbols.

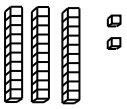
WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to represent 52 (or any 2-digit number) with
  - ten-frames
  - tallies
  - coins
  - base-ten blocks
  - a picture
  - an expression
- Ask students to choose a 2-digit number. Ask them to represent their chosen 2-digit numbers with base-ten blocks in as many ways as they can.
- Ask students, Which of the expressions in the box represent 36?

30 + 6	28 + 8
3 + 6	66 – 30
40 – 4	20 + 26
35 + 2	

- Ask students to create two more expressions that would also equal 36. (This activity could initially be assessed using models, but students may be able to complete it symbolically after working on SCO N09.)
- Give students two representations of the same number, point to one and ask, Is this number more, less, or are they the same? Have students explain their thinking.



- Ask students to represent the number of students in their class as many different ways as they can.
- Ask students to use words to describe a number (e.g., thirty-six is four less than forty) using comparative language.
- Ask students to read a given number presented in symbolic form (e.g., 47) or in word form (e.g., forty-seven).
- Ask students to identify where they might see the numeral 26 in their neighbourhood.
- Say a number or show a number of base-ten blocks or ten-frames and ask students to record that number in symbolic form.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2009)
- Grade 3 Checkpoint 3, pp. 26–27

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Use a variety of representations to explore numbers. Different representations of the same number can be compared for equivalency.
- Have available a class or individual chart / word wall that matches pictorial, and/or symbolic representations with the words to support students as they learn number words.

SUGGESTED LEARNING TASKS

- Give each student a different number (e.g., 25, 36, 42, 48, …) of counters. Ask students to arrange their counters in groups to make it easier for a classmate to count them; for example, 25 might be grouped as 5 groups of 5 or as 2 groups of 10 and 1 group of 5. Have students rotate around the room determining the number of counters each classmate has displayed.
- Ask students to represent a given 2-digit number, for example 53, using ten-frames. Ask them to make as many statements about the number as possible. For example, 53 is 3 more than 50; 53 is 7 less than 60; 53 is the same as 50 and 3 more; 53 is 2 less than 55; 53 is 13 more than 40. This may also be done using a hundred chart.
- Begin with the first day of school in September and do "Number of the Day" tasks. Have students express a given number in as many ways as they can. For example, Day 26, probably sometime in October, may be expressed as 5 + 5 + 5 + 5 + 5 + 1; 20 + 6; 10 + 10 + 6; 10 + 16; 26 ones; 2 tens and 6 ones; 1 quarter, 1 penny, etc.
- Have students choose a price card, for example 37 cents, and represent that amount with coins (create a context by setting up a class "store"). Ask, How many ways can you pay for one item with coins?
- Ask students to work in a group of four. Students take turns drawing a numeral card. Each student in the group takes a turn representing the chosen number with base-ten blocks and with symbols. Each block display within the group must be different. For example, if 46 was selected, it might be represented as 4 rods and 6 small cubes, 3 rods and 16 small cubes, 25 small cubes and 21 small cubes, 3 rods and 3 groups of 5 small cubes and 1 small cube.
- Ask students to select a 2-digit number and describe it in different ways. Have a student read his or her description to the class and ask other students to identify the number.

SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- coins
- counters
- hundred charts
- ten-frames

MATHEMATICAL LANGUAGE

Teacher	Student
▪ coins: penny, nickel, dime, quarter ▪ expression ▪ hundred chart, number line ▪ number words: zero to twenty (zero, one, two, …., twenty) ▪ numeral ▪ parts ▪ partition numbers/quantities ▪ represent ▪ tally ▪ ten-frames, base-ten blocks	▪ coins: penny, nickel, dime, quarter ▪ expression ▪ hundred chart, number line ▪ number words: zero to twenty (zero, one, two, …., twenty) ▪ numeral ▪ parts ▪ quantity ▪ tally ▪ ten-frames

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 138–140, 142–143, 146
- *Making Math Meaningful to Canadian Students, K–8,* Second Edition (Small 2013), pp. 194–196, 198–199, 202
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 47–51, 54–55, 56–58, 122–125, 146–148

Notes

SCO N05 Students will be expected to compare and order numbers up to 100. [C, CN, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N05.01 Compare and order a given set of numbers in ascending or descending order and verify the result using a hundred chart, number line, ten-frames, or by making references to place value.
N05.02 Identify errors in a given ordered sequence.
N05.03 Identify missing numbers in a given hundred chart.
N05.04 Identify errors in a given hundred chart.

Scope and Sequence

Mathematics 1	**Mathematics 2**	**Mathematics 3**
N05 Students will be expected to compare sets containing up to 20 objects to solve problems using referents and one-to-one correspondence.	**N05** Students will be expected to compare and order numbers up to 100.	**N03** Students will be expected to compare and order numbers to 1000.

Background

Students should encounter a variety of numbers in context. These contexts help them develop an understanding of number size. Students will be able to order a set of numbers in ascending and descending order. They will be able to justify their solutions using benchmarks, hundred charts, number lines, ten-frames, and/or place value. Visual models encourage reasoning**,** as students consider how to compare and order numbers. As with all concepts, begin with concrete models. Initially, use groupable materials such as beans or wooden stir sticks, and then move to pre-grouped materials such as base-ten blocks or ten-frames. Students should be given many experiences with concrete materials before moving to more pictorial and symbolic representations.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

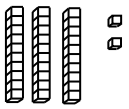
Tasks such as the following could be used to determine students' prior knowledge.

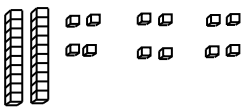
- Provide two different sets of counters, each with 20 or fewer. Ask students which set has more and which has fewer (e.g., one set has 19 and the other has 12) and to explain how they know.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Show a number of base-ten unit cubes (e.g., 34). Beside the cubes, show 5 rods and 6 unit cubes. Ask, Which has more? Is one easier to count than the other? Explain.
- Give students two representations of the same number, point to one and ask, Is this number more, less, or are they the same? Have students explain their thinking.





- Ask students to explain using concrete and/or pictorial representations, why 42 is greater than 29.
- Ask students to use base-ten materials to show why 24 is less than 42.
- Ask students to record a number in the blank so that the three numbers are in order. For example, 25, __, 31 or 75, __, 68.
- Ask students to choose three numbers, each less than 100. Ask them to put their numbers in order from smallest to largest. Ask them to explain their thinking.
- Ask students, Is a number with a 7 in it always greater than a number with a 6 in it? Ask students to explain their thinking.
- Show a number line with some numbers placed incorrectly. Ask students to identify the errors, explain their thinking, and place the numbers correctly.
- Ask the students to rearrange the numbers below to correct the errors in the following ascending sequence: 7, 13, 20, 32, 28, 56, 69, 71, 44
- Ask students to explain how they compare two numbers.
- Provide students with a hundred chart and cubes of different colours. Ask them to place cubes on the hundred chart as directed. For example, ask students to place
  - a red cube on a number less than 36
  - a green cube on a number greater than 84
  - a blue cube on a number between 53 and 60
- Provide students with a hundred chart with missing numbers. Ask them to explain what number would be placed in each of the empty spaces and to explain their thinking.
- Provide students with a hundred chart containing errors. Ask them to correct the errors and to explain their thinking.
- Ask students to record a number that is greater than 34 but less the 43.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)
- Grade 2 Checkpoint 4, Task 2, pp. 51–52 (Line Master 4.2)

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Compare benchmark quantities of the same item (e.g., 50 counters and 10 counters) to provide a visual of the relative size of numbers.
- Ask students to compare numbers during classroom routines (e.g., Are there more boys than girls in class today?).
- Expect students to compare and order numbers in a variety of ways including concrete, pictorial, and symbolic representations and to explain their thinking.
- Encourage students to use number lines and open number lines to compare and order numbers.

SUGGESTED LEARNING TASKS

- Ask students to order a given set of numbers, such as 34, 43, 17, and 21, in ascending or descending order and verify the result using a hundred chart, number line, or ten-frames. Verification may also make reference to place value when working on SCO N07.
- Prepare a set of 100 stir sticks numbered from 1 to 100. Select a group of five students. Ask each student in the group to draw out a stir stick and then to work co-operatively to order themselves in ascending (or descending) order.

- Provide students with 9 rods and 9 small cubes (base-ten blocks). Ask students to create two different amounts, each using exactly 5 of the base-ten blocks. Ask, Which amount is greater than the other? How is it possible to use the same number of blocks, but still have one amount greater than the other?
- Provide a teacher-made flyer in which prices are less than one dollar. Ask students to circle the item that costs the most, the least, more than 50 cents, etc.
- Provide each student with a deck of 2-digit numeral cards. Students work in pairs. Each student turns over the top card in his or her deck. The student with the greatest number collects both cards and sets them aside. Play continues until all cards have been compared. The student who has collected the greatest number of cards wins.
- Ask students to identify missing numbers or errors in a given hundred chart or ordered sequence. Ask them to fill in the missing numbers or correct the errors and to explain their thinking.
- Play "Guess My Number." The teacher selects a target number and identifies the range for students (e.g., "I am thinking of a number between 1 and 60."). Students take turns asking questions in order to identify the number. The teacher can only answer "yes," "no," "too small," or "too large." It is important to model effective questions rather than have students guess random numbers.
  Example: Is the number greater than ___? Is it less than ___? Is it an even or odd number? A number line may be used to record the information determined after each question. In the example, the game would start with two markers placed at 1 and the 60. If the first student asks if the number is 20 and the teacher answers, "too small," the first marker can then be moved from the 1 to the 20. The range is then narrowed to between 20 and 60.
- Use labels on which numerals from 0 to 100 are written. Place a label on each student's back. Each student then moves around the room and asks classmates questions similar to those above in order to identify the numeral written on his or her label. Each question may only be answered with "yes," "no," or "I don't know." Once the student has identified the numeral written on his or her label, he or she works with other students to place themselves in order creating a number line.

SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- hundred chart
- number line
- ten-frames

MATHEMATICAL LANGUAGE

Teacher	Student
▪ ascending, descending order ▪ compare, order numbers ▪ hundred chart, number line, ten-frame, ▪ identify missing numbers, errors ▪ place value: tens	▪ compare, order numbers ▪ hundred chart, number line, ten-frame ▪ tens

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 142–143, 145
- *Making Math Meaningful to Canadian Students, K–8,* Second Edition (Small 2013), pp. 198–199, 201
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 38, 42–47, 54–55, 56–58, 142–143

Notes

SCO N06 Students will be expected to estimate quantities to 100 by using referents. [C, ME, PS, R]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N06.01 Estimate a given quantity by comparing it to a referent (known quantity).
N06.02 Estimate the number of groups of ten in a given quantity using 10 as a referent.
N06.03 Select between two possible estimates for a given quantity and explain the choice.

Scope and Sequence

Mathematics 1	Mathematics 2	Mathematics 3
N06 Students will be expected to estimate quantities to 20 by using referents.	**N06** Students will be expected to estimate quantities to 100 using referents.	**N04** Students will be expected to estimate quantities less than 1000 using referents.

Background

The ability to estimate, a key reasoning skill in mathematics, should develop with regular practice over the course of the year. To develop estimation skills, students should be provided with collections of objects and be asked to estimate the size of the group, using a referent. Estimation helps students develop flexible, intuitive ideas about numbers, further developing number sense. Estimating is difficult for students; therefore, time must be spent developing an understanding of the language associated with estimating such as “more than,” “less than,” “closer to,” and “about.” Model this language and encourage students to use it when talking about estimating experiences.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Place between 10 and 20 counters on the table. Ask students to record an estimate for the number of counters shown. Then, move 5 counters (referents) to the side of the table. Tell students, Here are 5 counters. Do you want to adjust your estimate? If you want to adjust it, will it be more or less than your first estimate? Ask students to explain their thinking.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Place a pile of paper clips on a desk. Ask students to estimate the number of paper clips. Observe and/or interview students to determine if they are using a referent. A guiding question might include, How did you choose that number?
- Give students a train of four linking cubes. Ask them to estimate the number of cubes in a longer train that is on display in the room. Have students explain their thinking.
- Show students a group of items and ask them to choose between two given estimates. Have them explain their reasoning for their choice.
- Show students a jar containing about 100 beads. Tell them that one student estimated there were 90 beads in the jar. Another student estimated there were 25 beads in the jar. Ask, Which estimate is closer to the actual number of beads in the jar? Explain your thinking.
- Ask students to estimate the number of steps it will take to walk a specified distance. (e.g., How many steps will it take to walk from the pencil sharpener to the classroom door?).

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)
- No Checkpoint for this outcome.

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?

- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Use the same unit (e.g., footsteps) to estimate and check the attributes of a variety of things. If students are able to explore these types of tasks in succession with the same unit, they refine their estimating skills.
- Use children's literature with an estimation focus, such as *Counting on Frank* by Rod Clement (1994) or *The Penny Pot* by Stuart Murphy (1998).
- Provide opportunities in the classroom for estimation. These may include
  - using estimation jars filled with different objects
  - determining the number of mathematics materials in a container
  - determining the number of pages in a book

SUGGESTED LEARNING TASKS

- Ask students to estimate how many footsteps
  - from the door to the window
  - from the door to the top of the stairs
  - from the door to your desk
- Show 10 paper clips on the overhead for the students to have a visual reference. Then display a larger group of paper clips. Ask students to estimate how many paper clips there are. Ask, Why do you think that?
- Give a small group of students a series of baggies with a number of bread ties. In each baggie place a card asking, Is it closer to __ or __? (e.g., is it closer to 20 or 50?). Bags are considered one at a time by the group, with students explaining their choice. The group may then count the quantity to determine the closest estimate.
- Challenge the students to estimate how many times they can print their name in one minute. Individual considerations include length of name or speed of printing.
- Have students draw a card that indicates how many cubes they need to get from a container (less than 20, between 30 and 50, about 20). They must choose which size scoop to use to get that many items. Students count to check. **Variation:** Have only one scoop and several buckets of different-sized objects. Students have to decide which object to scoop to get their target range. Decide whether students can use the scoop only once or a smaller scoop more than once. Emphasis should be placed on estimating to scoop the target number.
- Provide three different sets of small objects such as centicubes, pennies, or beans. Ask student to take a handful of one of the objects, estimate the number of objects, and then count to check. Students should then self-assess and indicate whether their estimate was too small, just right, or too large. Students should record their results in a chart with labels Object, Estimate, Count, My Estimate Was Too Small, Just Right, Too Large. Students should then repeat the process with each of the remaining objects.

SUGGESTED MODELS AND MANIPULATIVES

- variety of containers and objects (baggies, beads, buckets, cubes, marbles, paper clips)

MATHEMATICAL LANGUAGE

Teacher	Student
▪ estimate ▪ group of ten ▪ more than, less than, closer to, about ▪ referent	▪ estimate ▪ group of ten ▪ more than, less than, closer to, about

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), p. 144
- *Making Math Meaningful to Canadian Students, K–8,* Second Edition (Small 2013), p. 144
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 132

Notes

SCO N07 Students will be expected to illustrate, concretely and pictorially, the meaning of place value for numerals to 100. [C, CN, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N07.01 Explain and show with counters the meaning of each digit for a given 2-digit numeral with both digits the same.
N07.02 Count the number of objects in a given set using groups of 10s and 1s, and record the result as a 2-digit numeral under the headings of 10s and 1s.
N07.03 Describe a given 2-digit numeral in at least two ways.
N07.04 Illustrate using ten-frames and diagrams that a given numeral consists of a certain number of groups of ten and a certain number of ones.
N07.05 Illustrate using proportional base-ten materials that a given numeral consists of a certain number of tens and a certain number of ones.
N07.06 Explain why the value of a digit depends on its placement within a numeral.
N07.07 Represent one unit if shown a pre-grouped model representing ten.

Scope and Sequence

Mathematics 1	**Mathematics 2**	**Mathematics 3**
—	**N07** Students will be expected to illustrate, concretely and pictorially, the meaning of place value for numerals to 100.	**N05** Students will be expected to illustrate, concretely and pictorially, the meaning of place value for numerals to 1000.

Background

The foundation for the development of the concept of place value centres around grouping tasks, and students' understandings of unitizing (the concept that ten ones equal one ten). In order to understand and use place value, students need to be able to think in groups or to unitize. Ten is both a single entity with a value of 10 and a collection of 10 single units each with a value of 1. Students should have many opportunities to explore counting and organizing larger sets of materials into groups of ten to build connections between 10 ones and 1 ten.

Groupable, proportional models should be used first; that is, materials that can be put together, or taken apart, to make, or unmake, tens that will be ten times the size of the ones. Suggested materials include wooden stir sticks that can be placed together in groups of 10 using rubber bands, linking cubes which can be connected to make strips of 10, or beans that can be bagged or placed in cups in groups of 10. It is important that this stage not be rushed. Many problems that students encounter with place value concepts are believed to stem from inadequate attention to these early place value tasks in which the students create the tens. When students make that important connection between all that they know about counting by ones and the concept of grouping by tens, they should notice how much easier it is to count.

Base-ten blocks are an efficient and valuable model, as they are proportional in size. This means that a rod is ten times the size of a small cube (the rod is scored to show the joining of 10 small cubes), and a flat is ten times the size of a rod, and one hundred times the size of a small cube (the flat is scored to show the 100 small cubes). This helps with developing number sense as a number like 80 is ten times as great as the number 8. While these multiplicative relationships among the blocks exist, it is unlikely that most students in Mathematics 2 will think in this way; they are more likely to count the number of small cubes they see joined together to make a rod and a flat, and they will need to place 10 rods on top of a flat to be convinced of that relationship.

When working with numbers, students should always have base-ten blocks, ten-frames, or other materials available to represent numbers, and should be expected to use those materials to support their thinking. This consistent use of manipulatives solidifies their understanding of number. It is essential that students be provided plenty of opportunity to manipulate and work with base-ten blocks and ten-frames. For example, these materials could be used daily in morning routines, such as calendars or number of the day, to reinforce base-ten concepts.

If they begin working with pre-grouped models prematurely, students can easily attach words such as tens to both materials and groups without realizing what the materials or symbols represent. Such students may learn by rote to represent a number, such as 46, by showing 4 rods and 6 small cubes without realizing the rods represent the 40 in the number. If they were given 46 counters and asked to show what the 4 and 6 each represent, they may very well point to 4 counters and 6 counters. These students have what is often referred to as a face-value interpretation of a number rather than a place-value interpretation.

The big idea must be to understand the counting of groups and the units within the groups of ten as the foundation of place value. Ensure that although physical models play a key role, they in themselves are not the concept. Students must construct the concept and relate it to the model. Using a variety of materials allows students to construct a deep understanding of place-value concepts.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to represent a given number up to 20 using ten-frames. Ask them to use ten-frames to represent another number that is greater than the first one they represented.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to choose a 2-digit number and to record it symbolically. Ask them to represent their 2-digit number using ten-frames. Ask them to explain the value of each digit in relation to the ten-frames.
- Ask students to represent a given 2-digit number using base-ten materials. Ask them to explain the value of each digit in relation to the base-ten materials.
- Show students a number represented with small cubes and the same number represented with rods and small cubes. For example, 23 represented as 23 small cubes and as 2 rods and 3 small cubes. Ask, Are these two numbers the same? Have students explain their thinking.
- Show students a rod from the base-ten materials. Tell them the rod is worth 10. Ask them to show you the base-ten material that would have a value of 1.
- Ask students to pick up a handful of counters and represent the total using ten-frames. Ask, How else could you show this number?
- Show the students a 2-digit numeral with both digits the same (e.g., 44). Have students model the value of each digit. Ask students to explain why these digits do not represent the same value.
- Provide students with a large number of objects (less than 100). Ask students to count the objects and to record the result as a 2-digit numeral. Observe whether students grouped the objects by tens or used other efficient strategies.
- Give students a place-value chart and a group of objects. Have them determine how many objects they have and record their results in the chart.
- Ask students to model numbers using pre-grouped materials, to record numerals from existing models, and to show how to count an amount that is modelled.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- No Checkpoint for this outcome.

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

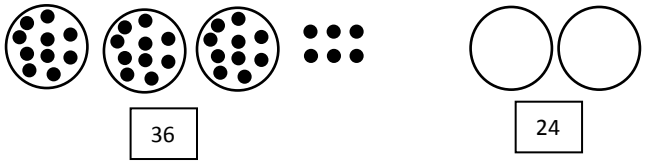
Consider the following strategies when planning daily lessons.

- Precede tasks with pre-grouped models with tasks using groupable models so students will know that a rod and ten small cubes are the same amount. The best base-ten models are proportional and can be classified as “groupable” or “pre-grouped.”
- Give students many opportunities to count and sort large numbers of objects within a relevant context (e.g., take inventory of classroom objects). Note whether students count the number of objects using groups of 10s and 1s. Ask them to record the result as a 2-digit numeral.
- Students should be expected to express numbers symbolically, verbally, and with pictures and models. They should translate among these representations.
- Ensure that although physical models play a key role, they are not the concept. Students must construct the concept and relate it to the model. Using a variety of materials allows students to construct a deep understanding of place-value concepts.

SUGGESTED LEARNING TASKS

- Place different amounts of wooden stir sticks into paper bags. Ask students to count the sticks by grouping them into bundles of 10s with elastic bands. Ask them to record the total as a 2-digit numeral under the headings of tens and ones. Ask, How many sticks are there? Is it easier to count the sticks by ones or by tens when they are grouped? How do you know there are that many sticks? (Note how the students respond. Do they count by groups of ten?)
- Give students a 2-digit numeral card. Ask them to use ten-frames to represent it. Ask them to show the ten-frames that represent the first digit in the numeral. Then, ask them to show the ten-frames that represent the second digit in the numeral.
- Give students a pre-grouped model and ask, If this is ten, what would one look like (what would 3 or 17 or 85 look like)?

- Set out a number of arrangements of beans grouped by tens (in cups, on sticks, on plates, ten-frames, etc.) and individual units. Also have some empty "stations" as shown below. Provide the students with a set of prepared 2-digit numeral cards. Direct them to place the matching card in front of the appropriate prepared display and to build the arrangements for the other cards they have. Use a large collection of numbers (e.g., 13, 16, 18, 24, 26, 28, 33, 36, 38, and 40).



- Show students a 2-digit numeral. Ask them to read the numeral and model it with base-ten materials.
- Provide students with a set of 33 counters. Ask them to record the 2-digit numeral that represents the set. Ask them to show the counters represented by the first digit in the numeral (thirty counters) and by the second digit (three counters).
- Have students work in groups. Give each group a piece of chart paper on which is written a 2-digit numeral. Ask students to record all the ways they can represent or describe the numeral. For example, students might record 24 as two 10s and four 1s, 20 and 4, two groups of 10 and 4 left over, one 10 and fourteen 1s, or twenty-four 1s.

SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- beans
- counters
- Cuisenaire rods
- ten-frames
- wooden stir stick

MATHEMATICAL LANGUAGE

Teacher	Student
▪ base-ten blocks, ten-frame ▪ numeral, digit ▪ ones, tens ▪ place value to 100 ▪ quantity ▪ small cube, rod, flat ▪ two-digit numeral	▪ base-ten blocks, ten-frame ▪ numeral, digit ▪ ones, tens ▪ quantity ▪ small cube, rod

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 138–144
- *Making Math Meaningful to Canadian Students, K–8,* Second Edition (Small 2013), pp. 194–200
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 127–141, 145–149

Videos

- *Teaching Place Value: 20 to 99* (29:08 min.) (ORIGO Education 2010)
- *Teaching Place Value: Teen Numbers* (8:30 min.) (ORIGO Education 2010)
- *Using a Hands-on Approach to Represent Tens and Ones* (10:05 min.) (ORIGO Education)

Notes

SCO N08 Students will be expected to demonstrate and explain the effect of adding zero to or subtracting zero from any number. [C, R]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N08.01 Add zero to a given number and explain why the sum is the same as the addend.

N08.02 Subtract zero from a given number and explain why the difference is the same as the given number.

Scope and Sequence

Mathematics 1	**Mathematics 2**	**Mathematics 3**
N08 Students will be expected to identify the number, up to 20, that is one more, two more, one less, and two less than a given number.	**N08** Students will be expected to demonstrate and explain the effect of adding zero to or subtracting zero from any number.	—

Background

Zero is conceptually different from any other number. Zero cannot be connected to an actual object or represented by a concrete item (until later grades when they explore negative numbers). Zero indicates an absence of quantity or the quantity before the count begins. Therefore, addition and subtraction with zero do not change the original value.

The no-change nature of adding and subtracting zero can be difficult for some students; therefore, it is important that students engage in discussion and tasks involving the number zero to further their understanding of this concept. Students should be encouraged to develop story problems involving zero to help them understand zero in addition and subtraction. Students can also role-play tasks where they receive or give away zero. They should model the addition and subtraction of zero frequently to develop a generalization that addition does not always increase the quantity and subtraction does not always decrease the quantity.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

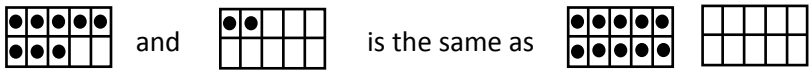
Tasks such as the following could be used to determine students' prior knowledge.

- Show students 5 counters on top of an inverted opaque container. Tell students, I have 5 counters in all. How many counters are hidden under the container?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Use ten-frames to construct addition and subtraction situations for the students that require them to use a blank ten-frame to complete the operation. How many counters will I need to add to make the two sides equal?



- Give students a prepared number line with a start point (e.g., 24). Ask students to show the hops on the number line as you dictate to them. For example, add 2, subtract 0. Where are you now? (e.g., 26) Explain your thinking.
- Ask students to create a word problem with 0 and 36 in it.
- Tell students, Billy said that he started with 16 and added a number to it. His sum was 16. Ask, What number did Billy add to 16 to get a sum of 16?

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- No Checkpoint for this outcome.

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Develop story problems involving zero to help students understand zero in addition and subtraction.
- Model the addition and subtraction of zero frequently to develop student's understanding that addition does not always increase the quantity and subtraction does not always decrease the quantity.
- Role-play tasks where students receive / give away zero.

SUGGESTED LEARNING TASKS

- Use a horizontal number line, hundred chart or a large floor chart with numbers 1 to 100. Ask a student volunteer to step on 57. Tell him or her to take zero steps forward. Tell him or her to take zero steps backward. Discuss the movements made by the student volunteer.
- Read children's literature such as, *A Place for Zero: A Math Adventure* by Angeline Sparagna LoPresti and Phyllis Hornug (2003) and discuss with students.
- Show students a number for a few seconds using dot patterns or your fingers. Tell students that a number has been added. Flash the sum (same as original value) and ask students to name that addend (0). This may also be done with ten-frames.
- Tell students, After I subtracted zero I have this many (illustrate value with dot cards, fingers, etc.). How many did I start with?
- Use a pan balance with equal amounts on each side and have the students explore what needs to be added or subtracted to keep the balance.
- Have students predict the answer when zero is added to a number. Use the constant function on a calculator and add zero several times to a number to show the constancy of the number. Repeat with different numbers. Repeat using the subtract function. Discuss the pattern that results.

SUGGESTED MODELS AND MANIPULATIVES

- calculator
- dot cards
- number lines
- pan balance
- ten-frames

MATHEMATICAL LANGUAGE

Teacher	Student
▪ adding, subtracting zero ▪ difference, take away ▪ no-change nature ▪ zero	▪ adding, subtracting zero ▪ difference, take away ▪ zero

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), p. 110
- *Making Math Meaningful to Canadian Students, K–8,* Second Edition (Small 2013), p. 166
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 75–77

Notes

SCO N09 Students will be expected to demonstrate an understanding of addition (limited to 1- and 2-digit numerals) with answers to 100 and the corresponding subtraction by

- using personal strategies for adding and subtracting with and without the support of manipulatives
- creating and solving problems that involve addition and subtraction
- explaining and demonstrating that the order in which numbers are added does not affect the sum
- explaining and demonstrating that the order in which numbers are subtracted matters when finding a difference

[C, CN, ME, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N09.01 Solve a given story problem of any type by modelling it with materials or a diagram, and write a number sentence that represents the thinking in the solution.

N09.02 Solve a given story problem of any type by writing a number expression and combining the numbers to complete the number sentences.

N09.03 Match a number sentence to a given story problem.

N09.04 Create an addition or a subtraction number sentence and a story problem for a given solution.

N09.05 Model addition and subtraction using concrete materials or visual representations and record the process symbolically.

N09.06 Add a given set of numbers in two different ways and explain why the sum is the same.

N09.07 Recognize and create equivalent addition and subtraction number sentences.

Scope and Sequence

Mathematics 1	Mathematics 2	Mathematics 3
N09 Students will be expected to demonstrate an understanding of the addition of two single-digit numbers and the corresponding subtraction, concretely, pictorially, and symbolically in join, separate, equalize/compare, and part-part-whole situations.	**N09** Students will be expected to demonstrate an understanding of addition (limited to 1- and 2-digit numerals) with answers to 100 and the corresponding subtraction by ▪ using personal strategies for adding and subtracting with and without the support of manipulatives ▪ creating and solving problems that involve addition and subtraction ▪ explaining and demonstrating that the order in which numbers are added does not affect the sum ▪ explaining and demonstrating that the order in which numbers are subtracted matters when finding a difference	**N06** Students will be expected to describe and apply mental mathematics strategies for adding two 2-digit numerals. **N07** Students will be expected to describe and apply mental mathematics strategies for subtracting two 2-digit numerals. **N08** Students will be expected to apply estimation strategies to predict sums and differences of 1-,2-, and 3-digit numerals in a problem-solving context. **N09** Students will be expected to demonstrate an understanding of addition and subtraction of numbers (limited to 1-, 2- and 3-digit numerals) with answers to 1000 by ▪ using personal strategies for adding and subtracting with and without the support of manipulatives ▪ creating and solving problems in context that involve addition and subtraction of numbers, concretely, pictorially, and symbolically

Background

Students develop an understanding of addition and subtraction of numbers by modelling, acting out, building, drawing, and using appropriate mathematics language in creating number sentences and solving number stories. Through these experiences, students will also begin to develop personal strategies for adding and subtracting. New vocabulary words can be added to the mathematics word wall to reinforce the use of terminology, such as **together**, **part**, **sum**, **difference**, **add**, **subtract**, and **take away**. Addition and subtraction should be taught simultaneously to enable students to see the relationship between the two operations.

Students should have experience with the many different types of addition and subtraction problems. The meanings of, and the relationship between, addition and subtraction are developed using situations (see chart below) that are first modelled concretely, then pictorially, and lastly, symbolically.

Join			Part-Part-Whole	Compare
Result Unknown	**Change Unknown**	**Start Unknown**	**Whole Unknown**	**Difference Unknown**
Pat has 8 marbles. Her brother gives her 4. How many does she have now? 8 + 4 = ?	Pat has 8 marbles but she would like to have 12. How many more does she need to get? 8 + ? = 12 or 12 – 8 = ?	Pat has some marbles. Her brother gave her 4 and now she has 12. How many did she have to start ? + 4 = 12 or 12 – 4 = ?	Pat has 8 blue marbles and 4 green marbles. How many does she have in all? 8 + 4 = ?	Pat has 8 blue marbles and 4 green marbles. How many more blue marbles does she have? 8 – 4 = ? or 4 + ? = 8
Separate			**Part-Part-Whole**	**Compare**
Result Unknown	**Change Unknown**	**Start Unknown**	**Part Unknown**	**Smaller or Larger Unknown**
Pat has 12 marbles. She gives her brother 4 of them. How many does she have left? 12 – 4 = ?	Pat has 12 marbles. She gives her brother some. Now she has 8. How many marbles did she give to her brother? 12 – ? = 8 or 12 – 8 = ?	Pat has some marbles. She gives her brother 4 of them. Now she has 8. How many marbles did she have to start? ? – 4 = 8 or 8 + 4 = ?	Pat has 12 marbles. Eight are blue and the rest are green. How many are green? 8 + ? = 12 or 12 – 8 = ?	Pat has 8 blue marbles and some green marbles. She has 4 more blue marbles than green ones. How many green marbles does she have? 8 – 4 = ? or ? + 4 = 8

The strategies and symbols that students use should reflect how they thought about the problem. For example, consider the story problem: Pat has 12 marbles. She gives her brother some. Now she has 8. How many marbles did she give to her brother? This problem represents a separate situation and could be solved by some students by starting with 12 counters, counting back as counters are removed until they reach 8, and counting the counters that were removed to get 4. These students are likely to write 12 – 4 = 8 to represent what they did. However, some students could start with 12 counters, remove the 8 that they knew were left, and count the remaining counters to get 4. These students are likely to write 12 – 8 = 4 to represent what they did. Whichever number sentence is used, it is essential that students understand how each part of the number sentence relates to the situation. This story problem illustrates that one situation can be represented in different ways symbolically.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Present students with a pictorial representation of a story problem involving two single-digit numbers. Ask them to tell a story that matches the picture. Ask them to solve the problem and to record a number sentence that matches the story problem and solution.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Give students an addition or subtraction sentence. Ask them to model the number sentence using concrete materials or pictures.
- Use base-ten blocks to model an addition or subtraction story problem. Ask students to record a number sentence that would match the base-ten model.
- Ask students to write the addition sentence that would help them solve the following:
  ? = 16 – 8
  18 – 9 = ?
  50 – ? = 20
- Ask students to add the following equations two different ways and explain why the sum is the same regardless of the order that they used.
  65 + 28 = ?
  7 + 4 + 3 + 6 = ?
- Ask students to solve problems, such as the following, and to record their work with pictures and numbers.
  - My dad made 43 chocolate chip cookies and some peanut butter cookies. There were 92 cookies on the cupboard. How many were peanut butter? Solve and explain your thinking.
  - My mom used 28 nails to make a birdhouse. There are 55 nails left in the box. How many nails were in the box before she started? Solve and explain your thinking.
- Tell students that the answer to a problem is 31 balloons. Ask students to make up a story problem and give the number sentence that matches this answer.
- Give students an addition and/or subtraction number sentence and ask them to show different strategies to solve it. Encourage them to show as many different ways as they can.
- Ask students to add a given set of numbers in two different ways and to explain why the sum is the same. For example, if asked to add 2 + 5 + 3 + 8, a student could add 2 + 3 + 5 + 8 or 8 + 2 + 5 + 3.
- Ask students to explain using models or pictures whether they would get the same answer for 9 – 3 and for 3 – 9.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2009)
- Grade 3 Checkpoint 3, pp. 25–27
- Grade 3 Checkpoint 5, pp. 33–34 (Line Masters 5.1 and 5.2)
- Grade 3 Checkpoint 6, pp. 36–37 (Line Master 6.1)

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

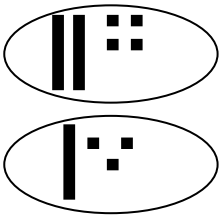
Consider the following strategies when planning daily lessons.

- Have students explore the fact that a story problem could be represented with an addition or a subtraction number sentence. For example: 2 + ? = 7 describes the same situation as 7 – 2 = ? Either of these is acceptable.
- Continue to use models and other representations. Encourage students to match various representations of the same problem.
- Write number sentences horizontally to encourage more divergent thinking and development of personal strategies. Students using personal strategies find solving problems with numbers requiring "regrouping" as easy as those problems that do not require "regrouping."
- Encourage students to create and solve the four different types of addition and subtraction problems: join, separate, part-part-whole, and compare (Van de Walle and Lovin, 2006, 67–69).

SUGGESTED LEARNING TASKS

- Tell students that Janet read 18 books and Fred read 42. Ask them to record a number sentence and then explain how to find the difference using an open number line (or other representation).
- Tell students that someone told you that you do not have to learn to subtract if you know how to add. Ask, Do you agree? Why or why not?
- Provide students with a variety of models and ask how they would add 42 + 29. Have students explore different ways to find the sum. Have students explore additional examples that require regrouping.
- Have students explore different ways to find the difference between 22 and 6.
- Have students make a booklet and on the front cover write, "The Answer is 25." Have students create their own addition and subtraction word problems that would result in an answer of 25. They

should record one problem on each page of their booklet. Students could illustrate each page to match the story problem.
- Ask students what the difference is between 6 and 12. Gradually extend the activity to use 2-digit numbers. Number lines are a good model to support students' thinking.
- Let students choose a favourite story and create addition and subtraction problems related to the story. These could be shared through dramatizations, pictures, or writing.
- Show two numbers modelled with base-ten blocks. After showing the students one pre-grouped model (top model) and the other pre-grouped model (bottom model), ask the students to record addition and/or subtraction sentences these represent and to explain their thinking (e.g., 37 – 23 = 14).



SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- hundred charts
- number lines
- open number lines
- ten-frames

MATHEMATICAL LANGUAGE

Teacher	Student
▪ add, together, plus, sum, total ▪ compatible numbers ▪ how many more, how many less ▪ number sentence ▪ ones, tens ▪ order and grouping addends ▪ part, whole ▪ story problem, situation ▪ strategy ▪ strip diagram ▪ subtract, minus, difference, take away	▪ add, together, plus, sum, total ▪ friendly numbers ▪ how many more, how many less ▪ number sentence ▪ ones, tens ▪ order and grouping ▪ part, whole ▪ story problem ▪ strip diagram ▪ subtract, minus, difference, take away

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 104–110, 162–173
- *Making Math Meaningful to Canadian Students, K–8,* Second Edition (Small 2013), pp. 159–166, 215–217
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 65–75, 157–172

Videos

- *Comparing Mental Strategies: Addition* (14:42 min.) (ORIGO Education 2010)
- *Questions for Developing Mental Computation Strategies* (13:42 min.) (ORIGO Education 2010)
- *Using a Hands-On Approach to Develop Mental Strategies for Addition* (11:04 min.) (ORIGO Education 2010)
- *Using a Hands-On Approach to Develop Mental Strategies for Subtraction* (6:45 min.) (ORIGO Education 2010)
- *Using Language Stages to Develop Addition Concepts* (15:38 min.) (ORIGO Education)
- *Using Language Stages to Develop Subtraction Concepts* (18:32 min.) (ORIGO Education 2010)
- *Using Mental Strategies to Add* (26:15 min.) (ORIGO Education 2010)
- *Using Static Problems to Relate Addition and Subtraction and Introduce Equality* (13:25 min.) (ORIGO Education 2010)
- *Using Static Problems to Relate Addition and Subtractions and Introduce Functions* (18:59 min.) (ORIGO Education 2010)

Notes

SCO N10 Students will be expected to apply mental mathematics strategies to quickly recall basic addition facts to 18 and determine related subtraction facts. [C, CN, ME, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N10.01 Explain the mental mathematics strategy that could be used to determine basic addition facts.
- Doubles Facts
- Plus-One Facts
- One-Apart (Near Doubles) Facts
- Plus-Two Facts
- Plus-Zero Facts
- Make-10 Facts
- Two-Apart Facts
- Plus-Three Facts

N10.02 Use and describe a personal strategy for determining a sum to 18.
N10.03 Quickly recall basic addition facts to 18 in a variety of contexts.
N10.04 Explain the think-addition strategy used to determine a basic subtraction fact.
N10.05 Use and describe a personal strategy for determining the subtraction facts.

Scope and Sequence

Mathematics 1	**Mathematics 2**	**Mathematics 3**
N10 Students will be expected to use and describe strategies to determine sums and differences using manipulatives and visual aids. Strategies include ▪ counting on or counting back ▪ one more or one less ▪ making ten ▪ doubles ▪ near doubles	**N10** Students will be expected to apply mental mathematics strategies to quickly recall basic addition facts to 18 and determine related subtraction facts.	**N10** Students will be expected to apply mental mathematics strategies and number properties to develop quick recall of basic addition facts to 18 and related basic subtraction facts.

Background

The development of mental mathematics needs to be a major goal of any mathematics program for two major reasons. First, in their day-to-day tasks, most people's computational, measurement, and spatial needs can be met by having well-developed mental mathematics strategies. Secondly, because technology has replaced paper-and-pencil as the major tool for complex tasks, people need to have well-developed mental mathematics strategies to be alert to the reasonableness of the results generated by this technology.

Fact learning refers to the acquisition of the 100 number facts related to the single digits 0 to 9 for each of the four operations. When students know these facts, they can quickly retrieve them from memory (usually in 3 seconds or less). Ideally, through practice, over time students will achieve automaticity; that is, they will have instant recall without using strategies.

Initially, students develop and use strategies to get quick recall of the facts. These strategies and the facts themselves are the foundations for the development of other mental mathematics strategies. When the facts are automatic, students are no longer employing strategies to retrieve them from memory. In turn, the facts and mental mathematics strategies are the foundations for computational estimation strategies. Actually, attempts at computational estimation are often thwarted by the lack of knowledge of the related facts and mental mathematics strategies.

In general, a strategy should be introduced in isolation from other strategies, a variety of different reinforcement tasks should be provided until students have become proficient, the strategy should be assessed in a variety of ways, and then it should be combined with other previously learned strategies.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to explain what strategies they might use to solve each of the following:
  7 + 1 5 – 1 7 + 2 8 – 2 8 + 9 8 – 4 6 + 4 6 – 4 6 + 7 5 – 1 9 + 6

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to describe in as many ways as possible why 15 – 8 = 7.
- Ask, What other facts could 4 + 4 = 8 help you know?
- Ask students to list three other facts (addition or subtraction) that would be easier to remember if they know that 6 + 5 = 11.

- Have students list all the subtraction questions they can for which both numbers are less than ten and their difference is 3 (or other similar problems).
- Ask the student to tell why □ + 5 has to be 2 greater than □ + 3. (This assumes that the □ represents the same number in both expressions.)
- Ask students to explain how they could find the sum of 5 + 5 or any other addition fact.
- Give students a set of addition and subtraction fact cards and a set of strategy cards. Ask students to sort the fact cards under the strategy headings. Ask students to explain how they would use the strategy to arrive at the answer.
- Record observations of student's explanations of their strategies that they use daily to solve computational problems. This can also be done through individual interviews, which can provide insights into a student's thinking and help identify groups of students that can all benefit from the same kind of instruction and practice.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)
- Grade 2 Checkpoint 6, pp. 68–69 (Line Masters 6.1 and 6.2)

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ask students to use as many representations as possible to determine sums and differences, including dramatization, drawing pictures, verbally explaining their ideas, using concrete materials, and writing number sentences.
- Use ten-frames to develop the concept of bridging through 10 (adding and subtracting).

- Facilitate the learning of addition and subtraction facts by having students solve problems with familiar contexts. Encourage students to create their own problems.
- Encourage ongoing discussion and exploration of the most efficient ways to add and subtract numbers. The discussion should focus more on how students obtained their answer rather than the actual sum or difference. The strategies will vary depending on the problem and individual students.
- Use "missing part" (e.g., 6 + □ = 8) and "join type" problems to develop the connection between addition and subtraction (e.g., Sally had some marbles. Sonya gave her 5 more. Now she has 11 marbles.).
- Provide opportunities for strategy practice, using games and meaningful context as much as possible, rather than only requiring the memorization of facts in isolation. Board games in which students are required to find the sum of two number cubes to determine how far to move are good examples.
- Provide students with time to learn basic facts, so they understand the operation and can invent their strategies rather than memorizing.

SUGGESTED LEARNING TASKS

- Use ten-frames to develop number relationships. For example, to solve 9 + 4, students can use the ten-frame to see that 9 is 1 less than 10, and they can take 1 from the 4 to fill the ten-frame. They can then see that the sum is 10 and 3 more or 13.
- Use dot cards and dominoes to practise addition facts. For example, show a dot card representing "4" and ask the students to say the ten fact that goes with it (4 + 6 = 10).
- Use multiple representations of numbers (ten-frames, dot cards, etc.) to reinforce number relationships.
- Have students work in pairs to sort addition facts into groups of facts that are related. Give them time to share with others and to explain their sorting based upon the strategy used to solve them.

SUGGESTED MODELS AND MANIPULATIVES

- addition chart
- counters
- dominoes
- dot cards
- linking cubes
- number cubes
- ten-frames
- visual cue cards to show doubles

MATHEMATICAL LANGUAGE

Teacher	Student
▪ 1-apart facts—near doubles	▪ 1-apart, near doubles
▪ 2-apart facts—smaller, larger	▪ 2-apart, smaller, larger
▪ addition facts, subtraction facts	▪ facts
▪ double facts—double	▪ double
▪ make-10 facts	
▪ make-10 facts with a 7	▪ make-10
▪ mental mathematics strategy	
▪ plus one facts—plus one, next number, number after	▪ plus one, next number, number after
▪ plus three facts	
▪ plus two facts—skip count by 2, next even number, next odd number	▪ skip count by 2, next even number, next odd number
▪ plus zero facts	▪ plus zero
▪ think addition	▪ think addition

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 110–115
- *Making Math Meaningful to Canadian Students, K–8,* Second Edition (Small 2013), pp. 166–170
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 94–110

Videos

- *An Introduction to Teaching Addition Number Facts* (15:51 min.) (ORIGO Education 2010)
- *Comparing Mental Strategies: Addition* (14:42 min.) (ORIGO Education 2010)
- *Questions for Developing Mental Computation Strategies* (13:42 min.) (ORIGO Education 2010)
- *Teaching the Bridge-to-10 Strategy for Addition Number Facts* (17:11 min.) (ORIGO Education 2010)
- *Teaching the Count-on Strategy for Addition Number Facts* (17:49 min.) (ORIGO Education 2010)
- *Teaching the Think-Addition Subtraction Fact Strategy* (13:41 min.) (ORIGO Education 2010)
- *Teaching the Use-Doubles Strategy for Addition Number Facts* (14:20 min.) (ORIGO Education 2010)
- *Using a Hands-On Approach to Develop Mental Strategies for Addition* (11:04 min.) (ORIGO Education 2010)
- *Using a Hands-On Approach to Develop Mental Strategies for Subtraction* (6:45 min.) (ORIGO Education 2010)
- *Using Language Stages to Develop Addition Concepts* (15:38 min.) (ORIGO Education)
- *Using Language Stages to Develop Subtraction Concepts* (18:32 min.) (ORIGO Education 2010)
- *Using Mental Strategies to Add* (26:15 min.) (ORIGO Education 2010)

Notes

Patterns and Relations

GCO: Students will be expected to use patterns to describe the world and solve problems.

GCO: Students will be expected to represent algebraic expressions in multiple ways.

Specific Curriculum Outcomes

Process Standards

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

PR01 Students will be expected to demonstrate an understanding of repeating patterns (three to five elements) by describing, extending, comparing, and creating patterns using manipulatives, diagrams, sounds, and actions. [C, CN, PS, R, V]

PR02 Students will be expected to demonstrate an understanding of increasing patterns by describing, extending, and creating numerical patterns (numbers to 100) and non-numerical patterns using manipulatives, diagrams, sounds, and actions. [C, CN, PS, R, V]

PR03 Students will be expected to demonstrate and explain the meaning of equality and inequality by using manipulatives and diagrams (0 to 100). [C, CN, R, V]

PR04 Students will be expected to record equalities and inequalities symbolically, using the equal symbol or not equal symbol. [C, CN, R, V]

SCO PR01 Students will be expected to demonstrate an understanding of repeating patterns (three to five elements) by describing, extending, comparing, and creating patterns using manipulatives, diagrams, sounds, and actions. [C, CN, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR01.01 Identify the core of a given repeating pattern.
PR01.02 Describe and extend a given double attribute pattern.
PR01.03 Create a repeating non-numerical pattern and explain the rule.
PR01.04 Predict an element of a given repeating pattern using a variety of strategies and extend the pattern up to the tenth element to verify the prediction.
PR01.05 Translate a repeating pattern from one mode to another.
PR01.06 Compare two given repeating patterns, and describe how they are alike/different.

Scope and Sequence

Mathematics 1	**Mathematics 2**	**Mathematics 3**
PR01 Students will be expected to demonstrate an understanding of repeating patterns (two to four elements) by describing, reproducing, extending, and creating patterns using manipulatives, diagrams, sounds, and actions. **PR02** Students will be expected to translate repeating patterns from one representation to another.	**PR01** Students will be expected to demonstrate an understanding of repeating patterns (three to five elements) by describing, extending, comparing, and creating patterns using manipulatives, diagrams, sounds, and actions.	**PR01** Students will be expected to demonstrate an understanding of increasing patterns by describing, extending, comparing, and creating numerical (numbers to 1000) patterns and non-numerical patterns using manipulatives, diagrams, sounds, and actions. **PR02** Students will be expected to demonstrate an understanding of decreasing patterns by describing, extending, comparing, and creating numerical patterns (numbers to 1000) and non-numerical patterns using manipulatives, diagrams, sounds, and actions.

Background

In Mathematics 1, students examined repeating patterns with a core of two to four elements. This patterning concept is essential to help students understand repeating patterns as they continue to study patterning with a core of up to five elements and work with double attributes in Mathematics 2. In Mathematics 1, students also had experiences translating patterns to other modes, such as letters. For example, the pattern red, blue, green, red, blue, green, red, blue, green could be also written as ABCABCABC. This led them to the convention of describing repeating patterns using letters to describe the elements in a core, such as an AB pattern, an AAB pattern, or an ABC pattern. Students in Mathematics 2 will continue using these letters to describe repeating patterns and extend them to cores of five elements.

The foundation of algebraic thinking is investigating patterns and their representations. Contextual, open-ended, and situation-specific problem solving should be an integral part of everyday mathematics instruction, tasks, and assessment. Patterns could be integrated in physical education, music, visual arts, science and other subject areas to provide a context. Students need to recognize and extend many different forms of the same pattern, including those constructed or in their environment.

Students should be able to identify the core of the repeating pattern, and predict elements in repeating patterns. These patterns may be found in a variety of contexts, such as a hundred chart, calendars, number lines, and tiling patterns. Students' predictions should be verified by extending the pattern concretely, pictorially, and symbolically. Students should describe, extend, compare, and create

- sound patterns, such as clap, snap, tap, clap, snap, tap, clap, snap, tap, …
- action patterns, such as sit, sit, stand, hop, sit, sit, stand, hop, sit, sit, stand, hop, …
- concrete and pictorial shape patterns, such as

…

- concrete and pictorial size patterns, such as large, small, small, large, small, small, large, small, small, ...

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

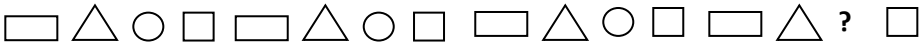
Tasks such as the following could be used to determine students' prior knowledge.

- Show students a repeating pattern that you have begun. Ask them to continue the pattern in two different ways and explain the different pattern rules they used to continue the pattern.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to use three colours of tiles to create a repeating pattern.
- Ask students to create a repeating pattern that has a core of three elements. Ask them to describe the pattern.
- Present students with a repeating pattern. Ask them to identify the core of the pattern.
- Present students with a repeating pattern. Ask them to describe the rule used to create that pattern.
- Ask students to create a repeating pattern where a specific element is to be identified (e.g., Create a pattern in which the 4th element is green.).
- Present the following pattern: –|||–|||–||| Ask students to translate this pattern into a different mode (sounds, shapes, etc.).
- Ask students to determine the missing element in a given repeating pattern. For example,



- Present students with a repeating pattern. Ask them to predict the position of a specific element. Ask them to extend the pattern to verify their answer.
- Show students three patterns, two of which have the same repeating pattern represented using different elements. Ask students to look at the patterns, and to explain which of the patterns are alike. Then, ask students to choose the pattern that is different from the others, and to represent it using actions.
- Present a double attribute pattern to students. Ask them to describe the pattern and identify the core. Ask them to extend the pattern (e.g., use a double attribute pattern involving colour and size).

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Grade 2 Checkpoint 7, Task 1, pp. 76–77

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?

- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Include concrete materials in patterning tasks.
- Integrate patterns in physical education, music, visual arts, science, and other subject areas to provide a context.
- Encourage students to see that patterns may continue in many different ways unless a pattern rule is defined.
- Provide opportunities for students to make predictions about patterns using a variety of materials.
- Use concrete materials for students to build double attribute patterns that can be described in more than one way.
- Expect students to communicate their thinking about patterns, verbally and in writing.

SUGGESTED LEARNING TASKS

- Provide the start of a pattern. Ask students to continue the pattern in more than one way and to describe the pattern rule in each case.
- Provide students with a repeating pattern to extend. Before students begin to extend the pattern, ask them to predict exactly what element will be in a specific position (e.g., What would be in the sixth position?). Have students provide a reason for their prediction before extending their pattern to check their prediction. If their prediction is incorrect, have them examine their reasoning and try to figure out why the prediction was not accurate.
- Ask students to describe and extend a given double attribute pattern, such as

- Ask the students to create two patterns that are similar, but not exactly the same. Ask them to comment on the similarities and differences.
- Tell students that the first two elements in a pattern are a red square and a blue triangle. Ask for several different ways in which the pattern might be continued.
- Tell students that another child continued the pattern blue square, red triangle, blue circle, red square, … , by saying “blue triangle, red circle.” Ask, Do you think that the other child is incorrect or is there a rule that might explain continuing the pattern in this way?

SUGGESTED MODELS AND MANIPULATIVES

- addition chart
- attribute blocks
- base-ten blocks
- calendar
- colour tiles
- hundred chart
- number line
- pattern blocks

MATHEMATICAL LANGUAGE

Teacher	Student
▪ core, element ▪ describe, extend, compare, create, predict ▪ mode ▪ pattern rule ▪ repeating pattern	▪ core, element ▪ describe, extend, compare, create, predict ▪ pattern rule ▪ repeating pattern

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 568–570, 571–576
- *Making Math Meaningful to Canadian Students, K–8,* Second Edition (Small 2013), pp. 605–612
- *Teaching Student-Centered Mathematic, Grades K–3* (Van de Walle and Lovin 2006), pp. 276–280, 313–315

Notes

SCO PR02 Students will be expected to demonstrate an understanding of increasing patterns by describing, extending, and creating numerical patterns (numbers to 100) and non-numerical patterns using manipulatives, diagrams, sounds, and actions. [C, CN, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR02.01 Identify and describe increasing patterns in a variety of given contexts.
PR02.02 Represent a given increasing pattern concretely and pictorially.
PR02.03 Identify errors in a given increasing pattern.
PR02.04 Explain the rule used to create a given increasing pattern.
PR02.05 Create an increasing pattern and explain the pattern rule.
PR02.06 Represent a given increasing pattern using another mode.
PR02.07 Solve a given problem using increasing patterns.
PR02.08 Identify and describe increasing patterns in the environment.
PR02.09 Determine missing terms in a given concrete, pictorial, or symbolic increasing pattern and explain the reasoning.

Scope and Sequence

Mathematics 1	**Mathematics 2**	**Mathematics 3**
—	**PR02** Students will be expected to demonstrate an understanding of increasing patterns by describing, extending, and creating numerical patterns (numbers to 100) and non-numerical patterns using manipulatives, diagrams, sounds, and actions.	**PR01** Students will be expected to demonstrate an understanding of increasing patterns by describing, extending, comparing, and creating numerical patterns (numbers to 1000) and non-numerical patterns using manipulatives, diagrams, sounds, and actions. **PR02** Students will be expected to demonstrate an understanding of decreasing patterns by describing, extending, comparing, and creating numerical patterns (numbers to 1000) and non-numerical patterns using manipulatives, diagrams, sounds, and actions.

Background

Increasing patterns is a new concept for students in Mathematics 2. An increasing pattern is a growing pattern where each term is increasing by a specific quantity. The following pattern shows each term increasing by one zero: 0, 00, 000, 0000, …

Through examples and discussions, students should understand how an increasing pattern is different from a repeating pattern. It should be noted that any growing pattern involving numbers should not exceed the number 100.

The foundation of algebraic thinking is investigating patterns and their representations. Contextual, open-ended, and situation-specific problem solving should be an integral part of everyday mathematics instruction, tasks, and assessment. Students need to recognize and extend many different forms of the same pattern, including those constructed or in their environment. They must be able to predict a term in increasing patterns using many strategies within a variety of contexts, such as a hundred chart, calendars, number lines, and tiling patterns. These predictions should be verified by extending the pattern concretely, pictorially, and symbolically.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

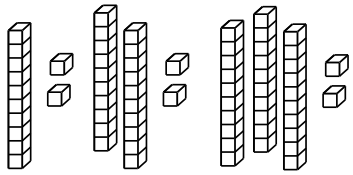
- Present students with two patterns. One should be a repeating pattern and one should be an increasing pattern. Ask them to point to the repeating pattern and explain the pattern rule. Ask them to tell why the other pattern is not a repeating pattern.

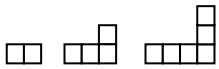
WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to use tiles to create an increasing pattern. Ask them to explain their pattern rule.
- Ask students to identify an increasing pattern they see in the world and to describe that pattern.
- Ask students to create an increasing pattern in which a specific element is identified (e.g., the 5th element is 50).
- Present students with an increasing pattern and ask them to represent it in a different mode.

- Present students with the following base-ten blocks.



- Ask students to identify what would be next in the pattern and explain how they know.
- Ask students to use a number line to show an increasing pattern. Ask them to describe the pattern.
- Present an increasing pattern in which there is an error. Ask students to identify and correct the error, and explain their thinking.
- Present the following increasing pattern:  Ask students to identify what would come next in the pattern and to explain the pattern rule.
- Ask students to create an increasing pattern that begins 1, 2, 3, … Ask them to explain how they developed their patterns.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets 3 (Bauman 2009)
- Grade 3 Checkpoint 10, Task 2, pp. 50–51

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

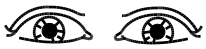
Consider the following strategies when planning daily lessons.

- Include concrete materials in patterning tasks.
- Integrate patterns in physical education, music, visual arts, science, and other subject areas to provide a context.
- Encourage students to see that patterns may continue in many different ways unless a pattern rule is defined. For example, a pattern that begins 1, 2, 3, … might continue in the following ways: 1, 2, 3, 1, 2, 3, 1, 2, 3, … (repeating pattern) or 1, 2, 3, 4, 5, 6, 7, ... (increasing pattern).
- Provide opportunities for students to make predictions about patterns using a variety of materials.
- Expect students to communicate their thinking about patterns, verbally and in writing.

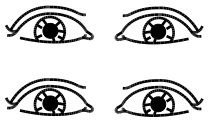
SUGGESTED LEARNING TASKS

- Provide students with a pattern rule for an increasing pattern (e.g., each number is two more than the one before). Ask students to build the pattern with concrete materials.
- Ask students to work in small groups to identify and describe increasing patterns that they see in the hundred chart or the calendar. Have each group present their discoveries to the class.
- Ask students to represent skip counting by 2s by showing jumps on a number line. Ask students to describe the increasing pattern that results.
- Provide the start of an increasing pattern using base-ten blocks. Ask students to continue the pattern and explain the pattern rule.
- Ask students to create two different increasing patterns with concrete materials that are similar, but not exactly the same. Ask them to comment on the similarities and differences.
- Tell students that the first two numbers in an increasing pattern are 5 and 10. Ask them to describe how the pattern could continue.
- Show students an increasing pattern such as 1, 4, 7, 10, … Ask them to recreate the pattern with pictures of counters using a computer drawing program and to continue the pattern.
- Show students the following increasing pattern: 2, 12, 22, 32, 42, 52, … Ask students to describe the pattern and to model it with base-10 blocks.
- Tell students that they are going to explore increasing patterns using the people in the classroom. Ask, “How many eyes are there in our classroom? How can we use increasing patterns to find the answer?” Model the process using digital pictures of students’ eyes and then translate them to numbers. For example,

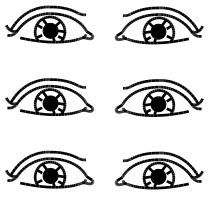
1 Student



2 Students



3 Students



Number of Students	1	2	3	4
Number of Eyes	2	4	6	8

SUGGESTED MODELS AND MANIPULATIVES

- base-ten blocks
- calendar
- colour tiles
- counters
- hundred chart
- number lines

MATHEMATICAL LANGUAGE

Teacher	Student
▪ describe, reproduce, extend, create, predict ▪ growing pattern ▪ increases, grows ▪ increasing pattern ▪ mode ▪ pattern rule ▪ start number, amount of increase ▪ term	▪ describe, reproduce, extend, create, predict ▪ growing pattern ▪ increases, grows ▪ increasing pattern ▪ pattern rule ▪ start number, amount of increase ▪ term

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 570–577
- *Making Math Meaningful to Canadian Students, K–8,* Second Edition (Small 2013), pp. 608–609, 613–614
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 281–282, 284–288

Notes

SCO PR03 Students will be expected to demonstrate and explain the meaning of equality and inequality by using manipulatives and diagrams (0 to 100). [C, CN, R, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR03.01 Determine whether two given quantities of the same object (same shape and mass) are equal by using a balance scale.

PR03.02 Construct and draw two unequal sets using the same object (same shape and mass) and explain the reasoning.

PR03.03 Demonstrate how to change two given sets, equal in number, to create inequality.

PR03.04 Choose from three or more given sets the one that does not have a quantity equal to the others and explain why.

Scope and Sequence

Mathematics 1	**Mathematics 2**	**Mathematics 3**
PR03 Students will be expected to describe equality as a balance and inequality as an imbalance, concretely and pictorially (0 to 20).	**PR03** Students will be expected to demonstrate and explain the meaning of equality and inequality by using manipulatives and diagrams (0 to 100).	**PR03** Students will be expected to solve one-step addition and subtraction equations involving symbols representing an unknown number.

Background

Balance tasks form a basis for understanding equality. Working with balance scale problems involving concrete materials and pictures helps students build foundations for further study in operations, algebra, and equation solving. Using concrete materials will help students see the equality or inequality relationship between the quantities on each pan of the balance scale.

Throughout balance tasks, students should answer questions, such as

- What is the relationship between the two quantities if the scale is balanced? (This provides opportunities to use the phrases “is the same as” and “is equal to”.)
- What is the relationship between the quantities if the left balance pan is lower than the right pan? (This provides opportunities to use the phrases “is more than” and “is not equal to,” and to discuss the “is-less-than” relationship between the right and left quantities on the pans.)
- What is the relationship between the quantities if the left balance pan is higher than the right pan? (This provides opportunities to use the phrases “is less than” and “is not equal to,” and to discuss the is-more-than relationship between the right and left quantities on the pans.)

This outcome should be addressed in conjunction with the development of the symbolic representations in outcome PR04.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to create two sets of cubes (each less than 20) that are equal. Ask them to use a pan balance to prove that they are equal. Then, ask them to change the sets so that they are not equal.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Show students three different balance scales with opaque bags of cubes placed on them. The first balance should represent an equality. The second balance should represent an inequality with the left pan lower than the right. The third balance should represent an inequality with the right pan lower than the left. Tell students that the bags contain cubes of the same mass and shape.
  - **Task 1:** Ask students to identify which balance represents equal sets and to explain their thinking. Ask them to tell you how many cubes might be in each bag.
  - **Task 2:** Ask students to identify which balance shows unequal sets. Ask them to explain their thinking. Ask them to tell you how many cubes might be in each of the bags.
  - **Task 3:** If during Task 2 the student identified only one of the two remaining balances as showing unequal sets, ask them to tell you about the remaining balance.
- Use a set of dominoes. Select three dominoes—two that are equal in value and one that is unequal. Ask students to identify the domino that is not equal to the others and to explain how they know.
- Ask students to represent two numbers (each less than 100) on a balance scale using counters. Ask them to tell if the two quantities are equal and to explain how they know.
- Have students build an equal or an unequal relationship using Cuisenaire rods and explain their thinking.
- Show students a pan balance with 25 cubes on one side and 52 cubes on the other side. Ask students to tell if the two sets are equal or not equal. Ask them to explain their thinking. Ask students to explain what to do to make the two sets equal.

- Ask students to use concrete materials and a pan balance to prove that $9 + 5 = 7 + 7$ and that $4 \neq 5 - 3$.
- Ask students to use concrete materials to prove that $35 \neq 20 + 20$.
- Ask students to use base-ten blocks to represent two numbers that are not equal. Ask them to explain their thinking.
- Ask students to use base-ten blocks to represent two numbers that are equal. Ask them to explain their thinking.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)
- No Checkpoint for this outcome.

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Use of the words “the same as” for the equal sign and “not the same as” for the unequal sign will help students see that the symbols represent a relationship. Students should become familiar with the terminology that “inequality” means “is greater than” or “is less than” and “equality” means “the same as.”
- Give students opportunities to develop their own equalities and inequalities on a pan balance.
- Give students many opportunities to construct equal and unequal sets and to translate their findings to diagrams and then to number sentences.

SUGGESTED LEARNING TASKS

- Present students with a pan balance on which you have placed two opaque bags containing quantities of cubes. Ask students to tell which bag contains the greatest number of cubes and to explain how they know. Students should answer questions, such as
  - What is the relationship between the two quantities if the scale is balanced? (This provides opportunities to use the phrases "is the same as" and "is equal to.")
  - What is the relationship between the quantities if the left balance pan is lower than the right pan? (This provides opportunities to use the phrases "is more than" and "is not equal to," and to discuss the "is-less-than relationship" between the right and left quantities on the pans.)
  - What is the relationship between the quantities if the left balance pan is higher than the right pan? (This provides opportunities to use the phrases "is less than" and "is not equal to," and to discuss the "is-more-than" relationship between the right and left quantities on the pans.)
- Use True or False tasks. Present various relationships, such as 8 + 4 is the same as 12 + 5, or 13 = 13, or 16 is the same as 25 – 9. Have students use pan balances to determine whether the statement is true or false and to justify their answers.
- Students work in pairs. One partner uses ten-frames to represent two 2-digit numbers. The other partner must determine whether the ten-frame display represents equal or unequal sets.
- Students work in pairs. Each pair is given a set of cards with numerals to 100 and a set of word cards "is the same as" and "is not the same as." One student draws a numeral card and creates a set of cubes to represent that number. The same student draws a word card and places it to the right of the set of cubes. The second student must create a second set of cubes to make the statement true. Both students record the statement in pictorial and/or symbolic form.
- Ask students to find dominoes that satisfy relationships such as "is the same as" or "is not the same as." For example,

- Have students compare three or more given sets to determine which are equal/not equal and explain their reasoning.
- Ask students to write a problem that involves the number 12 as shown below in the number sentence (the blanks are numbers): ___ + 12 = ___ and ___ – 12 = ___.

SUGGESTED MODELS AND MANIPULATIVES

- Cuisenaire rods
- dominoes
- dot cards
- five-frames and ten-frames
- pan balance

MATHEMATICAL LANGUAGE

Teacher	Student
▪ balance ▪ balance scale ▪ equal sets, unequal sets ▪ equality, inequality ▪ is more than, is less than ▪ is not the same as, is not equal to ▪ is the same as, is equal to	▪ balance ▪ balance scale ▪ equal sets, unequal sets ▪ is more than, is less than ▪ is not the same as, is not equal to ▪ is the same as, is equal to

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 585–586
- *Making Math Meaningful to Canadian Students, K–8,* Second Edition (Small 2013), p. 625
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 70, 125, 133–134, 299–301

Notes

SCO PR04 Students will be expected to record equalities and inequalities, symbolically, using the equal symbol or not equal symbol. [C, CN, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR04.01 Determine whether two sides of a given number sentence are equal (=) or not equal (≠). Write the appropriate symbol and justify the answer.
PR04.02 Model equalities using a variety of concrete representations and record the equality.
PR04.03 Model inequalities using a variety of concrete representations and record the inequality.

Scope and Sequence

Mathematics 1	**Mathematics 2**	**Mathematics 3**
PR04 Students will be expected to record equalities using the equal symbol.	**PR04** Students will be expected to record equalities and inequalities, symbolically, using the equal symbol or not equal symbol.	**PR03** Students will be expected to solve one-step addition and subtraction equations involving symbols representing an unknown number.

Background

Students need to understand that the equal sign (=) is a symbol that represents a relationship between quantities and does not mean "the answer is." It tells us the quantity on the left-hand side is the same as the quantity on the right-hand side; that is, the two sides are balanced. Often, when students only have experience using the equal sign in addition and subtraction number sentences, they mistakenly believe the sign (=) means is the answer to and, therefore, do not accept as possible, number sentences such as 8 = 5 + 3, 6 = 6, and 4 + 5 = 2 + 7. However, through balance tasks, students see that such number sentences express equality relationships between quantities on the two sides of a balance scale, or in a one-to-one correspondence. As such, these number sentences are not only possible, but are necessary to express the relationships.

This outcome should be addressed in conjunction with the development of the meaning of equality and inequality addressed in outcome PR03.

The focus of this outcome is to have students interpret the = and ≠ signs. This is the first time students will have experiences with the ≠ sign and it means two quantities are not equal. They must be able to write these symbols to describe the relationship between quantities with or without an operation, such as 72 = 72, 7 ≠ 5, 21 = 18 + 3, and 10 – 6 ≠ 3 + 3. It may be easier balancing one addition or subtraction sentence first, such as 10 + 4 = 14 and 12 – 7 ≠ 7, and then moving to balancing two addition or subtraction sentences, such as 1 + 4 ≠ 7 + 3 and 5 – 2 = 6 – 3. Lastly, students should be ready to deal with addition and subtraction sentences in combination, such as 3 + 9 = 13 – 1.

When the focus is on the number relationships between sides of number sentences, students become more flexible in their thinking and may find more efficient ways to solve problems. Exploring the relationship between the expressions on either side of an equal sign enables students to develop an understanding of relationships that may exist between these expressions. It is important that students discover these relationships on their own. For example, they may recognize that 16 + 18 = 18 + 16 without adding both sides because they realize both sides involve the addition of the same two numbers. They may recognize that 4 + 8 = 3 + 9 without adding both sides because they realize 1 has been subtracted from 4 and added to 8 to create 3 and 9. Such thinking is called relational thinking, and it should be encouraged and explored; however, it will likely not be typical of most students in Mathematics 2. One way to encourage relational thinking is by providing examples that encourage students to use their number sense and to examine the numbers in a computation to determine if it can be simplified. For example, in the sentence 8 + 7 – 7 =___, students may recognize that adding 7 and then subtracting 7 will leave 8 unaffected; therefore, the computation is not necessary to know the solution. Other equations, such as 8 + 4 = __ + 5, can be solved only if students have a broad understanding of the equal sign. Students can solve this sentence using relational thinking by noticing that 5 is 1 more than 4, so the unknown number has to be one less than 8.

Informal work with inequalities and equalities may lead students to discover that there could be more than one possible answer to satisfy a relationship, such as 5 + ? + ? = 9, 6 + 7 ≠ ? + 5.

Note: A common error made by students in solving 8 + 4 = __ + 5 is to place 12 in the blank. These students have not yet internalized that the equal sign means is the same as; rather, they think it is a call for an answer to 8 + 4. Students need to regularly explore four types of equality relationships: whole is equal to whole (6 = 6); whole is equal to part-part (6 = 4 + 2); part-part is equal to whole (4 + 2 = 6), and part-part is equal to part-part (4 + 2 = 5 + 1).

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to fill in the blank (with a whole number or a combination of numbers showing an operation) to complete the number sentence. _____ = 7 + 6 or 4 + 2 = _____ or 5 – 3 = _____ or _____ = 5 – 1

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to build number sentences to demonstrate an understanding of equality using ten-frames. Place a ten-frame on either side of the equal sign and have students place different coloured counters to show more than one combination (e.g., 5 + 5 = 8 + 2 or 10 = 2 + 2 + 6 or 6 + 4 = 7 + 3).
- Complete the number sentence so that it is true:
  5 + 1 = __ + 2; or 4 + __ = 2 + 2 + 2; or __ + 0 = 30 – 1; or 16 + 5 ≠ __ – 7
- Provide students with a number sentence representing equality and ask students to model the number sentence using a pan balance and explain their thinking.
- Have students build an equal or an unequal relationship using Cuisenaire rods and explain their thinking to a partner.
- Provide students with the following expressions:
  3 + 7 6 + 4 9 + 1 8 + 2 4 + 7 5 + 5
  Ask students to identify, using concrete materials, which expression does not have a quantity equal to the others and explain their thinking.
- Ask a student to write number sentences using the equal sign (=) and/or the not equal sign (≠) and then explain their reasoning.
- Ask a student to use concrete materials and pictures to determine whether the following statements are true:
  3 + 2 = 9 – 4 0 ≠ 1 + 0 0 = 8 – 8 7 – 4 ≠ 6 – 3

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- No Checkpoint for this outcome.

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

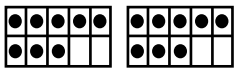
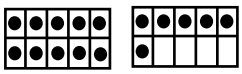
CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Introduce students to a variety of equation types. For example: 6 + 7 = 13 or 68 = 61 + 7 or 8 – 5 = 7 – 4.
- Use of the words "the same as" for the equal sign and "not the same as" for the unequal sign will help students see that the symbols represent a relationship. Students should become familiar with the terminology that **inequality** means "is greater than" or "is less than" and **equality** means "the same as."
- Give students opportunities to develop their own equations and inequalities.
- Give students many opportunities to construct equal and unequal sets and to translate their findings to diagrams and then to number sentences.

SUGGESTED LEARNING TASKS

- Challenge students to find different ways to express a particular number encouraging the use of different operations or relationships. For example: 10 ≠ 10 + 2, 10 = 2 + 8, 10 is less than 24. Include examples with more than two numbers on the same side. For example:
  10 = 4 + 1 + 5 17 – 2 – 5 = 10
  Ask students to use concrete materials to prove that they are correct.
- Provide students with pictures of ten-frames showing equalities and inequalities. Ask students to write number sentences that match the pictures. For example, if shown

 is the same as  ,

  students would record 8 + 8 = 10 + 6.
- Provide each pair of students with a sheet of chart paper and a 2-digit number. Ask each pair to work co-operatively to decide whether they will create representations that are equal to or not equal to their number. Pairs of students then work to create a pictorial display for their number in accordance with the decision they made. After all students have completed their charts, post them. Ask students to read each of the charts and to decide whether the display shows "is equal to" or "is not equal to."
- Use True or False tasks. Present various relationships, such as 8 + 4 = 12 + 5 or 13 = 13 or 16 = 25 – 9. Have students tell whether it is true or false and justify their answers using concrete materials and/or pictures .
- Ask students to write true sentences in several different forms. For example, __ + __ = __ + __ ; or __ – __ = __ – __, or __ + __ = __ – __ , or __ + __ ≠ __ – __, or __ = __ – __, or __ = __ + __.
- Have students compare three or more given sets to determine which are equal/not equal and explain their reasoning.
- Ask students to write a problem that involves the number 12 as shown below in the number sentence (the blanks are numbers): ___ + 12 = ___, ___ + 12 ≠ ___, ___ – 12 = ___, ___ – 12 ≠ ___

SUGGESTED MODELS AND MANIPULATIVES

- Cuisenaire rods
- dominoes
- dot cards
- five-frames and ten-frames
- pan balance

MATHEMATICAL LANGUAGE

Teacher	Student
▪ addition sentence, subtraction sentence ▪ balance ▪ equal sign, not equal sign (= and ≠) ▪ equal, not equal ▪ equality, inequality ▪ is not the same as, is not equal to ▪ is the same as, is equal to ▪ number sentence	▪ balance ▪ equal sign, not equal sign (= and ≠) ▪ equal, not equal ▪ is not the same as, is not equal to ▪ is the same as, is equal to ▪ number sentence

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 585–586
- *Making Math Meaningful to Canadian Students, K–8,* Second Edition (Small 2013), pp. 624, 626–627
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 70, 125, 133–134, 299–301

Notes

Measurement

GCO: Students will be expected to use direct and indirect measure to solve problems.

Specific Curriculum Outcomes: Measurement (M)

Process Standards

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

M01 Students will be expected to demonstrate an understanding of the calendar and the relationships among days, weeks, months, and years. [C, CN, PS, R]

M02 Students will be expected to relate the size of a unit of measure to the number of units (limited to non-standard units) used to measure length and mass. [C, CN, ME, R, V]

M03 Students will be expected to compare and order objects by length, height, distance around, and mass using non-standard units and make statements of comparison. [C, CN, ME, R, V]

M04 Students will be expected to measure length to the nearest non-standard unit by using multiple copies of a unit and using a single copy of a unit (iteration process). [C, ME, R, V]

M05 Students will be expected to demonstrate that changing the position of an object does not alter the measurements of its attributes. [C, R, V]

SCO M01 Students will be expected to demonstrate an understanding of the calendar and the relationships among days, weeks, months, and years. [C, CN, PS, R]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

M01.01 Read a calendar.
M01.02 Name and order the days of the week and months of the year.
M01.03 Communicate the number of days in a week and the number of months in a year.
M01.04 Solve a given problem involving time which is limited to the number of days in a week and the number of months in a year.

Scope and Sequence

Mathematics 1	**Mathematics 2**	**Mathematics 3**
—	**M01** Students will be expected to demonstrate an understanding of the calendar and the relationships among days, weeks, months, and years.	**M01** Students will be expected to relate the passage of time to common activities using non-standard and standard units (minutes, hours, days, weeks, months, years). **M02** Students will be expected to relate the number of seconds to a minute, the number of minutes to an hour, the number of hours to a day, and the number of days to a month in a problem-solving context.

Background

Although calendar work has not been an outcome in previous grades, some students will have some previous knowledge with a calendar either in school or at home. Calendar tasks should be incorporated into daily routines, helping students make connections between their experiences and real-world scenarios. It is important that students in Mathematics 2 be given opportunities to explore and learn different calendar units (days, weeks, and months) and relationships among these units. Each month brings a new calendar page to explore. It is expected that students will communicate their understanding of calendar skills through questioning, games, problem solving, journal writing, and group discussions.

In the real world, the calendar is used to plan, keep track of appointments, and measure time. This is how it should be used in the classroom. In order to focus on the structure of the month and numerical patterns, engage students, at the beginning of the month, in a teacher-directed activity where students build and create their own monthly calendar. They will need to write the month, write the days of the week in order, number the days, and fill in any special dates for that month, such as class trips and holidays. Having a one page, year-long calendar nearby, will help students see and understand where the current month fits into a year's progression.

Additional Information

- See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students, What day of the week is it today?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Point to a date on the calendar. Ask students to read the date. They should tell you the month, the day of the week, and the date (e.g., Tuesday, April 6).
- Sequence a set of events occurring in a familiar story involving days or months.
- Ask students to name the days in order beginning with Sunday.
- Ask students to name the months of the year beginning with January.
- Tell students, Today is Wednesday. Ask, What day was it yesterday? What day will it be tomorrow?
- Tell students, Billy says that 7 days is longer than a week. Ask students to explain whether Billy is correct or not and to explain their thinking.
- Have students solve problems involving time, such as
  a) If Grandma eats an apple every second day for three weeks, how many apples will she eat?
  b) It is April 16th. In two weeks, Bethany is going to the dentist. On what date will Bethany go to the dentist?
  c) You go swimming in the lake in July. What might you be doing at the lake seven months later?
  d) It is the 11th day of the month. If our class is going on a field trip in 16 days, what is the date of our field trip?
- Ask students, How might you use a calendar to help subtract 14 from a number?

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)
- No Checkpoint for this outcome.

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Use a calendar daily to explore days and months.
- Give students problematic situations to work on, such as pointing out the date on the calendar and asking how many days/weeks until a specific holiday or event.
- Provide students with experiences involving comparisons between events of differing durations to help them understand time. For example, How many days in a school week as compared to a calendar week? How many months until your friend's birthday compared to your birthday?
- Use school activity calendars and lunch menus to explore days and months.
- Read children's literature such as *Chicken Soup with Rice: A Book of Months* by Maurice Sendak (1991) to your students. Provide students with a random set of the sequence of months from the poem and have them order the months. This activity could be duplicated with the days of the week using *Today Is Monday* by Eric Carle (2002) or a poem such as "Monday's Child" (Mother Goose nursery rhyme).
- Assign two students each day to the calendar routine. Routines could include, Write today's date and be prepared to give the dates for yesterday and tomorrow; highlight any special events listed for the day or locate today's date on the year-long calendar.

SUGGESTED LEARNING TASKS

- List the days of the week in order along a seven-section number line. Attach the ends of the number line to complete a circle. This unit (one week) demonstrates the cyclical property of how 7 days equals 1 week (unitizing). This activity can be extended to include several weeks. A similar activity can be developed to show the months of the year.
- Show students a calendar for the year. Ask him or her to point out the day's date and to find out what date it will be in 6 weeks.
- Show students a calendar for the year and ask him or her to identify ways in which months are the same and ways in which they differ.
- Have students solve problems where they must determine what day it would be 3 days from now. This activity could be varied by using months instead of days and changing the number.
- Have students prepare a schedule on a blank calendar starting on a particular day/date, for a specific job (e.g., recycling pick up), that is repeated regularly (e.g., biweekly).
- Ask different types of calendar questions such as, Is the date of the third Wednesday an even or an odd number? On which day of the week is the first 2-digit number? On which day of the week is the last 1-digit number? On which day of the week is the 9th? How many full weeks are in this month? Which day of the week is 9 days after the 6th? Find the second week. What is Friday's date that week?

SUGGESTED MODELS AND MANIPULATIVES

- calendar
- number lines

MATHEMATICAL LANGUAGE

Teacher	Student
▪ calendar	▪ calendar
▪ date	▪ date
▪ day, week, month, year	▪ day, week, month, year
▪ days of the week: Sunday, Monday, …	▪ days of the week: Sunday, Monday, …
▪ months of the year: January, February, …	▪ months of the year: January, February, …
▪ next, before	▪ next, before
▪ today's, yesterday's, tomorrow's date	▪ today's, yesterday's, tomorrow's date

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 573–575
- *Making Math Meaningful to Canadian Students, K–8,* Second Edition (Small 2013), pp. 612–613

Notes

SCO M02 Students will be expected to relate the size of a unit of measure to the number of units (limited to non-standard units) used to measure length and mass. [C, CN, ME, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

M02.01 Explain why one of two given non-standard units may be a better choice for measuring the length of an object.

M02.02 Explain why one of two given non-standard units may be a better choice for measuring the mass of an object.

M02.03 Select a non-standard unit for measuring the length or mass of an object and explain why it was chosen.

M02.04 Estimate the number of non-standard units needed for a given measurement task.

M02.05 Explain why the number of units of a measurement will vary depending upon the unit of measure used.

Scope and Sequence

Mathematics 1	**Mathematics 2**	**Mathematics 3**
M01 Students will be expected to demonstrate an understanding of measurement as a process of comparing by ▪ identifying attributes that can be compared ▪ ordering objects ▪ making statements of comparison ▪ filling, covering, or matching	**M02** Students will be expected to relate the size of a unit of measure to the number of units (limited to non-standard units) used to measure length and mass. **M03** Students will be expected to compare and order objects by length, height, distance around, and mass using non-standard units and make statements of comparison.	**M03** Students will be expected to demonstrate an understanding of measuring length (cm, m) by ▪ selecting and justifying referents for the units centimetre and metre (cm, m) ▪ modelling and describing the relationship between the units centimetre and metre (cm, m) ▪ estimating length using referents ▪ measuring and recording length, width, and height **M04** Students will be expected to demonstrate an understanding of measuring mass (g, kg) by ▪ selecting and justifying referents for the units gram and kilogram (g, kg) ▪ modelling and describing the relationship between the units gram and kilogram (g, kg) ▪ estimating mass using referents ▪ measuring and recording mass

Background

In previous grades the measurement outcomes focused on direct comparisons of attributes of objects. Students realized that an object has several attributes that can be compared to other objects or figures. For example, a 3-D object has mass; length, width, and height; volume; and capacity. Any one of these attributes can be directly compared to the same attribute of another object. This can be done by lifting both objects to compare masses; placing objects side-by-side to compare lengths, widths, or heights; filling one object with sand or water and pouring into the other object to compare capacities; and placing objects side-by-side to compare volumes.

In this Mathematics 2 outcome, students will begin to measure the attributes of mass and length using non-standard units, getting numerical measurements that vary according to the size of the unit used. This relationship between the size of the unit and the number of units is an inverse relationship (the larger the non-standard unit, the smaller the number of units or the smaller the non-standard unit, the larger the number of units). While this inverse relationship is obvious to adults, it is often a difficult concept for young students and will need to be carefully developed.

Once the attribute to be measured is understood, a unit of measure can be chosen to measure that attribute. Students should understand that they must use multiple uniform copies of their chosen unit of measure to measure the attribute. For example, if they are going to measure the lengths of their desks using paper clips, all those paper clips have to be the same length. Students should concentrate on the measurement process. For example, to measure the length of their desks using paper clips, they must place the paper clips in a straight line against the length of the desks with no gaps or overlaps. Research has shown that using non-standard units first allows students to concentrate on the attribute of the object being measured and the measurement process, both of which seem to get lost when the focus is on a standard unit of measure. Estimation is a key component to the measurement process and should be developed while students are using non-standard units. Prior to actually measuring an attribute of an object with non-standard units, have students estimate how many of those units they think they will need to use.

Note: When addressing this outcome, note that the terms **mass** and **weight** are similar, but they are not the same. Weight measures how heavy an object is and depends upon gravity, so it will vary with height above sea level. For example, weight is the number showing on bathroom scales when you weigh yourself. On the other hand, mass measures the amount of matter in an object and will be the same at all heights above sea level. For example, mass is the value you get when an object is measured on a balance scale. Students should be exposed to the correct term **mass**; therefore, mass measurements should be made using non-standard units on balance scales.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with two objects such as an eraser and a book. Ask, Can you tell which of these two objects is longer? Can you tell which of these two objects is heavier?
  After each question, have students explain their thinking.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Give students an object to hold in one hand. Ask each student to estimate how many of another non-standard unit would have the same mass and explain their thinking.
- Provide students with containers filled with various sizes of paper clips. Ask students to estimate the length of a table using large paper clips. Then, ask them to estimate the length of the table using small paper clips. Note whether their estimates reflect the impact of the different sized units of measure. This can also be done using cubes of various masses. Students can be asked to estimate the mass of an object.
- Have students choose one of two non-standard units to measure a particular length/mass and to defend their choice (e.g., linking cubes, a toothpick, and a straw as choices of non-standard units to measure the length of a table).
- Ask questions to demonstrate the reasonableness of estimates (e.g., Is this pencil 6 paper clips or 60 paper clips long?).
- Ask students to use several different non-standard units to measure the same object. Ask, Which would give you the closest measure? How do you know? (e.g., if students are given paper clips, pencils, and linking cubes, they may identify the cubes as being the most accurate if they fit almost exactly on the item they are measuring.)
- Tell students that Mark and Eli measured the length of the teacher's desk. Mark said that the teacher's desk was 45 units long. Eli said that the teacher's desk was 15 units long. Both students measured correctly. Ask students to explain why they might have gotten different answers?

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)
- Grade Primary Checkpoint 11, pp. 105–106
- Grade 1 Checkpoint 11, pp. 107–108
- Grade 2 Checkpoint 10, Task 2, pp. 101–102 (Line Master 10.1)
- Grade 2 Checkpoint 11, Task 1, pp. 109–110

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide a variety of non-standard measurement tools, such as stir sticks, paper clips of various sizes, toothpicks, straws, unifix or linking cubes, colour tiles, pennies, and blocks.
- Provide students the opportunity to make their own simple measuring instruments using objects such as paper clips. The units are lined up and marked off on a length of paper. This will link the understanding that it is the space and not the mark that indicates a measure of length on a ruler.
- Give students many opportunities to use a variety of non-standard units to measure an object and then to discuss which is most appropriate and why.
- Students require practise making and accepting estimates. Model the use of referents at all times, (e.g., placing one paper clip beside a pencil before estimating).
- Encourage students to recognize that the mass of objects is not related to its size (e.g., compare a golf ball with an inflated balloon).

SUGGESTED LEARNING TASKS

- Have students work in small groups. Provide each group with a different non-standard unit, such as small paper clips, stir sticks, straws, counters, or toothpicks. Have each group use the given unit to measure the length of their table or desk and record their results. Debrief the activity with the whole class. After each group presents their results, ask questions such as, Did any group have problems using their measuring unit? Did you find your measuring unit easy to use? Why or why not? Each group measured the same object. Why did groups get a different number of units? Does the size of the measuring unit make a difference? This can be repeated for mass using different non-standard units.
- Provide students with a variety of non-standard units and an object to measure. Ask them to determine the best (most efficient and accurate) non-standard measurement unit to use and to justify their choice of unit.
- Ask students to search for items that have estimates of a given number of paper clips long or a mass of two bags of pennies.
- Provide students with a length of string and have them estimate how many paper clips long it is.
- Provide students with a large object such as an inflated balloon and a small object such as a rock. Ask them to estimate how many pennies it would take to match the mass of each of the objects.
- Have students develop a book on length and/or mass that they can add to over time. Students can draw a picture of the object they measured and record below it the number of non-standard units used to measure it. Alternately, they could create a book for a specific measurement, for example, “Objects That Are Four Paper Clips Long” or “Objects That Have a Mass of 10 Linking Cubes.”
- Read the book, *How Big Is a Foot?* by Rolf Myller (1991), and relate the story to non-standard measurement.

SUGGESTED MODELS AND MANIPULATIVES

- blocks
- everyday objects such, as string
- hand spans
- linking cubes
- paper clips
- paper strips
- pennies

MATHEMATICAL LANGUAGE

Teacher	Student
▪ about, little less than, little more than ▪ balance scale ▪ compare ▪ estimate, measure ▪ length, width, height, mass ▪ non-standard unit ▪ object	▪ about, little less than, little more than ▪ balance scale ▪ compare ▪ estimate, measure ▪ length, width, height, mass ▪ unit ▪ object

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 370–375, 430–433
- *Making Math Meaningful to Canadian Students, K–8,* Second Edition (Small 2013), pp. 414–415, 418–422, 478–479
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 58–59, 223–231, 241–242

Notes

SCO M03 Students will be expected to compare and order objects by length, height, distance around, and mass using non-standard units and make statements of comparison. [C, CN, ME, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

M03.01 Estimate, measure, and record the length, height, distance around, or mass of a given object using non-standard units.

M03.02 Compare and order the measure of two or more objects in ascending or descending order and explain the method of ordering.

Scope and Sequence

Mathematics 1	**Mathematics 2**	**Mathematics 3**
M01 Students will be expected to demonstrate an understanding of measurement as a process of comparing by ▪ identifying attributes that can be compared ▪ ordering objects ▪ making statements of comparison ▪ filling, covering, or matching	**M02** Students will be expected to relate the size of a unit of measure to the number of units (limited to non-standard units) used to measure length and mass. **M03** Students will be expected to compare and order objects by length, height, distance around, and mass using non-standard units and make statements of comparison.	**M03** Students will be expected to demonstrate an understanding of measuring length (cm, m) by ▪ selecting and justifying referents for the units centimetre and metre (cm, m) ▪ modelling and describing the relationship between the units centimetre and metre (cm, m) ▪ estimating length using referents ▪ measuring and recording length, width, and height **M04** Students will be expected to demonstrate an understanding of measuring mass (g, kg) by ▪ selecting and justifying referents for the units gram and kilogram (g, kg) ▪ modelling and describing the relationship between the units gram and kilogram (g, kg) ▪ estimating mass using referents ▪ measuring and recording mass **M05** Students will be expected to demonstrate an understanding of perimeter of regular, irregular, and composite shapes by ▪ estimating perimeter using referents for centimetre or metre (cm, m) ▪ measuring and recording perimeter (cm, m) ▪ creating different shapes for a given perimeter (cm, m) to demonstrate that many shapes are possible for a perimeter

Background

Children should recognize that length tells about the extent of an object along one dimension. Initially, they would compare lengths informally by simply viewing the two lengths. Later, they should investigate strategies to compare the lengths of two or more objects directly and indirectly. Direct comparison involves comparing lengths by lining up items side by side, beginning at a common base. Students should be led to see why a common starting point is important. Indirect comparison involves comparing lengths using another object. This is particularly useful when it is not possible to physically line up the objects. For example, to compare length of hand to wrist size, students may cut pieces of string the length of their hands, and then wind the strings around their wrists for comparison. Students are encouraged to compare and order the length and height using appropriate mathematical vocabulary, such as **length**, **height**, **longer than**, **shorter than**, and **number of units**.

Distance around refers to perimeter and circumference. **Distance around** is a term that students in Mathematics 2 may be more comfortable using instead of the formal terminology, **perimeter**. Estimating distance around, especially when that distance cannot be easily treated as linear parts (such as a circle), is very difficult for students. They need many experiences such as the one that follows. Initially, students will learn to measure the perimeter of a shape, such as their desk, by fitting a string around it and cutting the string to that length. Once students are comfortable measuring the distance around a linear object (one that has straight edges), students are more capable of understanding the notion of measuring around curves. When measuring round or curvy objects is first introduced, students should use material such as string, ribbon, or wool.

Students should recognize that mass tells about the heaviness of an object. They should explore methods to compare and order masses, including situations involving both direct and indirect comparisons. Direct comparisons involve, for instance, placing two objects on a balance simultaneously and comparing their masses. Indirect comparisons involve comparing the masses of two objects by using another object as a referent.

After objects are ordered by an attribute using either direct or indirect comparisons, students should estimate and measure the attribute using a non-standard unit. It is the numerical values assigned to the attribute that are the measurements of those attributes. If students estimate and measure the smallest or largest of the objects, they can use that result to refine their estimates of the other objects before they are measured.

While students are likely to use a variety of language phrases when referring to comparisons of attributes of objects, they should hear correct language they can begin to model. For example, one object may be said to be longer than another object because it has a greater measure. Students may say “bigger measure”; however, measures are numerical, and numbers are compared using greater and less.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with a set of ribbons that are of various lengths. Ask them to show you the ribbons that are the shortest and the longest. Point to a particular ribbon and ask them to tell show you a ribbon that is longer than yours. Point to a particular ribbon and ask them to show you a ribbon that is shorter than yours.
- Provide students with a set of objects that have different masses. Ask them to show you the object that is the heaviest and the object that is the lightest.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Give students an object to hold in one hand. Ask each student to show how many of another non-standard unit would have the same mass and to explain their thinking.
- Provide a set of objects for students. Ask them to measure the mass of each object using non-standard units and then to order them from heaviest to lightest.
- Ask students to estimate and measure the distance around a water bottle.
- Ask students how they would compare the height of a garbage can to the distance around the top of the can and explain.
- Ask students to prepare a set of ribbons for first, second, and third places in a race, so that the faster runner gets a longer ribbon.
- Ask students, If three kiwis have the same mass as one apple, what do you know about the mass of the apple?
- Provide a group of objects to be measured; for example, a book, a piece of paper, a length of string, and a pencil. Ask students to use non-standard units to measure each object and then to identify the object that is longest and the one that is shortest.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)
- Grade 2 Checkpoint 9, pp. 93–94
- Grade 2 Checkpoint 10, Task 1, pp. 101–102
- Grade 2 Checkpoint 11, Task 2, pp. 109–110

Numeracy Nets 3 (Bauman 2009)
- Grade 3 Checkpoint 12, Task 1, pp. 58–59
- Grade 3 Checkpoint 13, pp. 62–63

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide students with experiences using a broad measurement vocabulary. For example, they should hear and use words such as **heavier**, **lighter**, **has the same mass**, **longer**, **taller**, and **wider**.
- Ask students to order objects from longest to shortest. Include situations in which students are dealing with other attributes, such as objects that are not straight or that are also wide or thick.
- Ask students to estimate and measure objects in non-standard units for the purpose of answering relevant and practical questions (e.g., Do people with longer legs usually jump farther?)
- Focus should be on answering, in every measurement situation, questions that focus on the attributes, such as, Which book is the tallest? How do you know?
- Use the same non-standard unit to measure a variety of items in order to compare them (e.g., How many wooden blocks would balance a sneaker? a book? a grapefruit? Which object is heaviest? lightest?)

SUGGESTED LEARNING TASKS

- Ask students to search for items that are a given number of paper clips long or that have a mass of two bags of marbles.
- Provide students with a length of string and have them estimate how many paper clips long it is. Ask them to measure the string using the paper clips. Ask them to explain whether their estimates were reasonable or not. This can also be repeated for mass.

- Have students compare the masses of two objects using a pan balance.
- Create an activity centre where students estimate, measure, compare, and sort various objects based on length, distance around, or mass. Comparison should include longer, shorter or about the same length as a specified object. The comparison for mass would be heavier, lighter, or about the same mass.
- Collect cylinders of different sizes. Ask students to order them according to their height or according to the distance around.
- Provide students with opportunity to measure, in non-standard units, the distance around a variety of classroom objects such as water bottles, cylinders, pumpkins, apples, the recycling bin, 2-D shapes, and 3-D objects.
- Have students work with a partner to measure a variety of classroom objects and record their results.
- Provide small groups of students with three or four objects, each with a different length and mass. If possible, try to have one of the shorter objects have a mass greater than the others. Ask students to measure the length of the objects and then order them from longest to shortest. Students should record their results. Then ask students to find the mass of the objects and order them from heaviest to lightest and record their results. Have students compare their results. Ask questions such as, Is the longest object the heaviest? Is the tallest object the heaviest? Is the shortest object the lightest? Do you think you can predict the mass of an object by looking at its height? Why or why not? If someone told you the mass of a hidden object, do you think you could predict the height? Why or why not?
- Give students pieces of string to use to measure the distance around their heads. Ask each student to find an object in the classroom that is about the same length as the piece of string.

SUGGESTED MODELS AND MANIPULATIVES

- blocks
- everyday objects such as string
- hand spans
- linking cubes
- paper clips
- paper strips
- pennies

MATHEMATICAL LANGUAGE

Teacher	Student
▪ balance, referent ▪ compare, order ▪ estimate, measure ▪ length, height, distance around, mass, heaviness ▪ length: longer than, shorter than ▪ mass: heavier than, lighter than ▪ non-standard unit ▪ object	▪ balance, referent ▪ compare, order ▪ estimate, measure ▪ length, height, distance around, mass ▪ length: longer than, shorter than ▪ mass: heavier than, lighter than ▪ object ▪ unit

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 370–375, 430–433
- *Making Math Meaningful to Canadian Students, K–8,* Second Edition (Small 2013), pp. 414–415, 418–422, 478–479
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 58–59, 223–231, 241–242

Notes

SCO M04 Students will be expected to measure length to the nearest non-standard unit by using multiple copies of a unit and using a single copy of a unit (iteration process). [C, ME, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

M04.01 Explain why overlapping or leaving gaps does not result in accurate measures.

M04.02 Count the number of non-standard units required to measure the length of a given object using a single copy or multiple copies of a unit.

M04.03 Estimate and measure a given object using multiple copies of a non-standard unit and using a single copy of the same unit many times, and explain the results.

M04.04 Estimate and measure, using non-standard units, a given length that is not a straight line.

Scope and Sequence

Mathematics 1	**Mathematics 2**	**Mathematics 3**
M01 Students will be expected to demonstrate an understanding of measurement as a process of comparing by ▪ identifying attributes that can be compared ▪ ordering objects ▪ making statements of comparison ▪ filling, covering, or matching	**M04** Students will be expected to measure length to the nearest non-standard unit by using multiple copies of a unit and using a single copy of a unit (iteration process). **M05** Students will be expected to demonstrate that changing the position of an object does not alter the measurements of its attributes.	**M03** Students will be expected to demonstrate an understanding of measuring length (cm, m) by ▪ selecting and justifying referents for the units centimetre and metre (cm, m) ▪ modelling and describing the relationship between the units centimetre and metre (cm, m) ▪ estimating length using referents ▪ measuring and recording length, width, and height **M05** Students will be expected to demonstrate an understanding of perimeter of regular, irregular, and composite shapes by ▪ estimating perimeter using referents for centimetre or metre (cm, m) ▪ measuring and recording perimeter (cm, m) ▪ creating different shapes for a given perimeter (cm, m) to demonstrate that many shapes are possible for a perimeter

Background

In order to measure the length of an object, a series of uniform units must be used, or a single unit must be used repeatedly (iteration). The measure is dependent on the type of unit used, the placement of the units, the uniformity of the unit, and the unit's positioning. Students should have many experiences

measuring lengths of objects with multiple copies of non-standard units that they can correctly place and count before they attempt to measure lengths with a single unit as a measuring device.

It is essential that students understand how to measure length of an object using non-standard units. For example, to measure the length of a straw using toothpicks, students should start at one end of the straw placing congruent toothpicks tip to tip along the edge of the straw, making sure there are no gaps or overlaps between any of the toothpicks. The number of toothpicks used to match the edge of the straw will be the measure of the length of the straw. Students often struggle to understand that it is the number of intervals that are counted; therefore, they could place the straw and toothpicks on paper, use a pencil to mark the endpoints of the toothpicks, remove the toothpicks, and count the intervals between the pencil marks. Opportunities to explore and compare counting intervals will result in a greater understanding of measurement before standard units are introduced.

Students should measure length and height using non-standard units. Initially, use objects to measure that show a large difference from the object that is being measured, such as using a pencil to measure the length of a desk, then moving to objects similar in length or height, such as using a pencil to measure the length of a straw. Students should use everyday classroom objects to measure lengths, such as paper clips, crayons, straws, toothpicks, markers, or pencils. Students should be exposed to measuring larger objects in the classroom, such as doors, windows, or white boards, to help students further visualize length and height attributes.

Before any measurement activity, students should be encouraged to provide an estimate. Through continual estimating, measuring, and comparing estimates with actual measurements, followed by discussions of estimation strategies, students will develop their estimation abilities.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with two objects such as an eraser and a book. Ask, Can you tell which of these two objects is longer? heavier? bigger? takes up the most space? holds more? After each question, have students explain their thinking. (Children should recognize that capacity is an attribute that cannot be used to measure the objects.)

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to compare two or more crooked paths, made with skipping ropes, lengths of rope, string, etc., and to estimate and then determine which path is the longest and which is the shortest.
- Ask students to solve problems, such as
  a) Navaeh used a tile to measure the table in the classroom. Tyrel used the same tile to measure the same side of the table but they have different results. How is this possible? You may use a diagram to show your thinking.
  b) Susan measured the length of her hand with linking cubes on three different days. On the first day, she said that her hand was 9 cubes long. On the second day, she said that her hand was 5 cubes long. On the third day, she said that her hand was 3 cubes long. Ask students to explain possible measurement errors Susan was making.
- Show students a straw and tell them that it is one unit. Ask students to estimate the length of various objects (e.g., a book, a table, the width of a door, etc.). Have students check their estimates.
- Ask students to estimate the number of cubes it would take to measure the length of a big book. Provide students with a collection of linking cubes and ask them to measure the length of the big book (multiple copies of a non-standard unit). Then, ask them to measure the length of the big book again using only one cube (single copy of the same unit many times). Ask them to explain the results.
- As students observe you, use linking cubes to measure the width of the window ledge but make an error in your measurement. For example, leave gaps between the cubes if using multiple units or overlap if using a single cube. Tell students the width of the window ledge. Ask them to tell if your measurement is accurate or not and explain their thinking.
- Provide students with containers of objects that can be used to measure length. Ask students to select one of the objects and measure a line segment. Note how students measure. Do they select only one type of object to do the measurement? Do they select objects of the same size (multiple uniform copies of this unit) or do they mix the sizes of the objects? Do they overlap the objects or do they leave gaps between them?
- Ask students to draw a line segment that they think is 6 cubes long. Then, have them measure the line segment with the cubes.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)

- Grade 2 Checkpoint 10, Task 1, pp. 101–102
- Grade 2 Checkpoint 11, pp. 109–110

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ask students to predict the results prior to making their measurements. Students should understand that the way a measurement unit is used is as important as the attribute being measured.
- Provide students with opportunities to measure an object using multiple copies and using single copies of a non-standard unit. They should compare results and discuss why (or why not) their measure is the same or different.
- Use children's literature, such as *David's Father* by Robert Munsch (1983), to have students discuss non-standard measurement units and estimating lengths and distances.

SUGGESTED LEARNING TASKS

- Present students with the following problem. I want to measure the length of the table with this whiteboard brush, but I only have one brush. How do you think we can measure the table using only one brush? Brainstorm ideas and have students try their suggestions.
- Estimate and measure a given object using multiple copies of a non-standard unit and using a single copy of the same unit many times, and explain the results. For example, ask students to estimate and measure the length of their arms by first measuring with straws laid end to end and then by measuring using only one straw. Ask them to give reasons for discrepancies between the two measures.
- Have students work in pairs. One student will measure the length, width, height, or distance around an object using non-standard unit. He or she may choose to measure correctly or he or she may choose to make an error. His or her partner observes the measuring process and explains whether the measurement is accurate and correct or not, and explains the error made.
- Students work in pairs to determine an effective measure of the height of their partner. Students should begin by selecting a unit of measure, and then they should estimate their partner's height in that non-standard unit. Explain that it might be easier to consider measuring a classmate when he or she is lying down. Discuss the similarity or difference in results from various groups of students.

- Provide students with containers filled with various sizes of paper clips. Ask students to measure the length of a book using paper clips. Note how students measure. Do they select paper clips of the same size (multiple uniform copies of this unit) or do they mix the sizes of the paper clips? Do they overlap the paper clips or do they leave gaps between them? Do they measure from one edge of the book to the other? This task can also be done using cubes of various masses. Students can be asked to measure the mass of an object.
- Draw curved or zigzagged line segments on large pieces of paper or make them with masking tape on the floor. Ask students how they might measure the length of the line segments. Try their suggestions to determine effective ways of measuring the line segments.

SUGGESTED MODELS AND MANIPULATIVES

- colour tiles
- Cuisenaire rods
- linking cubes
- paper clips
- pattern blocks
- straws

MATHEMATICAL LANGUAGE

Teacher	Student
▪ estimate, measure, compare ▪ length ▪ multiple copies, single copies ▪ non-standard unit	▪ estimate, measure, compare ▪ length ▪ unit

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 370–375
- *Making Math Meaningful to Canadian Students, K–8,* Second Edition (Small 2013), pp. 418–422
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 58–59, 223–231

Notes

SCO M05 Students will be expected to demonstrate that changing the position of an object does not alter the measurements of its attributes. [C, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicator

Use the following indicator to determine whether students have achieved the corresponding specific curriculum outcome.

M05.01 Measure a given object, change the position, remeasure, and explain the results.

Scope and Sequence

Mathematics 1	Mathematics 2	Mathematics 3
M01 Students will be expected to demonstrate an understanding of measurement as a process of comparing by ▪ identifying attributes that can be compared ▪ ordering objects ▪ making statements of comparison ▪ filling, covering, or matching	**M04** Students will be expected to measure length to the nearest non-standard unit by using multiple copies of a unit and using a single copy of a unit (iteration process). **M05** Students will be expected to demonstrate that changing the position of an object does not alter the measurements of its attributes.	**M03** Students will be expected to demonstrate an understanding of measuring length (cm, m) by ▪ selecting and justifying referents for the units centimetre and metre (cm, m) ▪ modelling and describing the relationship between the units centimetre and metre (cm, m) ▪ estimating length using referents ▪ measuring and recording length, width, and height

Background

It is very common even for adults to misjudge the size of objects when those objects are in different positions. For example, a carpet hung on a wall can appear to be larger than one of the same size lying on the floor. It is important to provide many opportunities for students to play with, examine, and measure different objects in different positions and directions. By providing students with these experiences, students will have the opportunity to conclude that the position and direction of an object does not change its measurements. Students need to recognize that whether an object is standing up, lying flat, or tilted, its dimensions and mass will remain the same. Furthermore, students need to begin to realize that if a solid object is restructured so that its dimensions are altered, its mass will not be changed.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

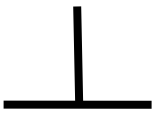
Tasks such as the following could be used to determine students' prior knowledge.

- Set up a measurement centre. Provide a collection of objects each with a different length and mass. Have students choose two objects and compare their length or their mass.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to measure the length of a piece of paper with a tile. Record the result. Ask students to make a second measurement along the same side of the paper after repositioning the paper. Record the result. Discuss your findings.
- Show students a big book and tell them, "I measured the length of this big book and it was 25 cubes long." Reposition the same big book and ask, If I measure the length of the big book now, how many cubes long will it be? Explain your thinking.
- Show students an apple and tell them, "I measured the mass of this apple and it has the same mass as 37 cubes." Turn the apple over and ask, If I measure the mass of the apple now, what will it be? Explain your thinking.
- Ask, If you change the position of an object (flip it, turn it, or slide it) do its measurements change? Explain your thinking.
- Show students a picture such as the one below.



  Ask students to predict and then measure to determine which line segment is longer.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)
- Grade 2 Checkpoint 11, Task 2, pp. 109–110

Numeracy Nets 3 (Bauman 2009)
- Grade 3 Checkpoint 12, Task 1, pp. 58–59

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ask students to predict the results prior to making their measurements.
- Provide students with opportunities to measure an object and then remeasure the same object after it has been reoriented. They should compare results and discuss whether their measure is the same or different. Indicate that sometimes reorientation assists in the ease but not the value of the measurement.

SUGGESTED LEARNING TASKS

- Ask students to measure the length of an object in non-standard units, such as linking cubes. Put the object on display, with the linking cubes along the side that was measured, and record the measure. Ask students to place the object in a new position and state the length of that object.
- Have students measure their height in non-standard units while standing up and while lying down. Ask them why the measure of their height didn't change. Ask them to explain which measurement was easier to complete.
- Ask students to use linking cubes to measure the distance around a triangle. Record the measure. Then, ask them to predict the measure of the distance around that triangle if they turn the triangle or flip it to a new position. Allow them to measure the distance around the triangle and to explain the new measure.

SUGGESTED MODELS AND MANIPULATIVES

- colour tiles
- Cuisenaire rods
- linking cubes
- paper clips
- pattern blocks
- straws

MATHEMATICAL LANGUAGE

Teacher	Student
▪ estimate, measure ▪ position, direction	▪ estimate, measure

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 370–375
- *Making Math Meaningful to Canadian Students, K–8,* Second Edition (Small 2013), pp. 412–415
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 58–59, 223–231

Notes

Geometry

GCO: Students will be expected to describe the characteristics of 3-D objects and 2-D shapes and analyze the relationships among them.

Specific Curriculum Outcomes

Process Standards

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

G01 Students will be expected to sort 2-D shapes and 3-D objects using two attributes and explain the sorting rule. [C, CN, R, V]

G02 Students will be expected to recognize, name, describe, compare, and build 3-D objects, including cubes and other prisms, spheres, cones, cylinders, and pyramids. [C, CN, R, V]

G03 Students will be expected to recognize, name, describe, compare, and build 2-D shapes, including triangles, squares, rectangles, and circles. [C, CN, R, V]

G04 Students will be expected to identify 2-D shapes as part of 3-D objects in the environment. [C, CN, R, V]

SCO G01 Students will be expected to sort 2-D shapes and 3-D objects using two attributes and explain the sorting rule. [C, CN, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

G01.01 Determine the differences between two given presorted sets and explain the sorting rule.
G01.02 Identify and name two common attributes of items within a given sorted group.
G01.03 Sort a given set of 2-D shapes (regular and irregular) according to two attributes and explain the sorting rule.
G01.04 Sort a given set of 3-D objects according to two attributes and explain the sorting rule.

Scope and Sequence

Mathematics 1	**Mathematics 2**	**Mathematics 3**
G01 Students will be expected to sort 3-D objects and 2-D shapes using one attribute and explain the sorting rule.	**G01** Students will be expected to sort 2-D shapes and 3-D objects using two attributes and explain the sorting rule.	**G01** Students will be expected to describe 3-D objects according to the shape of the faces and the number of edges and vertices. **G02** Students will be expected to name, describe, compare, create, and sort regular and irregular polygons, including triangles, quadrilaterals, pentagons, hexagons, and octagons, according to the number of sides.

Background

"Children need experiences with a rich variety of both two- and three-dimensional shapes. It is useful for students to be able to identify common shapes, notice likenesses and differences among shapes, become aware of the properties that different shapes have, and eventually use these properties to further define and understand their geometric world." (Van De Walle and Lovin 2006, 193) "The [Van Hiele] levels [of geometric thought] describe how we think and what types of geometric ideas we think about, rather than how much knowledge we have." (Van de Walle and Lovin 2006, 188)

Sorting tasks help develop visual discrimination. It is important to encourage students to look for alternative ways of sorting; this necessitates the further investigation of objects. According to the Van Hiele model, most students in Mathematics 2 are at the visualization level. They rely mostly on the appearance of the shape or object when doing sorting tasks. There are many different attributes or characteristics of shapes and objects. Students are most familiar with attributes that describe the whole shape, such as colour, size, shape, or texture.

Other attributes refer to parts of the shape or object, such as number of sides or faces, shapes of faces, number of vertices (corners), or lengths of sides or edges. Students in Mathematics 2 may need help focussing their attention on these attributes if their sorting is to include such attributes. Students will often use non-geometric language to describe the attributes, such as crooked or wavy. Students may also sort according to the position of the shape, such as a square turned may be seen as a diamond. Again, students may need help to be convinced that a shape or object in a different position in space is not a different one.

In earlier grades, students will have had many opportunities to explore shapes through sorting by one attribute, patterning, and building tasks. In Mathematics 2, tasks that will further develop these skills should be provided. In particular, this outcome extends students' sorting of 2-D shapes and 3-D objects to sorts involving two attributes. Students will begin at different levels of development so it will be necessary to provide tasks with a range of complexity.

In Mathematics 2, the 2-D geometric shapes being explored include triangles, squares, rectangles, and circles, while 3-D geometric objects being explored include, spheres, cones, cylinders, cubes and other prisms, and pyramids with rectangular and triangular bases. However, other shapes and objects may also be used in sorting tasks without an emphasis on naming them. When discussing 3-D objects with students, it is important to refer to them as simply objects or 3-D solids. This will assist students in differentiating between 2-D shapes and 3-D objects.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with a collection of 2-D shapes or 3-D objects. Tell them a sorting rule based on one attribute. Ask them to sort the collection based on your sorting rule.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Ask students to describe a given 2-D shape or 3-D object. Note the attributes described by the students.
- Provide several different 3-D objects. Ask the student to sort them using two attributes and to explain the sorting criteria. Ask him or her to sort them again, using different criteria.
- Show students a set of 3-D objects or 2-D shapes that have been sorted. Ask students to explain the differences between the two sets and to explain the sorting rule.
- Provide several different 2-D shapes. Ask the student to sort them using two attributes and to explain the sorting criteria. Ask him or her to sort them again, using different criteria.
- Show students a set of 2-D shapes or 3-D objects that have been sorted, but which contains a shape or an object that does not belong. Ask students to remove the shape or object that does not belong and to explain their thinking.
- Show students a set of 3-D objects that you have sorted based on two common attributes. Have additional 3-D objects available that have yet to be sorted. Ask students to select an object and to add it to the set that you sorted. Ask them to explain their thinking.
- Note the language students use to describe their sorting rules. As students develop a higher level of thinking, they will begin to use more geometric language and focus more on the properties of the shapes.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)
- No Checkpoint for this outcome.

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Include a variety of sorting and classifying tasks. Use collections of real objects, such as cans, boxes, or balls, for students to sort.
- Focus on how shapes and objects are alike and how they are different. Provide many opportunities for students to use oral and written language to describe the attributes of 2-D shapes and 3-D objects.
- Make a collection of 2-D shapes cut from poster board. These should include not only shapes with which students are familiar, but also some less common shapes that would fit in potential sorting categories (e.g., a semi-circle or a jellybean shape could be classified under "things with curved edges"). Because it is important for students to share their thinking with others, it is recommended that they work in small groups, talking about their ideas and strategies.
- Challenge students to test their ideas about shapes and objects. For example, Can you find a triangle that has one square corner?

SUGGESTED LEARNING TASKS

- Hold up a geometric solid, like a cone, and ask the students to find other objects that could be sorted into the same group as the cone. When a student adds an object to the group, have them explain what attribute it shares with the original object.
- Provide a collection of about eight shapes or objects. Have students take turns sorting the shapes while the other group members try to guess the sorting rule being used. Each student in the group should have a turn to sort the shapes. As a class, discuss all the ways that the shapes were sorted and try to determine if there are any ways that were missed.
- Ask students to select two 2-D shapes or 3-D objects and tell how they are the same or how they are different.
- Create a set of 3-D objects or 2-D shapes (about five) that have a secret sorting rule based on two attributes. Ask the students to add to your set (a drawing or a real item) and explain the rule.
- Play game of "One Way Different." Select an object or shape as a starter piece. Students take turns placing objects on either side of the original shape. These objects must differ by one attribute from the one it is placed beside. Students need to explain why their piece works. For example, if the original object was a pyramid, the next object placed beside it could be a cone and the student could justify that the cone has curved faces.
- Sort a small number of students into two groups without telling the class how they are being sorted. One at a time, the remaining students go to the group to which they think they belong. Tell each student whether he or she is in the correct group. When all students are correctly sorted in groups, ask, What's my sorting rule? Repeat using other attributes.
- Sort a set of buttons using two attributes; for example, buttons that are round and have two holes; and buttons that are not round and do not have two holes. Have students guess the sorting rule.

SUGGESTED MODELS AND MANIPULATIVES

- attribute blocks
- dot and/or grid paper
- geoboards
- geometric solids
- pattern blocks
- pentominoes
- polydrons
- tangrams

MATHEMATICAL LANGUAGE

Teacher	Student
▪ 2-D shapes: triangle, square, rectangle, circle ▪ 3-D objects: cube, sphere, cone, cylinder, pyramid, prism ▪ attribute ▪ regular, irregular shapes ▪ sides, edges, corners/vertices, faces ▪ sorting rule ▪ sorting sets of shapes ▪ straight, curved, large, small, points, square, roll, stack	▪ 2-D shapes: triangle, square, rectangle, circle ▪ 3-D objects: cube, sphere, cone, cylinder, pyramid, prism ▪ sides, edges, corners/vertices, faces ▪ sorting rule ▪ sorting sets of shapes ▪ straight, curved, large, small, points, square, roll, stack

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 284–291, 516–520
- *Making Math Meaningful to Canadian Students, K–8,* Second Edition (Small 2013), pp. 340–347, 560–564
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 192–195, 206–208

Notes

SCO G02 Students will be expected to recognize, name, describe, compare, and build 3-D objects, including cubes and other prisms, spheres, cones, cylinders, and pyramids. [C, CN, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

G02.01 Sort a given set of 3-D objects and explain the sorting rule.
G02.02 Identify common attributes of cubes and other prisms, spheres, cones, cylinders, and pyramids from given sets of the same 3-D objects.
G02.03 Identify and describe given 3-D objects with different dimensions.
G02.04 Identify and describe given 3-D objects with different positions.
G02.05 Create and describe a representation of a given 3-D object using materials such as modelling clay.
G02.06 Identify and name examples of cubes and other prisms, spheres, cones, cylinders, and pyramids found in the environment.

Scope and Sequence

Mathematics 1	**Mathematics 2**	**Mathematics 3**
G01 Students will be expected to sort 3-D objects and 2-D shapes using one attribute and explain the sorting rule. **G02** Students will be expected to replicate composite 2-D shapes and 3-D objects. **G03** Students will be expected to identify 2-D shapes in 3-D objects.	**G02** Students will be expected to recognize, name, describe, compare and build 3-D objects, including cubes and other prisms, spheres, cones, cylinders, and pyramids.	**G01** Students will be expected to describe 3-D objects according to the shape of the faces and the number of edges and vertices. **G02** Students will be expected to name, describe, compare, create, and sort regular and irregular polygons, including triangles, quadrilaterals, pentagons, hexagons, and octagons, according to the number of sides.

Background

Students will continue to develop their depth of understanding of 3-D objects. Students in Mathematics 2 need many varied opportunities to manipulate 3-D objects. Tasks in which they describe, compare, and build 3-D objects and discuss their observations help to develop essential geometric skills. In order to describe, compare, and construct 3-D objects, students must first have multiple opportunities to explore concrete models of cubes and other prisms with rectangular faces, spheres, cones, cylinders, and pyramids. Students should be exploring prisms and pyramids with triangular and rectangular bases. As part of the exploration process, students should be able to touch, feel, build, and observe a wide variety of geometric solids in the classroom as well as familiar 3-D objects in their home and school environment. It is through such tasks that students will learn the names of 3-D objects and begin to recognize their characteristics.

"As students develop mathematically, they are increasingly able to identify and name a shape by examining its properties and using reasoning." (Small 2009, 287) However, initially students recognize a 3-D object by its overall appearance and through association with objects like it in their environment or in books. It is not unusual for students at this level of geometric development to state, for example, "This object is a cube because it looks like a cube," or "This object is a cube because it looks like a box."

Through hands-on exploration, students will discover the various components used to classify 3-D solids. These components would include faces, curved surfaces, edges, and vertices. While it is very important to encourage students to use accurate language when naming objects such as cube, prism, sphere, cone, cylinder, and pyramid, it is important to accept the language that students use to describe these components; however, you should consistently model the appropriate mathematical terminology and display these words in the classroom environment. They should also discover that an object has certain attributes regardless of its position or size. For example, even though a pyramid may look different lying on one of its triangular faces, it is still a pyramid with the same attributes. They should also recognize that it is still a pyramid whether it is tall or short.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with a collection of 3-D objects. Ask them to sort the 3-D objects and to explain their sorting rule.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Provide students with a set of 3-D objects that share common attributes. Ask them to describe how the objects are the same and how they are different. Note the attributes that students describe.
- Ask students to sort a given set of 3-D objects and to explain their sorting rule. Ask them to sort the set a second time using a new sorting rule.

- Ask students to choose a 3-D object and tell how they would describe the object to someone who did not know what it was.
- Provide students with modelling clay. Ask them to create a particular 3-D object, such as a sphere or a pyramid.
- Ask students to look around the classroom and identify at least three different rectangle-based prisms that they see and to explain why the objects they identify are all rectangle-based prisms.
- Ask students to describe how a cube and square-based pyramid are alike and how they differ. Repeat the question using two other 3-D objects such as a cone and cylinder.
- Place a set of 3-D objects in front of students. There should be a mix of pyramids and prisms. Ensure that, within the set, the triangle-based pyramids are placed in different positions (e.g., resting on the base, resting on a face). Ask students to identify all the triangle-based pyramids in the set. Note whether students select only those triangle-based pyramids that are positioned on their bases.
- Tell students that you traced around one of the faces of a 3-D object, and the shape that you drew was a square. Ask students to tell what the object could be and to explain their thinking.
- Show students a picture of a 2-D shape (circle, square, or triangle). Provide a variety of 3-D objects and ask the student to choose one that has a face that matches the picture.
- Label each side of a cube with the words: “cube,” “sphere,” “cone,” “cylinder,” “pyramid” and “free choice.” Have students roll the cube and find, in a book or in the environment, an example of something that is the shape shown on the cube.
- Provide students with a blackline master that has drawings of a variety of 2-D shapes. Ask students to use a familiar 3-D object from their environment (toy or article from home) and match one of its faces to one or more of the shapes from the sheet.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)
- Grade Primary Checkpoint 12, Task 2, pp. 114–115
- Grade 2 Checkpoint 12, Task 2, pp. 118–119

Numeracy Nets 3 (Bauman 2009)
- Grade 3 Checkpoint 15, Task 1, pp. 70–71 (Line Master 15.1–15.5)
- Grade 3 Checkpoint 16, Task 1, pp. 73–74 (Line Master 16.1)

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Have students bring in 3-D objects that could be described as a cube, sphere, cone, cylinder, or pyramid. Students should explain their reasons for its classification focusing on particular properties. For example, a student may bring in a funnel and classify it as both a cone and a cylinder.
- Have the students explore how they can create a cone, sphere, or cylinder using materials in the classroom. Expect to see use of modelling clay, play dough, pipe cleaners, toothpicks, paper, and circles from the attribute blocks. Have the students describe their methods to a partner, and present to the class, or write about their methods in their journals.
- Provide opportunities for students to build with 3-D objects by following oral directions; for example, Place the cube between the small cylinder and the cone, and place the large cylinder behind the cube. Invite students to make their own designs and then challenge their classmates to build the same structure from their oral directions.
- "Students need ample opportunities to draw, build, make, put together, and take apart shapes. These activities should be built around specific characteristics or properties so that students develop an understanding of geometric properties …" (Van de Walle and Lovin, 2006, 192)
- Ensure that students are exposed to 3-D objects in a variety of sizes and positions so they can discover that an object has certain attributes regardless of its position or size.

SUGGESTED LEARNING OPPORTUNITIES

- Give the student some toothpicks and clay and ask him or her to build a cube or a pyramid. Ask how many toothpicks were needed.
- Provide pairs of students with a small collection of 3-D objects. Have them build a creature or robot or something else of their choosing. Have them present their creation to the class and describe it using the names of the 3-D objects they used in their creation.
- Tell students that you have a 3-D object in a bag. One of its faces is round (a circle). Ask what the object could be. Follow up with having the students find an object in the class that has the same face.
- Have students trace one of the faces of a 3-D object and then search for another 3-D object in the classroom that that has the same face.
- Challenge students to build your "secret object." Give them clues such as, My object uses 10 cubes and has a rectangle base and it looks like stair steps, or My object uses 8 cubes and has a square base and it looks like the letter L.
- Ask students to look around the room and see what 3-D shapes they can see and name. List the shape and the object name on chart paper.
- Put students in small groups and have each group create a book of 3-D shapes by cutting pictures from catalogues and magazines.
- Provide students with sets of cubes and other prisms, spheres, cones, cylinders, or pyramids. Each set should contain objects of different sizes. Have students compare the objects and identify common attributes. Record attributes on a chart.

SUGGESTED MODELS AND MANIPULATIVES

- geometric solids
- polydron

MATHEMATICAL LANGUAGE

Teacher	Student
▪ 3-D objects: cube, sphere, cone, cylinder, pyramid, prism ▪ attributes: faces/surfaces, edges, vertices/corners, ▪ different positions: slide, flip, turn ▪ different sizes/dimensions ▪ recognize, name, describe, compare, build	▪ 3-D objects: cube, sphere, cone, cylinder, pyramid, prism ▪ faces/surfaces, edges, vertices/corners, ▪ slide, flip, turn ▪ different sizes ▪ recognize, name, describe, compare, build

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 287–292, 303–304
- *Making Math Meaningful to Canadian Students, K–8,* Second Edition (Small 2013), pp. 343–346, 358–359
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 196–200, 204–206

Notes

SCO G03 Students will be expected to recognize, name, describe, compare, and build 2-D shapes, including triangles, squares, rectangles, and circles. [C, CN, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

G03.01 Sort a given set of 2-D shapes and explain the sorting rule.
G03.02 Identify common attributes of triangles, squares, rectangles, and circles from given sets of the same type of 2-D shapes.
G03.03 Identify given 2-D shapes with different dimensions.
G03.04 Identify given 2-D shapes with different positions.
G03.05 Identify and name examples of triangles, squares, rectangles, and circles found in the environment.
G03.06 Create a model to represent a given 2-D shape.
G03.07 Create a pictorial representation of a given 2-D shape.

Scope and Sequence

Mathematics 1	**Mathematics 2**	**Mathematics 3**
G01 Students will be expected to sort 3-D objects and 2-D shapes using one attribute and explain the sorting rule. **G02** Students will be expected to replicate composite 2-D shapes and 3-D objects. **G03** Students will be expected to identify 2-D shapes in 3-D objects.	**G03** Students will be expected to recognize, name, describe, compare, and build 2-D shapes, including triangles, squares, rectangles, and circles.	**G01** Students will be expected to describe 3-D objects according to the shape of the faces, and the number of edges and vertices. **G02** Students will be expected to name, describe, compare, create, and sort regular and irregular polygons, including triangles, quadrilaterals, pentagons, hexagons, and octagons, according to the number of sides.

Background

A student's ability to conceptualize shape develops gradually. Initially, younger students identify and name 2-D shapes by their overall appearance and by association with objects in their environments, not by examining their parts or properties. For example, they may know a 2-D shape they are examining is a square because it looks like a geoboard that they know is a square. In this early stage, however, students may not recognize a square that has been rotated, thinking instead that it is a diamond. As students develop mathematically, they are able to name shapes according to properties and develop spatial abilities to recognize a shape regardless of its position or size. Tasks in which they sort, make, and recognize patterns, build and draw shapes, and talk about what is happening help to develop these essential geometric skills.

It is very important to encourage students to use accurate language when naming 2-D shapes. Students should be comfortable using the names **triangle**, **square**, **rectangle**, and **circle**. Describing shapes allows students to focus on their basic characteristics. Use questioning to focus student thinking; for example, What can you say about its sides? Is there anything special about its corners? What other shapes are like this one? In what ways are they alike?

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Provide a set of 2-D shapes that have been sorted based on one attribute. Ask students to explain the sorting rule. Then ask them to resort the shapes and to explain their new sorting rule.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Listen to students' observations as they work on tasks to determine whether they understand common attributes of shapes.
- Provide students with a set of 2-D shapes. Ask students to sort the shapes and to explain their thinking.
- Show students a set of triangles in various sizes and positions. Tell them that one student thinks these shapes are all triangles but another student doesn't agree. Ask them to explain who is correct.
- Ask students to describe or draw different triangles (rectangles).
- Have students create a picture using only two shapes (e.g., circles and squares). Have them share their pictures with the class and discuss what they used.
- Make a rectangle (or triangle or square) on a geoboard and have students make two different rectangles on their geoboards.
- Provide students with a set of tangrams. Have students use two or more of the pieces to create a square or a triangle.

- Provide students with a design that includes a variety of 2-D shapes, such as a tangram puzzle. Ask them to identify for you the different shapes they can find. They can use different colours to trace around each shape. Real-world examples that include various 2-D shapes could also be used (e.g., a hockey rink or a curling rink).
- Give students 10 triangular pattern blocks. Ask them to make as many different-sized triangles as they can and to record their answers by making a drawing of each one.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)
- Grade Primary Checkpoint 12, Task 1, pp. 114–115 (Line Master 12.1)
- Grade 1 Checkpoint 12, Task 1, pp. 116–117
- Grade 2 Checkpoint 12, Task 1, pp. 118–119 (Line Master 12.2–12.5)
- Grade 2 Checkpoint 13, pp. 126–127 (Line Master 13.3)

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Use a variety of models of the 2-D shapes to ensure students are provided with many different examples of triangles, squares, rectangles, or circles.
- Use 5 × 5 geoboards for students to explore how many different squares, rectangles, or triangles they can make.
- Encourage students to provide explanations beyond simple naming of shapes. For example, if the student says it is a triangle because it looks like a triangle, ask them to describe the characteristics of triangles.

- Include tasks that focus on the entire class of shapes (e.g., all rectangles) and their likenesses and differences.
- Provide opportunities for students to create 2-D shapes by following oral directions (e.g., Place a square below a triangle). Invite students to make their own designs and then challenge their classmates to build the same design from their oral directions.
- Ensure that students are exposed to 2-D shapes in a variety of sizes and positions so they can discover that a shape has certain attributes regardless of its position or size.
- Explore a variety of tangram puzzles.

SUGGESTED LEARNING TASKS

- Provide students with the large triangle, square, and rectangle of an attribute-block set. Ask them to work in pairs, comparing the three shapes, and listing all the ways in which they are the same and how they differ. They should be able to properly identify the shapes and to compare such things as number of sides or length of sides.
- Provide a drawing that incorporates many shapes in different positions and of various sizes. Ask the student to point out the triangles among the shapes.
- Give students a combination of 2-D shapes and 3-D objects. Ask them to investigate which of the 2-D shapes appears most often as a face of a 3-D object.
- Include tasks in which students are required to find shapes in pictures of objects and their environment.
- Have children hunt around the school to find various shapes (e.g., squares, triangles, circles, rectangles). Have them share their findings and discuss why certain shapes are more common than others.
- Put students in small groups and have each group create a book of 2-D shapes by cutting pictures from catalogues and magazines.
- Have students use shapes to spell the name of the shape or make the shape. For example, use a collection of triangles to form the letters that spell “triangle” or create the sides of a larger triangle.
- Have students physically build the shapes using their hands or fingers or in groups using their bodies.

SUGGESTED MODELS AND MANIPULATIVES

- attribute blocks
- dot and/or grid paper
- geoboards
- geometric solids
- pattern blocks
- pentominoes
- polydrons
- tangrams

MATHEMATICAL LANGUAGE

Teacher	Student
▪ 2-D shapes: triangle, square, rectangle, circle ▪ attributes: sides, corners, square corners ▪ different positions: slide, flip, turn ▪ different sizes/dimensions ▪ recognize, name, describe, compare, build	▪ 2-D shapes: triangle, square, rectangle, circle ▪ sides, corners, square corners ▪ slide, flip, turn ▪ different sizes ▪ recognize, name, describe, compare, build

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 287–290, 292–294, 303–304, 309–314, 516–520
- *Making Math Meaningful to Canadian Students, K–8,* Second Edition (Small 2013), pp. 343–346, 348–351, 560–567
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 196–200, 202–204

Notes

SCO G04 Students will be expected to identify 2-D shapes as part of 3-D objects in the environment. [C, CN, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

G04.01 Compare and match a given 2-D shape, such as a triangle, square, rectangle, or circle, to the faces of 3-D objects in the environment.

G04.02 Name the 2-D faces of a given 3-D object.

Scope and Sequence

Mathematics 1	Mathematics 2	Mathematics 3
G02 Students will be expected to replicate composite 2-D shapes and 3-D objects. **G03** Students will be expected to identify 2-D shapes in 3-D objects.	**G04** Students will be expected to identify 2-D shapes as part of 3-D objects in the environment.	**G01** Students will be expected to describe 3-D objects according to the shape of the faces and the number of edges and vertices.

Background

Students have had opportunities to explore shapes through sorting, patterning, and building tasks. Considering the attributes of various 2-D shapes and 3-D objects, students identified, compared, and sorted them by one attribute. Students had many varied opportunities to manipulate both 2-D shapes and 3-D objects to help make connections to objects in their environment.

Many of the 3-D shapes students see or explore have faces (pyramids and cubes), but others have curved surfaces (cylinders, spheres, and cones). Students should begin to include in their descriptions of a 3-D shape the surfaces and faces that make it up. For example, a cylinder has two circle faces and a curved surface, and a sphere has one curved surface. Through experiences with 3-D objects, students should be comfortable naming a cylinder, sphere, cone, pyramid, and cube and naming their curved surfaces and their 2-D faces that are squares, triangles, rectangles, or circles.

Pressing the faces of 3-D objects in sand to see the 2-D footprints made by the objects helps students focus on these faces. They should also trace on paper the faces of a 3-D solid to see these footprints. Pasting paper copies of 2-D shapes on 3-D objects also focuses students' attention on the faces. For example, if students are provided with six different coloured 10 cm x 10 cm squares and a large cube from base-ten blocks, they could tape those squares to the faces of the cube and describe the result.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with a set of 3-D objects. Ask them to sort the objects by the shape of their faces. Ask them to explain their thinking as they sort the objects.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Tell students that you traced around one of the faces of a 3-D object and the shape that you drew was a square. Ask students to tell what the object could be and to explain their thinking.
- Show students a picture of a 2-D shape (circle, square, rectangle, or triangle). Provide a variety of 3-D objects and ask the student to choose one object that has a face that matches the picture.
- Provide students with a blackline master that has drawings of a variety of 2-D shapes. Ask students to use a familiar 3-D object from their environment (toy or article from home) and match one of its faces to one or more of the shapes from the sheet.
- Show students a given 3-D object. Ask them to name the faces of the object.
- Show students a rectangle-based prism. Ask them to find other 3-D objects in the set they are given that have at least one face that is the same. Ask them to explain their thinking.
- Provide students with 3-D objects. Ask them to write a description of the faces of the object they've been given.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)
- No Checkpoint for this outcome.

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Have students bring in 3-D objects that could be described as a cube, sphere, cone, cylinder, or pyramid. Students should explain their reasons for its classification focusing on particular properties, including faces.
- When creating 3-D solids using a variety of materials such as modelling clay, pipe cleaners, toothpicks, paper, and circles from the attribute blocks, have students focus on the shapes of the faces. Have students describe the faces of the 3-D objects to a partner, present their findings to the class, or write about their discoveries in their journals.
- Provide opportunities for students to build with 3-D objects by following oral directions; for example, Make a 3-D object that has two faces that are circles. Invite students to make their own objects and then challenge their classmates to build the same structure from their oral directions.
- "Students with need ample opportunities to draw, build, make, put together, and take apart shapes in both two and three dimensions. These activities should be built around specific characteristics or properties so that students develop an understanding of geometric properties …" (Van de Walle and Lovin 2006, 192).
- Ensure that students are exposed to 3-D objects in a variety of sizes and positions so they can discover that an object has certain attributes regardless of its position or size.

SUGGESTED LEARNING TASKS

- Give students a combination of 2-D shapes and 3-D objects. Ask them to investigate which of the 2-D shapes appears most often as a face of a 3-D object.
- Include tasks in which students are required to find shapes in pictures of objects and their environment. For example, provide students with a chart showing 2-D shapes. Take students on a walk around the school or neighbourhood and ask students to record in the chart the number of times they see each shape. Discuss why some certain shapes occur more frequently than others.
- Have students identify all of the faces for a triangular prism and a triangular pyramid by pressing the prisms and pyramids in sand, coating them in paint, and making face prints or by tracing them on paper. Ask them to compare the impressions, face prints, or drawings and to describe how they are the same and how they are different.

- Place a variety of 3-D objects in a bag. Invite students to reach in the bag, feel the objects, and pull out an object with a square face. Ask them to name the object they pull out of the bag. Ask, Are there other 3-D objects that could have the same shaped face?
- Provide students with a set of 3-D objects. Ask them to select only those 3-D objects that have rectangular faces.
- Have students create riddles that focus on the faces of 3-D objects. For example, I am a 3-D object. Four of my faces are triangles. My base is a square. Who am I?

SUGGESTED MODELS AND MANIPULATIVES

- attribute blocks
- cones
- cubes and other prisms
- cylinders
- modelling clay
- pipe cleaners
- pyramids
- spheres
- toothpicks

MATHEMATICAL LANGUAGE

Teacher	Student
▪ 2-D shapes: square, triangle, rectangle, circle ▪ 3-D objects: cube, sphere, cone, cylinder, pyramid ▪ faces, footprint, curved surface, prism	▪ 2-D shapes: square, triangle, rectangle, circle ▪ 3-D objects: cube, sphere, cone, cylinder, pyramid ▪ faces, footprint, curved surface, prism

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 288–289, 292
- *Making Math Meaningful to Canadian Students, K–8,* Second Edition (Small 2013), pp. 344–347
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 193–195, 216–217

Notes

Statistics and Probability

GCO: Students will be expected to collect, display, and analyze data to solve problems.

GCO: Students will be expected to use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Specific Curriculum Outcomes: Statistics and Probability (SP)

Process Standards

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

SP01 Students will be expected to gather and record data about self and others to answer questions. [C, CN, PS, V]

SP02 Students will be expected to construct and interpret concrete graphs and pictographs to solve problems. [C, CN, PS, R, V]

SCO SP01 Students will be expected to gather and record data about self and others to answer questions. [C, CN, PS, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

SP01.01 Formulate a question that can be answered by gathering information about self and others.
SP01.02 Organize data as it is collected using concrete objects, tallies, checkmarks, charts, or lists.
SP01.03 Answer questions using collected data.

Scope and Sequence

Mathematics 1	**Mathematics 2**	**Mathematics 3**
—	**SP01** Students will be expected to gather and record data about self and others to answer questions. **SP02** Students will be expected to construct and interpret concrete graphs and pictographs to solve problems.	**SP01** Students will be expected to collect first-hand data and organize it using tally marks, line plots, charts, and lists to answer questions. **SP02** Students will be expected to construct, label, and interpret bar graphs to solve problems.

Background

Students often find themselves collecting and organizing data. Even though data management has not been formally taught in mathematics prior to grade 2, it is reasonable to expect that students will have had experiences collecting data in other content areas. The focus of data management explorations should be to answer questions. The contexts should be realistic and of interest to the students.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- How could we find out the favourite flavour of ice cream for our class? Explain your thinking.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Provide students with a set of 2-D shapes. Ask them to create a list or chart to show the types of shapes in the given set and how many of each type of shape there are.
- Provide students with a collection of about 25 linking cubes in three or four different colours. Ask them to organize the cubes and record the data in a chart using tally marks or another method. Ask them to write two questions that the tally marks would answer.
- Show students tallies or a chart on a topic of interest to students. Have them answer questions about the tallies or chart. Ask them to make up their own question that the tally marks or chart would answer.
- Have students formulate their own question that can be answered by collecting information within the school. Students should be able to write their question, gather their data, and organize it. In SCO SP02, students will be expected to display their information in the form of a concrete or pictograph and write about what they have learned.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)
- Grade 2 Checkpoint 14, pp. 135–136 (Line Master 14.1)
- Grade 2 Checkpoint 15, pp. 143–144

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
- Yearly plan involving this outcome
- Unit plan involving this outcome

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Emphasis should be placed on answering real questions and ways to organize and interpret the data collected.
- Model the formulation of questions such as, I wonder…, How can we find out?, or Whom shall we ask?
- Use everyday events to formulate questions and collect data about the children's environment such as How do you travel to school? Which kind of pizza did you order? Which author should we read this week?
- Model questions on the same topic in several ways and allow students to choose the best question for its purpose; for example, How did you travel to school today? or Did you walk to school today? or How many students in our class used the school bus today?
- Ensure that data management tasks are relevant and of interest to the students in your class. Each class member should be able to participate and contribute data to the investigation.
- Encourage students to conduct small surveys to collect data. Encourage students to organize the data as they collect it using tallies, checkmarks, charts, or lists.
- Make use of opportunities to integrate data collection into other subject areas, such as You and Your World.
- Read a book such as *The Best Vacation Ever* by Stuart J. Murphy (1997) or *Charlie's Checklist* by Rory S. Lerman (1997). Both books have characters that formulate questions and gather data. Discuss the questions chosen and the methods used to gather and record the data.

SUGGESTED LEARNING TASKS

- Ask students to collect data about the type of snacks brought for recess. Display that data as tally marks. Ask students questions that relate to the tally marks; for example, How many students brought an apple for recess? or How many more students brought apples than brought granola bars?
- Ask students to collect data about a question of interest to them. This may include questions such as, What is your favourite day of the week? What is your favourite type of book? What is your favourite colour? Ask students to display the data they collect using charts, tally marks, checkmarks, or lists. Ask students to create questions that relate to the data they have collected. Students can share their work with a partner, and the partner can interpret the data and answer the questions.
- Ask students to create a chart to show the colours of tops/shirts/blouses the students are wearing and how many students are wearing each colour. Complete the activity again but ask students to organize the data using tally marks instead of a chart. Ask students to compare the two data displays and to explain how they are the same and how they are different.
- Ask the student what is wrong with the following conclusion drawn from the tally marks below:
  Just as many kids like pizza as burgers.
  Pizza: ~~𝍸 𝍸~~
  Burgers: ~~𝍸~~

- Show students a set of data represented in two ways, such as tally marks and as a chart. Ask students to describe how the data is the same and how it is different.
- Show students a sheet of 2-D shapes including squares, triangles, rectangles, and circles. Show them three sets of tally marks meant to represent the number of each type of shape on the sheet. Ask students to identify which set of tally marks correctly represents the sheet of 2-D shapes.

SUGGESTED MODELS AND MANIPULATIVES

- colour tiles
- linking cubes
- links

MATHEMATICAL LANGUAGE

Teacher	Student
▪ collect/gather, organize, record ▪ data ▪ tallies, checkmarks, charts, lists, tables	▪ data ▪ tallies, checkmarks, charts, lists, tables

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 525–528
- *Making Math Meaningful to Canadian Students, K–8,* Second Edition (Small 2013), pp. 568–570
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 310–312, 317–319

Notes

SCO SP02 Students will be expected to construct and interpret concrete graphs and pictographs to solve problems. [C, CN, PS, R, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

SP02.01 Determine the common attributes of concrete graphs by comparing a given set of concrete graphs.
SP02.02 Determine the common attributes of pictographs by comparing a given set of pictographs.
SP02.03 Answer questions pertaining to a given concrete graph or pictograph.
SP02.04 Create a concrete graph to display a given set of data and draw conclusions.
SP02.05 Create a pictograph to represent a given set of data using one-to-one correspondence.
SP02.06 Solve a given problem by constructing and interpreting a concrete graph or pictograph.

Scope and Sequence

Mathematics 1	**Mathematics 2**	**Mathematics 3**
—	**SP01** Students will be expected to gather and record data about self and others to answer questions. **SP02** Students will be expected to construct and interpret concrete graphs and pictographs to solve problems.	**SP01** Students will be expected to collect first-hand data and organize it using tally marks, line plots, charts, and lists to answer questions. **SP02** Students will be expected to construct, label, and interpret bar graphs to solve problems.

Background

Concrete graphs organize objects into carefully aligned rows or columns in order that one-to-one correspondence can be used to compare quantities of objects in those rows or columns. The objects used can progress from the real objects to toy models of the real objects to representative concrete objects such as counters or cubes. These concrete graphs can lead to pictographs as pictures of objects are used rather than the real objects, and as a way to record concrete graphs on paper. When constructing concrete graphs and pictographs, model and discuss the importance of aligning objects accurately.

It is important to draw students' attention to the different layouts and formats of graphs. Students should create and interpret graphs that run horizontally and those that run vertically. It would be helpful to show the same data in two different formats.

Additional Information

See Appendix A: Additional Information.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Provide a set of prisms and pyramids. Ask students to sort the shapes into two groups. Ask students, Are there more prisms or pyramids in the set? Ask them to explain how they know.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Have students use colour tiles to create a concrete graph showing the number of boys and girls in the classroom. Ask them to use the graph to determine whether there are more girls or boys in class.
- Have students create a pictograph to show how many students in the class know how to swim (or skate).
- Provide students with a set of data about a topic of interest to them. Ask them to create a pictograph and a concrete graph that represents the data.
- Ask students to explain how they would use a set of data to create a pictograph.
- Show students a concrete graph or pictograph on a topic of interest to students. Have them answer questions about the graph and have them make up their own question that the graph would answer.
- Provide students with a set of concrete graphs or a set of pictographs, one of which has an error in it. Ask students to identify the error and to explain how to correct it.
- Present students with a pictograph or a concrete graph. Ask them to write three questions that could be answered by reading the graph. Ask them to write one question that could not be answered by reading the graph.
- Show students two graphs, one horizontal and one vertical, that represent the same data. Ask students to compare the two displays and to explain how they are the same.
- Provide students with a collection of about 25 linking cubes in three or four different colours. Ask them to organize the cubes and record the data in a chart using tally marks or another method. Provide them with grid paper and have them create a graph to display the data.
- Have students formulate their own question that can be answered by collecting information within the school. Students should be able to write their question, gather their data, display their information in the form of a concrete or pictograph, and write about what they have learned.

FOLLOW-UP ON ASSESSMENT

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

RESPONDING TO ASSESSMENT

Numeracy Nets K–2 (Bauman 2011)
- No Checkpoint for this outcome.

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning
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- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Emphasis should be placed on answering real questions and ways to present data and how to interpret the data that is collected. Provide meaningful opportunities for students to collect, represent, and interpret data; for example, ask students to vote on a class book to read, collect data on the number of sunny, cloudy, rainy, or snowy days in a particular month, or vote on games to play during indoor recess.
- Ensure that data management tasks are relevant and of interest to the students in your class. Each class member should be able to participate and contribute data to the investigation.
- Encourage students to conduct small surveys to collect and display data.
- Use a piece of vinyl and tape to create a floor mat grid on which children can stand to form a graph. Have students place name cards on the grid before they step off it so everyone can see the whole graph that was created.
- Make use of opportunities to integrate graphing concepts into other subject areas, such as You and Your World.

SUGGESTED LEARNING TASKS

- Construct two different pictographs. Have students compare the graphs and identify the common attributes.

- Construct two different concrete graphs. Have students compare the graphs and identify the common attributes.
- Ask students to arrange themselves into a concrete people graph to compare the number of students who are wearing shoes with laces to those wearing shoes with Velcro.
- Have students create three different representations of the same set of data. For example, ask them to glue coloured pasta on the first graph. On the second graph, ask them to draw and colour pasta pieces to represent the same information that is on the first graph. Lastly, ask them to represent the same data with tallies. Discuss how the three representations are the same and how they are different.
- Ask the student what is wrong with the following conclusion drawn from the graph below.
  Just as many kids like pizza as burgers.
  Pizza: □ □ □ □ □ □
  Burgers: □□ □ □
- Ask the students to create a pictograph to show the number of children in the class who play various games (sports) or musical instruments. Have them write two questions that their graph will answer.
- Show students a graph without a title or labels. Ask them to create different sets of data that the graph could represent.
- Show students a graph without a title or labels and have them match the graph to a set of tally marks.
- Attach a photograph of each child to a strip of magnetic tape. Have each student place his or her photograph on the whiteboard to create a graph.

SUGGESTED MODELS AND MANIPULATIVES

- colour tiles
- grid paper
- linking cubes
- links
- various objects to use in a concrete graph

MATHEMATICAL LANGUAGE

Teacher	Student
▪ compare, similar, different ▪ graphs: concrete/people, pictographs/picture ▪ one-to-one correspondence ▪ title, labels, columns, rows	▪ graphs: people, picture ▪ title

Resources/Notes

Print

- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 471–478
- *Making Math Meaningful to Canadian Students, K–8,* Second Edition (Small 2013), pp. 517–523
- *Teaching Student-Centered Mathematics, Grades K–3* (Van de Walle and Lovin 2006), pp. 310–312, 317–319

Notes

Appendices

Appendix A: Additional Information

Number (N)

SCO N01 Students will be expected to say the number sequence by
- 1s, forward and backward, starting from any point to 200
- 2s, forward and backward, starting from any point to 100
- 5s and 10s, forward and backward, using starting points that are multiples of 5 and 10 respectively to 100
- 10s, starting from any point, to 100

[C, CN, ME, R]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N01.01 Extend counting sequence (by 1s), forward and backward.
N01.02 Extend a given skip counting sequence (by 2s, 5s, or 10s) forward and backward.
N01.03 Skip count by 10s, given any number as a starting point.
N01.04 Identify and correct errors and omissions in a given skip counting sequence.
N01.05 Count a given sum of money with pennies, nickels, or dimes (to 100¢).
N01.06 Count quantity using groups of 2s, 5s, or 10s and counting on.

Performance Indicator Background

N01.01 It is important for students to learn the sequence of number names in advance of attaching meaning to these numbers. Students should be provided with experiences that enable them to rote count beyond 100 to 200. They should examine how the patterns of words said in the number sequence 100 to 200 are similar to the patterns from 0 to 100. A hundred chart can easily be extended to 200 to aid exploration of the counting sequence to 200.

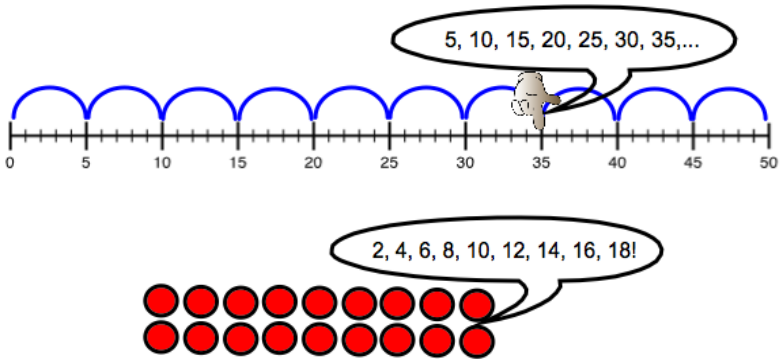
Students should be able to count forward and backward by 1s from any number in the interval 0 to 200, and extend a given forward and backward sequence. This will help to reinforce understandings of what comes next and what comes before in a given counting sequence. When exploring numbers between 0 and 200, considerable time should be spent focusing on the numbers between 100 and 200, so that students see that the pattern of the decades 10 to 90 repeats and the 1 to 9 patterns within each decade repeats. For example, they should see that 110, 120, 130, …, 190 is the same pattern as 10, 20, 30, …, 90 and that 130, 131, 132, 133, …, 139 is the same as 30, 31, 32, 33, …, 39. In reciting the numbers over 100, a common misconception for students is to think that 200 comes after 109 because of their experience with numbers to 100 where they observed that the next number after the one that ends in 9 is the next big number name, such as 30 after 29 and 50 after 49.

N01.02 Proficiency with skip counting will support later work with operations. In Mathematics 2, it is expected that students will develop the following skip counting skills:
- Counting by 2s, forward and backward, starting at any number within 0 to 100.

- Counting by 5s and 10s, forward and backward, starting from multiples of 5 and 10 within 0 to 100.
- Counting using coins (pennies, nickels, dimes).

Not only should students be able to respond to requests to skip count in any of these three ways, they should also be able to detect patterns in given sequences and extend those sequences. For example, if presented with the sequence 75, 70, 65, … , students recognize that there is a skip count by 5 backward and continue the sequence 60, 55, 50, 45, 40, …

It is helpful to students to use models, such as a number line or counters, as they are developing skip counting skills. With number lines, students need to realize that they are counting spaces, while with counters they are counting quantities.

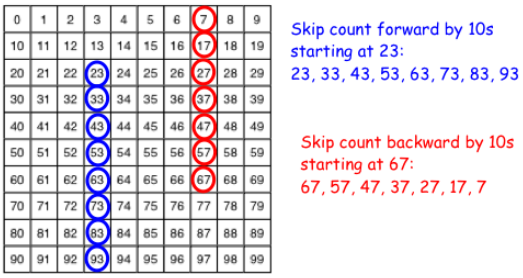


Students should be given frequent opportunities to develop these skills so they become proficient and are able to skip count without models.

When planning opportunities for skip counting, consider using appropriate related children's literature; having students walk on a number line or on a 100 mat; and using rhythmic skip counting activities such as clapping, marching, drumming with hands on the desk, and striking instruments. Students may also use the constant function on a calculator (press 0, +, 2, =, =, =, …) to skip count to a target number.

Students should be asked reflective questions to solidify their understanding of skip counting. For example, if you start at 0 and want to end at 40, by which number(s) could you skip count? (2, 5, 10) What if you started at a different point? What if you wanted to end at a different point?

N01.03 Students should be able to start at any number between 0 and 100 and skip count by 10s, forward or backward. This will help to develop strategies for addition and subtraction that will be used in later outcomes. A hundred chart is an especially helpful tool for supporting the development of skip counting by 10 from any number because they are able to see that from any starting point they simply continue to count down or up the column.



N01.04 Students should be presented with incorrect skip counting sequences and be asked to identify and correct errors; for example, Mary was saying the numbers 2, 4, 6, 8, 10, 11, 12, 14, 17, 20. Ask if Mary made any errors and, if so, what were they and what should she have said instead. Similarly, students should be presented with skip counting sequences with missing terms.

N01.05 One use of skip counting is for counting coins. Students need sufficient experience counting pennies, nickels, and dimes separately using skip counting by 5s and 10s.

N01.06 It is important that students understand that skip counting is a way to find out how many objects are in a set, and that they will get the same quantity as counting by 1s. It is important that students have many and varied experiences with materials that they can group and count in a variety of ways. These activities should be purposeful, and presented throughout the year as students build their sense of number.

"As students get older, the numbers they deal with in their everyday lives become more complex. Students need strategies for representing and making sense of these greater numbers. Although it is possible to count, say, 87 items individually, it is not practical. When items are grouped, counting is made easier and probably more accurate." (Small 2009, 138).

Provide students with a variety of activities that require students to recognize when it is more efficient to skip count quantities. For example, a student may be given a quantity of counters or blocks and asked to find a way to count them efficiently. Teachers should watch to see if students group in 2s, 5s, or 10s to make counting easier. Encouraging students to group to count will help them to appreciate grouping ideas that will be foundational to later place-value concepts.

SCO N02 Students will be expected to demonstrate if a number (up to 100) is even or odd. [C, CN, PS, R]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

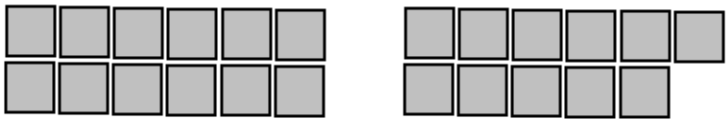
Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N02.01 Use concrete materials or pictorial representations to determine if a given number is even or odd.
N02.02 Identify even and odd numbers in a given sequence, such as on a hundred chart.
N02.03 Sort a given set of numbers as even numbers and odd numbers.

Performance Indicator Background

N02.01 It is important to provide students with a variety of concrete and pictorial representations of even and odd numbers. These concepts can be shown using counters. If all of the counters can be paired up, that number is even; if all the counters but one can be paired up, then the number is odd. In other words, students should discover

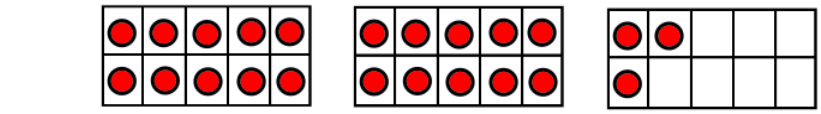
- even numbers can be used to make groups of two with none leftover or two equal whole number groups
- odd numbers can be used to make groups of two with one left over or cannot be shared into two equal whole number groups



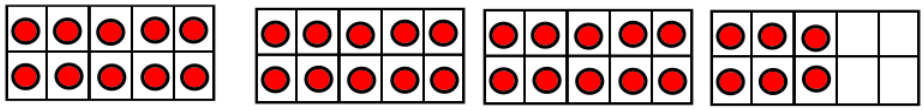
N02.02 Once students have experienced small even and odd numbers using concrete and pictorial models, the concept can be extended to larger numbers. Students should come to discover that even-number quantities have numerals that end in the digits 0, 2, 4, 6, or 8, and odd-number quantities have numerals that end in the digits 1, 3, 5, 7, or 9. Students should arrive at these generalizations through hands-on experiences. Students will learn that even numbers are the numbers they say when they count by 2s from 0. Odd numbers are the numbers they say when they count by 2s from 1.

Often students will mistakenly consider the tens digit when looking at the symbolic representation of numbers, such as 14, 23, and 55, to decide whether it is even or odd. An important concept in understanding even and odd is that the ones digit is the determining feature. By examining the hundred chart and using models to consider whether quantities can be shared in two equal groups, students will come to discover that the tens digit does not affect the evenness of a number; the ones digit will be the deciding factor.

A possible way to clarify the misconception regarding the tens digits in 2-digit numbers is to encourage students to represent the number with ten-frames or square tiles to determine if it can be shared in two equal parts.



23 is odd, it cannot be organized into two equal rows.



36 is even, it can be organized into two equal rows.

Note: Since using counters to convince students that zero is even is too abstract, noting its position on the hundred chart in an even column and having an odd number as a neighbour introduces this idea more convincingly.

N02.03 Students should be able to take a given set of numbers and sort them into even and odd numbers and be able to explain how they determined the sort.

SCO N03 Students will be expected to describe order or relative position using ordinal numbers (up to tenth). [C, CN, R]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N03.01 Indicate a position of a specific object in a sequence by using ordinal numbers up to tenth.
N03.02 Compare the ordinal position of a specific object in two different given sequences.

Performance Indicator Background

N03.01 Students should be able to identify orally, in words, and using symbols, the relative position of objects or events in sets that have been ordered by different criteria. They should also identify the object or event that corresponds to a stated ordinal number.

While calendars show cardinal numbers, they are actually ordinal numbers because the numbers refer to the days' positions within the month; for example, we write November 10, but say November tenth. Students will experience ordinal numbers up to 31st through exposure to the calendar; however, they need only be assessed on ordinal numbers to tenth.

N03.2 Students should be asked to identify the position of the same object in different sequences so that they realize the ordinal number is not dependent upon the object itself, rather its position in a sequence. Therefore, the ordinal number used will change if the sequence changes.

SCO N04 Students will be expected to represent and partition numbers to 100. [C, CN, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N04.01 Represent a given number using concrete materials, such as ten-frames and base-ten materials.
N04.02 Represent a given number using coins (pennies, nickels, dimes, and quarters).
N04.03 Represent a given number using tallies.
N04.04 Represent a given number pictorially.
N04.05 Find examples of a given number in the environment.
N04.06 Represent a given number using expressions (e.g., 24 + 6, 15 + 15, 40 – 10)
N04.07 Read a number (0–100) given in symbolic or word form.
N04.08 Record in words a given number (0–20).
N04.09 Record, symbolically, any number (0–100).

Performance Indicator Background

N04.01 Students need to represent and describe numbers using a variety of concrete materials, such as wooden stir sticks, straws, beans, counters, ten-frames, and books. Students may bundle a quantity of stir sticks or toothpicks to show partitions of a given quantity. For example, a student may take 50 stir sticks and show this can be bundled in five groups of 10, 10 + 10 + 10 + 10 + 10. They may also group the bundles of ten to show that it could be expressed as 30 + 20 or 10 + 40. They may also open one bundle and show that 50 could be expressed as 25 + 25 or 21 + 29.

After experiences with groupable materials bundled in 10s, students can be introduced to base-ten blocks. These commercially made materials differ from the counters and bundled materials because they involve trading a set of ten 1s (small cubes) for a single representation of ten (rod). Take time to discuss this difference with students and introduce these materials carefully.

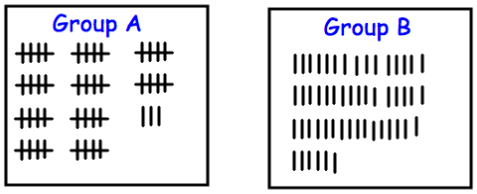
N04.02 Counting coins is an important skill for students to develop and also provides an everyday context for representing and partitioning a number. Students will need to be introduced to the names and values of pennies, nickels, dimes, and quarters in order to be able to use them in their representations.

Students could be asked to show different ways to make 78¢ with coins. They can describe how the coins show different partitions of the number. Students should be given numerous opportunities to count coins in context and discuss various ways to partition amounts using coins. The emphasis should be on using whole numbers with the ¢ symbol and not decimal numbers with the $ symbol.

Note: Most materials used to represent numbers have a visible one-to-one correspondence with the quantity they represent. Coins do not. Each coin represents a value that is not easily seen or proportional to its value.

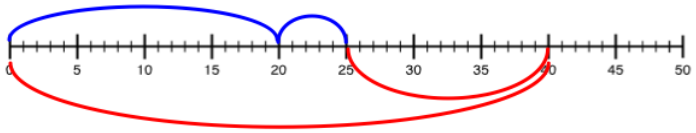
N04.03 Students should be introduced to tally marks as a way to count in groups of 5. This can be especially useful when students are counting larger objects that cannot be easily moved into nice arrangements of equal groups. For example, students may use tally marks while counting cars in a

parking lot, posters in the school hallways, or windows in the school. Students should be shown how to make tally marks and should discuss why counting in this way makes counting easier. For example, students may be asked to talk about why the 53 tally marks in group A are easier to count than those in group B.

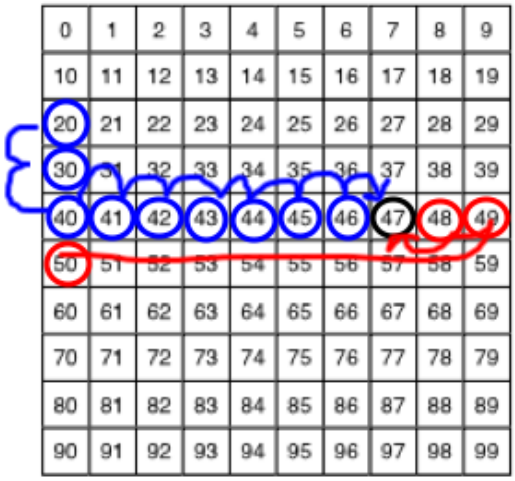


N04.04 Students should be able to use pictorial models to represent and partition number. Initially, students will use pictures to record their concrete models. Additional pictorial representations can be added throughout the year.

Both number lines and hundred charts are important pictorial models for students to use when partitioning numbers. For example, a student may show that 25 can be represented on the number line by a jump of 20 followed by a jump of 5 (20 + 5) or a jump of 40 followed by a jump back of 15 (40 – 15).



Similarly, a student may show that to get to 47 in a hundred chart a student may start at 20 and jump down 20 and over 7, showing that 47 can be represented by 20 + 27. Similarly a student may say that if you start at 50 and go back 3 you will also get to 47, which can be represented by the expression, 50 – 3.



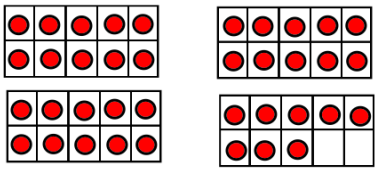
0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

N04.05 An important part of number development is to be able to identify and explain how numbers and quantities are used in everyday contexts. Students should be able to identify real-world examples of where they may see quantities of 0 to 100 objects. For example, they may be able to say that there were about 50 fans at the hockey game on the weekend, or they saw 10 to 15 ducks on a pond.

Students should also discuss numbers they see being used in their environment that are not connected to quantity, such as house numbers, sports jerseys, and clothing sizes.

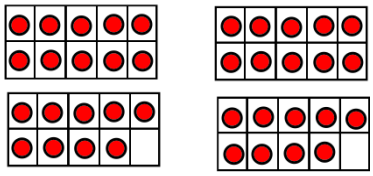
N04.06 Students should represent a quantity in a variety of ways using partitioning. For example, a student may be asked to show 38 in two separate parts using counters and ten-frames and may show this in one of the two ways below:

a)



38 can be shown as 20 and 18.

b)



38 can be shown as 19 and 19.

Addition and subtraction symbols were introduced to students in Mathematics 1. Students should be encouraged to use words and symbolic notation to describe what they have done concretely or pictorially. There is no expectation that students should be able to partition numbers symbolically without having used concrete or pictorial models first. For example, a student may partition 63 objects, saying and writing that 63 can be shared in a group of 30 and a group of 33 before recording this symbolically as 30 + 33. Also a student may use a number line and say and write that jumping forward 70 and back 7 will also get us to 63 before writing 70 – 7.

N04.07 Students should be given lots of exposure to the written form of numerals to 100. This may be in the form of written instructions on how to complete tasks, charts displayed in the classroom with pictorial and symbolic representations, or as entries into a personal mathematics dictionary or on a classroom mathematics word wall.

N04.08 Provide ample opportunities for students to record number words up to 20. For example, there are seventeen boys in the classroom. Number words are often written in contexts of language arts, science, and social studies.

SCO N05 Students will be expected to compare and order numbers up to 100. [C, CN, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N05.01 Compare and order a given set of numbers in ascending or descending order and verify the result using a hundred chart, number line, ten-frames, or by making references to place value.
N05.02 Identify errors in a given ordered sequence.
N05.03 Identify missing numbers in a given hundred chart.
N05.04 Identify errors in a given hundred chart.

Performance Indicator Background

N05.01 Students should develop strategies for ordering numbers. Seeing the relative position of numbers on number lines and in hundred charts will help students to solidify their understanding of the order and size of numbers. Students should know that as they count forward, the numbers increase and the quantity they represent is greater. Students may use a hundred chart to show that 87 is more than 63 because 87 is in the 80s and 80 is more than 60.

An open number line (no marked increments) provides opportunity for students to refine their knowledge of number relationships. Students may be asked to place the numbers 49, 18, 25, 37, and 42 on a number line where 0 and 50 are the only numbers shown. Students will need to use benchmarks and what they know about the relative position of numbers to accurately place numbers on that number line.

Another strategy for ordering numbers is to consider place value. Students may refer to the number of tens when ordering numbers. For example, 47 is more than 21 since all numbers in the 40s are greater than all numbers in the 20s. Students should focus on the fact that the digit 4 in 47 has a value of 40 and the digit 2 in 21 has a value of 20. This knowledge will emerge from having represented numbers using ten-frames and groupable materials.

N05.02 Students should be able to identify when a given sequence of numbers is not in the correct order and be able to correct it. For example, they may be told that a student was asked to sort numbers in ascending order and gave the following sequence: 13, 17, 26, 24, 28, 42, 38, 56. They should be able to identify which numbers are not in the correct order and rearrange them. They should be encouraged to talk about how they made their corrections.

N05.03 Students should have enough familiarity with the hundred chart that they are able to identify the values of missing numbers. A student could be given a hundred chart with numbers missing and be asked to fill in the missing values and to explain how they decided what number went in each empty position. They may also be given a hundred chart, cut into pieces, and be asked to put it back together in the correct way.

N05.04 Students should be able to identify errors in a hundred chart and explain why they believe that there are errors. For example, as a routine morning activity, display a hundred chart with individual numbers in the wrong positions, or entire decades in the wrong order. Have students come up one at a time and identify the error and explain to the class why it is wrong. Students should correct the hundred chart as they explain the error.

SCO N06 Students will be expected to estimate quantities to 100 by using referents. [C, ME, PS, R]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

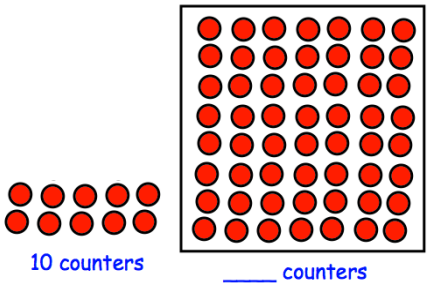
Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N06.01 Estimate a given quantity by comparing it to a referent (known quantity).
N06.02 Estimate the number of groups of ten in a given quantity using 10 as a referent.
N06.03 Select between two possible estimates for a given quantity and explain the choice.

Performance Indicator Background

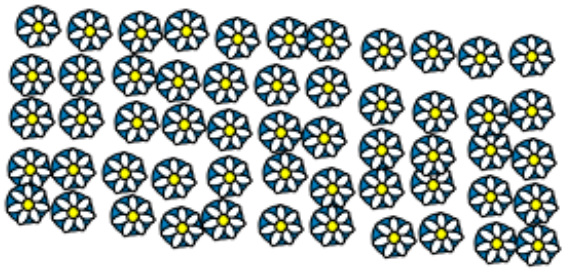
N06.01 A referent (a known quantity) is useful as a benchmark or anchor in the development of estimation skills. Students use referents to determine the amount in a large group of similar objects. For example, if I know what a group of 5 people looks like, I can estimate the number of people in a classroom. Ask students to explain their decision. Dot cards or similar models are helpful in providing visual referents and building estimation skills. Subitizing quantities addressed in Mathematics Primary and Mathematics 1 should be continued as they provide helpful referents.

N06.02 To estimate quantities to 100, 10 is an important referent. For example, if the small picture shows 10 counters, how many counters are in the larger picture?



Students need a strong sense of 10-ness in order to use ten as a referent. Provide opportunities for students to see 10 in a variety of different contexts and arrangements; for example, 10 people, 10 chairs, 10 counters.

N06.03 Students should also be able to judge the reasonableness of an estimate and be able to explain the reasons for their choice. For example, they may be asked to say whether 50 or 80 would be a good estimate for the collection of flowers below. To justify their estimate, a student may circle the first two columns (ten flowers) and use this as a referent.



Students should be provided with numerous opportunities to choose between two possible estimates for a given set of objects and to explain how they have chosen their estimate. The more they do this, the more refined their estimates will become.

SCO N07 Students will be expected to illustrate, concretely and pictorially, the meaning of place value for numerals to 100. [C, CN, R, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

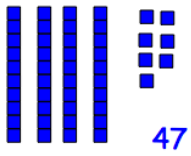
Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N07.01 Explain and show with counters the meaning of each digit for a given 2-digit numeral with both digits the same.
N07.02 Count the number of objects in a given set using groups of 10s and 1s, and record the result as a 2-digit numeral under the headings of 10s and 1s.
N07.03 Describe a given 2-digit numeral in at least two ways.
N07.04 Illustrate using ten-frames and diagrams that a given numeral consists of a certain number of groups of ten and a certain number of ones.
N07.05 Illustrate using proportional base-ten materials that a given numeral consists of a certain number of tens and a certain number of ones.
N07.06 Explain why the value of a digit depends on its placement within a numeral.
N07.07 Represent one unit if shown a pre-grouped model representing ten.

Performance Indicator Background

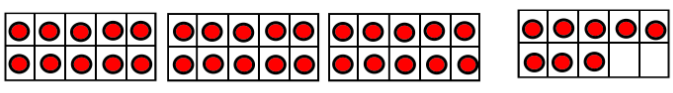
N07.01 Students should be given many opportunities to build groupable models of 2-digit numbers in which the digits are the same and explain which parts of the model represent the tens and which parts represent the ones. For example, a student may build 22 by showing 2 full ten-frames and 2 extra counters on an additional ten-frame. They should be able to explain that the 2 full ten-frames represents the 20, shown by a 2 in the tens place, and the 2 counters represent the 2 in the ones place. They may also use bundled sticks to show 22 by showing that there are two full bundles of 10 and 2 extra sticks.

N07.02 Students need to be able to count quantities by groups of tens and ones and be able to explain how many tens and how many ones the number represents. For example, if students are shown the base-ten model below they should be able to identify the quantity by counting the rods in 10s (10, 20, 30, 40) and then counting the small cubes (41, 42, 43, 44, 45, 46, 47). They should have opportunities to record this in a chart under 10s and 1s, and as the number 47.



10s	1s
4	7

Students need to see different models of 2-digit numbers and become comfortable with using counting strategies learned in SCO N01 to count the quantity and discuss what this tells them about the place value of each digit. For example, they may be presented with ten-frame models or bundled sticks. They should be able to identify the quantity of tens and ones in each model.

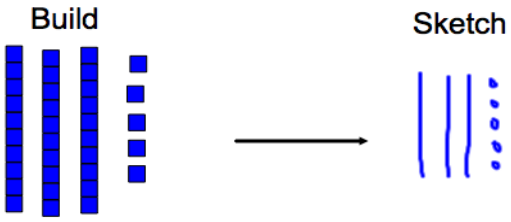


38 is 3 sets of 10 and 8 more.

N07.03 When students understand place value they are able to think of a number, such as 37, not only as 3 tens and 7 ones or 30 + 7, but also as 2 tens and 17 ones or 20 + 17, and as 37 ones. Students should be asked to model 2-digit numbers in at least two different ways. This connects place value concepts to work done in SCO N04 with partitioning. This flexibility with numbers will provide good preparation for later work with operations. For example, a student should be able to show and explain why 43 can be shown using 4 tens and 3 ones or as 3 tens and 13 ones. This may be needed later in subtraction situations such as 43 – 37 or 43 – 15.

N07.04 Students should be able to sketch models and diagrams to show how a certain numeral can be shown in groups of ten and groups of one. Ten-frames can be very effective ways for students to sketch a model for a 2-digit number, such as the model for 38 shown above. These ten-frame models clearly show groups of ten and ones.

Students should also develop strategies for sketching base-ten models. Rather than spending time drawing ten small cubes attached together to represent a rod, students can simply draw a line. To represent a small cube, they can draw a circle or a large dot.



Of course, such representational pictures often do not visually show the relationship between a rod and a small cube. Therefore, careful consideration should be given as to the best time to introduce such pictures.

N07.05 Students should have plenty of opportunities to manipulate and work with base-ten blocks to represent numbers to 100. Have students model 2-digit numbers in various ways, such as 46 as 46 ones, or 4 tens and 6 ones, or 3 tens and 16 ones. Ask students to find out how many ways they can make a given number using base-ten blocks.

N07.06 Students should be able to say that 45 can be modelled with four groups of 10 and five 1s. They should be able to say that the number in the tens place represents 40 and the number in the ones place represents 5. This will enable students to develop the understanding that the position of a digit matters and they will be able to explain why the value of a digit depends on its placement within a numeral. For example, they should be able to use this reasoning to explain the difference between the 2 in 42 and the 2 in 27.

SCO N08 Students will be expected to demonstrate and explain the effect of adding zero to or subtracting zero from any number. [C, R]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N08.01 Add zero to a given number and explain why the sum is the same as the addend.
N08.02 Subtract zero from a given number and explain why the difference is the same as the given number.

Performance Indicator Background

N08.01 To experience the no-change nature of adding zero, students should be encouraged to think about building a set of a certain amount and then adding zero to it to see that the amount remains unchanged. They can also stand on a certain spot on a number line and take no steps forward; they have not moved position. These experiences show that adding zero to a quantity results in no change, such as 12 + 0 = 12 and 25 + 0 = 25.

N08.02 Students need similar experiences with subtracting zero. They should be able to explain that if they build a set of a certain amount and take away zero, the quantity stays the same. If a student is standing on a number line at 6 and does not take any steps backwards, he or she stays on the same spot, and 6 – 0 = 6 represents this action. Students should be able to record subtraction of zero as a no-change nature such as 37 – 0 = 37 and 21 – 0 = 21. In subtraction, students can also consider the difference between a number and zero. This difference meaning of subtraction provides an alternative way for students to think about subtracting zero. For example, if one student is standing at 0 on the number line and another student is standing at 6, the students are 6 units apart, and 6 – 0 = 6.

SCO N09 Students will be expected to demonstrate an understanding of addition (limited to 1- and 2-digit numerals) with answers to 100 and the corresponding subtraction by

- using personal strategies for adding and subtracting with and without the support of manipulatives
- creating and solving problems that involve addition and subtraction
- explaining and demonstrating that the order in which numbers are added does not affect the sum
- explaining and demonstrating that the order in which numbers are subtracted matters when finding a difference

[C, CN, ME, PS, R, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N09.01 Solve a given story problem of any type by modelling it with materials or a diagram, and write a number sentence that represents the thinking in the solution.

N09.02 Solve a given story problem of any type by writing a number expression and combining the numbers to complete the number sentences.

N09.03 Match a number sentence to a given story problem.

N09.04 Create an addition or a subtraction number sentence and a story problem for a given solution.

N09.05 Model addition and subtraction using concrete materials or visual representations and record the process symbolically.

N09.06 Add a given set of numbers in two different ways and explain why the sum is the same.

N09.07 Recognize and create equivalent addition and subtraction number sentences.

Performance Indicator Background

N09.01 Students should be presented with many opportunities to model story problems of all types (see chart), and find solutions while modelling using counting, addition, and/or subtraction. They should be able to write appropriate number sentences that reflect how they thought about the problems. Through sharing of strategies and discussions, students will begin to realize that the same situation, such as a join (change unknown), can be thought about as addition by some students and subtraction by others.

For example, consider the following story problem. Pat has 8 marbles but she would like to have 12. How many more does she need to get? Some students may count out 8 marbles and add on until they reach 12, counting the 4 that they added on. These students would write 8 + 4 = 12 to represent how they thought about the problem. Other students may count out 12 marbles, remove the 8 they knew they had, and count the 4. These students would write 12 – 8 = 4 to represent how they thought about the problem. Either sentence would be acceptable, but they should be able to explain the number sentence they have written in relation to the situation they are modelling.

After students have modelled and solved a number of addition and subtraction situations, they may be introduced to pictorial representations such as those described below in performance indicator 9.05. As well, Van de Walle and Lovin offer four possible pictorial representations for addition and subtraction.

Refer to *Teaching Student-Centered Mathematics, Grades K–3*, Volume 1, page 67 (Van de Walle and Lovin 2006) for an example.

In addition to the pictorial representations described by Van de Walle and Lovin, students may be introduced to strip diagrams as another way to represent the situations. For example, Bobby was given 6 green stamps. He already had 12 stamps. How many does he have now? The strip diagram for this problem is

12	6
?	

For example, Bobby had 9 stamps. He was given more by his friend. Then he had 13 stamps. How many stamps did his friend give him? The strip diagram for this problem is

9	?
13	

Because students have to decide where to place in the diagram the two given numbers in the story problem, they have to carefully read the problem to determine whether each given quantity is a part or a whole. If the quantity is a part, it would be placed in one section of the top rectangle; if the quantity is a whole it would be placed in the bottom rectangle. They should put a question mark in the bottom rectangle or one of the sections in the top rectangle, depending upon what is missing (what they are asked to find).

The principal use of strip diagrams is as a strategy to help students interpret story problems. Students will solve the problems using their personal strategies; however, through extensive use of strip diagrams some students may generalize that subtraction is the operation that will always find a missing part and that addition will always find a missing whole.

Students in Mathematics 2 may be less familiar with the types of comparison problems than the other types of problems. As such, these comparison problems should be the subject of specific lessons and discussions. Subtracting two quantities to find how much more one quantity is than another is conceptually difficult for many students because they associate “moreness” with addition, and subtraction with “take away” rather than “difference.”

N09.02 By the end of Mathematics 2, after many experiences modelling story problems of all types, students should be able to read some story problems, write open number sentences, and work with the numbers to solve the problem. For example, consider the following story problem. Pat has 28 marbles. Her brother gives her 14. How many does she have now? Students should be able to read this problem, write 28 + 14 = __ without modelling the story, and combine the 28 and 14 using their personal strategies to get the answer 42.

N09.03 When presented with a story problem of any type, students should be able to match a number sentence to the story problem. For example, read students a story problem that includes a missing addend such as, There were 9 students that drank milk during recess. If 3 students drank white milk, how many had chocolate milk? Then display two or three number sentences on the board or on chart paper.

9 + 3 = 12
12 – 3 = 9
9 – 3 = 6

Have them match the appropriate number sentence with the problem. Encourage students to discuss their strategies to solve the problem.

N09.04 Creating story problems is an important part of the development of students' understanding of addition and subtraction. As students create their own number stories, they may use different materials, such as snap cubes, ten-frames, toys, or other students to model the situations. If they are told that the solution to a story problem is a specified number, such as 12, they should be able to create addition and subtraction number sentences and corresponding story problems involving this number as the solution.

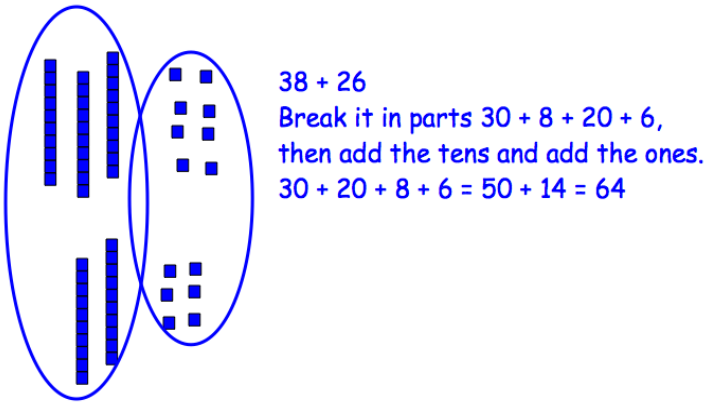
Students will need directed lessons to create story problems of various types. For example, students could be given four different story problems of one type, such as join (start unknown), asked to solve them, and discuss how the four problems are alike. Then they should create other story problems that are like these four.

N09.05 It is essential that students model addition and subtraction using a variety of models that may include counters, base-ten materials, number lines, and hundred charts. The more models students become comfortable with, the more flexibility they have with open number sentences. Students should not be taught a variety of strategies and be expected to memorize them; rather, they should develop their own personally meaningful strategies through exploration, modelling, and sharing. The strategies should emerge from the models students use through focused questions and discussions.

Initially, students will get the answer using models but may not be clear on what strategy they used. Encourage students to use symbols to record what they did with the models, so there is a strong connection between the symbolic and concrete representations. This will encourage students to reflect on what they are doing in order to realize that they are using a strategy that could be applied to the addition or subtraction of any two numbers.

While there are many strategies that could be used for addition and subtraction, two possible strategies for each operation are illustrated below.

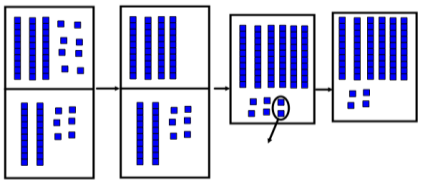
A student may use base-ten blocks to model 38 + 26 as shown below.



This method requires place value understanding and should be developed with consistent use of concrete models.

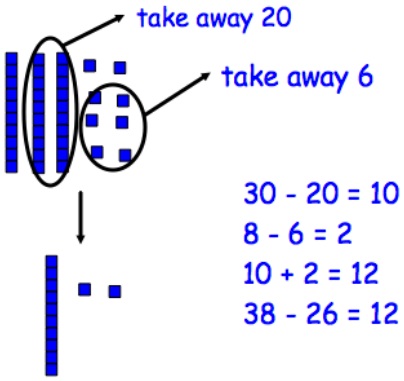
Another student may use a compensating (making nice or friendly numbers) strategy.

Students need to use their knowledge of compatible number pairs for 10 to use this strategy.

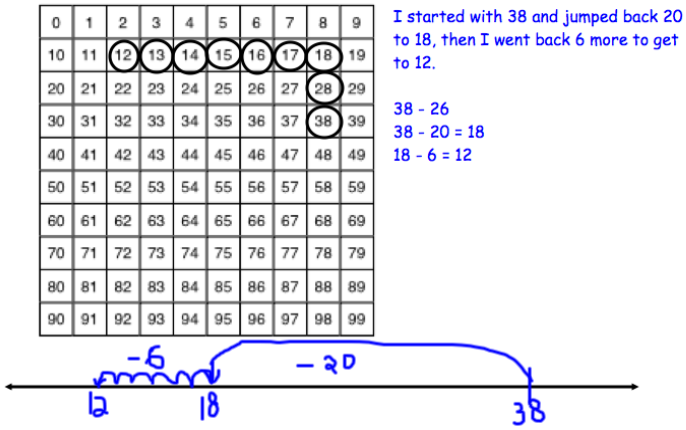


I noticed that 38 is almost 40 so I added 40 + 26, but this was 2 too many so I took 2 away.
38 + 26 = 64 because 40 + 26 = 66 and 66 - 2 = 64

Students may use base-ten blocks and place-value strategies to subtract the tens and subtract the ones. This strategy is most efficient when no regrouping is required. When regrouping is required, students will need to recognize that a trade of one ten for ten ones may be needed. In the example to the right, the student is modelling 38 – 26 using base-ten models, and started by showing 38. They have recorded the calculation to the side.



Another approach to solving this problem involves taking away 10s, then 1s. This can be modelled using the number line or hundreds chart.



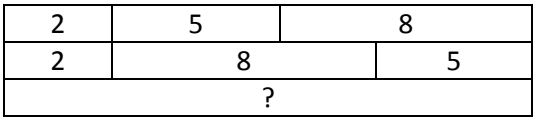
0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

I started with 38 and jumped back 20 to 18, then I went back 6 more to get to 12.

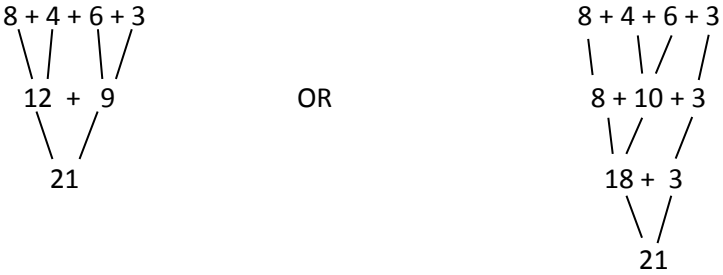
38 - 26
38 - 20 = 18
18 - 6 = 12

When two 2-digit numbers are added and subtracted, standard algorithms may become part of the discussion, if students share how their parents do it. While these standard algorithms may be shared and discussed, there is no expectation that all students should use these strategies for all addition and subtraction questions. The goal is to have students develop a repertoire of personal strategies and be flexible, efficient, and accurate in their application. In Mathematics 2, strategies that use the quantity represented by the tens digit (as in the examples above) should be given emphasis.

N09.06 Students should understand and use the fact that two numbers can be added in any order (commutative property of addition). For example, once they find 45 + 36 is 81, they should realize that 36 + 45 is also 81. Students should extend this understanding that two addends can be combined in any order to situations that have three or four addends. The focus should continue to be that the order in which you add does not matter. For example, 2 + 5 + 8 could be added left to right, 2 + 5 is 7 and 7 + 8 is 15 or it can be thought of as 2 + 8 + 5, so again going left to right 2 + 8 = 10 and 10 + 5 is 15. This may be modelled with Cuisenaire rods or illustrated with a strip diagram to see that the order does not matter.



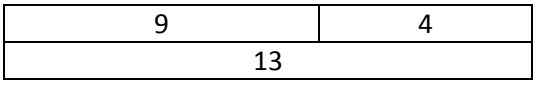
Students should be able to explain that, in addition, the addends can be added in any order and the result will be the same. The associative property tells us that three or more addends can be grouped in different ways to make adding easier. For example, 8 + 4 + 6 + 3 can be thought of in both of the following ways:



As with order, students should be able to explain that different groupings will not change the sum.

It is not necessary that students use the vocabulary **commutative property** or **associative property**; they should understand and apply these to make adding more efficient. For example, they may combine these two properties by changing the order to group friendly (compatible) numbers.

N09.07 Through sharing and discussion of story problems, students should realize that often the same problem may be thought of as an addition situation by some students and as a subtraction situation by others. Strip diagrams provide a graphic illustration of the relationship between addition and subtraction. There are four number sentences that can be written from a given strip diagram.



For example, for this strip diagram, the possible number sentences would be 9 + 4 = 13, 4 + 9 = 13, 13 – 4 = 9, 13 – 9 = 4. These four number sentences that represent the relationships among the three numbers are said to be equivalent.

When presented with a set of number sentences, students should be able to recognize those that are equivalent and those that are not. Given a number sentence, students should be able to write other number sentences that are equivalent to it.

SCO N10 Students will be expected to apply mental mathematics strategies to quickly recall basic addition facts to 18 and determine related subtraction facts. [C, CN, ME, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N10.01 Explain the mental mathematics strategy that could be used to determine basic addition facts.
- Doubles Facts
- Plus-One Facts
- One-Apart (Near Doubles) Facts
- Plus-Two Facts
- Plus-Zero Facts
- Make-10 Facts
- Two-Apart Facts
- Plus-Three Facts

N10.02 Use and describe a personal strategy for determining a sum to 18.
N10.03 Quickly recall basic addition facts to 18 in a variety of contexts.
N10.04 Explain the think-addition strategy used to determine a basic subtraction fact.
N10.05 Use and describe a personal strategy for determining the subtraction facts.

Performance Indicator Background

N10.01 , N10.02, and N10.03 The following strategies are presented in a sequence for introduction, reinforcement, and subsequent integration with previous strategies. In this provided sequence, all facts involving zeros are left until 51 other facts are known. This approach is partly for motivational reasons because students get to add 19 relatively easy facts to their charts to get a total of 70. This approach is also suggested to help counter a common misconception that causes many students to make errors with these facts—they overgeneralize that addition involves an action that results in a larger quantity; thus, they are reluctant to give a response that shows no change. As well, addition sentences with zeros are not likely encountered in response to story problems or in situations in their everyday lives.

DOUBLES FACTS

There are nine *doubles* facts: 1 + 1, 2 + 2, 3 + 3, 4 + 4, 5 + 5, 6 + 6, 7 + 7, 8 + 8, 9 + 9.
The strategy for the double facts is to use association to common things with which students have experience. The table below gives examples of possible associations. Use these or others with which students can strongly identify.

Double Fact	Possible Association
1 + 1	Number of tires on two unicycles.
2 + 2	Number of tires on two bicycles. Number of eyes on two people.
3 + 3	Number of tires on two tricycles. Number of sides on two triangles. Six pack of pop.
4 + 4	Number of tires on two cars. Number of sides on two squares. Number of legs on a spider.
5 + 5	Number of fingers on two hands.
6 + 6	Dozen eggs in a carton.
7 + 7	Number of days in two weeks.
8 + 8	Number of crayons in two rows in a box. Number of legs on two octopuses.
9 + 9	Number of tires on an 18-wheeler truck.

PLUS-ONE FACTS

There are 16 *plus-one* facts: 2 + 1, 3 + 1, 4 + 1, 5 + 1, 6 + 1, 7 + 1, 8 + 1, 9 + 1 and their commutative pairs 1 + 2, 1 + 3, 1 + 4, 1 + 5, 1 + 6, 1 + 7, 1 + 8, 1 + 9 (**Note:** 1 + 1 was already a double but could also be thought of as next number.)

These strategies are learned with a call for the next number. That is, whenever students see a 1 in an addition phrase, they should look at the other number and think, What number comes after this number? For example, for 6 + 1, think, The number after 6 is 7.

Time should be spent first with the set of ___ + 1 facts, and then with the set of 1 + ___ facts, being careful that students are convinced about the commutative nature of addition. Finally, mix both sets.

ONE-APART (NEAR-DOUBLES) FACTS

There are 14 *near-doubles* facts: 2 + 3, 3 + 4, 4 + 5, 5 + 6, 6 + 7, 7 + 8, 8 + 9 and their commutative pairs 3 + 2, 4 + 3, 5 + 4, 6 + 5, 7 + 6, 8 + 7, 9 + 8. (**Note:** 1 + 2 and 2 + 1 were already included in the plus-one facts but could also be thought of as 1-aparts.)

This strategy is a combination of the doubles facts and the plus-one facts. It involves doubling the smaller number and adding one. (Some students may double the larger and subtract 1.) Students need to be convinced that the larger number can be partitioned without changing the sum. For example, they need to see that for 4 + 5, the 5 can be partitioned into 4 and 1, and that the 4 can be combined with the other 4 before adding the 1. For example, for 2 + 3, think, double 2 is 4 and the next number is 5; or 2 and 2 is 4, and plus 1 is 5.

PLUS-TWO FACTS

There are 12 *plus-two* facts: 4 + 2, 5 + 2, 6 + 2, 7 + 2, 8 + 2, 9 + 2 and their commutative pairs 2 + 4, 2 + 5, 2 + 6, 2 + 7, 2 + 8, 2 + 9**.** (**Note:** 1 + 2, 2 + 2, 3 + 2, 2 + 1, and 2 + 3 have already been included in other strategies but could also be thought of as next even or odd numbers.)

This strategy involves associating the addition of 2 with skip counting by 2, or with getting the next even or odd number. You could help students get ready for this strategy by conducting skip-counting chants starting at different numbers. For example, for 5 + 2, think, 5 is an odd number and the next odd number is 7; or skipping 2 starting at 5 is 7.

PLUS-ZERO FACTS

There are 19 *plus-zero* facts: 0 + 0, 1 + 0, 2 + 0, 3 + 0, 4 + 0, 5 + 0, 6 + 0, 7 + 0, 8 + 0, 9 + 0 and their commutative pairs 0 + 1, 0 + 2, 0 + 3, 0 + 4, 0 + 5, 0 + 6, 0 + 7, 0 + 8, 0 + 9.

This strategy involves the association of adding zero with making no change to the other addend. This should be introduced by reference to story problems that would be represented by the addition of zero. For example, I had 5 stickers. I met my friend who would not give me any more stickers. How many stickers did I have after I met my friend? Stories, such as this one, are not likely to have been encountered before, and while students probably think of them as silly, they do make the point about the role of zero in addition. For example, for 5 + 0, think, adding zero will make no change to 5, so the answer is 5.

When reinforced in isolation, these facts are not problematic; however, when integrated with other facts, some students treat them as if they were plus-one facts because they overgeneralize that addition is an action that makes a larger quantity and are reluctant to record no change as a result. Consequently, you will probably not have to spend much time reinforcing these 19 facts in isolation; rather, more time should be spent reinforcing the total of 70 facts.

MAKE-10 FACTS

There are 10 *Make-10* facts involving 9: 9 + 3, 9 + 4, 9 + 5, 9 + 6, 9 + 7 and their commutative pairs 3 + 9, 4 + 9, 5 + 9, 6 + 9, 7 + 9.

There are 8 *Make-10* facts involving 8: 8 + 3, 8 + 4, 8 + 5, 8 + 6 and their commutative pairs 3 + 8, 4 + 8, 5 + 8, 6 + 8.

There are 6 *Make-10* facts involving 7: 7 + 3, 7 + 4, 7 + 5 and their commutative pairs 3 + 7, 4 + 7, 5 + 7.

This strategy involves taking 1 or 2 or 3 from one addend to make the 9 or 8 or 7 addend a 10, and then adding this 10 to what was left from the other addend. A strategy is to visualize a ten-frame and note how many it would take to make 10, and then add on any leftovers from the other number.

Students need to be convinced that adding 9 or 8 or 7 is as easy as adding 10. Start with the facts that involve 9s, and reinforce them. Once the response time is achieved for these facts, move on to isolate the facts involving 8s. Once the response time for these facts is achieved, move on to isolate the facts involving 8s. Finally, isolate the facts involving 7s.

Instead of this Make-10 strategy, some students may naturally add 10, instead of 9, to the other number and then adjust the answer by subtracting 1. It is not necessary that students know the name of this strategy; however, it is the Compensation Strategy.

TWO-APART FACTS

There are 6 *two-apart* facts: 3 + 5, 4 + 6, 5 + 7 and 5 + 3, 6 + 4, 7 + 5.

If 1 from the larger of the two numbers is transferred to the smaller, a double is produced; this double is the number between the two given numbers. If students made Unifix towers for the two numbers, they could move one cube from the larger to the smaller and see the resultant double. Some students have trouble with this strategy because both numbers change so they are doubling a number that they don't actually see in front of them.

Another strategy for these facts involves removing 2 from the larger to make a double of the smaller, so the smaller is doubled and the 2 is added back on. Again, if students used Unifix cubes to model this strategy, they would see the double plus 2.

PLUS-THREE FACTS

There are 6 *plus-three* facts: 5 + 3, 6 + 3, 7 + 3 and 3 + 5, 3 + 6, 3 + 7.

A strategy for these facts is to add the 3 in two steps: first add 2 and then add 1.

N10.04 and N10.05 In Mathematics 1, through visualization of ten-frames, students learned to subtract with minuends of 10 or less. This strategy involves visualizing the ten-frame dot configuration for a number, mentally removing the required number of dots, visualizing the resultant dot configuration, and naming this number. For some facts with minuends of 5 or 10, some students may just visualize the ten-frame configuration of the subtrahend, and know the difference is the number of empty cells in first row or the second row. A full top row that represents 5 and a full ten-frame that represents 10 provide students with anchors for visualization of the numbers and operations.

THINK ADDITION

This strategy involves students finding answers to subtraction facts by using their knowledge of addition facts and their understanding of the inverse relationship of addition and subtraction. Instead of counting back to subtract, students ask themselves what they would have to add to the subtrahend to get the minuend.

Knowing the subtraction facts with a quick recall has been assigned to Mathematics 3 mental mathematics; however, the strategies should be discussed and applied with no time constraints in Mathematics 2.

Patterns and Relations (PR)

SCO PR01 Students will be expected to demonstrate an understanding of repeating patterns (three to five elements) by describing, extending, comparing, and creating patterns using manipulatives, diagrams, sounds, and actions. [C, CN, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR01.01 Identify the core of a given repeating pattern.
PR01.02 Describe and extend a given double attribute pattern.
PR01.03 Create a repeating non-numerical pattern and explain the rule.
PR01.04 Predict an element of a given repeating pattern using a variety of strategies and extend the pattern up to the tenth element to verify the prediction.
PR01.05 Translate a repeating pattern from one mode to another.
PR01.06 Compare two given repeating patterns, and describe how they are alike/different.

Performance Indicator Background

PR01.01 As students identify the core of a pattern in activities, be sure to use appropriate patterning vocabulary with students, such as **core** (the repeating part of the pattern) and **elements** (the actual objects used in the pattern). It is important to create patterns that have the core repeating at least three times, and to remind students that repeating patterns can be extended in both directions. Encourage students to reference the position of the elements using ordinal numbers.

The core of this shape pattern is—circle, square, triangle.
There are three elements in this pattern, namely a circle, a square, and a triangle.

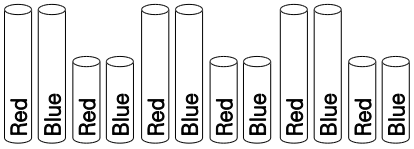
The pattern to the right is also a three-element pattern.

The core of this three-element pattern is heart (1st element), heart (2nd element), star (3rd element). To help students identify the pattern core, it is suggested students highlight, or isolate, the core each time it repeats.

PR01.02 Although students have had experiences with repeating patterns in earlier grades, it is important to provide them with more challenging repeating patterns in Mathematics 2, such as double attribute patterns. A double attribute pattern (or a two attribute pattern) is a pattern that consists of like objects with two different attributes, such as colour and size, or shape and position. For example, this pattern is made with 2-D shapes that have two different attributes (size and shape):

In describing this pattern in relation to size, it is an ABB pattern—small, big, big. In describing it in relation to shape, it is an ABC pattern—square, circle, triangle.

Another example of a double attribute with like objects (straws) with two different attributes (colour and length):



Introduce double attribute patterns with your class by using student attributes as the elements of the pattern. You may wish to create a pattern by using students with blonde, brown, and black hair, as well as, students with glasses and no glasses. Ask students to describe the pattern and invite them to choose classmates to extend the pattern.

PR01.03 Students should create their own repeating patterns without a given pattern rule using manipulatives, sounds, diagrams, and actions. Some possible manipulatives include snap cubes, counters, coloured tiles, rubber stamps, pattern blocks, and collections of small items. Students should explain how the pattern is repeating by identifying the core of the pattern and the particular way the pattern repeats. To help students identify the pattern, ask students questions such as, What comes first in your pattern? and What comes next?

Although students find it easier to demonstrate rather than articulate patterns, they must learn to explain, in both words and symbols, the rule used to create a repeating pattern.

PR01.04 Students should have experiences predicting missing elements within the pattern and also elements at the end of the repeating pattern. It is important that students have practice predicting the next element in a pattern up to the tenth element. Some strategies that would encourage students to predict and verify elements of the pattern would be to

- build the pattern with manipulatives or act it out (if applicable)
- identify the core of the pattern before they predict the element
- say the pattern aloud

PR01.05 When students are given a repeating pattern in one mode, they should be able to represent that pattern in other modes. For example, if presented the action pattern up, down, around, up, down, around, up, down, around, …, some students may use cubes to make a colour pattern—red, blue, green, red, blue, green, red, blue, green, …

Translating two or more modes of the same pattern to a common format helps children see beyond the materials making up the pattern. In fact, repeating patterns are sometimes described using letter codes that help students to name and compare patterns. Students should be provided with many experiences describing repeating patterns containing three to five elements such as AAB, ABB, ABC, AABB, ABBBC, and other combinations.

PR01.06 When comparing two repeating patterns, students describe the similarities and differences between number of elements, attributes of the elements, and the core of the pattern.

SCO PR02 Students will be expected to demonstrate an understanding of increasing patterns by describing, extending, and creating numerical patterns (numbers to 100) and non-numerical patterns using manipulatives, diagrams, sounds, and actions. [C, CN, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

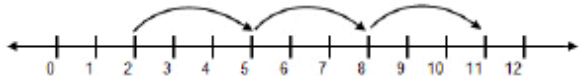
Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR02.01 Identify and describe increasing patterns in a variety of given contexts.
PR02.02 Represent a given increasing pattern concretely and pictorially.
PR02.03 Identify errors in a given increasing pattern.
PR02.04 Explain the rule used to create a given increasing pattern.
PR02.05 Create an increasing pattern and explain the pattern rule.
PR02.06 Represent a given increasing pattern using another mode.
PR02.07 Solve a given problem using increasing patterns.
PR02.08 Identify and describe increasing patterns in the environment.
PR02.09 Determine missing terms in a given concrete, pictorial, or symbolic increasing pattern and explain the reasoning.

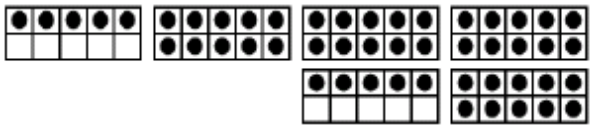
Performance Indicator Background

PR02.01 There are a variety of contexts in which increasing patterns can be demonstrated.

- On a number line, you can identify a start number and a jump number. For example, the start number could be 2, and the jump number could be 3.



  Students should find the pattern in these numbers (an increase of three).
- On a hundred chart, students could colour start-and-jump sequences, making a visual pattern. Students can be encouraged to investigate how patterns change when only the start number changes, or how they change when only the skip number changes. They should investigate which skip counts make diagonal patterns and/or which make column patterns.
- On ten-frames, build an increasing pattern by placing counters on the ten-frames and have students identify how the pattern is growing. For example, for the pattern 5, 10, 15, 20, ... These ten-frames show that the numbers increase by 5 because another full row of 5 is filled each time.



- On a calendar, students can be shown how the days of the week form an increasing pattern of 1, by looking at the rows, and an increasing pattern of 7 by looking at the columns. The calendar can be used to find other kinds of number patterns, such as repeating digits, skip counting, and patterns in rows, columns, and diagonals.

PR02.02 When presenting increasing patterns to students, always provide at least the first three terms. Some students have difficulty identifying an increasing pattern, and may think of the first term as being the core of a repeating pattern. Although students may use other language to describe patterns, it is important to model mathematical language and thinking.

Students may represent increasing patterns using base-ten blocks. For example, for the number pattern 20, 40, 60, …, they could make the capital letter T with two rods, then make the T grow bigger by adding one more rod to the top of the T and one to the bottom, and finally make it grow again by adding one more rod to the top and one more to the bottom. Students could also be asked to predict how big their 7th T would be if they continue this pattern.

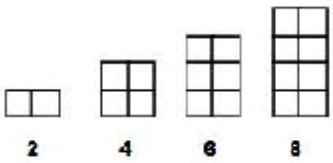
Students may also use coins to represent increasing patterns. For example, they could use pennies and dimes to represent the pattern 11, 22, 33, 44, …, where the pattern grows by one penny and one dime.

Students could be provided with increasing number patterns and asked to represent them using interlocking cubes.

Students will need many concrete experiences representing increasing patterns before they begin to represent them using pictures, numbers, and words, orally and written. It is suggested that students first create patterns as a whole class and be shown how to model them pictorially.

PR02.03 When students are trying to find any possible errors in an increasing pattern, encourage them first to identify the pattern rule to describe how the pattern is increasing and then to check that each term is increasing by the stated rule.

PR02.04 Students must be able to explain, in both words and symbols, the rule used to create a given non-numerical or numerical increasing pattern, and to represent the pattern in another mode, such as materials, actions, or sounds. Because students find it easier to demonstrate rather than articulate patterns, they must learn that when describing the rule of an increasing pattern, they need to identify the starting term and how it is increasing. Without indicating these, the pattern rule is incomplete. Students should learn that a pattern rule must describe how each and every term of the pattern is increasing. For example, the following block pattern starts with 2 blocks and increases by 2 blocks each time.



PR02.05 Students may find it easier to begin creating increasing patterns using small increments of one or two. Encourage students to first build their pattern and then record. They may create their increasing patterns in many ways, such as using a variety of concrete materials, sounds, actions, pictures, or numbers. Regardless of the mode of representation, when explaining the rule, remind students the pattern rule should state the start term and by how the pattern increases.

PR02.06 Students need to see that they are able to recreate patterns using different materials, and in different ways (modes). For example, a student could make an increasing pattern using numbers and then show this pattern using hand clapping, or vice versa. Students should be encouraged to look for other students' patterns that are the same as theirs but represented in different ways.

PR02.07 Give students many opportunities to solve problems using increasing patterns. This real-world connection using increasing patterns may help them to better understand this concept. For example, tell students that Tommy brought 4 candies to school on Monday, 6 candies on Tuesday, and 8 candies on Wednesday. If he continues in this same way of bringing more candies each school day, how many candies will he bring next Monday?

PR02.08 Provide students with opportunities to recognize naturally occurring increasing patterns in the world around them, and encourage them to look for patterns in their environment at all times. For example, students could be taken on a pattern scavenger hunt around the school and invited to be pattern detectives, observing and recording patterns that they find.

PR02.09 To help students determine missing terms in an increasing pattern, encourage them to first determine the pattern rule. Once they know how much it should grow each time, they can make sure the missing term(s) grows according to that rule.

SCO PR03 Students will be expected to demonstrate and explain the meaning of equality and inequality by using manipulatives and diagrams (0 to 100). [C, CN, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR03.01 Determine whether two given quantities of the same object (same shape and mass) are equal by using a balance scale.

PR03.02 Construct and draw two unequal sets using the same object (same shape and mass) and explain the reasoning.

PR03.03 Demonstrate how to change two given sets, equal in number, to create inequality.

PR03.04 Choose from three or more given sets the one that does not have a quantity equal to the others and explain why.

Performance Indicator Background

PR03.01 Using concrete materials, students can examine how a balance operates. Construct two equal sets using the same objects (same shape and mass) and demonstrate their equality of number using a balance scale. Prepare a set of identical covered containers, each one holding 6 cubes in different colour combinations, such as 6 red cubes, 4 yellow, and 2 blue cubes; 3 red and 3 blue cubes; 5 red and 1 yellow cubes. Have students place a container on each pan of the balance. Ask, Are the containers equal? How do you know? Then have students open the containers, count the number of different coloured cubes in each, and describe the equality, saying, for example," six red cubes is the same as four yellow cubes and two blue cubes or six red cubes is equal to four yellow cubes and two blue cubes." Repeat with other combinations of containers.

PR03.02 A balance scale can also be used to demonstrate inequality relationships; for example, place 6 red cubes on the left pan of a balance scale and four yellow cubes on the right pan without actually showing students the quantities. Ask them to explain on which pan they think there are more cubes and why they think so. Have them count the quantities on both pans. Model how they could compare the quantities on the pans, saying, "six red cubes is more than four yellow cubes and six red cubes is not the same as four yellow cubes or six red cubes is not equal to four yellow cubes.

When comparing sets, many students may recognize that 6 cubes is more than 4 cubes, but not automatically realize that 4 cubes is less than 6 cubes. Both sides of the relationship need to be considered. Therefore, ask students what they could say about the 4 yellow cubes compared to the 6 red cubes, bringing out the relationship is less than and is not the same as or is not equal to.

After students have had many concrete experiences interpreting the relationships between quantities on two pans of a balance scale, they could be asked to interpret and draw balance scale pictures. Finally, they can draw two unequal sets of the same object and, using their understanding of inequality of quantities, can explain their reasoning.

PR03.03 Students should be able to demonstrate changing an equality situation to make it an inequality. For example, place 12 cubes on both pans of a balance scale to show equality. Ask students to remove two cubes from the left pan, ask them to describe the relationship between the quantities on the pans and to explain their reasoning.

PR03.04 Student should be able to distinguish which set from multiple given sets is not like the others. For example, place 6 cubes in each of three small paper bags and 8 cubes in a fourth bag. Ask the students, using a balance scale, to find the bag that is not equal to the others, and to explain how they know if it is more or less.

SCO PR04 Students will be expected to record equalities and inequalities, symbolically, using the equal symbol or not equal symbol. [C, CN, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR04.01 Determine whether two sides of a given number sentence are equal (=) or not equal (≠). Write the appropriate symbol and justify the answer.
PR04.02 Model equalities using a variety of concrete representations and record the equality.
PR04.03 Model inequalities using a variety of concrete representations and record the inequality.

Performance Indicator Background

PR04.01 Present the two sides of equal and unequal number sentences with a (?) where the sign should be. Have students indicate if they think the number sentences are equal or unequal, and to explain how they made their decisions. Use opportunities to encourage relational thinking in decision making.

PR04.02 and PR04.03 Manipulatives, such as ten-frames, snap cubes, balance scales, and marbles, can all be used to help students develop an understanding of equal and unequal number sentences. Allowing students to model number sentences and situations with manipulatives, particularly with balance scales, gives them a visual to see whether, or not, the two sides of the number sentence are equal.

Measurement (M)

SCO M01 Students will be expected to demonstrate an understanding of the calendar and the relationships among days, weeks, months, and years. [C, CN, PS, R]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

M01.01 Read a calendar.
M01.02 Name and order the days of the week and months of the year.
M01.03 Communicate the number of days in a week and the number of months in a year.
M01.04 Solve a given problem involving time which is limited to the number of days in a week and the number of months in a year.

Performance Indicator Background

M01.01 Through modelling and much practise, students should be able to identify the day of the week, the month, and the year when a day on a calendar is pointed to, or when a special event that is marked on a calendar is referenced. Similarly, when given a date, such as December 12, 2013, students should be able to locate it on a calendar and state that it is a Thursday. As well, if students are told today's date, such as October 24, 2013, they should be able to spontaneously state yesterday's and tomorrow's dates.

M01.02 and M01.03 By the end of Mathematics 2, students should know the days of the week and the months of the year in order. Learning the days of the week and months of the year can be taught simultaneously and not as separate entities. It is important to involve students in daily problem-solving activities with the calendar in order to help students gain a deeper understanding of how the days of the week and months of the year are organized.

Students may have more difficulty with the months for which they have less experience in their own lives. Ask questions about the months regularly, such as, It is March. Which month comes next? Start reciting the months beginning with a month other than January and have students continue. Recite a sequence of five months with an error, such as an incorrect order or an omission, and ask students to detect the error.

The calendar is one of the first places where young students are exposed to a ready-made pattern; that is, the days of the week form a seven-element repeating pattern, and the months of the year is the core of a 12-element repeating pattern.

M01.04 Students should move beyond just identifying the day and date by engaging in problem-solving activities to make deeper connections between the calendar, patterning, and their own lives. For example, If it is now May 25, then is last or next Christmas closer?

SCO M02 Students will be expected to relate the size of a unit of measure to the number of units (limited to non-standard units) used to measure length and mass. [C, CN, ME, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

M02.01 Explain why one of two given non-standard units may be a better choice for measuring the length of an object.

M02.02 Explain why one of two given non-standard units may be a better choice for measuring the mass of an object.

M02.03 Select a non-standard unit for measuring the length or mass of an object and explain why it was chosen.

M02.04 Estimate the number of non-standard units needed for a given measurement task.

M02.05 Explain why the number of units of a measurement will vary depending upon the unit of measure used.

Performance Indicator Background

M02.01 and M02.03 Provide a selection of non-standard measuring units for students to explore. Have them work in pairs to choose appropriate non-standard units to measure the lengths of various objects found in their environment. Ask students to present their findings to their classmates, explaining what was measured, which non-standard unit was used, how many units they used, and why it was an appropriate unit.

M02.02 Students should see the relationship between the mass of the object that is being measured and the mass of the non-standard measuring unit. For example, if students want to find the mass of a rock, they must first understand the rock is a heavy object and it would be best to choose a non-standard unit that is also heavy, such as a marble as opposed to a toothpick.

M02.03 Students should be provided with opportunities to discover why certain non-standard units may be more efficient and more accurate to use when measuring the mass of an object. Through guided instruction and exploration, students can further develop thinking about choosing appropriate non-standard units of measure. Provide groups of students with two types of non-standard units of measurement that clearly differ in size, such as toothpicks and blocks. Instruct them to measure, on a balance scale, the mass of an eraser using both units and to record their answers. Discuss what students discovered during this activity. Ask, If you wanted to know the mass of your shoe, would it be best to use toothpicks or blocks? Why? Through activities, such as this one, students will develop their abilities to select the more appropriate non-standard unit when given two choices.

M02.04 There is always value to use estimation in teaching measurement because estimation is an effective way to get students to focus on the attribute being measured. Sometimes an estimate, or approximate measurement, is all you need, whereas other times it is a useful check on the reasonableness of a more precise answer. Students should be encouraged to estimate the number of non-standard units they will use before they actually measure the attribute of an object. After they measure the objects and get actual numbers of units, they should compare these to their estimates.

With time and experiences like this, students will refine their estimation abilities. When talking to students, try to use phrases such as, The desk is about 3 unsharpened pencils long, or The tile is a little less than two unsharpened pencils.

M02.05 Students should understand that the size of the non-standard unit used to measure an object's length will affect the results of the measurement. For example, have each student measure the length of his or her desk using two different non-standard units, such as paper clips and straws. Students should discover that more paper clips than straws were used because straws are a larger unit. Similarly, with mass, have students work in pairs to measure the mass of a small book on a balance scale, one student using pennies and the other student using boxes of paper clips. Have them record their answers on paper, compare the numbers, and discuss what they notice. Try to get students to generalize that the larger or heavier the unit used, the fewer the number of units and the smaller or lighter the units used, the greater the number of units.

SCO M03 Students will be expected to compare and order objects by length, height, distance around, and mass using non-standard units and make statements of comparison. [C, CN, ME, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

M03.01 Estimate, measure, and record the length, height, distance around, or mass of a given object using non-standard units.

M03.02 Compare and order the measure of two or more objects in ascending or descending order and explain the method of ordering.

Performance Indicator Background

M03.01 In measurement, approximations are important. When talking to students about measuring, use phrases such as, The length of the desk is about 12 pencils long; The mass of the eraser is about 3 blocks; and The distance around my head is about 3 hand spans. Students need some understanding of measuring using non-standard units, prior to estimating. Once they start estimating, they should always estimate prior to doing a measurement.

M03.02 Students should be given opportunities to order measurements from least to greatest and greatest to least. They should be able to explain why and how they have ordered the measurements. For example, given a set of objects, students could measure different attributes using non-standard units and order those objects in different ways. They could order the objects by length from least to greatest numbers of non-standard units, such as paper clips; or they could order the same objects by mass from greatest to least numbers of non-standard units, such as yellow pattern bocks.

SCO M04 Students will be expected to measure length to the nearest non-standard unit by using multiple copies of a unit and using a single copy of a unit (iteration process). [C, ME, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

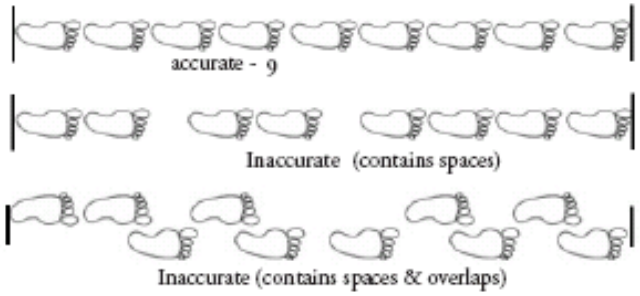
Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

M04.01 Explain why overlapping or leaving gaps does not result in accurate measures.
M04.02 Count the number of non-standard units required to measure the length of a given object using a single copy or multiple copies of a unit.
M04.03 Estimate and measure a given object using multiple copies of a non-standard unit and using a single copy of the same unit many times, and explain the results.
M04.04 Estimate and measure, using non-standard units, a given length that is not a straight line.

Performance Indicator Background

M04.01 Activities should be provided where students are shown measurements, some of which are correct and others of which have obvious gaps and overlapping, and asked to explain which measurements are accurate and which are not. For example, students could be shown these cut outs of feet measuring the length of a closet and be asked to explain how two students who both measured the length using the same cut outs of feet could have gotten completely different answers.



M04.02 It is suggested when using non-standard units to measure length, students should first be provided with multiple copies of a non-standard unit before using a single copy of a unit. For example, to measure the length of a desk, a student should first use several cubes placed along the edge of a desk before using a single cube repeatedly. Using a single copy of a unit to measure is the same as using a measuring tool and there are many possible sources of error, such as not advancing in a straight path, or unwittingly leaving gaps or overlapping.

M04.03 Before students make any measurement of length, have them examine the object that they are going to measure and the non-standard unit they will use, and get them to commit in writing their estimates of the number of units they will use. After they actually measure the length, get them to compare it to their estimates, and have them discuss strategies that could be used to get closer estimates.

M04.04 Give students common objects found in the classroom that can be easily bent into curvy lines, such as pipe cleaners, modelling clay, and wool. Have students first estimate and measure the objects straight and then curvy. They could also measure all around an object, such as their desk or a picture frame. Through these experiences, students will realize that measuring objects that are not straight requires them to make estimation decisions as they measure since the rigid non-standard units can't always bend to match the object, and they have to decide what to count when they turn and go in a different direction.

SCO M05 Students will be expected to demonstrate that changing the position of an object does not alter the measurements of its attributes. [C, R, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicator

Use the following indicator to determine whether students have achieved the corresponding specific curriculum outcome.

M05.01 Measure a given object, change the position, remeasure, and explain the results.

Performance Indicator Background

M05.01 Students need to recognize that whether an object is standing up, lying flat, or tilted, its dimensions, and mass will remain the same. For example, display two congruent cereal boxes, one standing in normal position and one lying down, ask students which box is larger, and explain how they decided. Then have students check their decisions by handling the two boxes.

Furthermore, students need to begin to realize that if a solid object is restructured so that its dimensions are altered, its mass will not be changed.

Geometry (G)

SCO G01 Students will be expected to sort 2-D shapes and 3-D objects using two attributes and explain the sorting rule. [C, CN, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

G01.01 Determine the differences between two given presorted sets and explain the sorting rule.
G01.02 Identify and name two common attributes of items within a given sorted group.
G01.03 Sort a given set of 2-D shapes (regular and irregular) according to two attributes and explain the sorting rule.
G01.04 Sort a given set of 3-D objects according to two attributes and explain the sorting rule.

Performance Indicator Background

G01.01 Students should be able to determine differences between two given presorted sets of 2-D shapes or 3-D objects. Sorting rules for 2-D shapes may include colour but ideally should have an emphasis on geometric attributes such as

- straight sides / curved sides
- large shapes / small shapes
- shapes with / without points
- shapes with / without square corners
- 4-sided shapes / 3-sided shapes
- squares / rectangles

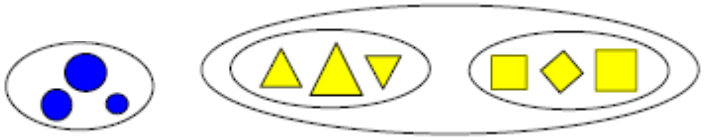
To help focus attention on these attributes, careful consideration should be given to the materials that are used in creating the presorted set. For instance, if a set is comprised of 2-D attribute blocks, students may automatically focus on colour and shape as attributes as opposed to observing the sides or corners of the shape. Another learning material to use could be paper cutouts all of the same colour.

G01.02 If students are presented with a set of shapes or objects that have been sorted by two attributes, they need to be able to analyze the set to identify the two common attributes and describe how the shapes or objects were sorted. For example, if they are given a single set with both blue squares and red rectangles, they should realize that this set of shapes could not have been sorted by colour (two colours in the set) and type of shape (both squares and rectangle), but may have been sorted by their number of sides (4) and by their corners being square. Their understanding could be further checked by asking them to place other shapes in the set and explain why the new shapes are members of the set.

G01.03 Regular 2-D shapes are those that have all equal sides and all equal corners, such as the triangle in the pattern blocks and squares. Irregular shapes are those that have sides that are not all equal and corners that are not all equal, such as the triangles in a set of tangrams. Students are not expected to know the definitions of regular and irregular shapes but should be exposed to sorting and identifying attributes of these shapes. Students have already had many experiences with sorting shapes and naming

common attributes. To further solidify their conceptual understanding of sorting and naming common attributes, it is important to include shapes that are not as common, such as hearts, arrows, and stars.

In Mathematics 2, students should build on their prior experiences to sort objects and shapes using two attributes. If students find it difficult to sort by two attributes at the same time, they may find it helpful to sort by one attribute and combine two of the sorted groups. For example, in the diagram below, students may have first sorted the shapes by number of sides and then combined the last two groups to make a single group that includes yellow shapes and straight sides. It is important that students are able to explain their sorting rule to others.



G01.04 When presented with a set of 3-D objects, students should be challenged to identify different sorting rules for the same set of objects. For example, the sorting rule for a set of spheres and cylinders could be those that roll and have curved edges or those that don't stack and have no vertex. By exposure to many different ways to sort 3-D objects, students will expand their repertoires of sorting rules.

SCO G02 Students will be expected to recognize, name, describe, compare, and build 3-D objects, including cubes and other prisms, spheres, cones, cylinders, and pyramids. [C, CN, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

G02.01 Sort a given set of 3-D objects and explain the sorting rule.
G02.02 Identify common attributes of cubes and other prisms, spheres, cones, cylinders, and pyramids from given sets of the same 3-D objects.
G02.03 Identify and describe given 3-D objects with different dimensions.
G02.04 Identify and describe given 3-D objects with different positions.
G02.05 Create and describe a representation of a given 3-D object using materials such as modelling clay.
G02.06 Identify and name examples of cubes and other prisms, spheres, cones, cylinders, and pyramids found in the environment.

Performance Indicator Background

G02.01 This performance indicator is directly linked to outcome G06; however, students' sorting rules should extend to include the names of the objects. For example, they may sort classroom objects into sets of cubes or sets of spheres.

G02.02 Attributes that students may observe in 3-D objects are
- square, circle, or triangle faces
- number of faces or edges
- identical faces or faces that are the same shape
- number of vertices
- rounded or curved surfaces
- more vertices than faces
- ability to slide/roll/stack

G02.03 and **G02.04** Students should identify and describe 3-D objects with different dimensions and in different positions. Dimensions refer to the length, width, and height of a 3-D object, each of which could be measured. It is not necessary to engage students in measuring these dimensions; they can do direct comparisons. The point of these performance indicators is to be sure that students recognize cubes and other prisms, spheres, cones, cylinders, and pyramids of all sizes and in all positions. Concrete and pictorial 3-D objects should not be considered as interchangeable when assessing students because pictorial representations are a great deal more difficult for students to process.

G02.05 By making models of 3-D objects, students focus on the shape attributes of different solids.

Describing shapes allows students to focus on their basic characteristics. Use questioning to focus student thinking; for example,
- What other shapes are like this one? In what way are they alike?
- What does this object look like?
- Does your shape have vertices or corners?

- Does the object have flat faces or curved surfaces?
- Pick two of the shapes and tell how they are alike and how they differ.

Students should be given many opportunities to build 3-D objects using a variety of materials as it increases their visualization skills. It is also important to provide students with these experiences and focus less on the accuracy of the end product. Students should also be encouraged to make constructions using a combination of 3-D objects.

G02.06 Observing objects in their environment that look like a cube, prism, sphere, cone, cylinder, or pyramid is a critical aspect in learning their geometric names. Caution will have to be exercised because so many of those objects will not be exactly one of these named objects. Students should be encouraged to recognize why an object may be, for example, almost a cube. It is difficult to find pyramids in the classroom or at home, so examples will likely have to be constructed. Looking for pictures of these 3-D objects in storybooks or on the computer should also be encouraged.

SCO G03 Students will be expected to recognize, name, describe, compare, and build 2-D shapes, including triangles, squares, rectangles, and circles. [C, CN, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

G03.01 Sort a given set of 2-D shapes and explain the sorting rule.
G03.02 Identify common attributes of triangles, squares, rectangles, and circles from given sets of the same type of 2-D shapes.
G03.03 Identify given 2-D shapes with different dimensions.
G03.04 Identify given 2-D shapes with different positions.
G03.05 Identify and name examples of triangles, squares, rectangles, and circles found in the environment.
G03.06 Create a model to represent a given 2-D shape.
G03.07 Create a pictorial representation of a given 2-D shape.

Performance Indicator Background

G03.01 This performance indicator is related to outcome G06; however, students' sorting rules should include sorts by shape names, as well as by number of sides or corners, and square corners.

G03.02 Most students will be familiar with many of the 2-D shapes from their experiences, but they may not necessarily know the attributes. While at this level of development they need to be encouraged to focus on the common attributes of any shape for which they have concrete examples, they are not likely yet identifying these attributes as properties of the class of that shape, nor are they likely able to talk about the attribute of a shape without seeing an example of that shape. For example, through explorations of, and focussed questions about, a set of squares, students may state that all these squares have four sides and four square corners.

G03.03 Students in Mathematics 2 will need experiences exploring shapes of different sizes and dimensions. Students usually easily identify squares and circles of different sizes because these two 2-D shapes maintain their overall appearance; however, students may find it difficult to recognize triangles and rectangles with dimensions that are different from typical examples. For example, students may not readily recognize a 1 cm x 30 cm rectangle as a rectangle or a triangle with a 170 degree angle as a triangle.

G03.04 It is important to be aware of how shapes are presented to students. Many times they are positioned in stereotypical ways that can lead students to think that the way a shape is positioned is part of what defines it. For example, triangles, squares, and rectangles are often only presented sitting on their bases. If these 2-D shapes are rotated so they are not sitting on their bases, students may think they are different shapes. The ability to recognize a shape regardless of how it is positioned is called position-in-space perception. Activities where students trace a shape, such as a rectangle from a set of attribute blocks, on a piece of paper, move it in a variety of ways by sliding, flipping, and turning it and retracing it until the paper is full of images of that rectangle, help students develop this ability.

G03.05 Associating a 2-D shape with examples in their home and school environments is a critical part of the development of this outcome. While many examples they select may come from pictures on the walls or in books, there will also be examples that are faces of 3-D objects (see SCO G09). Therefore, students should explain which part of the 3-D object is the square, rectangle, triangle, or circle.

G03.06 and **G03.07** Students may use a variety of media to create models of 2-D shapes. Suggested materials would include yarn, pipe cleaners, stir sticks, toothpicks, straws, and elastics on geoboards. Freehand drawings should be discouraged because students should produce exemplars of squares, triangles, rectangles, and circles that truly represent these shapes. Templates of these shapes could be provided, so students can trace the shapes as needed. Triangular and square dot paper used with straight edges and pencils will enable students to produce good exemplars of triangles, squares, and rectangles.

SCO G04 Students will be expected to identify 2-D shapes as part of 3-D objects in the environment. [C, CN, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

G04.01 Compare and match a given 2-D shape, such as a triangle, square, rectangle, or circle, to the faces of 3-D objects in the environment.

G04.02 Name the 2-D faces of a given 3-D object.

Performance Indicator Background

G04.01 A number of different 3-D objects, such as aluminum cans, balls, die, and boxes, should be collected to be available for students to identify faces of a particular named 2-D shape. They should also get experience locating pictures of 3-D objects on charts and in picture books that have faces that are triangles, rectangles, squares, or circles. In addition to a number of visual experiences, students should also have some tactile experiences. For example, if a 3-D object is placed in a bag, students should be asked to reach in and find by touching a face of a particular type, or if three 3-D objects, two of which are the same and one different, are placed in a bag, students should feel the objects inside the bag to identify the one that is different and to explain how it is different.

G04.02 Through many explorations and focused questions, students will be able to recognize and name the 2-D faces and/or surfaces of a given 3-D object. If pictures of 3-D objects are used, it is recommended that students initially also have concrete examples available to examine. Students will not necessarily see the 3-D object that is intended in a picture without experiences making pictorial connections to concrete objects. For example, if students work with a spinner containing pictures of the geometric solids, they could take turns spinning, selecting the correct concrete object, and naming and describing the faces of that solid. After many such opportunities, students may be able to name and describe faces of an object when just given its picture.

SCO SP01 Students will be expected to gather and record data about self and others to answer questions. [C, CN, PS, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

SP01.01 Formulate a question that can be answered by gathering information about self and others.
SP01.02 Organize data as it is collected using concrete objects, tallies, checkmarks, charts, or lists.
SP01.03 Answer questions using collected data.

Performance Indicator Background

SP01.01 When students formulate questions of interest to themselves and about their communities, the data they gather will be more meaningful. Encourage students to start with questions that require a yes or no answer as this data is much easier to collect and organize. For example, Do you have a pet? It is suggested to go beyond classroom limits when formulating questions and gathering data, such as surveying neighbouring classes, parents, and siblings.

SP01.02 Students should be encouraged to organize the data they collect in the course of classroom investigations. Tally marks are one of the simplest ways to record and organize data and it also promotes skip counting by 5s and 10s; however, students may also use checkmarks or objects to record their data. When creating charts and lists, it is important to provide tables to ensure proper alignment of data. This will guard against misrepresentation of data caused by varying penmanship and size of student printing.

SP01.03 At this point, not only may the organized data answer the original question, it can also provide answers to other questions and provide stimulus for the collection of additional data. For example, after students collect the data related to “Do you have a pet?” they could answer questions such as, How many more students have pets than do not?, and they may want to investigate the types of pets. Encourage students to formulate questions that can be answered from the data collected.

SCO SP02 Students will be expected to construct and interpret concrete graphs and pictographs to solve problems. [C, CN, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

SP02.01 Determine the common attributes of concrete graphs by comparing a given set of concrete graphs.
SP02.02 Determine the common attributes of pictographs by comparing a given set of pictographs.
SP02.03 Answer questions pertaining to a given concrete graph or pictograph.
SP02.04 Create a concrete graph to display a given set of data and draw conclusions.
SP02.05 Create a pictograph to represent a given set of data using one-to-one correspondence.
SP02.06 Solve a given problem by constructing and interpreting a concrete graph or pictograph.

Performance Indicator Background

SP02.01 and SP02.02 The common attributes of graphs, such as titles, labels, columns, and rows should be explored. Once a graph is completed, students need to discuss what the graph shows and what other questions it may answer. Encourage students to discuss the similarities and differences that they can see, between two concrete graphs, a concrete graph and a pictograph, or two pictographs.

SP02.03 When answering questions about data, encourage students to provide support for their answers by referring to the data. For example, they can ask, How do you know? When interpreting concrete graphs and pictographs, model good questioning that requires students to think on both a literal level (How many students have a dog? How many students have a cat?) and on interpretive or evaluative levels (How many more cats than dogs are there? Or if dog treats come in bags of ten, how many bags would we need to buy? Would we have any treats left over?).

SP02.04 The most basic type of concrete graph is a people graph, in which the students themselves form the graph. It is important initially to allow students to form lines on their own so they learn the importance of all lines starting at the same level and of matching students in the lines in some form of one-to-one correspondence. Eventually, it is helpful to use some form of graphing mat on which each student stands in one square. If the students tape their name tags to their boxes before stepping out, they will be able to view the graph as a whole. With these initial experiences with concrete graphs involving themselves, students should further develop using concrete real objects and representative objects.

SP02.05 Pictographs are bar graphs that use a drawing of some type to represent what is being graphed. The pictographs should be based on a one-to-one correspondence (i.e., a picture represents one item, not a group of items. Students should create and interpret graphs that run horizontally and vertically using magnetic board and pictures, felt board and pictures, cutting pictures from magazines or using students' photos.

SP02.06 Students should be able to solve a given problem by constructing and interpreting a concrete graph or a pictograph. Discuss the data displayed in a random, unorganized manner. This discussion should lead students to suggest organizing the data more clearly. Suggestions may include displaying

data on a chart with headings, groups of ten circled on a chart, horizontal pictograph with data shown on sticky notes, vertical graph using cubes to represent the data, or vertical object graph with clothespins to represent the data or tally charts. Students further organize this data by constructing their own pictographs. In their journals, students may write all that they know about the graph they constructed.

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