Mathematics 9

Guide

NOVA SCOTIA
NOUVELLE-ÉCOSSE

2015

Mathematics 9

June 2015

Website References

Website references contained within this document are provided solely as a convenience and do not constitute an endorsement by the Department of Education and Early Childhood Development of the content, policies, or products of the referenced website. The Department does not control the referenced websites and subsequent links, and is not responsible for the accuracy, legality, or content of those websites. Referenced website content may change without notice.

School boards and educators are required under the Department's *Public School Network Access and Use Policy* to preview and evaluate sites before recommending them for student use. If an outdated or inappropriate site is found, please report it to links@EDnet.ns.ca.

Mathematics 9

© Crown Copyright, Province of Nova Scotia, 2015, 2026
Prepared by the Department of Education and Early Childhood Development

The contents of this publication may be reproduced in part provided the intended use is for non-commercial purposes and full acknowledgment is given to the Nova Scotia Department of Education and Early Childhood Development. Where this document indicates a specific copyright holder, permission to reproduce the material must be obtained directly from that copyright holder. Please note that all attempts have been made to identify and acknowledge information from external sources. In the event that a source was overlooked, please contact Education Innovation, Programs and Services, Nova Scotia Department of Education, eps@EDnet.ns.ca.

Please note that all attempts have been made to identify and acknowledge information from external sources. In the event that a source was overlooked, please contact Education Program Services, Nova Scotia Department of Education and Early Childhood Development, eps@EDnet.ns.ca.

Acknowledgements

The Nova Scotia Department of Education and Early Childhood Education wishes to express its gratitude to the following organizations for granting permission to adapt their mathematics curriculum in the development of this guide.

- Manitoba Education
- New Brunswick Department of Education
- Newfoundland and Labrador Department of Education
- The Western and Northern Canadian Protocol (WNCP) for Collaboration in Education

We also gratefully acknowledge the contributions of the following individuals toward the development of the Nova Scotia Mathematics 7 curriculum.

Darryl Breen
Strait Regional School Board

Trudy Comeau
Tri-County Regional School Board

Bob Crane
Mi'kmaw Kina'matnewey

Paul Dennis
Chignecto-Central Regional School Board

Trisha Demone
South Shore Regional School Board

Darlene MacKeen Hudson
Chignecto-Central Regional School Board

Mark MacLeod
South Shore Regional School Board

Sonya O'Sullivan
Halifax Regional School Board

Brad Pemberton
Annapolis Valley Regional School Board

Fred Sullivan
Strait Regional School Board

Marlene Urquhart
Cape Breton-Victoria Regional School Board

Tom Willis
Tri-County Regional School Board

Contents

Introduction 1
Background and Rationale 1
Purpose 1

Program Design and Components 3
Assessment 3
Time to Learn for Mathematics 4

Outcomes 5
Conceptual Framework for K–9 Mathematics 5
Structure of the Mathematics Curriculum 5
Mathematical Processes 14
Nature of Mathematics 18
Curriculum Document Format 20

Contexts for Learning and Teaching 23
Beliefs about Students and Mathematics Learning 23

Strands
Number 29
Patterns and Relations 69
Measurement 132
Geometry 149
Statistics and Probability 181

References 204

Introduction

Background and Rationale

Mathematics curriculum is shaped by a vision that fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in society. It is essential that the mathematics curriculum reflect current research in mathematics instruction. To achieve this goal, Western and Northern Canadian Protocol's (WNCP) *The Common Curriculum Framework for K–9 Mathematics* (2006) has been adopted as the basis for the new mathematics curriculum in Nova Scotia.

The Common Curriculum Framework was developed by the seven ministries of education in the Western provinces (Alberta, British Columbia, Manitoba, Northwest Territories, Nunavut, Saskatchewan, and Yukon Territory) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators, and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and performance indicators agreed upon by the seven jurisdictions. The outcomes and performance indicators have been adapted for Nova Scotia. This document is based on both national and international research by the WNCP and the National Council of Teachers of Mathematics (NCTM).

There is an emphasis in the Nova Scotia curriculum on particular key concepts at each grade that will result in greater depth of understanding and, ultimately, stronger student achievement. There is also a greater emphasis on number sense and operations concepts in the early grades to ensure students develop a solid foundation in numeracy.

Purpose

This document provides sets of outcomes and performance indicators to be used as a mandated common base for defining mathematics curriculum expectations. This common base should result in consistent student outcomes in mathematics within the Province of Nova Scotia. It should also enable easier transfer for students moving within the Province or from any jurisdiction that has adopted the WNCP framework. This document is intended to clearly communicate to all education partners across the province the high expectations for students' mathematical learning.

Program Design and Components

Assessment

Ongoing assessment for learning is essential to effective teaching and learning. Research has shown that assessment *for* learning (formative assessment) practices produce significant and often substantial learning gains, close achievement gaps, and build students' ability to learn new skills (Black and Wiliam 1998; OECD 2006). Student involvement in assessment promotes learning. Timely and effective teacher feedback and student self-assessment allow students to reflect on and articulate their understanding of mathematical concepts and ideas.

Assessment in the classroom includes

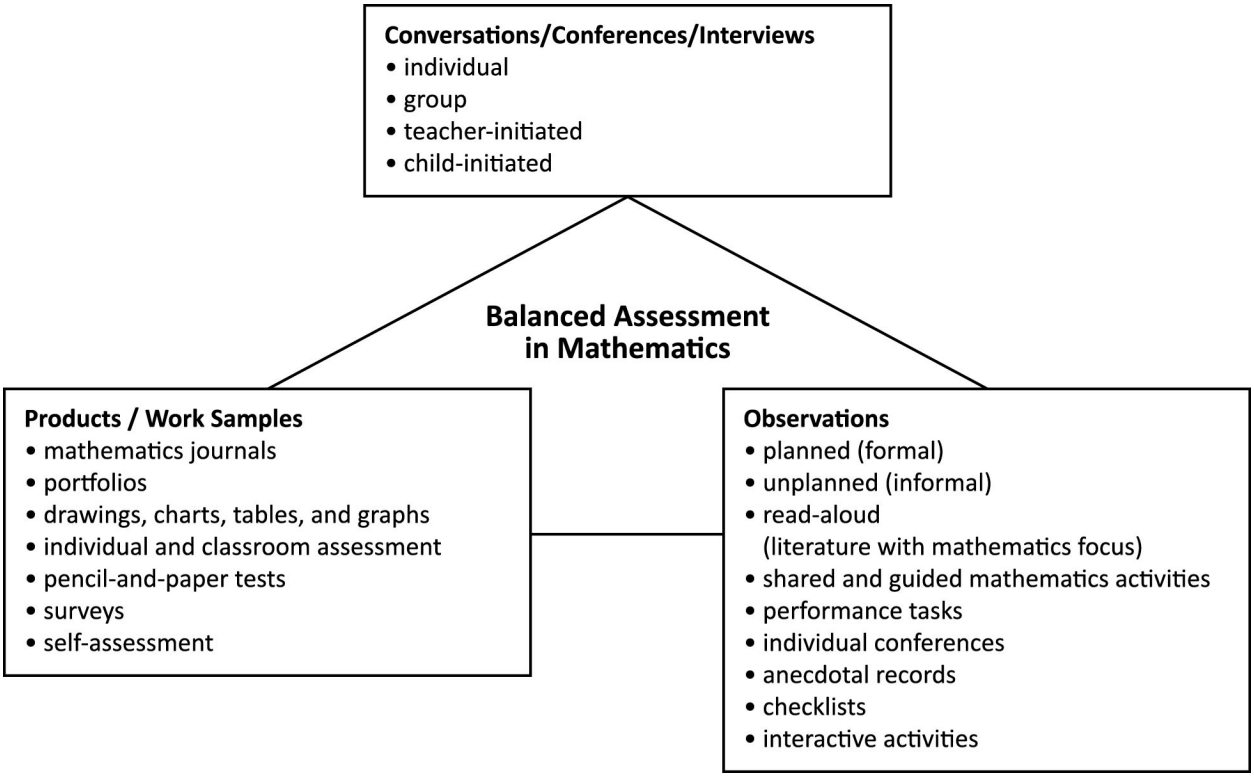
- providing clear goals, targets, and learning outcomes
- using exemplars, rubrics, and models to help clarify outcomes and identify important features of the work
- monitoring progress towards outcomes and providing feedback as necessary
- encouraging self-assessment
- fostering a classroom environment where conversations about learning take place, where students can check their thinking and performance and develop a deeper understanding of their learning

(Davies 2000)

Assessment for learning practices act as the scaffolding for learning, which only then can be measured through assessment of learning (summative assessment). Assessment of learning tracks student progress, informs instructional programming, and aids in decision making. Both forms of assessment are necessary to guide teaching, stimulate learning, and produce achievement gains.

Assessment *of* student learning should

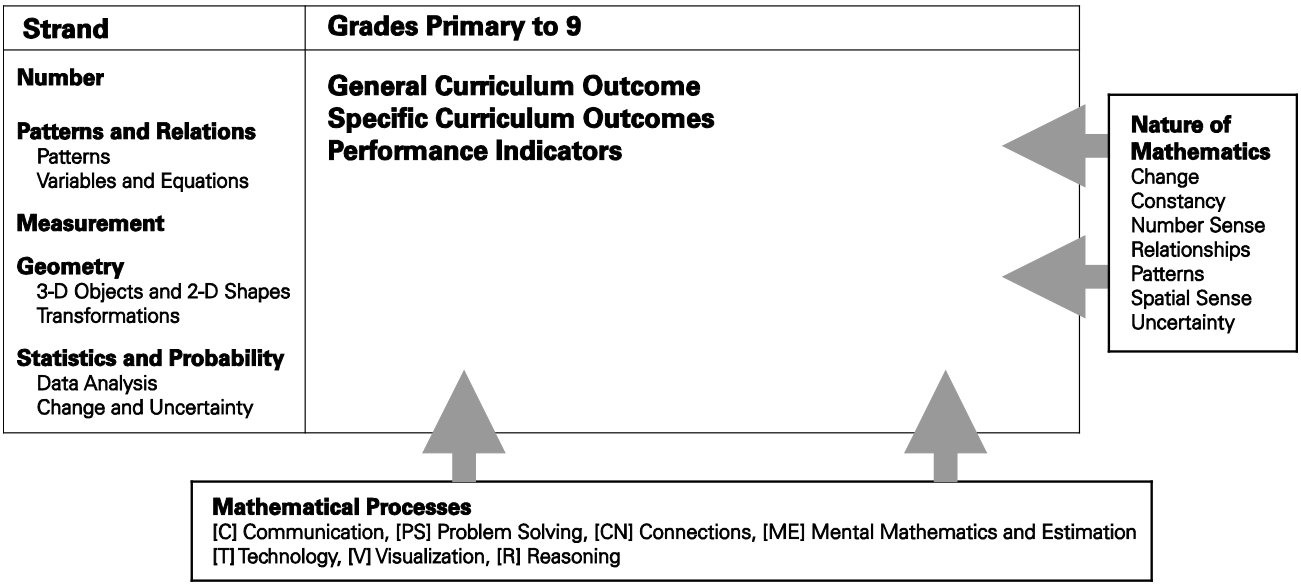
- align with curriculum outcomes
- clearly define criteria for success
- make explicit the expectations for students' performance
- use a wide variety of assessment strategies and tools
- yield useful information to inform instruction


Conversations/Conferences/Interviews
• individual
• group
• teacher-initiated
• child-initiated
Balanced Assessment in Mathematics
Products / Work Samples
• mathematics journals
• portfolios
• drawings, charts, tables, and graphs
• individual and classroom assessment
• pencil-and-paper tests
• surveys
• self-assessment
Observations
• planned (formal)
• unplanned (informal)
• read-aloud (literature with mathematics focus)
• shared and guided mathematics activities
• performance tasks
• individual conferences
• anecdotal records
• checklists
• interactive activities

Outcomes

Conceptual Framework for Mathematics Primary–9

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



Adapted from *The Common Curriculum Framework for K–9 Mathematics* (Western and Northern Canadian Protocol, 2005, 5). All rights reserved.

Structure of the Mathematics Curriculum

Strands

The learning outcomes in the Nova Scotia Framework are organized into five strands across grades primary to 9.

- Number (N)
- Patterns and Relations (PR)
- Measurement (M)
- Geometry (G)
- Statistics and Probability (SP)

General Curriculum Outcomes (GCO)

Some strands are further subdivided into sub-strands. There is one general outcome (GCO) per sub-strand. GCOs are overarching statements about what students are expected to learn in each strand/sub-strand. The general curriculum outcome for each strand/sub-strand is the same throughout the grades.

NUMBER (N)

- GCO: Students will be expected to develop number sense.

PATTERNS AND RELATIONS (PR)

Patterns
- GCO: Students will be expected to use patterns to describe the world and solve problems.

Variables and Equations
- GCO: Students will be expected to represent algebraic expressions in multiple ways.

MEASUREMENT (M)

- GCO: Students will be expected to use direct and indirect measure to solve problems.

GEOMETRY (G)

3-D Objects and 2-D Shapes
- GCO: Students will be expected to describe the characteristics of 3-D objects and 2-D shapes and analyze the relationships among them.

Transformations
- GCO: Students will be expected to describe and analyze position and motion of objects and shapes.

STATISTICS AND PROBABILITY (SP)

Data Analysis
- GCO: Students will be expected to collect, display, and analyze data to solve problems.
-

Chance and Uncertainty
- GCO: Students will be expected to use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Grade 9 Specific Curriculum Outcomes (SCOs) and Performance Indicators

Specific curriculum outcomes (SCOs) are statements that identify the specific conceptual understanding, related skills, and knowledge students are expected to attain by the end of a given grade.

Performance indicators are statements that identify specific expectations of the depth, breadth, and expectations for the outcome. Teachers use performance indicators to determine whether students have achieved the corresponding SCO.

Process Standards Key

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

NUMBER (N)

N01 Students will be expected to demonstrate an understanding of powers with integral bases (excluding base 0) and whole number exponents by

- **representing repeated multiplication using powers**
- **using patterns to show that a power with an exponent of zero is equal to one**
- **solving problems involving powers**

[C, CN, PS, R]

Performance Indicators

N01.01 Demonstrate the differences between the exponent and the base by building models of a given power, such as 2^3 and 3^2.

N01.02 Explain, using repeated multiplication, the difference between two given powers in which the exponent and base are interchanged.

N01.03 Express a given power as a repeated multiplication.

N01.04 Express a given repeated multiplication as a power.

N01.05 Explain the role of parentheses in powers by evaluating a given set of powers.

N01.06 Demonstrate, using patterns, that a^0 is equal to 1 for a given value of a ($a \neq 0$).

N01.07 Evaluate powers with integral bases (excluding base 0) and whole number exponents.

N02 Students will be expected to demonstrate an understanding of operations on powers with integral bases (excluding base 0) and whole number exponents:

- $\left(a^m\right)\left(a^n\right) = a^{m+n}$
- $a^m \div a^n = a^{m-n},\ m > n$
- $(a^m)^n = a^{mn}$
- $(ab)^m = a^m b^m$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n},\ b \neq 0$

[C, CN, PS, R, T]

Performance Indicators

N02.01 Explain, using examples, the exponent laws of powers with integral bases (excluding base 0) and whole number exponents.

N02.02 Evaluate a given expression by applying the exponent laws.

N02.03 Determine the sum of two given powers and record the process.

N02.04 Determine the difference of two given powers and record the process.

N02.05 Identify the error(s) in a given simplification of an expression involving powers.

N03 Students will be expected to demonstrate an understanding of rational numbers by comparing and ordering rational numbers and solving problems that involve arithmetic operations on rational numbers. [C, CN, PS, R, T, V]

Performance Indicators

N03.01 Order a given set of rational numbers in fraction and decimal form by placing them on a number line.

N03.02 Identify a rational number that is between two given rational numbers.

N03.03 Solve a given problem involving operations on rational numbers in fraction or decimal form.

N04 Students will be expected to explain and apply the order of operations, including exponents, with and without technology. [PS, T]

Performance Indicators

N04.01 Solve a given problem by applying the order of operations without the use of technology.

N04.02 Solve a given problem by applying the order of operations with the use of technology.

N04.03 Identify the error in applying the order of operations in a given incorrect solution.

N05 Students will be expected to determine the exact square root of positive rational numbers. [C, CN, PS, R, T]

Performance Indicators

N05.01 Determine whether or not a given rational number is a square number and explain the reasoning.

N05.02 Determine the square root of a given positive rational number that is a perfect square.

N05.03 Identify the error made in a given calculation of a square root (e.g., is 3.2 the square root of 6.4)?

N05.04 Determine a positive rational number, given the square root of that positive rational number.

N06 Students will be expected to determine an approximate square root of positive rational numbers. [C, CN, PS, R, T]

Performance Indicators

N06.01 Estimate the square root of a given rational number that is not a perfect square, using the roots of perfect squares as benchmarks.

N06.02 Determine an approximate square root of a given rational number that is not a perfect square, using technology (e.g., a calculator, a computer).

N06.03 Explain why the square root of a given rational number as shown on a calculator may be an approximation.

N06.04 Identify a number with a square root that is between two given numbers.

PATTERNS AND RELATIONS (PR)

PR01 Students will be expected to generalize a pattern arising from a problem-solving context using a linear equation and verify by substitution. [C, CN, PS, R, V]

Performance Indicators

PR01.01 Write an expression representing a given concrete, pictorial, oral, and/or written pattern.

PR01.02 Write a linear equation to represent a given context.
PR01.03 Describe a context for a given linear equation.
PR01.04 Solve, using a linear equation, a given problem that involves concrete, pictorial, oral, and/or written linear patterns.
PR01.05 Write a linear equation representing the pattern in a given table of values, and verify the equation by substituting values from the table.

PR02 Students will be expected to graph a linear relation, analyze the graph, and interpolate or extrapolate to solve problems. [C, CN, PS, R, T, V]

Performance Indicators

PR02.01 Describe the pattern found in a given graph.
PR02.02 Graph a given linear relation, including horizontal and vertical lines.
PR02.03 Match given equations of linear relations with their corresponding graphs.
PR02.04 Extend a given graph (extrapolate) to determine the value of an unknown element.
PR02.05 Interpolate the approximate value of one variable on a given graph, given the value of the other variable.
PR02.06 Extrapolate the approximate value of one variable from a given graph, given the value of the other variable.
PR02.07 Solve a given problem by graphing a linear relation and analyzing the graph.

PR03 Students will be expected to model and solve problems, where *a*, *b, c, d, e,* and *f* are rational numbers, using linear equations of the form

- $ax = b$
- $\frac{x}{a} = c,\ a \neq 0$
- $ax + b = c$
- $\frac{x}{a} + b = c,\ a \neq 0$
- $ax = b + cx$
- $a(x + b) = c$
- $ax + b = cx + d$
- $a(bx + c) = d(ex + f)$
- $\frac{a}{x} = b,\ x \neq 0$

[C, CN, PS, V]

Performance Indicators

PR03.01 Solve the given linear equation, using concrete and pictorial representations, and record this process symbolically.
PR03.02 Verify by substitution whether a given rational number is a solution to a given linear equation.
PR03.03 Solve a given linear equation symbolically.
PR03.04 Identify and correct an error in a given incorrect solution of a linear equation.
PR03.05 Represent a given problem, using a linear equation.
PR03.06 Solve a given problem, using a linear equation, and record the process.

PR04 Students will be expected to explain and illustrate strategies to solve single variable linear inequalities with rational coefficients within a problem-solving context. [C, CN, PS, R, V]

Performance Indicators

PR04.01 Translate a given problem into a single variable linear inequality, using the symbols ≥, >, <, or ≤.

PR04.02 Determine if a given rational number is a possible solution of a given linear inequality.

PR04.03 Generalize and apply a rule for adding or subtracting a positive or negative number to determine the solution of a given inequality.

PR04.04 Generalize and apply a rule for multiplying or dividing by a positive or negative number to determine the solution of a given inequality.

PR04.05 Solve a given linear inequality algebraically, and explain the process orally or in written form.

PR04.06 Compare and explain the process for solving a given linear equation to the process for solving a given linear inequality.

PR04.07 Graph the solution of a given linear inequality on a number line.

PR04.08 Compare and explain the solution of a given linear equation to the solution of a given linear inequality.

PR04.09 Verify the solution of a given linear inequality, using substitution for multiple elements in the solution.

PR04.10 Solve a given problem involving a single variable linear inequality, and graph the solution.

MEASUREMENT (M)

M01 Students will be expected to solve problems and justify the solution strategy, using the following circle properties:

- **The perpendicular from the centre of a circle to a chord bisects the chord.**
- **The measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc.**
- **The inscribed angles subtended by the same arc are congruent.**
- **A tangent to a circle is perpendicular to the radius at the point of tangency.**

[C, CN, PS, R, T, V]

M01.01 Demonstrate that

- the perpendicular from the centre of a circle to a chord bisects the chord
- the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc
- the inscribed angles subtended by the same arc are congruent
- a tangent to a circle is perpendicular to the radius at the point of tangency

M01.02 Solve a given problem involving application of one or more of the circle properties.

M01.03 Determine the measure of a given angle inscribed in a semicircle, using the circle properties.

M01.04 Explain the relationship among the centre of a circle, a chord, and the perpendicular bisector of the chord.

GEOMETRY (G)

G01 Students will be expected to determine the surface area of composite 3-D objects to solve problems. [C, CN, PS, R, V]

Performance Indicators

G01.01 Determine the area of overlap in a given composite 3-D object, and explain the effect on determining the surface area (limited to right cylinders, right rectangular prisms, and right triangular prisms).

G01.02 Determine the surface area of a given composite 3-D object (limited to right cylinders, right rectangular prisms, and right triangular prisms).
G01.03 Solve a given problem involving surface area.

G02 Students will be expected to demonstrate an understanding of similarity of polygons. [C, CN, PS, R, V]

Performance Indicators
G02.01 Determine if the polygons in a given presorted set are similar, and explain the reasoning.
G02.02 Model and draw a polygon similar to a given polygon, and explain why the two are similar.
G02.03 Solve a given problem using the properties of similar polygons.

G03 Students will be expected to draw and interpret scale diagrams of 2-D shapes. [CN, R, T, V]

Performance Indicators
G03.01 Identify an example of a scale diagram in print and electronic media.
G03.02 Draw a diagram to scale that represents an enlargement or a reduction of a given 2-D shape.
G03.03 Determine the scale factor for a given diagram drawn to scale.
G03.04 Determine if a given diagram is proportional to the original 2-D shape, and if it is, state the scale factor.
G03.05 Solve a given problem that involves the properties of similar triangles.

G04 Students will be expected to demonstrate an understanding of line and rotation symmetry. [C, CN, PS, V]

Performance Indicators
G04.01 Classify a given set of 2-D shapes or designs according to the number of lines of symmetry.
G04.02 Complete a 2-D shape or design, given one half of the shape or design and a line of symmetry.
G04.03 Determine if a given 2-D shape or design has rotation symmetry about the point at its centre, and if it does, state the order and angle of rotation.
G04.04 Rotate a given 2-D shape about a vertex, and draw the resulting image.
G04.05 Identify the type of symmetry that arises from a given transformation on a Cartesian plane.
G04.06 Complete, concretely or pictorially, a given transformation of a 2-D shape on a Cartesian plane, record the coordinates, and describe the type of symmetry that results.
G04.07 Identify and describe the types of symmetry created in a given piece of artwork.
G04.08 Determine whether or not two given 2-D shapes on a Cartesian plane are related by either rotation or line symmetry.
G04.09 Draw, on a Cartesian plane, the translation image of a given shape using a given translation rule such as R2, U3, or → →, ↑↑↑; label each vertex and its corresponding ordered pair; and describe why the translation does not result in line or rotation symmetry.
G04.10 Create or provide a piece of artwork that demonstrates line and rotation symmetry, and identify the line(s) of symmetry and the order and angle of rotation.

STATISTICS AND PROBABILITY (SP)

SP01 Students will be expected to describe the effect on the collection of data of bias, use of language, ethics, cost, time and timing, privacy, and cultural sensitivity. [C, CN, R, T]

Performance Indicators

SP01.01 Analyze a given case study of data collection; and identify potential problems related to bias, use of language, ethics, cost, time and timing, privacy, or cultural sensitivity.

SP01.02 Provide examples to illustrate how bias, use of language, ethics, cost, time and timing, privacy, or cultural sensitivity may influence data.

SP02 Students will be expected to select and defend the choice of using either a population or a sample of a population to answer a question. [C, CN, PS, R]

Performance Indicators

SP02.01 Identify whether a given situation represents the use of a sample or a population.

SP02.02 Provide an example of a situation in which a population may be used to answer a question, and justify the choice.

SP02.03 Provide an example of a question where a limitation precludes the use of a population, and describe the limitation.

SP02.04 Identify and critique a given example in which a generalization from a sample of a population may or may not be valid for the population.

SP02.05 Provide an example to demonstrate the significance of sample size in interpreting data.

SP03 Students will be expected to develop and implement a project plan for the collection, display, and analysis of data by

- **formulating a question for investigation**
- **choosing a data collection method that includes social considerations**
- **selecting a population or a sample**
- **collecting the data**
- **displaying the collected data in an appropriate manner**
- **drawing conclusions to answer the question**

[C, PS, R, T, V]

Performance Indicators

SP03.01 Create a rubric to assess a project that includes the assessment of

- a question for investigation
- the choice of a data collection method that includes social considerations
- the selection of a population or a sample and the justification for the selection
- the display of collected data
- the conclusions to answer the question

SP03.02 Develop a project plan that describes

- a question for investigation
- the method of data collection that includes social considerations
- the method for selecting a population or a sample
- the methods for display and analysis of data

SP03.03 Complete the project according to the plan, draw conclusions, and communicate findings to an audience.

SP03.04 Self-assess the completed project by applying the rubric.

SP04 Students will be expected to demonstrate an understanding of the role of probability in society. [C, CN, R, T]

Performance Indicators

SP04.01 Provide an example from print and electronic media where probability is used.

SP04.02 Identify the assumptions associated with a given probability, and explain the limitations of each assumption.

SP04.03 Explain how a single probability can be used to support opposing positions.

SP04.04 Explain, using examples, how decisions may be based on a combination of theoretical probability, experimental probability, and subjective judgment.

Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics.

Students are expected to

- communicate in order to learn and express their understanding of mathematics (Communication [C])
- develop and apply new mathematical knowledge through problem solving (Problem Solving [PS])
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines (Connections [CN])
- demonstrate fluency with mental mathematics and estimation (Mental Mathematics and Estimation [ME])
- select and use technologies as tools for learning and solving problems (Technology [T])
- develop visualization skills to assist in processing information, making connections, and solving problems (Visualization [V])
- develop mathematical reasoning (Reasoning [R])

The Nova Scotia curriculum incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning. The key to these process standards is presented in a box, as shown below, with each specific curriculum outcome within the strands.

Process Standards Key

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing, and modifying ideas, knowledge, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics.

Students also need to communicate their learning using mathematical terminology. Communication can help students make connections between and among the different representational modes—contextual, concrete, pictorial, linguistic/verbal, and symbolic—of mathematical ideas. Students must communicate *daily* about their mathematics learning. This enables them to reflect, to validate, and to clarify their thinking and provides teachers with insight into students' interpretations of mathematical meanings and ideas.

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, How would you ... ? or How could you ... ? the problem-solving approach is being modelled. Students develop their own problem-solving strategies by being open to listening, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement, perseverance, and collaboration.

Problem solving is also a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for and engage in finding a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive, mathematical risk takers.

When students are exposed to a wide variety of problems in all areas of mathematics, they explore various methods for solving and verifying problems. In addition, they are challenged to find multiple solutions for problems and to create their own problem.

Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to one another or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

"Because the learner is constantly searching for connections on many levels, educators need to *orchestrate the experiences* from which learners extract understanding. … *Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching*." (Caine and Caine 1991, 5).

Mathematics should be viewed as an integrated whole rather than as the study of separate strands or units. Connections must also be made between and among the different representational modes—contextual, concrete, pictorial, linguistic/verbal, and symbolic. The process of making connections, in turn, facilitates learning. Concepts and skills should also be connected to everyday situations and other curricular areas. For example, when developing literacy skills students learn to make text-to-world, text-to-text, and text-to-self connections. Students can also make connections to make mathematics come alive through math-to-world, math-to-math, and math-to-self connections.

Mental Mathematics and Estimation [ME]

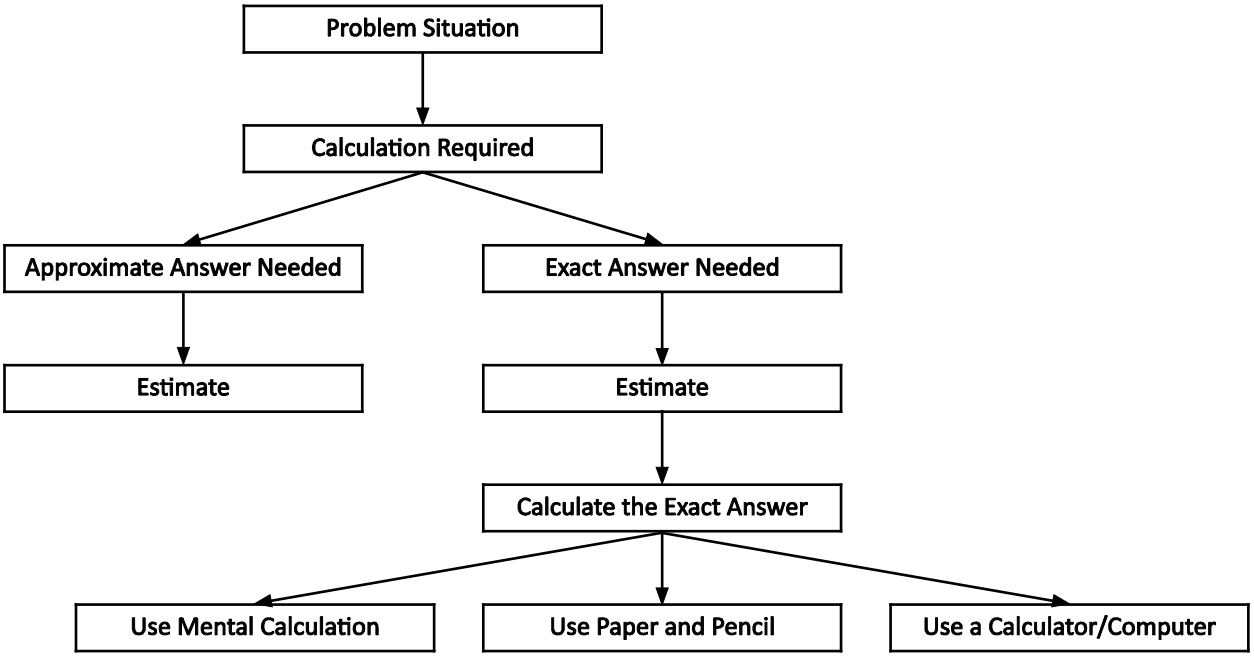
Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external aids. Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy, and flexibility. "Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental math." (National Council of Teachers of Mathematics, May 2005).

Students proficient with mental mathematics "become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers, and are more able to use multiple approaches to problem solving." (Rubenstein 2001) Mental mathematics "provides a cornerstone for all

estimation processes, offering a variety of alternative algorithms and nonstandard techniques for finding answers." (Hope et al. 1988, v)

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and what strategy to use when estimating. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision-making process as illustrated below.



The skill of estimation requires a sound knowledge of mental mathematics. Both are necessary to many everyday experiences, and students should be provided with frequent opportunities to practise these skills.

Technology [T]

Technology can be effectively used to contribute to and support the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Technology can be used to

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts and test properties

- develop personal procedures for mathematical operations
- create geometric displays
- simulate situations
- develop number sense

The use of calculators is recommended to enhance problem solving, to encourage discovery of number patterns, and to reinforce conceptual development and numerical relationships. They do not, however, replace the development of number concepts and skills. Carefully chosen computer software can provide interesting problem-solving situations and applications.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels. While technology can be used in grades primary to 3 to enrich learning, it is expected that students will achieve all outcomes without the use of technology.

Visualization [V]

Visualization "involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world." (Armstrong 1999). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers. These mental images are needed to develop concepts and understand procedures. Images and explanations help students clarify their understanding of mathematical ideas in all strands.

Being able to create, interpret, and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure, when to estimate and to know several estimation strategies. (Shaw and Cliatt 1989)

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics. Mathematical experiences in and out of the classroom provide opportunities for inductive and deductive reasoning. Inductive reasoning occurs when students explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Deductive reasoning occurs when students reach new conclusions based upon what is already known or assumed to be true.

Mathematics reasoning involves informal thinking, conjecturing, and validating—these help students understand that mathematics makes sense. Students are encouraged to justify, in a variety of ways,

their solutions, thinking processes, and hypotheses. In fact, good reasoning is as important as finding correct answers.

Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that define the nature of mathematics, and these are woven throughout this document. These components include change, constancy, number sense, relationships, patterns, spatial sense, and uncertainty.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as

- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen 1990, 184)

Constancy

Different aspects of constancy are described by the terms **stability**, **conservation**, **equilibrium**, **steady state**, and **symmetry** (American Association for the Advancement of Science 1993, 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems. Examples of constancy include the following:

- The area of a rectangular region is the same regardless of the methods used to determine the solution.
- The sum of the interior angles of any triangle is 180°.
- The theoretical probability of flipping a coin and getting heads is 0.5.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (British Columbia Ministry of Education 2000, 146). A true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms. Number sense develops when students connect numbers to real-life experiences and use benchmarks and referents. This results in students who are computationally fluent, flexible with numbers, and have intuition about numbers. The evolving number sense typically comes as a by-product of learning rather than through

direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, and concepts. The search for possible relationships involves the collection and analysis of data, and describing relationships visually, symbolically, orally, or in written form.

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all strands, and it is important that connections are made among strands. Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with an understanding of their environment. Patterns may be represented in concrete, visual, or symbolic form. Students should develop fluency in moving from one representation to another. Students must learn to recognize, extend, create, and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving problems. Learning to work with patterns in the early grades helps develop students' algebraic thinking, which is foundational for working with more abstract mathematics in higher grades.

Spatial Sense

Spatial sense involves visualization, mental imagery, and spatial reasoning. These skills are central to the understanding of mathematics. Spatial sense enables students to reason and interpret among and between 3-D and 2-D representations and identify relationships to mathematical strands. Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes. Spatial sense offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations. Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions. For example,

- knowing the dimensions of an object enables students to communicate about the object and create representations
- the volume of a rectangular solid can be calculated from given dimensions
- doubling the length of the side of a square increases the area by a factor of four

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Curriculum Document Format

This guide presents the mathematics curriculum so that a teacher may readily view the scope of the outcomes that students are expected to achieve during the year. Teachers are encouraged to examine teaching and learning that precedes and following this grade level to better understand how students' learning at a particular grade level is part of a bigger picture of concept and skill development.

The order of the outcomes does not prescribe a preferred order of presentation in the classroom, but provides the specific curriculum outcomes in relation to the overarching general curriculum outcomes (GCOs).

The table on the next page provided the structure of each specific curriculum outcomes section. When a specific curriculum outcome (SCO) is introduced, it is followed by the mathematical processes and performance indicators for that outcome. A scope and sequence is then provided, which relates the SCO to previous and next grade SCOs. Also for each SCO, there is background information, assessment strategies, suggested instructional strategies, suggested models and manipulatives, mathematical language, and a section for resources and notes. For each section, the guiding questions should be used to help with unit and lesson preparation.

Assessment Strategies

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Follow-up on Assessment

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcome and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SCO		
Mathematical Processes		
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections
[ME] Mental Mathematics and Estimation		
[T] Technology	**[V]** Visualization	**[R]** Reasoning

Performance Indicators

Describes observable indicators of whether students have achieved the specific outcome.

Scope and Sequence

Previous grade or course SCOs	Current grade SCO	Following grade or course SCOs

Background

Describes the "big ideas" to be learned and how they relate to work in previous grade and work in subsequent courses.

Assessment, Teaching, and Learning

Assessment Strategies

Assessing Prior Knowledge
Sample tasks that can be used to determine students' prior knowledge.

Whole-Class/Group/Individual Assessment Tasks
Some suggestions for specific activities and questions that can be used for both instruction and assessment

Follow-up on Assessment

Planning for Instruction

Choosing Instructional Strategies

Suggested strategies for planning daily lessons.

Suggested Learning Tasks

Suggestions for general approaches and strategies suggested for teaching this outcome.

Suggested Models and Manipulatives

Mathematical Language

Teacher and student mathematical language associated with the respective outcome.

Resources/Notes

Contexts for Learning and Teaching

Beliefs about Students and Mathematics Learning

"Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge." (National Council of Teachers of Mathematics 2000, 20)

The Nova Scotia mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning that have grown out of research and practice. These beliefs include the following:

- Mathematics learning is an active and constructive process.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.
- Learners are individuals who bring a wide range of prior knowledge and experiences and who learn via various styles and at different rates.
- Learning is most likely to occur when placed in meaningful contexts and in an environment that supports exploration, risk taking, and critical thinking, and that nurtures positive attitudes and sustained effort.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Students develop a variety of mathematical ideas before they enter school. Children make sense of their environment through observations and interactions at home and in the community. Mathematics learning is embedded in everyday activities, such as playing, reading, storytelling, and helping around the home. Such activities can contribute to the development of number and spatial sense in children. Curiosity about mathematics is fostered when children are engaged in activities such as comparing quantities, searching for patterns, sorting objects, ordering objects, creating designs, building with blocks, and talking about these activities. Positive early experiences in mathematics are as critical to child development as are early literacy experiences.

Students learn by attaching meaning to what they do and need to construct their own meaning of mathematics. This meaning is best constructed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of models and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students and enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with and translating through a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful discussions can provide essential links among concrete, pictorial, contextual, and symbolic representations of mathematics.

The learning environment should value and respect all students' experiences and ways of thinking, so that learners are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. Learners must realize that it is acceptable to solve problems in different ways and that solutions may vary.

Goals for Mathematics Education

The main goals of mathematics education are to prepare students to

- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- make connections between mathematics and its applications
- become mathematically literate adults, using mathematics to contribute to society

Students who have met these goals will

- gain understanding and appreciation of the contributions of mathematics as a science, a philosophy, and an art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity about mathematics and situations involving mathematics

Opportunities for Success

A positive attitude has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for student success help develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations, and engage in reflective practices.

To experience success, students must be taught to set achievable goals or assess their progress as they work toward these goals. Striving toward success and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessing of personal goals.

Engaging All Learners

*"*No matter how engagement is defined or which dimension is considered, research confirms this truism of education: *The more engaged you are, the more you will learn."* (Hume 2011, 6)

Student engagement is at the core of learning. This is critical for teachers to take into account when planning and implementing instruction. Effective instruction engages, embraces, and supports all learners through a range of learning experiences. This curriculum is designed to provide learning opportunities that reflect culturally proficient instructional and assessment practices and are equitable, accessible, and inclusive of the multiple facets of diversity represented in today's classrooms.

Engagement in learning occurs when students are provided with opportunities to become more invested in their learning. When teachers know their students as individual learners and as individual people, their students are more likely to be motivated to learn, participate in classroom activities, persist in challenging situations, and engage in reflective practices. Students often become more engaged when teachers demonstrate a genuine belief in each student's potential to learn.

SUPPORTIVE LEARNING ENVIRONMENTS

A supportive and positive learning environment has a profound effect on students' learning. In classrooms where students feel a sense of belonging, are encouraged to actively participate, are challenged without being frustrated, and feel safe and supported to take risks with their learning, students are more likely to experience success. It is realized that not all students will progress at the same pace or be equally positioned in terms of their prior knowledge of and skill with particular concepts and outcomes. Teachers provide all students with equitable access to learning by integrating a variety of instructional approaches and assessment activities that consider all learners and align with the following key principles:

- Instruction must be flexible and offer multiple means of representation.
- Students must have opportunities to express their knowledge and understanding in multiple ways.
- Teachers must provide options for students to engage in learning through multiple ways.

Teachers who know their students well become aware of individual learning differences and infuse this understanding into planned instructional and assessment decisions. They organize learning experiences to accommodate the many ways in which students learn, create meaning, and demonstrate their knowledge and understanding. Teachers use a variety of effective teaching approaches that may include

- providing all students with equitable access to appropriate learning strategies, resources, and technology
- offering a range of ways students can access their prior knowledge to connect with new concepts
- scaffolding instruction and assignments so that individual or groups of students are supported as needed throughout the process of learning
- verbalizing their thinking to model comprehension strategies and new learning
- balancing individual, small-group, and whole-class approaches to learning activities
- involving students in the co-creation of criteria for assessment and evaluation
- providing students with choice in how they demonstrate their understanding according to learning styles and preferences, building on individual strengths, and including a range of difficulty and challenge
- providing frequent and meaningful feedback to students throughout their learning experiences

LEARNING STYLES AND PREFERENCES

The ways in which students make sense of, receive, and process information, demonstrate learning, and interact with peers and their environment both indicate and shape learning preferences, which may vary widely from student to student. Learning preferences are influenced also by the learning context and purpose and by the type and form of information presented or requested. Most students tend to favour one learning style and may have greater success if instruction is designed to provide for multiple learning styles, thus creating more opportunities for all students to access learning. The three most commonly referenced learning styles are

- auditory (such as listening to teacher-presented lessons or discussing with peers)
- kinesthetic (such as using manipulatives or recording print or graphic/visual text)
- visual (such as interpreting information with text and graphics or viewing videos)

While students can be expected to work using all modalities, it is recognized that one or some of these modalities may be more natural to individual students than the others.

A Gender-Inclusive Curriculum

It is important that the curriculum respects the experiences and values of all students and that learning resources and instructional practices are not gender-biased. Teachers promote gender equity and inclusion in their classrooms when they

- articulate equally high expectations for all students
- provide equal opportunity for input and response from all students
- model gender-fair language and respectful listening in their interactions with students

Valuing Diversity: Teaching with Cultural Proficiency

Teachers understand that students represent diverse life and cultural experiences, with individual students bringing different prior knowledge to their learning. Therefore, teachers build upon their knowledge of their students as individuals and respond by using a variety of culturally-proficient instruction and assessment strategies. “Instruction that is embedded in socially meaningful contexts, and tasks that are meaningful and relevant to the lives of students, will engage students in high-level problem-solving and reasoning and enhance students’ engagement (Frankenstein 1995; Gutstein 2003; Ladson-Billings 1997; Tate 1995).” (Herzig 2005)

Students with Language, Communication, and Learning Challenges

Today’s classrooms include students who have diverse backgrounds, abilities, levels of development, and learning challenges. By observing and interacting with students as they work on assigned activities, teachers can identify areas where students may need additional support to achieve their learning goals. Teachers can then respond with a range of effective instructional strategies. Students who have English as an Additional Language (EAL) may require curriculum outcomes at different levels, or temporary individualized outcomes, particularly in language-based subject areas, while they become more proficient in their English language skills. For students who are experiencing difficulties, it is important that teachers distinguish between students for whom curriculum content is challenging and students for whom language-based issues are at the root of apparent academic difficulties.

Students who Demonstrate Gifted and Talented Behaviours

Some students are academically gifted and talented with specific skill sets or in specific subject areas. Most students who are gifted and talented thrive when challenged by problem-centred, inquiry-based learning and open-ended activities. Teachers may challenge students who are gifted and talented by adjusting the breadth, the depth, and/or the pace of instruction. Learning experiences may be enriched by providing greater choice among activities and offering a range of resources that require increased cognitive demand and higher-level thinking at different levels of complexity and abstraction. For additional information, refer to *Gifted Education and Talent Development* (Nova Scotia Department of Education 2010).

Connections across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the students' understanding of mathematical concepts and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating mathematics in health education, literacy, music, physical education, science, social studies, and visual arts.

Number (N)

GCO: Students will be expected to develop number sense.

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

GUIDING QUESTIONS

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Follow-up on Assessment

The evidence of learning that is gathered from student participation and work should be used to inform instruction.

GUIDING QUESTIONS

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan available at Mathematics Learning Commons: Grades 7–9: http://nsvs.ednet.ns.ca/nsps/nsps26/course/view.php?id=3875.

GUIDING QUESTIONS

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SCO N01 Students will be expected to demonstrate an understanding of powers with integral bases (excluding base 0) and whole number exponents by
- representing repeated multiplication using powers
- using patterns to show that a power with an exponent of zero is equal to one
- solving problems involving powers

[C, CN, PS, R]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N01.01 Demonstrate the differences between the exponent and the base by building models of a given power, such as 2^3 and 3^2.
N01.02 Explain, using repeated multiplication, the difference between two given powers in which the exponent and base are interchanged.
N01.03 Express a given power as a repeated multiplication.
N01.04 Express a given repeated multiplication as a power.
N01.05 Explain the role of parentheses in powers by evaluating a given set of powers.
N01.06 Demonstrate, using patterns, that a^0 is equal to 1 for a given value of a ($a \neq 0$).
N01.07 Evaluate powers with integral bases (excluding base 0) and whole number exponents.

Scope and Sequence

Mathematics 8	Mathematics 9	Mathematics 10
N01 Students will be expected to demonstrate an understanding of perfect squares and square roots, concretely, pictorially, and symbolically (limited to whole numbers).	**N01** Students will be expected to demonstrate an understanding of powers with integral bases (excluding base 0) and whole number exponents by ▪ representing repeated multiplication using powers ▪ using patterns to show that a power with an exponent of zero is equal to one ▪ solving problems involving powers	**AN03** Students will be expected to demonstrate an understanding of powers with integral and rational exponents.

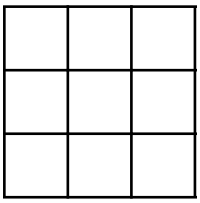
Background

The use of exponents as a means of expressing factors in a compact form was introduced in Mathematics 8, as students explored perfect squares and worked with the Pythagorean theorem. This previous exposure was limited to square numbers. In this unit, powers with whole number exponents will be developed through repeated multiplication.

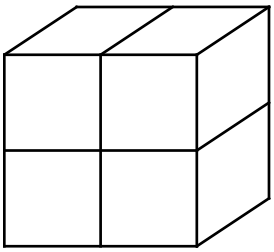
Students will be expected to identify the base, the exponent, and the power in an expression in exponential form. For example, the power 6^4 (where 6 is the base and 4 is the exponent), can be read as "six to the exponent of four," or "six to the fourth" and not "six to the power of four." The term 'power' is not to be used in the same sense as the term "exponent." Just as repeated addition can be represented as multiplication (e.g., 3 + 3 + 3 + 3 + 3 = 5 x 3), repeated multiplication can be represented as a power (e.g., $3\times3\times3\times3\times3=3^5$).

In Mathematics 6 students learned to link the term "squared" with a 2-D area model and "cubed" with a 3-D volume model. From Mathematics 8, students are familiar with representing a power as a square region. This will help students connect units for area and volume (e.g., square centimetres as cm^2, cubic metres as m^3) to measurement and geometry.

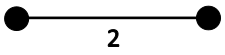
Ask students to draw a square with side lengths of 3 units. The base is represented by the side length of the square. A whole number base raised to the exponent of 2 results in a square number. They should conclude the area of this square is 3^2 or 9 $units^2$.



Ask students to use multi linking cubes to build and then draw a cube with side lengths of 2 units. The base is represented by the edge of the cube. A whole number base raised to the exponent of 3 results in a cube number. They should conclude the volume of this cube is 2^3 or 8 $units^3$.



Students should observe that 3^2 yields a two-dimensional **image** (length, width) and that 2^3 yields a three-dimensional **image** (length, width, and height). Ask students what type of image models the expression 2^1 and what measurements are involved. This should lead to a discussion of one-dimensional **images** which possess only length. The value 2^1 would produce:



Ultimately, students should recognize that powers with an exponent of 1 will have a value equal to the base ($3^1=3$, $a^1=a$). If students do not see an exponent, it is implied that the exponent is 1 ($8=8^1$, $b=b^1$).

As they explore such expressions, students should become aware that interchanging the base and the exponent will most likely not result in a power having the same value. For example, through the use of models and repeated multiplication, students should conclude that $3^2 \neq 2^3$ and that exponents and bases are not interchangeable. Ask them if they can think of an example of powers where the exponent and base interchanged result in the same value (i.e., 2^4 and 4^2 both have a value of 16). It should be

emphasized that sometimes the same number can be expressed in multiple ways using powers (e.g., 64 = 8^2 or 4^3 or 2^6).

Students have used parentheses to evaluate expressions involving the order of operations. They should have an understanding that the placement of parentheses in a mathematical statement sometimes changes the value of the expression (e.g., $2+3\times4\neq(2+3)\times4$).

Similarly, the placement of parentheses in an exponential expression can affect the value of the expression. Ask students to simplify the expression -3^2 and then $(-3)^2$ on their calculator and explain why their answers are different.

Parentheses are used when a power has a negative base to show that the negative sign is part of the base. As in the above example, -3^2 is not equal to $(-3)^2$. Students should be able to explain the role of parenthesis in powers by evaluating a given set of powers. For example:

$(-2)^4=(-2)\times(-2)\times(-2)\times(-2)=16$, where the base is –2

$-(2)^4=-(2\times2\times2\times2)=-16$, where the base is 2

$-2^4=-(2\times2\times2\times2)=-16$, where the base is 2

One way to demonstrate that a^0 is equal to 1 is through the use of patterns. Ask students to simplify the following powers to verify a power with an exponent of zero is equal to 1 (excluding base 0).

$2^5=32$	$3^5=343$
$2^4=16$	$3^4=81$
$2^3=8$	$3^3=27$
$2^2=4$	$3^2=9$
$2^1=2$	$3^1=3$
$2^0=1$	$3^0=1$

This can be revisited in the next outcome when students are introduced to the laws of exponents. At this point, it is sufficient that students understand that a^0=1 through the use of patterning.

Students should evaluate exponential expressions to determine their values in standard form. Such expressions may contain a single term or multiple terms. To continue to develop number sense, the use of mental mathematics should be encouraged whenever possible.

Assessment, Teaching, and Learning

Assessment Strategies

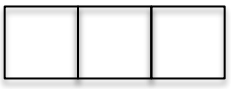
ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Draw a diagram to represent each square. State the value of each square.
  - 2^2

  - 5^2
  - 7^2

- The perimeter of a square is 28 cm. Three of these squares are joined together to make rectangle. What is the area of rectangle?



WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Demonstrate using patterns, that $8^0 = 1$.
- Express 25 as a power where the exponent is 2 and the base is:
  - positive
  - negative
- Explain why 6^2 is called a square number while 6^3 is called a cube number.
- Determine if $(-6)^2 = -6^2$ Explain why or why not. Determine if the same reasoning applies to $(-6)^3 = -6^3$.
- Identify the base in the expression $-(-7)^5$.
- Write as a power: $9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9$.
- Write 32 as a power with a base of 2.
- If three students with cold germs on their hands each shook hands with three other students, and those students each shook hands with three other students, how many students would be exposed to the cold germs?
- Respond to the following:
  - Explain why $-62 \neq (-6)^2$ but $-6^3 = (-6)^3$.
  - Explain why $(-3)^2 > 0$ but $(-3)^3 < 0$.
- Evaluate powers such as the following:
  - $3 + 2^0$
  - $3^0 + 2^0$
  - $(3 + 2)^0$
  - $-3^0 + 2$
  - $-3^0 + (-2)^0$
  - $-(3 + 2)^0$

Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide students with many opportunities to explore concrete and pictorial representations of 2-D and 3-D models in order to differentiate between the base and the exponent.
- Investigate the difference between pairs of powers such as: 6^2 and 2^6, 5^8 and 8^5.
- Investigate the pattern in powers using a given base and exponents from 4 to 0 such as: 3^4, 3^3, 3^2, 3^1, 3^0.
- Provide students with the opportunity to explore the difference between negative and positive bases, with or without parentheses.
- Explore the calculator to find the most efficient way to evaluate powers (e.g., y^x, x^y, x^y).

Suggested Learning Tasks

- Provide students with 25 tiles and 30 multi linking cubes. Have students explore the number of tiles required to make squares and the number of cube-a-links required to make cubes. Students should investigate squares with sides of 1, 2, 3, 4, and 5 tiles. They should also investigate cubes with sides of 1, 2, and 3 cube-a-links, while a cube with a side of 4 cube-a-links could be done as a whole group.
- Have students copy and complete the following table.

Repeated Multiplication	Power	Value
	6^2	
$3\times3\times3\times3\times3\times3$		
		64
	$(-3)^4$	
$-(5\times5\times5)$		
		-49

- Ask students to create models for 4^2 and 4^3 using multi linking cubes. They should describe how these models are the same, and also how they are different. Ask students if they can model 41 using multi linking cubes and to explain their reasoning.
- Teachers could create a set of flash cards containing pairs that display powers in exponential form on one card and repeated multiplication on the other. One card is distributed to each student in the room randomly. Ask students to find their partner with the matching card and to explain why they are a match. The goal is to be both quick and correct. This is a simple game to reinforce the meaning of exponents.
- Ask students to represent the numbers below using as many different powers as possible. Ask them if some of them can only be represented in one way. If so, they should identify which ones.
  - 144
  - –32
  - 64
  - 81
  - 125

- 70

SUGGESTED MODELS AND MANIPULATIVES

- Building Blocks*
- calculator
- colour tiles
- grid paper
- multi linking cubes

*also available in *Interactive Math Tools* (Pearson n.d.)

MATHEMATICAL LANGUAGE

Teacher	Student
▪ base ▪ cube ▪ exponent ▪ integral base ▪ power ▪ repeated multiplication ▪ square	▪ base ▪ cube ▪ exponent ▪ power ▪ repeated multiplication ▪ square

Resources

Digital

Interactive Math Tools. (Pearson. n.d.): http://nsvs.ednet.ns.ca/nsps/nsps26/course/view.php?id=3875.

Print

- *Math Makes Sense 9* (Baron et al. 2009; NSSBB #: 2001644)
  - Unit 2: Powers and Exponent Laws
    - > Section 2.1: What Is a Power?
    - > Section 2.2: Powers of Ten and the Zero Exponent
    - > Unit Problem: How Thick Is a Pile of Paper?
  - *ProGuide* (CD; Word Files; NSSBB #: 2001645)
    - > Assessment Masters
    - > Extra Practice Masters
    - > Unit Tests
  - *ProGuide* (DVD; NSSBB #: 2001645)
    - > Projectable Student Book Pages
    - > Modifiable Line Masters
- *Teaching Student-Centered Mathematics, Grades 5–8,* Volume Three (Van de Walle and Lovin 2006), pp. 131–132

SCO N02 Students will be expected to demonstrate an understanding of operations on powers with integral bases (excluding base 0) and whole number exponents

- $\left(a^m\right)\left(a^n\right)=a^{m+n}$
- $a^m \div a^n = a^{m-n},\ m>n$
- $(a^m)^n = a^{mn}$
- $(ab)^m = a^m b^m$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n},\ b\neq 0$

[C, CN, PS, R, T]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N02.01 Explain, using examples, the exponent laws of powers with integral bases (excluding base 0) and whole number exponents.
N02.02 Evaluate a given expression by applying the exponent laws.
N02.03 Determine the sum of two given powers and record the process.
N02.04 Determine the difference of two given powers and record the process.
N02.05 Identify the error(s) in a given simplification of an expression involving powers.

Scope and Sequence

Mathematics 8	Mathematics 9	Mathematics 10
–	**N02** Students will be expected to demonstrate an understanding of operations on powers with integral bases (excluding base 0) and whole number exponents ▪ $\left(a^m\right)\left(a^n\right)=a^{m+n}$ ▪ $a^m \div a^n = a^{m-n},\ m>n$ ▪ $(a^m)^n = a^{mn}$ ▪ $(ab)^m = a^m b^m$ ▪ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n},\ b\neq 0$	**AN03** Students will be expected to demonstrate an understanding of powers with integral and rational exponents.

Background

The primary focus in Mathematics 9 should be to develop an understanding of the exponent laws involving powers with integral bases (excluding base 0) and **whole number** exponents. Instruction should be designed so that students have the opportunity to discover rules and relationships and are able to confirm their discoveries to develop an understanding of the laws of exponents. By expanding powers and completing the calculation, students should be able to predict the exponent laws.

Operations with exponents can be efficiently performed with the use of the laws of exponents. In Mathematics 10, this will be extended to include powers with fractional exponents and negative exponents, as well as powers with rational and literal bases.

The chart below shows the relationship between repeated multiplication and exponent laws. Students who have an understanding of the relationship between all three components of this table will be able to manipulate numbers to solve problems using various strategies.

Sample Question	Repeated Multiplication	Simplification	Exponent Law
Product of Powers $3^2 \times 3^5$	$(3\times3)\times(3\times3\times3\times3\times3)$	$3^{2+5}=3^7$	$(a)^m \times (a)^n = a^{m+n}$
Quotient of Powers $\frac{5^6}{5^2}$	$\frac{5\times5\times5\times5\times5}{5\times5}=\frac{5}{5}\times\frac{5}{5}\times\frac{5}{1}\times\frac{5}{1}\times\frac{5}{1}\times\frac{5}{1}$	$5^{6-2}=5^4$	$\frac{a^m}{a^n}=a^{m-n}$
Power of a Power $(4^2)^3$	$4^2 \times 4^2 \times 4^2 = (4\times4)\times(4\times4)\times(4\times4)$	$4^{2\times3}=4^6$	$(a^m)^n=a^{mn}$
Product of a Power $(2\times4)^3$	$(2\times4)\times(2\times4)\times(2\times4)=(2\times2\times2)\times(4\times4\times4)$	$2^3\times4^3$	$(ab)^m=a^m\times b^m$
Quotient of a Power $\left(\frac{2}{5}\right)^3$	$\left(\frac{2}{5}\right)\times\left(\frac{2}{5}\right)\times\left(\frac{2}{5}\right)=\frac{2\times2\times2}{5\times5\times5}$	$\frac{2^3}{5^3}$	$\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}$

Students should be introduced to each rule separately and provided with other sample questions in a table so that they may predict the rule themselves. Use powers with positive *and* negative bases as they repeat the process. Naming the exponent law should not be a focus, but generating the law themselves should be.

For expressions in the form $a^m \div a^n = a^{m-n}$, ensure that $m > n$, as students are only expected to work with whole number exponents in Mathematics 9. They are already familiar with expressions where $m = n$ resulting in a power with an exponent of zero through patterning in outcome N01, but this is a good time to revisit why this is equal to a value of 1. Integral exponents will be introduced in Mathematics 10.

Whenever possible, instruction should be designed so that students discover rules/relationships and verify discoveries. Students can explore different solutions to problems to develop an appreciation for the efficiency that the exponent laws provide. For example, alternate solutions used to evaluate $\left(2^3 \times 2^2\right)\left(2^3 \times 2^2\right)$ are shown here:

$\left(2^3 \times 2^2\right)^2$	$\left(2^3 \times 2^2\right)^2$	$\left(2^3 \times 2^2\right)\left(2^3 \times 2^2\right)$
$=\left(2^{3+2}\right)^2$	$=\left(2^3\right)^2 \times \left(2^2\right)^2$	$=\left(2^5\right)\left(2^5\right)$
$=(2^5)^2$	$=(2^{6+4}$	$=(2^{5+5}$
$=2^{10}$	$=2^{10}$	$=2^{10}$
$=1024$	$=1024$	$=1024$

Students should be encouraged to use the laws as efficiently as possible. It is important that examples, such as those shown above, be done by applying the exponent laws instead of using a calculator. This provides an important foundation for multiplying and dividing a polynomial by a monomial.

Students sometimes think that addition and subtraction of powers with the same base apply the same types of rules as with multiplication and division. Have students evaluate to show that this is not the case.

For example:

$4^3 + 4^2$

$=(4 \times 4 \times 4) + (4 \times 4)$

$=64 + 16$

$=80$

Some common student errors with exponential calculations are highlighted here.

Incorrect:	Correct:
$5^3 – 3$	$5^2 – 3$
$=10 – 3$	$=25 – 3$
$=7$	$=22$

Students should be reminded that the order of operations must always be applied when evaluating a mathematical expression. The terms with exponents, therefore, are evaluated before the powers can be added or subtracted. Students often apply the order of operations incorrectly to expressions involving powers.

Incorrect:	Correct:
$7 + 2 \times 4^2 – 4$	$7 + 2 \times 4^2 – 4$
$= 9 \times 4^3 – 4$	$= 7 + 2 \times 16 – 4$
$= 1296 – 4$	$= 7 + 32 – 4$
$= 1292$	$= 35$

Students should be able to identify and correct errors such as those present in the following:

Incorrect	Correct
$(4 + 3)^2$	$(4 + 3)^2$
$= (4^2 + 3^2)$	$= (7)^2$
$= (16 + 9)$	$= 49$
$= 25$	

Sometime students confuse $(4 + 3)^2$ and $(4 \times 3)^2$ and attempt to evaluate both the same way. Students need to understand how to properly evaluate $(4 + 3)^2$, as they will need to expand expressions such as $(a + b)^2$ in Mathematics 10.

Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Fill in the missing exponent.
  - $5^n = 5$
  - $(-3)^n = -27$
  - $9^n - 1 = 80$
  - $15^n = 1$
  - $(-4)^n = 256$

- Complete the following table:

Base	Exponent	Power	Value
(–6)	3		
4			64
	2		64
(–2)			–32

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Explain why $(-5) \times (-5)^6 \times (-5)^2 = (-5)^9$.
- Show why $(4^2)^5 = 4^{10}$.
- Explain why $5^2 \times 5^4$ and $(5^2)^4$ are different. Show your work.
- Write the expression $6^5 \times 5^5$ using only one exponent.

- Write the expression $\frac{4^4 \times 4}{4^2}$ in simplified form, and then evaluate.
- Write the following expression as the division of two powers. $\left(\frac{-4}{7}\right)^4$
- Evaluate:
  - $-(-3)^5$
  - $(1-3)^4 \div 2^2$
- The prime factorization of 2048 is $2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2$. Write 2048 as the product of two powers of 2 in as many ways as possible.
- Yvan made an error in simplifying the following expression:

  $(15 \div 5)^4 + (2 + 5)^2$

  $= (3)^4 + 2^2 + 5^2$

  $= 81 + 4 + 25$

  $= 110$

  – Indicate where he made his mistake and describe it.
  – Show the correct procedure and determine the correct answer.

Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Ensure that there is a clear understanding of powers as a short way to represent a repeated multiplication of the same number.
- Have students show the relationship between repeated multiplication and exponent laws using repeated multiplication (see Elaboration for chart illustrating this relationship). This concept is taught in preparation for literal bases addressed in Mathematics 10.
- Place an emphasis on simplification of expressions using exponent laws before evaluation.
- Note special cases, such as when there is no exponent a "1" is understood: $5 = 5^1$.
- Have students demonstrate an understanding of exponent laws through explanations of inappropriate use of exponent laws, e.g.: $\left(2+3^2\right)^3 \neq 2^3+3^6$.

Suggested Learning Tasks

- Create a foldable with an entry for each of the exponent laws accompanied by examples.

Product of Powers $(a^m)(a^n) = a^{m+n}$
Quotient of Powers $\frac{a^m}{a^n} = a^{m-n}$
Power of a Power $(a^m)^n = a^{mn}$
Power of a Product $(ab)^m = a^m \times b^m$
Power of a Quotient $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

- Explain how to write a product or a quotient of powers as a single power.
- Write the following as a single power:
  - $\left(-5\right)^3 \times \left(-5\right)^7$
  - $\left(\frac{2}{5}\right)^5$
  - $\left[\left(-3\right)^2\right]^3$
  - $\left[5 \times \left(-4\right)\right]^3$
- The "9" key on your calculator is missing. Explain how you can find the value of 9^4 without using the "9" key.
- Explain two ways that $\left[\left(-4\right) \times 5\right]^3$ can be calculated. Which way is more efficient?
- Solve $7^9 \div \left(7^7 \times 7\right)$ mentally.
- Simplify the following expression by first rearranging the powers and putting like bases together: $2^4 \times 5^3 \times 2^6 \times 10^2 \times 10^3 \times 5^{10}$
- Express $2^4 \times 3^4$ with a single base using the law of exponents. Extend this by asking them to simplify $8^2 \times 2^5$ to a single base using the law of exponents.
- Provide students with an index card where they are asked to simplify expressions as a single power. The student should not place their name on the card since the index cards could be used for error analysis during the next class. This activity could be used as an exit card. A sample list is shown:
  - $\left(-4\right)^2 \left(-4\right)^2$
  - $3^6 \times 3^8$
  - $\frac{4^{10}}{4^9}$

- $(-3)^6 \div (-3)^4$
- $\left(\left(-9\right)^2\right)^5$
- $\left(3^7 \times 3^8\right) \div \left(3^3\right)^2$
- $\left(2^3 \times 2^7\right)^3 - \left(3^7 \times 3\right)^2$
- $2^5 \times 2 - 2^3 \times 2^2$

- Ask students to play the Exponent Bingo game. Give each student a blank Bingo card, and ask them to number the card in any fashion from 1 to 24. Allow for a free space. Ask students to simplify various expressions, such as $\left(2^4 \times 2^3\right) \div 2^6$, find the value on their Bingo cards and cross it off. Traditional Bingo alignments win—horizontal, vertical, and diagonal. Options such as four corners may also be used.

SUGGESTED MODELS AND MANIPULATIVES

- index cards

MATHEMATICAL LANGUAGE

Teacher	Student
▪ expand ▪ exponent ▪ evaluate ▪ power ▪ product ▪ quotient ▪ simplify	▪ expand ▪ exponent ▪ evaluate ▪ power ▪ product ▪ quotient ▪ simplify

Resources

Digital

- "Exploring Laws of Exponents: Use It," *Math Interactives* (Alberta Education, LearnAlberta.ca 2015): www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.NUMB&ID2=AB.MATH.JR.NUMB.EXPO&lesson=html/object_interactives/exponent_laws/use_it.html
- "Exploring Laws of Exponents: Explore It," *Math* Interactives (Alberta Education, LearnAlberta.ca 2015): www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.NUMB&ID2=AB.MATH.JR.NUMB.EXPO&lesson=html/object_interactives/exponent_laws/explore_it.html

Print

- *Math Makes Sense 9* (Baron et al. 2009; NSSBB #: 2001644)
  - Unit2: Powers and Exponent Laws
    - > Section 2.4: Exponent Laws I
    - > Section 2.5: Exponent Laws II
  - *ProGuide* (CD; Word Files; NSSBB #: 2001645)
    - > Assessment Masters
    - > Extra Practice Masters
    - > Unit Tests
  - *ProGuide* (DVD; NSSBB #: 2001645)
    - > Projectable Student Book Pages
    - > Modifiable Line Masters
- *Teaching Student-Centered Mathematics, Grades 5–8*, Volume Three (Van de Walle and Lovin 2006), pp. 131–132
- *Big Ideas from Dr. Small: Creating a Comfort Zone for Teaching Mathematics, Grades 9–12* (Small 2010), pp. 100–103

SCO N03 Students will be expected to demonstrate an understanding of rational numbers by comparing and ordering rational numbers and solving problems that involve arithmetic operations on rational numbers. [C, CN, PS, R, T, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N03.01 Order a given set of rational numbers in fraction and decimal form by placing them on a number line.
N03.02 Identify a rational number that is between two given rational numbers.
N03.03 Solve a given problem involving operations on rational numbers in fraction or decimal form.

Scope and Sequence

Mathematics 8	Mathematics 9	Mathematics 10
N04 Students will be expected to demonstrate an understanding of ratio and rate. **N05** Students will be expected to solve problems that involve rates, ratios, and proportional reasoning. **N06** Students will be expected to demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially, and symbolically.	**N03** Students will be expected to demonstrate an understanding of rational numbers by comparing and ordering rational numbers and solving problems that involve arithmetic operations on rational numbers.	**AN02** Students will be expected to demonstrate an understanding of irrational numbers by representing, identifying, simplifying, and ordering irrational numbers. **FM01** Students will be expected to solve problems that involve unit pricing and currency exchange, using proportional reasoning. **FM02** Students will be expected to demonstrate an understanding of income to calculate gross pay and net pay, including wages, salary, contracts, commissions, and piecework.

Background

The four operations on negative integers were introduced in Mathematics 8. These operations are extended to rational numbers in Mathematics 9.

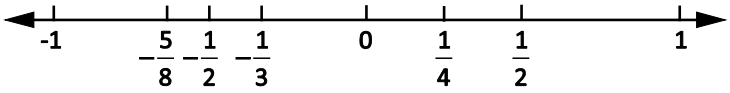
A rational number is any number that can be written as a ratio of two integers $\frac{a}{b}$, where b does not equal zero. Classifying numbers by sets is introduced in Mathematics 10.

Students have previous experience with ratios, integers, positive decimals, and fraction operations. In Mathematics 6, students were introduced to ordering integers. In Mathematics 7, students compared and ordered positive fractions, positive decimals, and whole numbers. In Mathematics 9 they will compare and order rational numbers, including negative fractions and negative decimals. They have worked with a variety of strategies for comparing fractions and decimals. Such strategies can be revisited here in the context of comparing rational numbers.

The placement of a negative sign in a fraction will be an extension of what students have learned in the past. It is important for students to understand that: $-\frac{6}{2}, \frac{6}{-2}$ and $-\frac{6}{2}$ are all equivalent. However the first ratio is the preferred placement of the negative sign. This becomes apparent when the division is completed and all equal – 3, regardless of where the negative sign is placed.

Comparing and ordering rational numbers largely draws upon students' number sense. Strategies for ordering numbers should include the following:

- understanding that a negative number is always less than a positive number
- developing a number line with zero marked, and with positioning of positive and negative benchmark fractions for comparison purposes, without conversion to decimals:



- comparing fractions with the same denominator
- comparing fractions with unlike denominator
- comparing fractions with the same numerator (Students should develop a variety of strategies to compare fractions in addition to finding common denominators).
- identifying fractions between any two given fractions, or decimals between any two decimals such as 0.3 and 0.4 , $\frac{1}{2}$ and $\frac{1}{3}$, $-\frac{1}{2}$ and $-\frac{1}{3}$
- reading fractions properly ($\frac{1}{4}$ is read as one-fourth and NOT as one over four, $-\frac{1}{8}$ is read as negative one-eighth and NOT as negative one over eight)

Students have a choice of strategies when they are asked to determine a rational number between a fraction and a decimal. They could convert the fraction to a decimal, or vice versa, and then use the appropriate method.

Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- What fractions are represented in the following diagram? Explain your reasoning.



- Order the following fractions without using decimal representations or converting all fractions to a common denominator: $\frac{3}{5}, \frac{8}{9}, \frac{1}{4}, \frac{9}{10}, \frac{13}{7}, \frac{13}{11}, \frac{7}{8}, \frac{1}{3}$
- Ask students which is greater:

$\frac{3}{10}$ or $\frac{17}{19}$ $\frac{2}{3}$ or 0.74

0.55 or 0.5

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Use estimation to determine which expression has the greatest quotient.

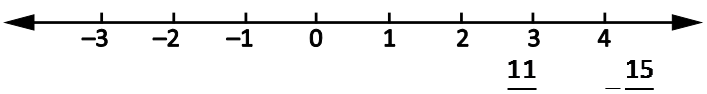
$\frac{9}{5} \div \frac{11}{12}$ $2\frac{1}{5} \div 1\frac{6}{8}$ $-3\frac{1}{10} \div \frac{5}{6}$ $-\frac{1}{4} \div -\frac{3}{8}$

- Find three rational numbers that lie between each of the following:

– -1 and 0 $\frac{1}{2}$ and $\frac{1}{3}$ -3.5 and -3.6 $-\frac{1}{3}$ and -0.4 $-\frac{2}{3}$ and -0.6

- Order the following rational numbers on the number line:

– 2.6 $-\frac{1}{2}$ $\frac{5}{3}$ 1.2 $-\frac{7}{8}$



$\frac{11}{5}$ $-\frac{15}{4}$

- Identify all integers that are between $\frac{11}{5}$ and $-\frac{15}{4}$.

Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Present examples of positive values with concrete models and pictures before moving to symbolic representation or introducing negative values.

- Generalize and apply a rule for determining the sign of the product or quotient of rational numbers through exploring patterns.
- Extend the common denominator method for division of fractions, taught in grade 8, to negative fractions. When denominators are the same, the numerators can be divided as in the following example:

$$\frac{5}{3} \div \frac{-1}{2} = \frac{10}{6} \div \frac{-3}{6} = \frac{10 \div (-3)}{6 \div 6} = \left(\frac{\frac{-10}{3}}{1}\right) = \frac{-10}{3}$$

The answer, read as "negative ten thirds," can be left as is, unless the context of the question requires it to be expressed in the mixed form: $-3\frac{1}{3}$.

- Compare the multiplication and division of fractions using the meanings of the operations in expressions such as:

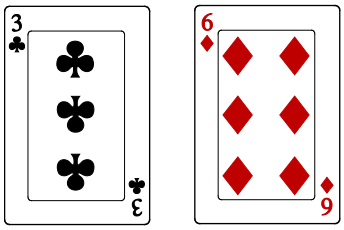
$8 \div -\frac{1}{2} = -16$ (How many halves in 8? How does the negative sign affect the answer?)

$8 \times -\frac{1}{2} = -4$ (What is half of 8? How does the negative sign affect the answer?)

- Use number lines as models for comparing and ordering rational numbers and for addition and subtraction of rational numbers.
- For this outcome, calculators should be discouraged, as careful attention should be given to the numbers used.

SUGGESTED LEARNING TASKS

- Prepare a set of cards with a variety of rational numbers. Give each student one card. Place them in teams. Students must compare their cards and arrange themselves so that they are in order (ascending or descending). The team who correctly orders the cards first is the winner. A variation of this activity is the Clothesline Game. Students are required to 'pin' the cards on a clothesline that has been prepared in advance. The clothesline would be marked off at regular intervals, representing integers on a number line.
- From several decks of cards remove all the cards that are numbered from 2 to 9. These will be the game cards. Pair students off and give each pair a set of game cards. Students should place the set of cards face down and turn over two cards at once.



- The black cards represent positives and the red cards represent negatives. Students will form two fractions using the indicated cards. In the case above, the fractions will be $-\frac{3}{6}$ and $-\frac{6}{3}$. The first student to determine which fraction is closest to zero wins that round. The game continues until all cards have been played. This activity can be modified to work with decimals.

- Use patterning to justify the result for a negative multiplied by a negative using rational numbers. Have students complete the following pattern:

$3 \times \frac{-1}{2} = -\frac{3}{2}$

$2 \times \frac{-1}{2} = -1$

$1 \times \frac{-1}{2} = \frac{-1}{2}$

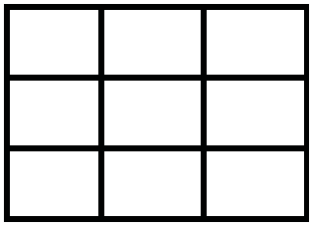
$0 \times \frac{-1}{2} = 0$

$-1 \times \frac{-1}{2} = \frac{1}{2}$

$-2 \times \frac{-1}{2} = ____$

$-3 \times \frac{-1}{2} = ____$

- Determine three rational numbers between two given decimal numbers, such as 0.6 and 0.61. Choose one number and explain why it is between the two given numbers.
- In a magic square, the sum of each row, column, and diagonal is the same:
  - Create a magic square that uses a mixture of positive and negative rational numbers written in decimal form.
  - Create a magic square that uses a mixture of positive and negative rational numbers written in fractional form.



- A children's wading pool has a small leak. During an afternoon, one-eighth of the water leaks out of the pool. What could the expression below describe about this situation?

$0.75 \times \left(-\frac{1}{8}\right)$

  - Using Think-Pair-Share, give individual students time to think about the question. Students then pair up with a partner to discuss their ideas. After pairs discuss, students share their ideas in a small-group or whole-class discussion.

SUGGESTED MODELS AND MANIPULATIVES

- fraction pieces
- integer tiles
- Number Lines (virtual Math Tools)

MATHEMATICAL LANGUAGE

Teacher	Student
▪ benchmarks ▪ compare ▪ greater, greatest ▪ integer ▪ less, least ▪ place value ▪ rational number ▪ simplify	▪ benchmarks ▪ compare ▪ greater, greatest ▪ integer ▪ less, least ▪ place value ▪ rational number ▪ simplify

Resources

Digital

"Ordering Rational Numbers," *SoftSchools.com* (Softschools.com 2015): www.softschools.com/math/rational_numbers/ordering_rational_numbers

Interactive Math Tools. (Pearson. n.d.): http://nsvs.ednet.ns.ca/nsps/nsps26/course/view.php?id=3875.

Print

- *Math Makes Sense 9* (Baron et al. 2009; NSSBB #: 2001644)
  - Unit3: Rational Numbers
    - > Section 3.1: What is a Rational Number?
    - > Start Where You Are: How Can I Learn from Others?
    - > Section 3.2: Adding Rational Number
    - > Section 3.3: Subtracting Rational Numbers
    - > Game: Closest to Zero
    - > Section 3.4: Multiplying Rational Numbers
    - > Section 3.5: Dividing Rational Numbers
    - > Unit Problem: Investigating Temperature Data
  - *ProGuide* (CD; Word Files; NSSBB #: 2001645)
    - > Assessment Masters
    - > Extra Practice Masters
    - > Unit Tests
  - *ProGuide* (DVD; NSSBB #: 2001645)
    - > Projectable Student Book Pages
    - > Modifiable Line Masters
- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 207–209
- *Teaching Student-Centered Mathematics, Grades 5–8*, Volume Three (Van de Walle and Lovin 2006), pp. 148–149

SCO N04 Students will be expected to explain and apply the order of operations, including exponents, with and without technology. [PS, T]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N04.01 Solve a given problem by applying the order of operations without the use of technology.
N04.02 Solve a given problem by applying the order of operations with the use of technology.
N04.03 Identify the error in applying the order of operations in a given incorrect solution.

Scope and Sequence

Mathematics 8	**Mathematics 9**	**Mathematics 10**
N06 Students will be expected to demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially, and symbolically. **N07** Students will be expected to demonstrate an understanding of multiplication and division of integers, concretely, pictorially, and symbolically.	**N04** Students will be expected to explain and apply the order of operations, including exponents, with and without technology.	**AN02** Students will be expected to demonstrate an understanding of irrational numbers by representing, identifying, simplifying, and ordering irrational numbers. **FM01** Students will be expected to solve problems that involve unit pricing and currency exchange, using proportional reasoning. **FM02** Students will be expected to demonstrate an understanding of income to calculate gross pay and net pay, including wages, salary, contracts, commissions, and piecework.

Background

Order of operations was introduced in Mathematics 6 and practiced in Mathematics 7 and Mathematics 8 when solving problems involving a variety of operations with integers, positive decimals and fractions. In Mathematics 9, students will extend the rules of order of operations to exponents and negative rational numbers. It is important that students have a solid foundation in operations with rational numbers, since it is fundamental to the study of algebra.

The order of operations is:
1. **Brackets**—Brackets group symbols so that we can treat them as a single term.
2. **Exponents**
3. **Multiplication and division** from left to right, in the order in which they appear. This means that multiplication and division are performed before addition or subtraction. It does not mean that multiplication should be done before division. Multiplication and division have the same priority. If both multiplication and division occur within an expression, these operations are performed from left to right.
4. **Addition and subtraction** from left to right, in the order in which they appear

It is important for students to demonstrate their understanding of the order of operations, with and without the use of calculators, and not simply memorize the mnemonic (BEDMAS). The mnemonic is solely a memory aid, and does nothing to promote the understanding that exponents are performed prior to multiplication because exponents represent repeated multiplication; multiplication is performed prior to addition because multiplication can represent repeated addition; and that division and subtraction are considered in the same manner, as there is an inverse relationship between multiplication and division, and between addition and subtraction. To help students gain an understanding of this concept, examine how grouping and order affects answers prior to going to the 'rules' of the mnemonic, by looking at the established order of precedence in which operations must be done. BEDMAS is simply a reminder of this order. Start by looking at an expression to see if order matters. For example, with $2+3\times5$:

Incorrect:	Correct:
$2+3\times5$	$2+3\times5$
$=5\times5$	$=2+15$
$=25$	$=17$

Since we can write $2+3\times5$ as $2+5+5+5$=17, it is reasonable to do multiplication before addition.

Exponents can be looked at in a similar manner. Since $2\times4\times4\times4=128$ can be written as 2×4^3, it is reasonable to do exponents before multiplication.

When there is more than one operation in an expression, it can cause confusion. Students need to understand that it matters what operation is done first. Have students examine expressions when order is important. For example:

Correct:	Incorrect:
$\frac{5}{6}\div\frac{1}{2}\times\left(-\frac{1}{4}\right)$	$\frac{5}{6}\div\frac{1}{2}\times\left(-\frac{1}{4}\right)$
$=\frac{5}{6}\times\frac{2}{1}\times\left(-\frac{1}{4}\right)$	$=\frac{5}{6}\div\left(-\frac{1}{8}\right)$
$=\frac{5}{3}\times\left(-\frac{1}{4}\right)$	$=\frac{5}{6}\times\left(-\frac{8}{1}\right)$
$=-\frac{5}{12}$	$=-\frac{20}{3}$

The answers are different. Operations need to be performed in the order that they appear, from left to right.

As an indication of understanding, students should be given steps towards an incorrect solution to a problem and be able to identify the step at which the error occurred.

Students should demonstrate a competence in evaluating expressions that include fractions, powers, decimals, and negative integers.

Note: Students can use calculators however, be aware that entry sequences of calculators will vary. An exploration of this variation could offer an opportunity to develop a better understanding of order of operations. It is important for students to know how their personal calculators process the input and that they are able to apply this knowledge.

Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Play Order of Operations Bingo from NCTM: http://illuminations.nctm.org/Lesson.aspx?id=2583.
- Which operations should be completed first in the following expressions:
  - $1\frac{2}{5} - \frac{3}{4} \times \frac{2}{5}$
  - $\left(\frac{2}{3} - \frac{1}{4}\right) \times \frac{7}{8}$
  - $3 + \frac{5}{6} - \frac{4}{5} \div \frac{4}{5}$
- Evaluate the expressions above, showing all steps.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Without the use of a calculator, simplify the expression and express your answer as a fraction.
  - By inserting one pair of brackets, how many different answers are possible?
  - By inserting two pairs of brackets, is it possible to receive a different answer?

  $\frac{1}{2} - \frac{1}{3} \times \frac{1}{2} - \frac{1}{3}$

- Use a calculator to simplify the following:

  $$\frac{56.3 - 22.5}{4.2 \times (5.9 - 10.5)}$$

- Arrange the solutions of the following from least to greatest:

$\frac{-3}{4}-\left(\frac{-3}{4}+\frac{4}{-5}\right)$ $\frac{-3}{5}-\frac{-3}{4}+\frac{9}{-10}$ $6\div\frac{-1}{5}-\frac{1}{-2}$ $\frac{3}{5}-\left(\frac{-3}{5}-\frac{-2}{3}\right)$

- Indicate at which step an error occurred and explain:

  $5-2(4+5)^2$

  $5-2(9)^2$ Step 1

  $3(9)^2$ Step 2

  $3(81)$ Step 3

  243 Step 4

- Use a calculator to convert the following Fahrenheit temperatures to Celsius, using the given formula: $^\circ C=\frac{5}{9}(^\circ F-32^\circ)$

  10°F 15°F –17.2°F

- Explain why it is essential that the rules for order of operations for rational numbers be the same as the order of operations for integers.

Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Explore a variety of expressions in which brackets, fractions, and negative numbers are used. Through these explorations students will demonstrate that the rules for order of operations ensure a consistent result.
- Students should be encouraged to perform operations without calculators as much as possible. However, some questions lend themselves to calculator use more than others. When evaluating with decimals, it is appropriate to use a calculator for more than 2-digit multipliers or more than 1-digit divisors.
- Have students compare their results when using calculators to simplify expression and, if different from one another, determine how different calculators interpret data inputs differently. This could be an opportunity to establish the importance of order of operations, and the importance of correct calculator use.
- For differentiation: Have students simplify expressions, showing each step in the order of operations. To the right of each step, identify the step as brackets, exponents, multiplication, division, addition, or subtraction.

SUGGESTED LEARNING TASKS

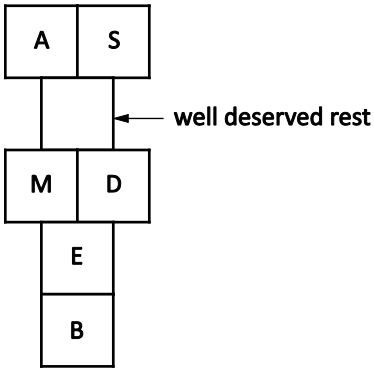
- Simplify expressions that include fractions and a mix of operations, without the use of a calculator, and express the answer as a fraction. Explore how the insertion of one or more sets of brackets at various positions affects the answer. For example:

$\frac{1}{2}\times\frac{2}{3}-\frac{4}{3}\times\frac{3}{2}=-\frac{5}{3}$ $\frac{1}{2}\times\left(\frac{2}{3}-\frac{4}{3}\right)\times\frac{3}{2}=-\frac{1}{2}$ $\left(\frac{1}{2}\times\frac{2}{3}-\frac{4}{3}\right)\times\frac{3}{2}=-\frac{3}{2}$ $\frac{1}{2}\times\left[\left(\frac{2}{3}-\frac{4}{3}\right)\times\frac{3}{2}\right]=-\frac{3}{2}$

- Add brackets where required to make this a true statement:

  $\frac{4}{3}$ x 10 + 5 + (–8.1) = 11.9

- Use a calculator to explore the use of brackets to simplify expressions with multiple terms in the numerator and denominator.
- You have been hired by a company to produce a skill-testing math question using order of operations. Create the question, with the solution, to be used to determine the prizewinner.
- Explain why the following hopscotch analogy works with respect to order of operations.



- Create a game where students are given a strip containing 4 rational numbers and ask students to use $+,-,\times,\div,(\ ),\sqrt{\ },x^2$ to create expressions. The individual operations could be placed on pieces of paper to allow students to move them around the numbers. Teachers may have students create the expression with the largest solution or solution closets to zero, or any other variation.

  For example, these numbers:

  Given

–1.86	–2	5.3	9

  Students create

  $(-1.86+2)^2\times-5.3-\sqrt{9}$

A variation of this activity is for a student to choose 4 numbers, create an expression, and ask their partner to simplify the expression.

SUGGESTED MODELS AND MANIPULATIVES

- calculator

MATHEMATICAL LANGUAGE

Teacher	Student
▪ brackets ▪ cube ▪ integers ▪ negative ▪ order of operations ▪ positive ▪ square	▪ brackets ▪ cube ▪ integers ▪ negative ▪ order of operations ▪ positive ▪ square

Resources

Digital

- "24 [Game]," *4 Numbers* (4 Numbers 2015): www.4nums.com/apps
- "Exploring Order of Operations: Use It," *Math Interactives* (Alberta Education, LearnAlberta.ca 2015) www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.NUMB&ID2=AB.MATH.JR.NUMB.INTE&lesson=html/object_interactives/order_of_operations/use_it.html
- "Order of Operations Bingo," *Illuminations: Resources for Teaching Math* (NCTM 2015): http://illuminations.nctm.org/Lesson.aspx?id=2583

Print

- *Elementary and Middle School Mathematics: Teaching Developmentally*, 8th Edition (Van de Walle, Karp, and Bay-Williams 2013), pp. 473–474
- *Math Makes Sense 9* (Baron et al. 2009; NSSBB #: 2001644)
  - Unit2: Powers and Exponent Laws
    - > Section 2.3: Order of Operations with Powers
    - > Game: Operation Target Practice
  - Unit 3: What Is a Rational Number?
    - > Section 3.6: Order of Operations with Rational Numbers
    - > Unit Problem: Investigating Temperature Data
  - *ProGuide* (CD; Word Files; NSSBB #: 2001645)
    - > Assessment Masters
    - > Extra Practice Masters
    - > Unit Tests
  - *ProGuide* (DVD; NSSBB #: 2001645)
    - > Projectable Student Book Pages
    - > Modifiable Line Masters
- *Teaching Student-Centered Mathematics, Grades 5–8*, Volume Three (Van de Walle and Lovin 2006), pp. 132, 134

SCO N05 Students will be expected to determine the exact square root of positive rational numbers. [C, CN, PS, R, T]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N05.01 Determine whether or not a given rational number is a square number and explain the reasoning.
N05.02 Determine the square root of a given positive rational number that is a perfect square.
N05.03 Identify the error made in a given calculation of a square root.
N05.04 Determine a positive rational number, given the square root of that positive rational number.

Scope and Sequence

Mathematics 8	**Mathematics 9**	**Mathematics 10**
N01 Students will be expected to demonstrate an understanding of perfect squares and square roots, concretely, pictorially, and symbolically (limited to whole numbers). **N02** Students will be expected to determine the approximate square root of numbers that are not perfect squares (limited to whole numbers).	**N05** Students will be expected to determine the exact square root of positive rational numbers.	**AN01** Students will be expected to demonstrate an understanding of factors of whole numbers by determining the prime factors, greatest common factor, least common multiple, square root, and cube root.

Background

Students in Mathematics 8 learned square roots of whole numbers up to $\sqrt{144}$, including both perfect squares and estimates of non-perfect squares. They would have seen various models of perfect squares, such as square shapes drawn on grid paper or constructed with colour tiles. Similarly, a fraction or a decimal is a perfect square if it can be represented as the area of a square. They would have found square roots of perfect squares by prime factorization, mental computation, estimation, and using the calculator. These strategies should be revisited accompanied by a discussion about when to use which strategy.

In Mathematics 9, the study of square roots is extended to finding the square root of positive rational numbers that are perfect squares, including whole numbers, fractions, and decimals. Mathematicians use $\sqrt{\ }$ to represent only positive roots, so the solution to $\sqrt{25}$, is 5, which is called the principal square root. However, when solving an equation such as $x^2 = 4$, there are two solutions, +2 and –2 :

$x^2 = 4$

$x = \pm\sqrt{4}$

$x = \pm 2$

Students should understand that in problem-solving situations the answer is almost always the positive square root of a number because it is the only value that makes sense in most contexts. It is useful, however, for students to recognize that both the positive and negative values exist. Although this is not a focus of attention at this level, it will be important for solving equations in higher grade levels.

Students should learn whole number perfect squares to 400 and be able to determine perfect squares beyond 400 through guess-and-check, using estimation strategies and/or prime factorization. For example, if a student knows that $\sqrt{144} = 12$ and that $\sqrt{400} = 20$, they could estimate that $\sqrt{256}$ lies somewhere between 12 and 20.

Fraction and decimal square roots will all be variations of whole number perfect squares. For example, students will be asked to find, $\sqrt{\frac{36}{25}}, \sqrt{0.25}, \sqrt{1.44}$. Students should also be able to explain why 25 and 0.25 are perfect squares but 2.5 is not.

In Mathematics 8, perfect square numbers were connected to the area of squares. When determining the square root of positive rational numbers, students should once again be encouraged to view the area as the perfect square, and either dimension of the square as the square root.

Students could calculate the side length of a square if the area is $\frac{4}{9}$ square units by creating a 3 by 3 square, resulting in an area of 9 units^2, and to shade 4 of those 9 regions.



They should observe that the side of the square has a length of 3 units and two of those units are shaded. The square with an area of $\frac{4}{9}$ units^2 has a side length of $\frac{2}{3}$ units. This should lead students to conclude that $\sqrt{\frac{4}{9}} = \frac{2}{3}$. Ask them to verify their answer by checking that $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$.

This method can also be used to determine the square root of a decimal. To determine the value of $\sqrt{0.64}$, for example, students must first convert 0.64 to the fraction $\frac{64}{100}$. Students should be thinking about a 10 × 10 grid with 64 blocks shaded. Using the square as a visual, students should determine that $\sqrt{0.64} = \frac{8}{10}$ or 0.8.

This pictorial method is an effective way to introduce square roots of rational numbers. Students should recognize a pattern emerging as they use the square models to determine square roots. The square root

of a rational number, or a quotient, equals the quotient of the square roots of the numerator and denominator. That is, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$. As students work through several examples of finding the square root of a positive rational number that is a perfect square, they should notice the value results in a terminating decimal.

To determine if a fraction is a perfect square without an area model, students should determine if the numerator and denominator are perfect squares. Students should notice that $\sqrt{\frac{36}{25}}$ is a perfect square, since both the numerator and denominator are perfect squares. If the numerator and denominator are not perfect squares, it may be possible for students to write an equivalent fraction so that the numerator and denominator are perfect squares. The rational number $\sqrt{\frac{8}{50}}$ is a perfect square, for example, since it is equivalent to both $\sqrt{\frac{4}{25}}$ and $\sqrt{\frac{16}{100}}$.

To determine if a rational number such as 1.44 is a perfect square, ask students to examine its fractional equivalent, $\frac{144}{100}$. Similar to the example above, they should conclude that both 144 and 100 are perfect squares therefore $\frac{144}{100}$ is a perfect square as well. Using the numerator and denominator may be students' preferred method for determining if a rational number is a perfect square. However, other methods do exist. They should confirm that 12 is the square root of 144 making 144 a perfect square. Since $12 \times 12 = 144$, students should realize that $1.2 \times 1.2 = 1.44$, and conclude that 1.44 is also a perfect square.

Some students may try to generalize the numerator and denominator approach to include mixed numbers. Caution them that $16\frac{4}{9}$ is not necessarily a perfect square simply because 16, 4, and 9 are all perfect squares. Converting $16\frac{4}{9}$ to the improper fraction $\frac{148}{9}$ shows that it is not a perfect square.

Engaging students in error analysis heightens awareness of common student errors. One of the perfect squares students frequently work with is 4. Students may incorrectly compute square roots by dividing the given number by 2 rather than finding its square root. The misconception probably arises from the fact that, in this case, one-half of 4 and the square root of 4 both have the same value. Therefore, numbers other than 4 should be used to help students overcome common errors.

Students may also struggle with the correct placement of decimals when finding square roots of rational numbers. Remind them that if a rational number is between 0 and 1, the square root will be greater than the number itself. Ask students to evaluate the following square roots and to observe the pattern of the place value.

$\sqrt{8100} = 90$
$\sqrt{81} = 9$
$\sqrt{0.81} = 0.9$
$\sqrt{0.0081} = 0.09$

Students should be able to determine a number given its square root. For example, the square root of a number is 0.7. What is the number? This relates to the fact that squares and square roots are inverse operations, which should be explored. If you find the square root of a number and then square it, you end up where you started.

Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

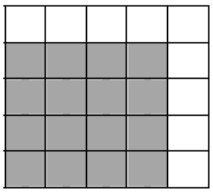
Tasks such as the following could be used to determine students' prior knowledge.

- Simplify:
  - 5^2
  - $\sqrt{196}$
  - $\sqrt{64}$
  - $\sqrt{10\,000}$
- Between which two consecutive numbers is each square root?
  - $\sqrt{115}$
  - $\sqrt{43}$
- Estimate the value of each square root in the above question, to two decimal places.

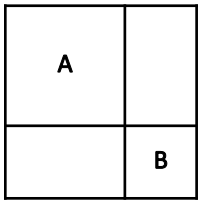
WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- If a dance floor has an area of 256 m^2, could the dance floor be a square?
- Determine what number and its square root can be represented by this grid if the whole square represents 1.



- Explain the term perfect square. Give an example of a whole number, a fraction, and a decimal that are perfect squares. Using diagrams, show why they are perfect squares.
- Find the square root of 289 using a "guess and test" strategy and prime factorization.
- Are 30, 1.6, and $\frac{2}{5}$ perfect squares? Explain your reasoning for each.
- Identify which of the following are perfect squares: 0.49, 4.9, and 0.0049. Explain your reasoning.
- Identify the error made in the following:
  - $\sqrt{16} = 8$
  - $\sqrt{0.036} = 0.6$
- Mary creates stained glass mosaics for a hobby. She wants to make one with an area of 3.24 cm^2. Determine if she can make it in the shape of a perfect square.
- A square auditorium is divided into 4 sections. Sections A and B are also squares. Section A has an area of 16 m^2 and Section B has an area of 9 m^2. Determine the combined area of the remaining space in the auditorium.



- Jim lives in the downtown area of a city where the houses are very close together. He wants to paint a windowsill on the second floor. The windowsill is 3.5 m above the ground. The only ladder available is 5 m long. The space between the houses is only 2 m, and the window is on the side of the house.
  - If he places a ladder at the height of the windowsill, how far away from the house will the base of the ladder need to be?
  - If he places the ladder as far away from the house as the house next door will allow, how far up the side of the house will the ladder reach?
  - Does the length of this ladder makes it suitable for painting the window?

Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

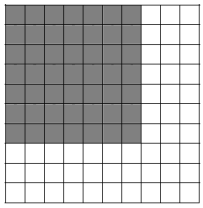
- Use area models to explore square root for fractions.
- Relate finding the square of a number to squaring the length of the sides of a square. Relate finding the dimensions of a square to finding the square root of a number.

SUGGESTED LEARNING TASKS

- Provide groups of students with of the following statements and identify them as “Always True,” “Sometimes True,” or “Never True.” Have students provide an example and where applicable a non-example for each statement and to explain their reasoning.

  – The square of a natural number is a natural number.
  – The square root of a natural number is a natural number.
  – The square of any number is bigger than the number.
  – The square root of a number is less than the number.
  – $\sqrt{n^2} = n$
  – Decimal numbers that are perfect squares have an even number of digits to the right of the decimal.
  – Perfect squares have an even number of prime factors.
  – If a mixed number consists of a whole number that is a perfect square, and a numerator and denominator that are perfect squares, the mixed number is also a perfect square.

- Challenge students to determine $\sqrt{\frac{8}{18}}$. What would be the first step? (Note that all equivalent fractions would have the same answer: $\sqrt{\frac{8}{18}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$)
- Have students investigate whether the square root of numbers greater than 1 is always less than the original number. For example, $\sqrt{64} = 8, \sqrt{1.21} = 1.1$, etc. This can lead to the misconception that this is always true for all numbers. Have students look at the square roots of perfect squares that are less than 1 , such as $\sqrt{\frac{1}{16}} = \frac{1}{4}$, or $\sqrt{0.01} = 0.1$. The area model can be used for decimals using a 10×10 grid. For example, $\sqrt{\frac{49}{100}}$ is modelled here:



- Connect the square roots of decimals to those of fractions. For example, $\sqrt{0.25} = \sqrt{\frac{25}{100}}$.
- Determine the value of each square root in simplest form:
  – $\sqrt{\frac{9}{36}}$ $\sqrt{\frac{196}{49}}$
- Provide students with the following scenario and ask them to decide who is correct, and to explain the mistakes made in the two incorrect solutions:
  – Katelyn calculated that the square root of 3.6 was 1.8. Nick determined that the answer was 0.6, and Renee estimated the answer to be 1.9.

Suggested Models and Manipulatives

- calculator
- colour tiles*
- geoboards*
- grid paper

*also available in *Interactive Math Tools* (Pearson n.d.)

Mathematical Language

Teacher	Student
▪ area model ▪ inverse operations ▪ non-perfect square ▪ perfect square ▪ rational number ▪ square root	▪ area model ▪ inverse operations ▪ non-perfect square ▪ perfect square ▪ rational number ▪ square root

Resources

Digital

Interactive Math Tools. (Pearson. n.d.): http://nsvs.ednet.ns.ca/nsps/nsps26/course/view.php?id=3875.

Print

- *Math Makes Sense 9* (Baron et al. 2009; NSSBB #: 2001644)
  - Unit 1: Square Roots and Surface Area
    - > Section 1.1: Square Roots of Perfect Squares
    - > Game: Making a Larger Square from Two Smaller Squares
  - *ProGuide* (CD; Word Files; NSSBB #: 2001645)
    - > Assessment Masters
    - > Extra Practice Masters
    - > Unit Tests
  - *ProGuide* (DVD; NSSBB #: 2001645)
    - > Projectable Student Book Pages
    - > Modifiable Line Masters
- *Teaching Student-Centered Mathematics, Grades 5–8*, Volume Three (Van de Walle and Lovin 2006), pp. 150–151

SCO N06 Students will be expected to determine an approximate square root of positive rational numbers. [C, CN, PS, R, T]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N06.01 Estimate the square root of a given rational number that is not a perfect square, using the roots of perfect squares as benchmarks.

N06.02 Determine an approximate square root of a given rational number that is not a perfect square, using technology (e.g., a calculator, a computer).

N06.03 Explain why the square root of a given rational number as shown on a calculator may be an approximation.

N06.04 Identify a number with a square root that is between two given numbers.

Scope and Sequence

Mathematics 8	**Mathematics 9**	**Mathematics 10**
N01 Students will be expected to demonstrate an understanding of perfect squares and square roots, concretely, pictorially, and symbolically (limited to whole numbers). **N02** Students will be expected to determine the approximate square root of numbers that are not perfect squares (limited to whole numbers).	**N06** Students will be expected to determine an approximate square root of positive rational numbers.	**AN01** Students will be expected to demonstrate an understanding of factors of whole numbers by determining the prime factors, greatest common factor, least common multiple, square root, and cube root.

Background

In Mathematics 8, students approximated the square root of non-perfect squares, up to $\sqrt{144}$. They used the perfect square benchmarks to enable them to identify, between which two whole numbers the square root fell. For example, since 27 is between 25 and 36, $\sqrt{27}$ lies between 5 and 6. They could also identify that the square root was closer to 5 because the $\sqrt{27}$ is closer to $\sqrt{25}$ than to the $\sqrt{36}$. Reference may have been made to the fact that the square root of non-perfect squares always result in non-terminating, non-repeating decimals, which are irrational numbers—numbers that cannot be expressed in the form $\frac{a}{b}$. Calculators may have been used to see the decimal approximations that remain an approximation no matter how many decimal places are retained in an irrational number.

In Mathematics 9, students will be required to estimate the square root of rational numbers in fraction and decimal form. Strategies used in Mathematics 8 to estimate square roots of non-perfect square whole numbers can be adapted to estimate square roots of non-perfect square rational numbers. Just as with whole numbers, students can use perfect squares as benchmarks to estimate a square root of fractions or decimals that are not perfect squares. As students estimate the square root of $\frac{14}{22}$, for example, they should identify the perfect squares closest to 14 and 22 as 16 and 25 respectively. Students should notice that $\frac{14}{22} \approx \frac{16}{25}$ and since $\sqrt{\frac{16}{25}} = \frac{4}{5}$, the square root of $\frac{14}{22}$ is approximately $\frac{4}{5}$. Students may have more difficulty determining the perfect square benchmarks when working with decimals. Encourage the use of number lines to help students visualize where the numbers lie in relation to each other. To estimate the square root of 1.30, the closest perfect square rational numbers on either side are 1.21 and 1.44. Students can then visualize where these numbers lie with respect to each other on a number line.



Since the square root of 1.21 is 1.1 and the square root of 1.44 is 1.2, students should conclude that the square root of 1.30 is between 1.1 and 1.2. Examining the location of 1.30 relative to the other two values should lead students to decide that its square root is closer to 1.1 than 1.2 and a reasonable estimate would be 1.14.

Two more examples follow:

$\sqrt{0.79} \approx \sqrt{0.81} \approx 0.9$ so $\sqrt{0.79} \approx 0.9$; students should understand that the answer is a little less than 0.9.

Fractions can be addressed in a similar manner and in two different ways:

Method 1: Since $\sqrt{\frac{8}{15}} \approx \sqrt{\frac{9}{16}}$, and $\sqrt{\frac{9}{16}} = \frac{3}{4}$, we can say that $\sqrt{\frac{8}{15}} \approx \frac{3}{4}$

Method 2: $\frac{8}{15}$ is a little more than $\frac{1}{2}$, and $\frac{1}{2} = 0.5$. Since $\sqrt{0.5} \approx \sqrt{0.49}$, and $\sqrt{0.49} = 0.7$, we can say that $\sqrt{0.5} \approx 0.7$

Once students are comfortable estimating without the aid of technology, a calculator can be used to approximate square roots. A calculator provides an efficient means of approximating square roots and it usually gives a closer approximation than an estimate does. As students use a calculator to determine their estimate, they should notice the square root of a non-perfect square results in a decimal that is non-repeating and non-terminating. This decimal value is approximate, not exact. This is a good place to discuss how accurate an approximation should be.

Often more decimal places results in a better approximation of a square root. However, one or two decimal places is acceptable.

Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Using a number line, determine if $\sqrt{12}$ is closer to $\sqrt{9}$ or $\sqrt{16}$.

- Without using a calculator, determine between which two whole numbers the $\sqrt{80}$ falls. Explain your method.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Estimate the following square roots using benchmarks. Explain your strategy.
  - $\sqrt{300}$
  - $\sqrt{0.45}$
  - $\sqrt{\frac{24}{65}}$
- Find the approximate square root of 1.4 using the guess-and-check strategy and then use your calculator to verify. Show each value you tried and the result of squaring each value.
- Using the $\sqrt{\ }$ symbol on his calculator, Paul found the square root of 87 to be exactly 9.327379053. Is he correct? Explain why or why not.
- Identify:
  - a whole number with a square root that lies between 6 and 7
  - a rational number with a square root that lies between 0.7 and 0.8
- Ask students to estimate the side length, to the nearest tenth, of a square with an area of 31.5 cm^2. Ask them to provide an explanation.
- How could you use estimation to determine that 0.7 and 0.007 are not reasonable values for $\sqrt{4.9}$?
- Ask students to explain why an answer displayed on a calculator may not be an exact answer.

Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Use perfect squares as benchmarks and have students estimate $\sqrt{0.24}$.
- Ensure that students are comfortable with the perfect square benchmarks from 1 to 400. The pattern in the difference between perfect squares can be explored to help students go beyond 144 to determine which numbers are perfect squares. Compare the differences of two perfect squares and investigate the patterns.

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, ___, ___, _____

+3 +5 +7 +9 etc

- Guide students through the algebraic process of identifying a rational number whose square root lies between two given numbers.

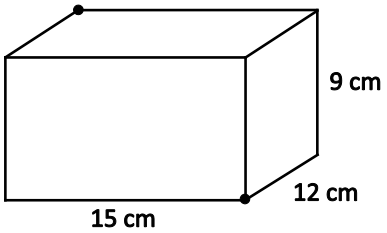
  $2.3 < \sqrt{n} < 2.31$

  $(2.3)^2 < n < (2.31)^2$

  $5.29 < n < 5.3361$

Suggested Learning Tasks

- A spider has taken up residence in a small cardboard box that has a length of 15 cm, a width of 12 cm, and a height of 9 cm. What is the length, in centimetres, of a spider web that will carry the spider in a straight line from the upper left back corner of the box to the lower right front corner of the box?



- Given the area of a circle, use the following formula to find the radius of that circle:

  $r = \sqrt{\frac{A}{x}}$

- Place the square roots of the non-perfect squares, $\sqrt{8}$, $\sqrt{2}$, and $\sqrt{6}$, on a number line using benchmarks of perfect squares.
- Ask students to complete the following activity:
  - Use a calculator to determine the value of $\sqrt{5}$. Record all the digits displayed on the calculator.
  - Clear your calculator and enter the number recorded in the above step.
  - Square this number and record the result.
  - Compare the answer to the number 5. Are they the same? Identify any differences and explain why they exist.
- Ask students to make a list of as many numbers with a square root between 18.1 and 18.2 as they can think of in two minutes. After the time is up, ask them to pass their list to another student. Then ask them, one at a time, to read an entry from the list in front of them. Everybody who has that number on his or her list will cross it off. At the end, the list with the most remaining entries is the winner. Students could be paired up for this activity.

SUGGESTED MODELS AND MANIPULATIVES

- calculator
- hundredths grids
- "Number Lines"*

* also available in *Interactive Math Tools* (Pearson n.d.)

MATHEMATICAL LANGUAGE

Teacher	Student
▪ inverse operations ▪ irrational number ▪ non-perfect square ▪ perfect square ▪ rational number ▪ square root	▪ inverse operations ▪ irrational number ▪ non-perfect square ▪ perfect square ▪ rational number ▪ square root

Resources

Digital

Interactive Math Tools. (Pearson. n.d.): http://nsvs.ednet.ns.ca/nsps/nsps26/course/view.php?id=3875.

Print

- *Math Makes Sense 9* (Baron et al. 2009; NSSBB #: 2001644)
  - Unit 1: Square Roots and Surface Area
    - > Section 1.2: Square Roots of Non-Perfect Squares
    - > Game: Making a Larger Square from Two Smaller Squares
  - *ProGuide* (CD; Word Files; NSSBB #: 2001645)
    - > Assessment Masters
    - > Extra Practice Masters
    - > Unit Tests
  - *ProGuide* (DVD; NSSBB #: 2001645)
    - > Projectable Student Book Pages
    - > Modifiable Line Masters
- *Teaching Student-Centered Mathematics, Grades 5–8*, Volume Three (Van de Walle and Lovin 2006), pp. 150–151

Patterns and Relations (PR)

GCO: Students will be expected to use patterns to describe the world and solve problems.

GCO: Students will be expected to represent algebraic expressions in multiple ways.

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

GUIDING QUESTIONS

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Follow-up on Assessment

The evidence of learning that is gathered from student participation and work should be used to inform instruction.

GUIDING QUESTIONS

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan available at Mathematics Learning Commons: Grades 7–9: http://nsvs.ednet.ns.ca/nsps/nsps26/course/view.php?id=3875.

GUIDING QUESTIONS

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SCO PR01 Students will be expected to generalize a pattern arising from a problem-solving context using a linear equation and verify by substitution. [C, CN, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR01.01 Write an expression representing a given concrete, pictorial, oral, and/or written pattern.
PR01.02 Write a linear equation to represent a given context.
PR01.03 Describe a context for a given linear equation.
PR01.04 Solve, using a linear equation, a given problem that involves concrete, pictorial, oral, and/or written linear patterns.
PR01.05 Write a linear equation representing the pattern in a given table of values, and verify the equation by substituting values from the table.

Scope and Sequence

Mathematics 8	**Mathematics 9**	**Mathematics 10**
PR01 Students will be expected to graph and analyze two-variable linear relations.	**PR01** Students will be expected to generalize a pattern arising from a problem-solving context using a linear equation and verify by substitution.	**RF04** Students will be expected to describe and represent linear relations, using words, ordered pairs, tables of values, graphs, and equations.

Background

This outcome should be taught in conjunction with PR02.

Students have been exposed to patterns through interpretation of graphs of linear relations in earlier grades. In Mathematics 7, students used algebraic expressions to describe patterns, and constructed graphs from the corresponding table of values. In Mathematics 8, students examined the various ways a relation can be expressed, including ordered pairs, table of values, and graphs. They also used patterns to find missing values in a linear relation. The algebraic expressions were often given to students.

In Mathematics 7, students formulated linear relations to represent the relationship in a given oral or written pattern. However, in Mathematics 8 most of the work was done with linear relations provided. In Mathematics 9, there is a focus on writing an expression or equation given the pictorial, oral, or written form of the relation. Students are expected to move interchangeably among the various representations that describe linear relations. They should be able to describe in words and use expressions and equations to represent patterns from tables, graphs, charts, pictures, and problem situations. Information presented in a variety of formats should be used to derive mathematical expressions and equations and to predict unknown values.

When a linear relation is represented using pictorial or written form, students should use patterns to derive the expression or equation. Students should examine the situation to determine what stays constant, what changes, and how it relates to the expression or equation. Once the equation has been created, students are expected to use it to find missing values of the independent and dependent variable. Students should make a connection between the constant change in the dependent variable and the equation. Students will use this connection to substitute values from the table to determine the equation. They can then verify this equation using substitution.

For example, when students are looking at the table of values below, they should look at the pattern and recognize a constant change between the values (an increase of 6 between the term values).

Term Number (n)	1	2	3	4	5
Term (t)	2	8	14	20	26

In this example, students should recognize that multiplying the term number, *n*, by 6 always results in four more than the associated term, *t*. Therefore, the term value (*t*) can be determined by subtracting 4 from 6*n*. As an equation, the pattern is represented by $t = 6n - 4$. Students should verify their equation by substituting values from the table (when $n = 5$, $t = 26$). Students should use their equation to solve for any value of *n* or *t*.

Students should also work with decreasing patterns that can be represented with linear equations.

As students analyze pictures, tables, and equations, they should recognize that each representational form is a viable way to solve a problem.

This understanding gives them a choice of representations to use and can lessen their reliance on procedural manipulation of the symbolic representation. Alternate representations can strengthen students' awareness of symbolic expressions and equations. For students to have this choice and this knowledge, they must have had experience with each type of representation.

In addition, students should connect their mathematical learning to contextual situations. Ask students to create a context to describe a given linear relation, such as $c = 3a + 1$.

Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- For each of the following problems, create a table of values to show the relationship, and construct a graph from the ordered pairs.
  - Grapefruits are $1.00 per grapefruit.
  - Movie tickets are $9.00 per ticket.
  - $y = 2x + 4$

– $y = \frac{x}{2} + 3$

- For the equation $y = -2x + 3$, find the missing values in the following ordered pairs.
  - (0, *y*)
  - (*x*, 1)
  - (5, *y*)
  - (*x*, –1)
- Fill in the missing values in the following table of values.

x	*y*
1	3
	7
4	
5	19

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Jules is getting in shape. The first day he does 9 sit-ups, the second day 13, the third day 17, the fourth day 21. Identify the variables and write an equation to represent this situation. If he continues in this way, how many sit-ups will he do on the 5th day? 6th day? 10th day? 20th day? 50th day? 60th day?
- Write a linear equation to represent the pattern in the given table of values. Describe a context for the equation.

x	*y*
1	10.50
2	11.00
3	11.50
4	12.00

- Given the equation, $c = 2t + 5$, describe this relation in words. Make up a problem that could be solved using this equation.
- Your class is planning a trip to the zoo. The school will have to pay $200 for the bus plus $5 per student. How much will it cost for 42 students?

- Jake is checking over his math assignment. He phones you to verify the equation for the following table of values:

x	y
3	8
4	10
5	12
6	14

  He thinks the equation is $y = 3x - 1$, since the point (3, 8) satisfies this equation. Is he correct? Justify your answer.

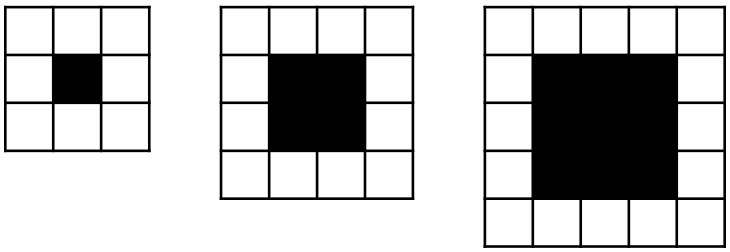
- Talisha pays a one-time fee of $6.00 to download songs plus $0.25 for each song.
  - Write an equation to represent this situation.
  - How much would it cost to download 16 songs?
  - How many songs can be downloaded for $13.00?

Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide students with opportunity to explore various patterns by explaining each pattern using words and writing an equation to represent a situation. For example, the relationship between the number of bricks (b) around a square fire pit with side lengths (s) is represented by the equation $4s + 4 = b$.

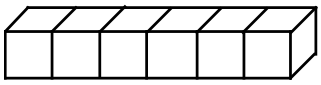


- Provide students with experiences to develop the ability to write equations for situations that are described in words. For example: "Ralph rents snowboards for $10.50 per hour, but requires a $25 non-refundable deposit. How much will it cost to rent Ralph's snowboard and use it for 2 hours? 3 hours? 6 hours? 10 hours?"
- From looking at the patterns that develop, or from the wording of the problem, students should be able to write equations from given contexts such as $25 deposit plus $10.50 per hour. This can be used to calculate cost depending on the number of hours rented.
- Develop equations from patterns expressed in table form. For example, given the following table of values, have students write an equation and then verify it by substituting values from the table.

Term Number (n)	1	2	3	4	5
Term (t)	39	35	31	27	23

SUGGESTED LEARNING TASKS

- To explore patterns, have students use multi linking cubes to determine the total number of visible faces found on 1 to 6 (or more) cubes linked together in a "train." Do not include the bottom, which is not visible.



Number of cubes (n)	1	2	3	4	5	6
Number of visible faces (f)	5	8	11	14	17	20

  To develop an understanding of the patterns found in linear equations, have students:
  - describe any patterns they see in the table and describe how the patterns are visible in the cubes
  - after the first few cubes, try to predict the total value for 5 cubes and for 6 cubes
  - complete: "If you tell me how many cars in the train, I can tell you the number of faces by …"
  - replace the words with mathematical symbols, describing the pattern in an equation and explaining the meaning of each of the coefficients in the equation.

  This activity can be repeated by making trains of pattern blocks.
- Ask students to discuss the benefits of describing a relation using
  - a pictorial model
  - an algebraic representation

  Ask them which representation they prefer and why.

- For the following sequence:

6, 14, 22, 30 …
  - Determine the next three terms.
  - Develop an expression that can be used to determine the value of each term in the number pattern.
  - Use the expression to find the 20th term in this pattern.
  - Determine which term has a value of 94.

SUGGESTED MODELS AND MANIPULATIVES

- multi linking cubes
- graph paper
- Pattern Blocks*

*also available in *Interactive Math Tools* (Pearson n.d.)

MATHEMATICAL LANGUAGE

Teacher	Student
▪ constant change	▪ constant change
▪ dependent variable	
▪ equation	▪ equation
▪ expression	▪ expression
▪ graph	▪ graph
▪ independent variable	
▪ numerical coefficient	
▪ linear relation	▪ linear relation
▪ pattern	▪ pattern
▪ ordered pair	▪ ordered pair
▪ substitution	▪ substitution
▪ table of values	▪ table of values
▪ term	▪ term
▪ term number	▪ term number
▪ unknown (values)	▪ unknown (values)

Resources

Digital

Interactive Math Tools. (Pearson. n.d.): http://nsvs.ednet.ns.ca/nsps/nsps26/course/view.php?id=3875.

Print

- *Elementary and Middle School Mathematics: Teaching Developmentally*, 8th Edition (Van de Walle, Karp, and Bay-Williams 2013), pp. 273–277
- *Math Makes Sense 9* (Baron et al. 2009; NSSBB #: 2001644)
  - Unit 4: Linear Relations
    - > Section 4.1: Writing Equations to Describe Patterns
    - > Unit Problem: Predicting Music Trends
  - *ProGuide* (CD; Word Files; NSSBB #: 2001645)
    - > Assessment Masters
    - > Extra Practice Masters
    - > Unit Tests
  - *ProGuide* (DVD; NSSBB #: 2001645)
    - > Projectable Student Book Pages
    - > Modifiable Line Masters

SCO PR02 Students will be expected to graph a linear relation, analyze the graph, and interpolate or extrapolate to solve problems. [C, CN, PS, R, T, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR02.01 Describe the pattern found in a given graph.
PR02.02 Graph a given linear relation, including horizontal and vertical lines.
PR02.03 Match given equations of linear relations with their corresponding graphs.
PR02.04 Extend a given graph (extrapolate) to determine the value of an unknown element.
PR02.05 Interpolate the approximate value of one variable on a given graph, given the value of the other variable.
PR02.06 Extrapolate the approximate value of one variable from a given graph, given the value of the other variable.
PR02.07 Solve a given problem by graphing a linear relation and analyzing the graph.

Scope and Sequence

Mathematics 8	**Mathematics 9**	**Mathematics 10**
PR01 Students will be expected to graph and analyze two-variable linear relations.	**PR02** Students will be expected to graph a linear relation, analyze the graph, and interpolate or extrapolate to solve problems.	**RF01** Students will be expected to interpret and explain the relationships among data, graphs, and situations. **RF05** Students will be expected to determine the characteristics of the graphs of linear relations, including the intercepts, slope, domain, and range. **RF06** Students will be expected to relate linear relations to their graphs, expressed in ▪ slope-intercept form ($y = mx + b$) ▪ general form ($Ax + By + C = 0$) ▪ slope-point form ($y - y_1 = m(x - x_1)$)

Background

This outcome is to be taught in conjunction with PR01.

In Mathematics 7 and Mathematics 8, students described the relationship between variables of a given graph. They also constructed and analyzed graphs of linear equations, with a focus on discrete data. In Mathematics 9, students will be asked to describe patterns from graphs. They will be expected to use terminology, such as increase and decrease, to describe the relationship between the two variables; they will be expected to work with both discrete and continuous data. In PR01 and PR02 students are informally working with the concept of slope. However, the actual term slope, is not introduced until Mathematics 10. Students have experience graphing linear relations from Mathematics 8. They will now create a table of values and use ordered pairs to graph linear relations. The "slope *y*-intercept method" is also not introduced until Mathematics 10 and therefore is not to be done in this grade. The intent of this outcome is to explore the patterns and represent them by linear equations with the use of graphs and tables only. This will create the foundation for rate of change (slope) and the slope *y*-intercept form of a linear equation ($y= mx + b$) that will be explored in Mathematics 10.

Exploration should quickly reveal that the graph of a linear relation is a straight line. Vertical and horizontal lines can be represented by single variable linear equations. This concept may be difficult for students to grasp at first, therefore multiple opportunities must be provided. In this case, students will realize that as one variable changes the other stays as a constant value. This will be an indication that the graph will be a horizontal or vertical line. The equations of horizontal and vertical lines contain only one of the variables. As a result, *x* or *y* is always constant. This results in a line that will be either perpendicular to the *x*-axis ($x = a$) or perpendicular to the *y*-axis ($y = a$).

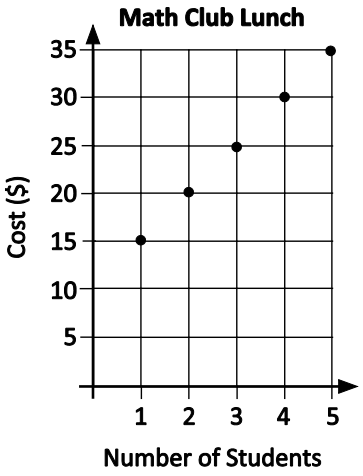
Students should graph horizontal, vertical, and oblique lines. Oblique, or slanted, lines are neither perpendicular nor parallel to the *x*- or *y*-axis. This is a new term for students, but they have previous experience with graphing oblique lines using the table of values method. When graphing oblique lines, students can substitute values of *x* into the equation, and use prior knowledge to solve the equation for *y*. At this point, they are not expected to rearrange equations when creating tables of values. They will work with more complex equations later in this course in the Linear Equations and Inequalities unit. For now, students can avoid having to solve equations with rational numbers by selecting convenient numbers to substitute for *x*. Situations may include either discrete or continuous data.

When graphing linear relations, students will be expected to distinguish between discrete and continuous data. Discrete data can only have a finite or limited number of possible values. Generally discrete data are counts: number of students in class, number of tickets sold, how many items were purchased. When graphing data points that represent discrete data, points are not connected. If there are no valid values between the plotted points, then no line is drawn.

Continuous data has an infinite number of values between data points. It makes sense to have fractions. When graphing points that represent continuous data, points are connected with a solid line.

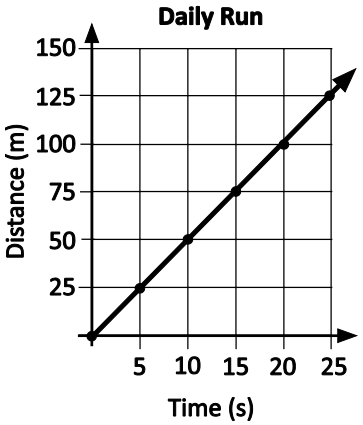
Contextual situations such as the following should make this idea more concrete for students:

Discrete Data



This graph has discrete data because it is not possible to have a fraction of a person. Since there are no valid data points between the plotted points, the points are not connected.

Continuous Data



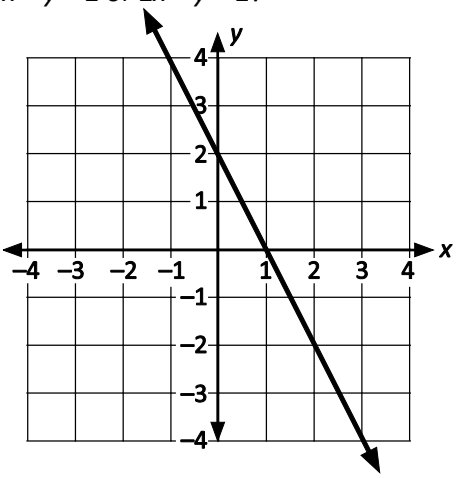
On this graph, the data is continuous because it makes sense to have fractional time. The points, therefore, are connected with a solid line.

Ask students to think about other situations involving discrete and continuous data. Examples for discrete data may include situations involving number of people, number of pizza toppings, number of concert tickets, etc. Examples of continuous data may include situations involving temperatures that occur over time, height or weight over age, distance over time, etc. The decision about whether or not to join points on a graph is necessary only in contextual situations. If students are graphing a linear relation from a given equation without context, points are connected.

Students should recognize that the graph, table of values, and ordered pairs show a relationship between two variables. To match graphs with their corresponding equations, selected ordered pairs from the graph can be tested to see if they satisfy the given equation. Students should be encouraged to select at least two points to verify, as they can incorrectly match graphs when just one point satisfies the equation. For example, if students choose the point (0, 2) when matching the graph below with the

correct equation, they may make an incorrect match. In this case, the ordered pair (0, 2) satisfies both equations. Testing a second ordered pair will ensure a correct match.

Which equation matches the graph below?
$x + y = 2$ or $2x + y = 2$?



Other methods to match the equation with the graph, such as comparing the graph's slope and y-intercept to the equation, will not be explored until Mathematics 10.

In Mathematics 7, students made predictions for unknown quantities by examining the graph and in Mathematics 8, from using the equation. The terms extrapolation and interpolation were not formally introduced. Students are now expected to make predictions by extending their graph. The focus here will be on interpreting the data and making predictions for unknown values. Interpolation is the prediction of a value between two known values on a graph. It is important for students to realize that when graphs display discrete data, interpolation is inappropriate because there are no data points between the known data points. Extrapolation is extending a graph to make a prediction about a value that goes beyond the data that is given. Generally, students are less comfortable with extrapolation than with interpolation. There is opportunity here for students to work with real-life applications. By extending the graph, assumptions are being made that the pattern will continue. Students need to be aware that this is not always applicable in contextual situations. As students make inferences from a graph, it is important that they justify their interpolations and extrapolations.

Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with a table of values, such as the following, that represents a linear relation.

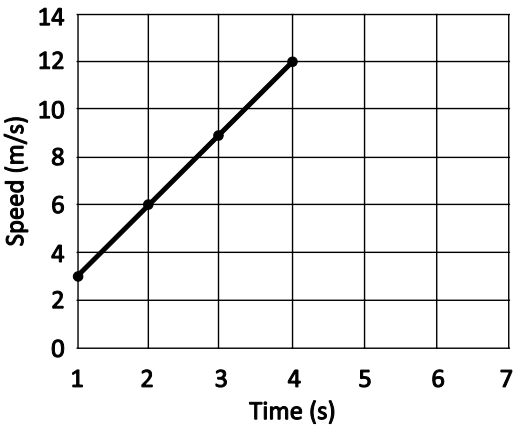
x	5	6	7	8	9
y	9	11	13	15	17

  - Have students graph the ordered pairs in the table of values.
  - Describe, in words, the relationship between the *x*-values and the *y*-values.
  - Write the linear equation using *x* and *y*.
- For each of the following problems, create a table of values to show the relationship, and construct a graph from the ordered pairs.
  - Apples are $1.00 per apple.
  - Movie tickets are $11.00 per ticket.
  - $y = 3x + 4$

Whole-Class/Group/Individual Assessment Tasks

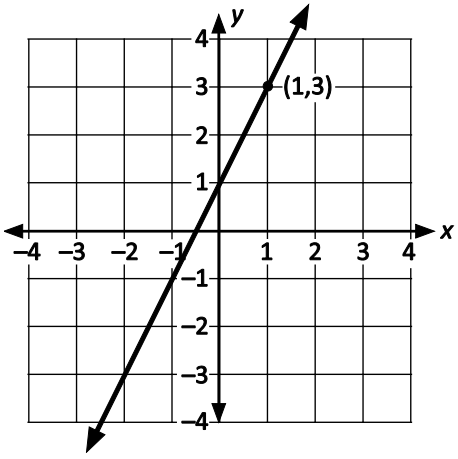
Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Create a table of values and sketch a graph for the linear equation: $y = 7x - 4$.
- You have just purchased a new cell phone. The phone plan costs $10 per month and $0.10 per text message. Create a graph to represent the situation. Estimate the cost of sending 100 text messages using the graph.
- Describe the pattern and write the equation for the following. Describe a situation that could result in the graph.

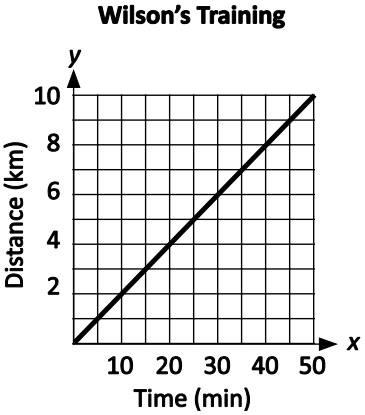


- Use examples and diagrams to help explain how horizontal and vertical lines and their equations are similar and how they are different.
- Create a table of values and a graph for the following linear equations:
  - $x = 4$
  - $4x + y = 5$
  - $y = 1$

- June stated that the equation for the graph below is $x + y = 4$, since the point (1,3) satisfies the equation. Is she correct? Justify your answer.



- Wilson is training for a 10 km race. The graph shows his times and distances at 10 minute intervals.



  - Determine how long it takes Wilson to run 3 km.
  - Determine how far he can run in 45 minutes.
  - Determine how fast he is running.
  - Can you use this graph to predict how far he will run in 200 minutes? Why or why not?

Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Provide students with a variety of problems in which they will graph a linear relation and use interpolation and extrapolation to solve the problem.
- Provide students with graphs of discrete data arranged horizontally and vertically. Students should create a table of values from the graph, write an equation by recognizing the pattern in the data and be able to describe a situation to represent each graph.
- Provide students with various graphs and linear relations and ask them to match the graph with the equation. Students could also be asked to describe the pattern within the graphs.

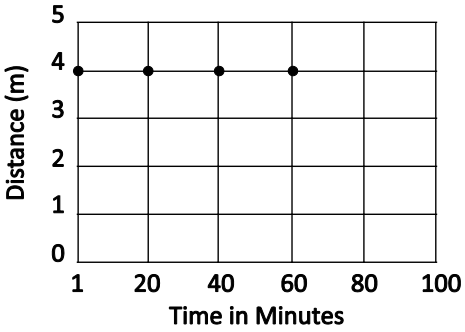
SUGGESTED LEARNING TASKS

- A taxi cab charges the following rates:

Length of trip (km)	5	10	15
Total cost ($)	9.25	15.50	21.75

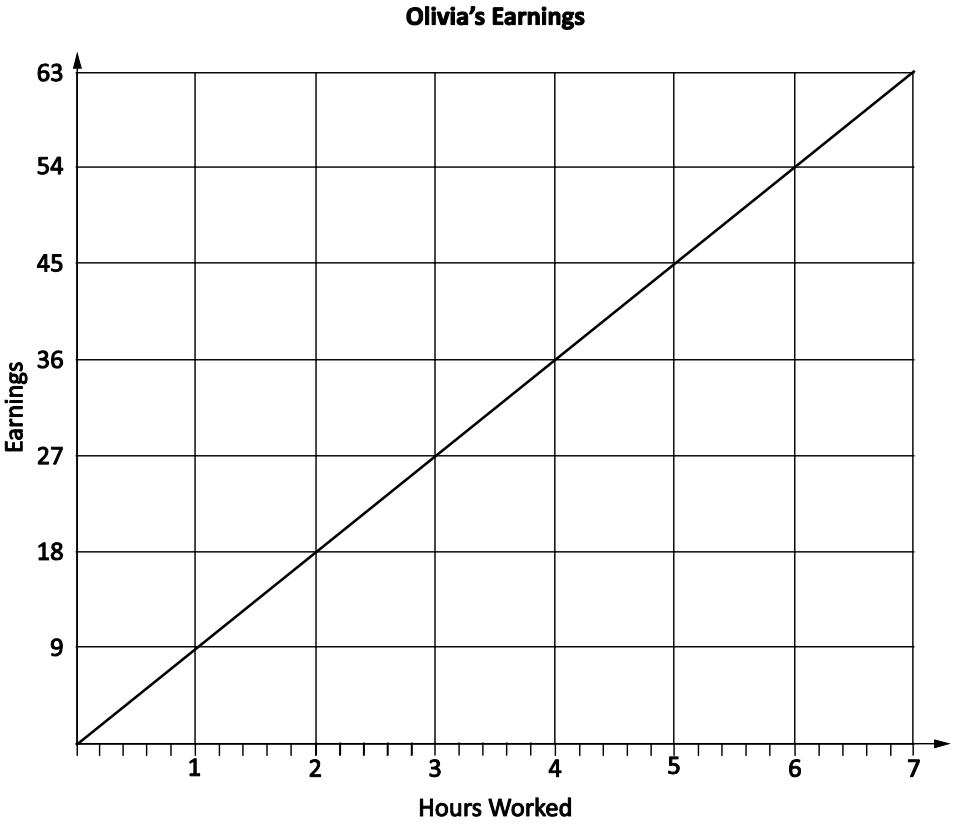
  - Plot these points on a coordinate system.
  - Discuss if these points should be joined.
  - Determine the equation.
  - Explain why the graph does not start at the origin.
  - From the graph, find the length of a trip that costs $25.
  - From the graph, find the cost of a trip of 12 km.

- Give students the following graph and have them complete the activities below.

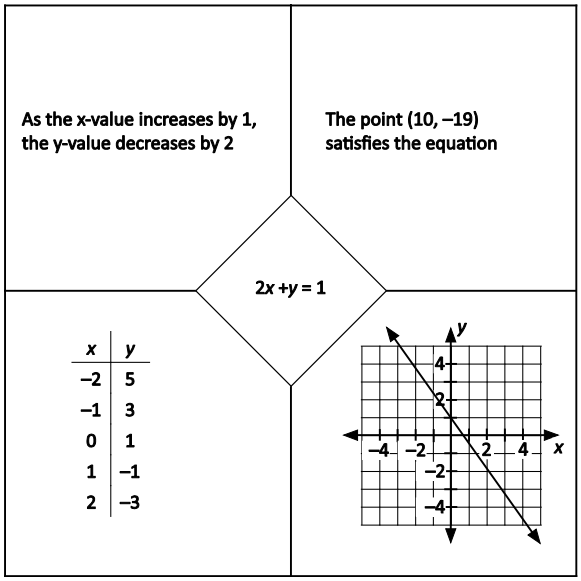


  - Create a table of values.
  - Describe the pattern found in the graph.
  - Describe a situation that the graph might represent.
  - Write a linear equation.

- Olivia works part time at a grocery store. Ask students to use the graph below to describe the pattern and explain what it represents.



- A common price for downloading apps is $0.99.
  - Write an equation that relates the total cost of buying apps, *C*, to the number of downloads, *d*.
  - Graph the equation. Should you connect the points on the graph? Explain.
  - What is the total cost for 100 downloads?
  - You have saved $24.75. How many apps can you download?
- Students can work in pairs to create the following graphic organizer. These will be collected and the teacher can create a puzzle investigating the characteristics and graphs of various linear relations. They should include an equation and four related characteristics. The puzzles will be cut up and used to correctly match the characteristics with each equation. A sample is shown below.



- Students can work in pairs to explore the relationship between the height of the top of a metre stick to the distance between the bottom of the stick and the wall. To begin, they stand a metre stick upright against a wall and record the measurements. They should then move the bottom end 10 cm away from the wall and measure the height of the top of the metre stick. This process continues until the metre stick is lying on the floor. Ask students to:
  - Record data in a table of values:

Distance between bottom of stick and wall (cm)	0	10	20	30	40	50	60	70	80	90	100
Height of top of stick (cm)											

  - Create a graph for this table of values.
  - Analyze the graph and describe any relationships that exist.
  - Write an equation for the relationship.
  - Use interpolation or extrapolation to make predictions for the given data.

SUGGESTED MODELS AND MANIPULATIVES

- graph paper
- graphing calculator

MATHEMATICAL LANGUAGE

Teacher	Student
▪ constant (rate) ▪ continuous data ▪ discrete data ▪ dependent variable ▪ independent variable ▪ extrapolate ▪ horizontal ▪ inference ▪ interpolate ▪ oblique line ▪ ordered pairs ▪ term ▪ variable ▪ vertical	▪ constant (rate) ▪ continuous data ▪ discrete data ▪ dependent variable ▪ independent variable ▪ extrapolate ▪ horizontal ▪ inference ▪ interpolate ▪ oblique line ▪ ordered pairs ▪ term ▪ variable ▪ vertical

Resources

Print

- *Elementary and Middle School Mathematics: Teaching Developmentally*, 8th Edition (Van de Walle, Karp, and Bay-Williams 2013), pp. 273–288
- *Math Makes Sense 9* (Baron et al. 2009; NSSBB #: 2001644)
  - Unit 4: Linear Relations
    - > Technology: Tables of Values and Graphing
    - > Section 4.2: Linear Relations
    - > Section 4.3: Another Form of the Equation for a Linear Relation
    - > Game: What's My Point?
    - > Section 4.4: Matching Equations and Graphs
    - > Section 4.5: Using Graphs to Estimate Values
    - > Technology: Interpolating and Extrapolating
    - > Unit Problem: Predicting Music Trends
  - *ProGuide* (CD; Word Files; NSSBB #: 2001645)
    - > Assessment Masters
    - > Extra Practice Masters
    - > Unit Tests
  - *ProGuide* (DVD; NSSBB #: 2001645)
    - > Projectable Student Book Pages
    - > Modifiable Line Masters

SCO PR03 Students will be expected to model and solve problems, where *a*, *b, c, d, e,* and *f* are rational numbers, using linear equations of the form

- $ax = b$
- $\frac{x}{a} = c,\ a \neq 0$
- $ax + b = c$
- $\frac{x}{a} + b = c,\ a \neq 0$
- $ax = b + cx$
- $a(x + b) = c$
- $ax + b = cx + d$
- $a(bx + c) = d(ex + f)$
- $\frac{a}{x} = b,\ x \neq 0$

[C, CN, PS, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR03.01 Solve the given linear equation, using concrete and pictorial representations, and record this process symbolically.

PR03.02 Verify by substitution whether a given rational number is a solution to a given linear equation.

PR03.03 Solve a given linear equation symbolically.

PR03.04 Identify and correct an error in a given incorrect solution of a linear equation.

PR03.05 Represent a given problem, using a linear equation.

PR03.06 Solve a given problem, using a linear equation, and record the process.

Scope and Sequence

Mathematics 8	Mathematics 9	Mathematics 10
PR02 Students will be expected to model and solve problems, concretely, pictorially, and symbolically, where *a*, *b,* and *c* are integers, using linear equations of the form ▪ $ax = b$ ▪ $\frac{x}{a} = b,\ a \neq 0$ ▪ $ax + b = c$ ▪ $\frac{x}{a} + b = c,\ a \neq 0$ ▪ $a(x + b) = c$	**PR03** Students will be expected to model and solve problems, where *a*, *b, c, d, e,* and *f* are rational numbers, using linear equations of the form: ▪ $ax = b$ ▪ $\frac{x}{a} = c,\ a \neq 0$ ▪ $ax + b = c$ ▪ $\frac{x}{a} + b = c,\ a \neq 0$ ▪ $ax = b + cx$ ▪ $a(x + b) = c$ ▪ $ax + b = cx + d$ ▪ $a(bx + c) = d(ex + f)$ ▪ $\frac{a}{x} = b,\ x \neq 0$	**RF10** Students will be expected to solve problems that involve systems of linear equations in two variables, graphically and algebraically.

Background

There are several important ideas that are addressed in this outcome:

- **Algebra** is used to represent and explain mathematical relationships as well as to describe and analyze change.
- **Equations** are used to express relationships between two quantities; both sides of the equal sign are equivalent expressions.
- **Variables** are a symbols that can stand for any one of a set of numbers or other objects and can be represented by boxes or letters.

In Mathematics 8, students had experience solving one- and two-step equations, where *a*, *b*, and *c* are integers, in the form of

- $ax = b$
- $\frac{x}{a} = b,\ a \neq 0$
- $ax + b = c$
- $\frac{x}{a} + b = c,\ a \neq 0$
- $a(x + b) = c$

This outcome builds on previous experience to now include rational number, equations with variables on both sides of the equal sign, and using more than two steps are required to solve the equation. Students have worked extensively with concrete and pictorial representations, and include the use of algebra tiles, inspection, and systematic trial (guess and check). Instruction should start with concrete materials and pictorial models and then move to symbolic representation. Research has shown that the

use of concrete models is critical in mathematics because most mathematical ideas are abstract. Students must move from the concrete to the pictorial to the symbolic, and part of instructional planning involves making informed decisions about where students are on the continuum of concrete to symbolic thinking.

Ultimately students should be able to solve equations without concrete or pictorial support. The progression from concrete to abstract and then from integral to rational should help scaffold the skills students learned in Mathematics 8 and help them to apply their learning to solve equations with rational coefficients and constants.

When solving equations, a balance scale is an appropriate model when the coefficients and constants are positive integers. Algebra tiles can be used to represent equations with any integer coefficient and constant. As students use a concrete model, they should also record the steps symbolically. Work with models leads to solving equations using inverse operations to gather like terms and balance the equation. Equations with rational numbers such as fractions or decimals cannot be solved easily using concrete models. Students, therefore, need to be able to solve equations symbolically.

It is expected that students will initially show all the steps as they solve equations. As they develop these skills, they may be able to reduce the number of steps.

Students may use different strategies when solving equations involving fractions. When solving an equation such as $\frac{x}{4}+\frac{1}{2}=\frac{x}{3}$, some students may eliminate the denominators by multiplying each term by the lowest common denominator. As they proceed to solve the equation, ask them the following questions:

- What is the lowest common denominator of 4, 2, and 3?
- What would happen if the lowest common denominator was multiplied on both sides of the equation? Why is this mathematically correct?
- What is the simplified equation?
- What is the solution?

Another strategy that can be used centers around the idea of undoing the operations that are being done to the variable. When students are provided with an equation such as $2x = 8$, since the inverse of multiplication is division, they divide both sides of the equation by 2, thereby undoing the multiplication operation. Similarly, if students are provided with the equation $\frac{x}{2}=8$, they multiply both sides of the equation by 2 since the inverse of division is multiplication.

When solving the equation $\frac{x}{4}+\frac{1}{2}=\frac{x}{3}$, students may decide to solve this equation using the “undo” process.

$$\frac{x}{4}+\frac{1}{2}=\frac{x}{3}$$
$$4\left(\frac{x}{4}\right)+4\left(\frac{1}{2}\right)=4\left(\frac{x}{3}\right)$$
$$x+2=\frac{4x}{3}$$
$$3(x)+3(2)=3\left(\frac{4x}{3}\right)$$
$$3x+6=4x$$
$$3x+6=4x-3x$$
$$x=6$$

After solving several examples, discuss with students the connection between this method and the process of multiplying each term by the lowest common denominator.

When solving an equation, it does not matter if the unknown variable is isolated on the left or right hand side of the equal sign. It is preferable to add or subtract the value of *x* that would result in a positive coefficient value. Students may show a preference to always isolate *x* on the left hand side, resulting in the step to multiply or divide by a negative coefficient. Understanding the role of the equal sign helps in knowing that the variable need only be isolated on one side of the equation, not necessarily the left hand side. As in the above equation, some students may have done the final steps in the following manner:

$$3x-4x+6-6=4x-4x-6$$
$$-x=-6$$

Have students consider in advance what might be a reasonable solution. They should be reminded that once they acquire a solution, it can be checked for accuracy by substitution into the original equation. Always encourage students to verify solutions, as this will lead to a better understanding of the process involved. Students should be provided with worked solutions of linear equations to verify. Along with providing the correct answers, they should identify any errors they find in solutions, and correct those errors. Common student errors include mistakes in using the distributive property, using sign rules incorrectly for multiplication and division, and errors in preserving equality. Error analysis reinforces the importance of verifying solutions and recording steps, rather than only producing the final answer.

To solve problems, it is necessary for students to make connections to previous work with linear equations. Students should be given the opportunity to solve problems with variables on both sides. Many problems can be solved using methods other than algebra, such as guess-and-check and systematic trial. It may, therefore, be necessary to specify the strategy to ensure that algebraic problem solving is being used. Students should be able to solve equations using rational numbers. Algebra can be used to solve problems that might otherwise be tedious to solve using methods such as guess-and-check. It would be manageable for students to solve a problem, such as the following, using guess-and check:

- John and Judy work part time. John earns $10 per day plus $6 per hour. Judy earns $8 per hour. Determine how many hours they have to work to earn the same daily pay.

It is more tedious to use guess-and-check to solve a problem such as:

- A cell phone company offers two different plans:

- Plan A: monthly fee of $45 and $0.20 per minute
- Plan B: monthly fee of $28 and $0.45 per minute

Determine how many minutes result in the same monthly cost for both plans.

In this case, students should realize that it is more efficient to solve the equation $0.2x + 45 = 0.45x + 28$ to determine the number of minutes.

Problem solving also requires communication through the application of a four-stage process, which was referenced in Mathematics 8:
• understand the problem by identifying given information
• make a plan to solve the problem
• carry out the plan and record the solution
• verify that the solution is correct for the information given in the problem

Assessment, Teaching, and Learning

Assessment Strategies

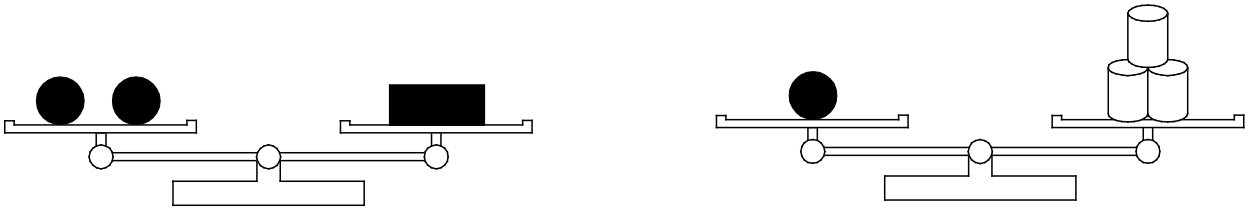
ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Explain what is the same and what is different about the way that the letter m has been used in the following cases:
  - Martin ran 100 m (the m represents an abbreviation for metre)
  - $2m + 15 = 23$ (m represents a single value)
  - $3m + 2$ (m represents an infinite number of values)
- Put each equation on an individual index card and ask students to sort the cards into the following categories:

  a. Easy to solve in my head
  b. Harder, but can be solved in my head
  c. Easier to use a model or pen and paper to solve
  - $n + 2 = 7$
  - $3n = 21$
  - $2n + 7 = 19$
  - $4n – 8 = 6n + 2$
  - $5 = 2n – 7$
  - $N – 2 = 7$
  - $3n = 21$
  - $3n – 5 = n + 1$
  - $6(n – 4) = 2n + 4$
  - $5n – 2 = 9$

Ask students to discuss the following questions with a partner or in a small group:
  - How did you decide which questions would be easy to solve mentally?
  - What process did you use to solve these equations?
  - What are the similarities and differences in the methods that each person used?
- A single shape has the same value or mass.
  - Which shape has the greatest mass?
  - Which shape has the least mass?
  - How do you know?



- Use algebra tiles to model and solve 3 *x* + 4 = 10. Record each step required to solve for the unknown variable.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Give the equation that shows that the perimeter of a rectangle is 36 m, if the length of the rectangle is 2 m less than its width. Solve for the length and width.
- Solve the following, using algebra tiles, and record each step algebraically.

  $2(-3x+1)=4(2x-3)$

  $-6x+2=8x-12$
- Estimate a solution to the following equation. Justify your estimate; solve and verify.

  $\frac{x}{2}-3=1\frac{1}{6}$
- Brenda and Thomas want to buy a $199 portable mp3 player. Brenda has $45 and saves $15 per week. Thomas has $70 and saves $12.50 per week. Who will be able to buy the mp3 player first? Solve using a linear equation.
- *The Chronicle Herald* can be delivered to your house for $0.70 per copy plus a $25.00 yearly subscription fee. *The Globe and Mail* can be delivered to your house for $0.75 per copy plus a $20.00 yearly subscription fee. Determine how many copies are delivered before the cost is the same.
- Leah solved the following equation. Check for any errors. If any were made, indicate where and make the necessary changes to correct them.

$\frac{1}{3}(x-2)=5(x+6)$

$3(x-2)=15(x+6)$

$3x-6=15x+90$

$3x-15x=15x-15x+96$

$-12x=96$

$\frac{-12x}{12}=\frac{96}{12}$

$x=-8$

- Solve the following:

_ $2(3x-6)=\frac{1}{2}(4x+2)$

_ $\frac{k}{3}-\frac{1}{2}=-1\frac{3}{4}$

_ $2.3(h-1.7)=4.2(h+1.3)$

_ $-2(1-c)=-3(2-c)$

_ $\frac{x}{4}=\frac{7}{10}$

_ $8(3d-2)=-12.32$

_ $\frac{x}{3}+\frac{1}{2}=\frac{5}{6}$

_ $\frac{t}{3}-\frac{3t}{4}=10$

_ $\frac{1}{3}x+8=-14$

_ $\frac{5}{m}+7=-3$

_ $\frac{1}{2}n-5=4-n$

_ $\frac{-5}{x}=-2$

Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

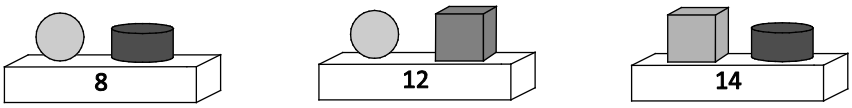
- Use diagrams and concrete materials to help students understand the steps needed to isolate the variable.

- Model the solving of equations with variables on both sides, using balancing strategies of balance scales (when terms are positive) and algebra tiles (when terms are negative). Examples are shown in the Whole-Class/Group/Individual Assessment section.
- Solve equations with rational numbers in fraction or decimal form (not easily modelled with balance scales or algebra tiles) by doing the same action on both sides of the equation.
- Use equations to model and then solve problems.

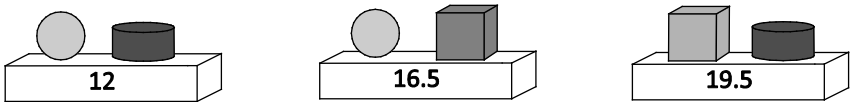
SUGGESTED LEARNING TASKS

- The combined mass for each pair of shapes is shown on the scale below the shapes. Ask students to determine the mass of each shape if the shapes that are the same colour and size have the same mass. Ask them to represent their answer in at least two ways or show at least two strategies. Set A has integral answers, while Set B has rational answers. Ask students to solve Set A first and share their solution strategies. Then provide them with Set B and ask them to solve for the unknowns. Was their strategy the same for both sets? Many students will use guess-and-check to solve the missing values. They may choose to represent the unknowns with letters like *x*, *y* and *z* or they may show their solution through guess-and-check. Students should share their process in solving these questions with the rest of the class. Differentiate these activities by using different sets of numbers, including natural and rational numbers.

Set A



Set B



- Have students model the solution to each of these equations using algebra tiles; then record the steps pictorially and numerically, show all the steps.

a) $3x = -9$

b) $2x + 3 = -1$

c) $2\ (x - 3) = 4$

d) $3x + 2 = x - 4$

e) $3\ (2x + 2) = 2\ (2x - 4)$

f) $\frac{x}{3} = 2$

g) $\frac{x}{3} + 5 = -2$

- Have students solve each of these equations, showing all steps. Have them verify the solution.

a) $\frac{1}{2}n = 12$

b) $-0.4n + 3.6 = 12.8$

c) $\frac{3}{4}(n-4)=5$

d) $\frac{3}{4}n-\frac{1}{2}=\frac{2}{3}n-\frac{2}{3}$

e) $0.8(6n+6)=0.6(4n-12)$

f) $\frac{x}{2}=0.8$

g) $\frac{2.5}{n}=0.5$

h) $\frac{x}{0.4}+8.2=12.8$

i) $\frac{12}{x}=3$

- Using two dice (one for positive numbers and one for negative), have students throw the dice twice. Substitute the value from each dice into the two boxes on the left-hand side of the equation on the first toss, and into the two boxes on the right-hand side of the equation on the second toss:

  □x+□=□x+□

  – Students should decide which dice represents positive values and which dice represents negative values. For example, if, on the first toss, the student throws a 3 and 4 this could represent –3 and 4; and if, on the second toss, the student throws a 2 and a 3, use –2 and 3.
  – Students decide which box contains which dice value. The first equation to solve could be: $-3x+4=-2x+3$
- Other equation formats that help students reinforce the concept of subtraction of negative values are:
  – □x–□=□x+□
  – □x+□=□x–□
  – □x–□=□x–□

- Consider using open-ended problems (with small-group discussion and then class sharing) to help students expand their repertoire of problem-solving approaches. You may also want to set up scenarios where students need to compare situations to arrive at a solution.

  For example: Yolanda's school is holding an evening fundraiser where students set up various games and competitions. There are two ways that students can participate: They can pay a $5 entry fee and play games for 75 cents each, or they can pay no entry fee and play games for $1.25 each.
  – If Yolanda has $15 to spend on games that evening, what choice will allow her to play the maximum number of games?
  – If Yolanda has an unlimited amount of money, would one choice be better than the other?
- Develop an equation for the following situations, and use it to answer the question(s):
  – To raise money the student council organized a dance. They hired a band and rented electronic equipment at a cost of $800. Participation and school spirit is important to the council so they charged only $5 per ticket. How many tickets would they need to sell to break even? To make a thousand dollars? To make two thousand dollars?

– Paul starts work 3 hours earlier than his sister Katie. Both work at the local grocery store. Katie earns $12 per hour and Paul earns $8 per hour. Paul wants to know how many hours he will have to work in order to earn the same amount of money as his sister.

- Three students are having a debate about the equation $y = \frac{1}{2}x + 5$. Student A thinks that every time *y* changes by 1, *x* changes by 2. Student B thinks that every time *y* changes by 1, *x* will change by $\frac{5}{2}$. Student C thinks that every time *y* changes by $\frac{1}{2}$, *x* changes by 1. Who do you think is correct and why? Use at least two representations to prove your case.
- Ask students to work in pairs to solve the following equations using algebra tiles. Students should take turns doing the following: Decide who will be the scribe and who will model the algebra tiles. The partner modelling with the tiles will tell the other person the steps to solve the equation. The scribe writes down the procedure algebraically.
  - $2a + 7 = 12$
  - $11 = 3 + 4x$
  - $9 - 3c = 15$

- Pass the Pen: Write a multi-step problem on the board and call on one student to come up and complete the first step and explain their reasoning for this step to the class. The student then calls on another student to complete the next step and "passes the pen." This continues until the problem is finished. When a question arises, the student holding the "pen" must answer the question, call on another student to help, or "pass the pen" to a different student. This activity can also be done on an interactive whiteboard, with hand held technology that can project, using a document camera, or in groups that pass the paper rather than the pen.

Equation	**Student Explanation**
2(*x*+3) = *x*–3(*x*–4)	
2*x*+6 = *x*–3(*x*–4)	I used the distributive property to expand
2*x*+6 = *x*–3*x*–12 continue….	I used the distributive property too. A question arises about the step. There is debate among students and the step is identified as a common error.

- Ask students to answer the following: Your class had to solve the equation 4(x – 2) = –3(2x + 6) on a recent math test. Below are two student solutions. Did either student make any errors?

Janicka's Solution	**Alison's Solution**
$4(x-2)=-3(2x+6)$ $4x-2=-6x+6$ $4x+6x=6+2$ $10x=8$ $\frac{10x}{10}=\frac{8}{10}$ $x=\frac{8}{10} or \frac{4}{5}$	$4(x-2)=-3(2x+6)$ $4x-8=-6x-18$ $4x-8=-6x-18$ $4x-6x=-18-8$ $-2x=-26$ $\frac{-2x}{-2}=\frac{-26}{-2}$ $x=13$

- Create a scavenger hunt around the school using QR codes. The questions should involve solving linear equations. Place a variety of QR codes around the classroom. Each group will select a question, scan it, and then write the steps of the solution. Provide each group with a scavenger hunt key where their solution informs them where to find the next code. The group is required to return all their solutions and workings to the teacher when they are finished with the activity.

QR Code Scavenger Hunt Key #1	
If your answer is :	Go to the entrance of the following rooms:
$\frac{7}{5}$	Art Room
–3	Main Office
–1	Music Room
$\frac{7}{8}$	Library
$\frac{-3}{10}$	Bulletin Board by Student Council Office
4	Main door of Cafeteria
5	Gym

- Ask students to complete the following: Two computer technicians both charge a fee for a home visit, plus an hourly rate for their work. Dawn charges a $64.95 fee plus $45 per hour. Alexi charges a $79.95 fee plus $40 per hour. For what length of service call do Dawn and Alexi charge the same amount?

SUGGESTED MODELS AND MANIPULATIVES

- 10 x 10 grid
- algebra tiles
- balance scales (pan or beam)
- dice
- number line
- Pan Balance Tool*

*also available in *Interactive Math Tools* (Pearson n.d.)

MATHEMATICAL LANGUAGE

Teacher	Student
▪ algebraic equation ▪ balance/balancing ▪ coefficient ▪ combine ▪ constant ▪ distributive property ▪ elimination process ▪ equality ▪ expression ▪ evaluate ▪ isolate the variable ▪ like terms ▪ simplify ▪ variable ▪ zero principle	▪ algebraic equation ▪ balance/balancing ▪ coefficient ▪ combine ▪ constant ▪ distributive property ▪ elimination process ▪ equality ▪ expression ▪ evaluate ▪ isolate the variable ▪ like terms ▪ simplify ▪ variable ▪ zero principle

Resources

Digital

- "Algebra Balance Scale [Unnamed]," *National Library of Virtual Manipulatives* (Utah State University 2015): http://nlvm.usu.edu/en/nav/frames_asid_324_g_4_t_2.html?open=instructions&from=category_g_4_t_2.html
- "Algebra Four," *Interactivate* (Shodor 2015): www.shodor.org/interactivate/activities/AlgebraFour
- "Algebra Tiles," *Illuminations Resources for Teaching Math* (National Council of Teachers of Mathematics 2015): http://illuminations.nctm.org/Activity.aspx?id=3482
- "Algebra Tiles [iPad App], *Apps for iPads* (Braining Camp 2013): www.brainingcamp.com/product/mobile.html
- "Solving Equations [iPad App], *Apps for iPads* (Braining Camp 2013): www.brainingcamp.com/product/mobile.html
- *Mathematics Planning Guide, Grade 9: Working with Linear Equations, Patterns and Relations (Variables and Equations)* (Alberta Education, LearnAlberta.ca 2012): www.learnalberta.ca/content/mepg9/html/pg9_linearequations/pdf/pg9_linearequations.pdf
- *Interactive Math Tools*. (Pearson. n.d.): http://nsvs.ednet.ns.ca/nsps/nsps26/course/view.php?id=3875.

Print

- "Using Error Analysis to Teach Equation Solving," *Mathematics Teaching in the Middle School* (Hawes 2007) pp. 238–242
- *Elementary and Middle School Mathematics: Teaching Developmentally*, 8th Edition (Van de Walle, Karp, and Bay-Williams 2013), pp. 258–276
- *Math Makes Sense 9* (Baron et al. 2009; NSSBB #: 2001644)
  - Unit 6: Linear Equations and Inequalities
    - > Section 6.1: Solve Equations by Using Inverse Operations
    - > Section 6.2: Solve Equations by Using Balance Strategies
    - > Game: Equation Persuasion
    - > Unit Problem: Raising Money for the Pep Club
  - *ProGuide* (CD; Word Files; NSSBB #: 2001645)
    - > Assessment Masters
    - > Extra Practice Masters
    - > Unit Tests
  - *ProGuide* (DVD; NSSBB #: 2001645)
    - > Projectable Student Book Pages
    - > Modifiable Line Masters
- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 207–209
- *Teaching Student-Centered Mathematics, Grades 5–8*, Volume Three (Van de Walle and Lovin 2006), p. 274

SCO PR04 Students will be expected to explain and illustrate strategies to solve single variable linear inequalities with rational coefficients within a problem-solving context. [C, CN, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR04.01 Translate a given problem into a single variable linear inequality, using the symbols ≥, >, <, or ≤.

PR04.02 Determine if a given rational number is a possible solution of a given linear inequality.

PR04.03 Generalize and apply a rule for adding or subtracting a positive or negative number to determine the solution of a given inequality.

PR04.04 Generalize and apply a rule for multiplying or dividing by a positive or negative number to determine the solution of a given inequality.

PR04.05 Solve a given linear inequality algebraically, and explain the process orally or in written form.

PR04.06 Compare and explain the process for solving a given linear equation to the process for solving a given linear inequality.

PR04.07 Graph the solution of a given linear inequality on a number line.

PR04.08 Compare and explain the solution of a given linear equation to the solution of a given linear inequality.

PR04.09 Verify the solution of a given linear inequality, using substitution for multiple elements in the solution.

PR04.10 Solve a given problem involving a single variable linear inequality, and graph the solution.

Scope and Sequence

Mathematics 8	**Mathematics 9**	**Mathematics 10**
PR02 Students will be expected to model and solve problems, concretely, pictorially, and symbolically, where *a*, *b,* and *c* are integers, using linear equations of the form ▪ $ax = b$ ▪ $\frac{x}{a} = b,\ a \neq 0$ ▪ $ax + b = c$ ▪ $\frac{x}{a} + b = c,\ a \neq 0$ ▪ $a(x + b) = c$	**PR04** Students will be expected to explain and illustrate strategies to solve single variable linear inequalities with rational coefficients within a problem-solving context.	**RF10** Students will be expected to solve problems that involve systems of linear equations in two variables, graphically and algebraically.

Background

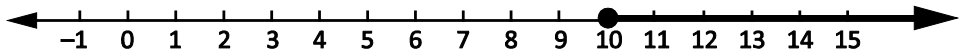
Solving linear inequalities is new at the Mathematics 9 level and will build on previous knowledge of linear equations. Students need to realize that unlike a single variable linear equation, which has a single solution, a single variable inequality will have many solutions. Students are familiar with the symbols < and > from their work with comparing two integers in Mathematics 6. The symbols $\leq$ and $\geq$ will have to be introduced.

Work with single variable inequalities should help students understand what the answer represents; that is, a set of values rather than a single number. If a container can hold no more than 45 kg, for example, different masses can be put in the container as long as they are less than or equal to 45, $x \leq 45$. Students should also recognize that the same inequality can be written in two different ways. For example, $x \leq 45$ and $45 \geq x$ represent the same set of numbers.

Similarly, graphing inequalities on a number line results in graphing part of the line rather than one specific point. Since there are too many points to graph when you have to consider rational numbers, shading of the number line is necessary. Ensure that students understand the difference between < and $\leq$, and > and $\geq$, and the effect these would have on the graph.

Although the majority of the work with inequalities includes rational numbers, some applications will involve discrete data. It is necessary to discuss how this affects the graph. The concept of continuous and discrete data was discussed in PR02.

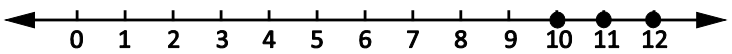
- For example, if we were to graph the admittance on a certain theme park ride as being at least 10 years old, the inequality $x \geq 10$ results in the following graph:



  Another application of this same inequality can modify this graph.

- Chantal's mom said she had to invite at least 10 people to the pool party.

  This would result in the following graph for $x \geq 10$:



An understanding of how various operations affect the truth of an inequality should be developed before introducing variables. Students can start with true sentences such as $-2 < 4$ and $5 > 1$. They can make a chart which shows each inequality and investigate how the truth of each is affected when the following operations are performed on both sides of the inequality:

- add a positive number, add a negative number
- subtract a positive number, subtract a negative number
- multiply by a positive number, multiply by a negative number
- divide by a positive number, divide by a negative number

Through investigation, students should recognize that adding or subtracting the same number from both sides of the inequality has no effect on the truth of the inequality.

- For example, if $-6 < -3$ is a true statement, then $-6 + 2 < -3 + 2$ or $(-4 < -1)$ is a true statement, and $-6 - 1 < -3 - 1$ or $(-7 < -4)$ is a true statement.

Through investigation, students should recognize that multiplying or dividing by a positive number results in a true inequality. If $10 > -2$ then $3(10) > 3(-2)$ is a true statement $(30 > -6)$ as is $\frac{10}{2} > \frac{-2}{2}$ a true statement $(5 > -1)$

However, multiplying or dividing by a negative number, results in a false inequality.

For example, if $-8 < 4$, then $\frac{-8}{-2} < \frac{4}{-2}$ or $(4 < -2)$ is a false statement. The inequality sign must be reversed to keep the truth of the inequality.

Once students have generalized these rules, they can apply them to solving inequalities. The process for solving inequalities is very similar to the process for solving equations. Both need to be balanced, through the use of inverse operations, to isolate the variable. When solving inequalities, however, emphasis should be placed on applying the generalized rule of reversing the sign when multiplying or dividing by a negative number. Provide students with ample practice to reinforce this concept.

As with equations, when solving inequalities the variable can be isolated on either side of the equation. When students solved equations, they could easily see that this did not affect the meaning of their solution (i.e., $x = 3$ and $3 = x$ are equivalent solutions). Students may be confused with the different ways of writing the solution for a linear inequality. Some work may need to be done so that students understand that the solution $x > 3$ is the same as $3 < x$.

Students should understand that the main difference in the solution of an equation as compared to an inequality is the value of the variable. A single variable linear equation has only one value of the variable that makes it true. There may be many values of the variable that satisfy an inequality.

As with equations, students should be aware that once they acquire a solution for an inequality, it can be checked for accuracy by substitution into the original inequality. To reinforce the concept that solutions to inequalities are sets of numbers, students should verify the solution by substituting multiple elements into the original inequality. Students are also required to apply the skills learned earlier in this unit to graph their solutions.

Where possible an effort should be made to have students describe a problem or situation as an inequality to be solved and then represented on a number line. This is a good opportunity for teachers to discuss with students that there may or may not be limits on these inequalities that are created, depending on the context of the problem. For example, if discussing speed of a vehicle as less than 60 km/h, or $v < 60$, students should realize that speed cannot drop below zero. Students should be reminded to distinguish between discrete and continuous data and the effect these have on graphing solutions.

Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Have students translate these statements into mathematical symbols:
  - My grandfather's age is greater than 45 years old.
  - Four or fewer students dropped the after-school yoga course.
  - Mario's golf score was less than 75.
  - My brother is older than my sister who turned 21 this month.
  - The speed on the highway between Truro and Halifax is less than or equal to 110 kilometres per hour.
  - Yarmouth is more than 150 km away from most larger towns in Nova Scotia.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

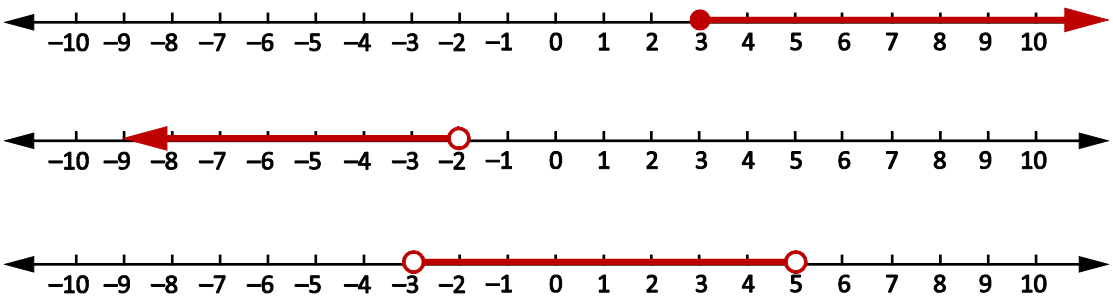
Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Using your own example, explain the impact of adding, subtracting, multiplying, and dividing an inequality by a positive number.
- Determine if the values satisfy the corresponding given inequality.

Inequality	Values
$x > 3$	5, –7, 9, 10
$-3x + 12 < 36$	–9, –10, –15.2
$\frac{x}{4} + 6 \geq -2$	–10, 15, $\frac{2}{3}$, 7

- Graph the solution to the following inequalities on a number line.
  - _ $3x - 2 \leq -20$
  - _ $7 - 3x \leq 22$
  - _ $2 + \frac{2}{3}x > \frac{1}{2}$
  - _ $2 - 5x > 2x + 16$
- Glen received grades of 75%, 82%, and 78% on his first three summative assessments. They are all equally weighted. Within what range must his next mark be on the next summative assessment to achieve an average of at least 80%? (Hint: To solve the problem, set up an inequality that would help you solve the problem, understanding that there is a maximum mark that Glen could get in the final assessment.)

- Julie bought a $50 prepaid card for her cell phone usage. She has to pay a monthly rate of $15 and then $0.15 per text message. If she only texts, how many text messages can she send this month? How would you graph the solution?
- Explain how you would know whether to use a closed circle or an open circle when representing an inequality on a number line.
- Give an example of a situation or problem that can be represented by an inequality. Write the inequality that represents the problem.
- Write an inequality for the following graphs:



Planning for Instruction

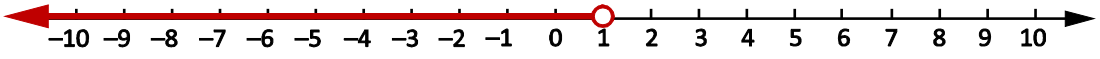
CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

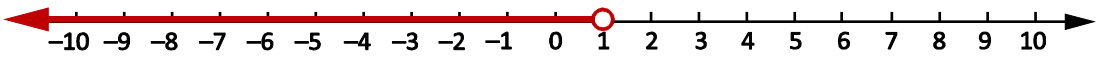
- Have students start with a statement they know is true, for example $5 > -2$. (Repeat this using a variety of statements so that students might generalize a rule.) Have them explore the operations of addition, subtraction, multiplication, and division of both positive and negative integers. Discuss the results. Use the outcomes of this activity to generalize rules for solving inequalities.
- A table such as the following could be used:

Inequality	Operation	Value of left side	Value of right side	Resulting Inequality
$5 > -2$	$+10$			
$5 > -2$	-10			
$5 > -2$	$\times 2$			
$5 > -2$	$\div 2$			
$5 > -2$	$\times(-2)$			
$5 > -2$	$\div(-2)$			

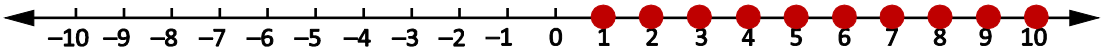
- Once students have an understanding of operation rules have them expand to inequalities involving a single variable, such as $-2x - 5 < 3$.
- Explore the difference between <, >, ≤, ≥ and how to represent them on the number line. For example $x < 1$ could look like:



Whereas $x \leq 1$, would look like:



- Introduce questions with answers that are discrete values and graph the solutions. For example: $x \geq 1, x \in Z$ (*Integers*)

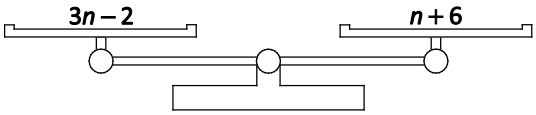


SUGGESTED LEARNING TASKS

- Have students work in pairs with one student completing the A questions and the other completing the B questions, and then have them verify each other's solutions and look for patterns in the exploration.

Student A. Solve. Show all steps. Verify your answer.	**Student B. Solve. Show all steps. Verify your answer.**	**How are the solutions to the two inequalities similar and how are they different? [Think of the solution, sign and graph of the solution.]**
$n+5>7$	$n+(-5)>7$	
$2n \leq -6$	$-2n \leq -6$	
$2n+(-3) \geq 7$	$2n+3 \geq 7$	
$-2n+3>-5$	$2n+3>-5$	
$2(n-3) \geq 6$	$-2(n-3) \geq 6$	
$3n+2<x-4$	$-3n+2<x-4$	
$\frac{n}{4} \geq 2$	$\frac{n}{-4} \geq 2$	

- Present students with models, such as the following, and ask the questions below:



  - What values of *n* will make this model tilt?
  - What values of *n* will make this model balance?
  - Write a mathematics statement to illustrate each situation.
  - Explain how equality is different from an inequality.

- Have students discuss the difference between $2x+1=5$ and $2x+1>5$.
- Work with a partner to complete the following activity:
  - Each partner chooses a different number.
  - Decide who has the greatest number and write an inequality that compares both numbers.
  - Choose the same mathematical operation to perform on each number.
  - Decide whose resulting number is greater and record an inequality that compares these new numbers.
  - Repeat this process with different mathematical operations.

  - Try different operations until you are able to predict which operations will reverse an inequality symbol and which ones will keep it the same.
  - Organize your observations and results.
- Respond to the following:
  - Jason says you can solve an inequality by replacing the inequality sign with an equal sign and putting it back in after solving the equation. Do you agree? Explain.
  - Explain why $3n - 2 > 8$ and $3n + 4 < 14$ do not have any solutions in common. Modify one of the inequalities so that they have exactly one solution in common.
- Journal writing prompt: Jamaal and Nancy are discussing the inequality $2x > -10$. Jamaal says, "The solution to the inequality is 6. When I substitute 6 for x, a true statement results". Nancy says, "I agree that 6 is a solution, but it is not the whole solution". Ask students to explain what Nancy means.
- Ask students to answer the following: Christy downloads music from two online companies. Tunes4U charges $1.50 per download plus a one-time membership fee of $15. YRTunes charges $2.25 per download with no membership fee.
  - Write an expression for the cost to download n songs from Tunes4U.
  - Write an expression for the cost to download n songs from YRTunes.
  - Write and solve an inequality to determine when it costs more to download songs from Tunes4U than from YRTunes. Verify and graph the solution.
  - Which site should Christy use to download music?

SUGGESTED MODELS AND MANIPULATIVES

- number line*
- pan balance models
- algebra tiles
- Pan Balance Tool*

*also available in *Interactive Math Tools* (Pearson n.d.) Mathematical Language

MATHEMATICAL LANGUAGE

Teacher	Student
▪ coefficient ▪ combine ▪ constant ▪ equality ▪ expression ▪ evaluate ▪ isolate the variable ▪ inequality ▪ like terms ▪ satisfy ▪ simplify ▪ solution set ▪ variable ▪ verify ▪ zero principle	▪ coefficient ▪ combine ▪ constant ▪ equality ▪ expression ▪ evaluate ▪ isolate the variable ▪ inequality ▪ like terms ▪ satisfy ▪ simplify ▪ solution set ▪ variable ▪ verify ▪ zero principle

Resources

Digital

- "Algebra Tiles," *Illuminations Resources for Teaching Math* (National Council of Teachers of Mathematics 2015): http://illuminations.nctm.org/Activity.aspx?id=3482
- "Algebra Tiles [iPad App], *Apps for iPads* (Braining Camp 2013): www.brainingcamp.com/product/mobile.html
- "Single Variable Linear Inequalities", *Planning Guide* (Alberta Education, LearnAlberta.ca 2012): www.learnalberta.ca/content/mepg9/html/pg9_singlevariablelinearinequalities/index.html
- "Solving Equations [iPad App], *Apps for iPads* (Braining Camp 2013): www.brainingcamp.com/product/mobile.html
- Interactive Math Tools. (Pearson. n.d.): http://nsvs.ednet.ns.ca/nsps/nsps26/course/view.php?id=3875.

Print

- *Elementary and Middle School Mathematics: Teaching Developmentally*, 8th Edition (Van de Walle, Karp, and Bay-Williams 2013), pp. 263–264
- *Math Makes Sense 9* (Baron et al. 2009; NSSBB #: 2001644)
  - Unit 6: Linear Equations and Inequalities
    - > Section 6.3: Introduction to Linear Inequalities
    - > Section 6.4: Solving Linear Inequalities by Using Addition and Subtraction
    - > Section 6.5: Solving Linear Inequalities by Using Multiplication and Division
    - > Unit Problem: Raising Money for the Pep Club
  - *ProGuide* (CD; Word Files; NSSBB #: 2001645)
    - > Assessment Masters
    - > Extra Practice Masters
    - > Unit Tests
  - *ProGuide* (DVD; NSSBB #: 2001645)
    - > Projectable Student Book Pages
    - > Modifiable Line Masters
- *Teaching Student-Centered Mathematics, Grades 5–8*, Volume Three (Van de Walle and Lovin 2006), p. 279

SCO PR05 Students will be expected to demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2). [C, CN, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR05.01 Create a concrete model and/or a pictorial representation for a given polynomial expression.
PR05.02 Write the expression for a given model of a polynomial.
PR05.03 Identify the variables, degree, and number of terms and coefficients, including the constant term, of a given simplified polynomial expression.
PR05.04 Describe a situation for a given first-degree polynomial expression.
PR05.05 Match equivalent polynomial expressions given in simplified form.

Scope and Sequence

Mathematics 8	**Mathematics 9**	**Mathematics 10**
–	**PR05** Students will be expected to demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2).	**AN04** Students will be expected to demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials, and trinomials), concretely, pictorially, and symbolically. **AN05** Students will be expected to demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially, and symbolically.

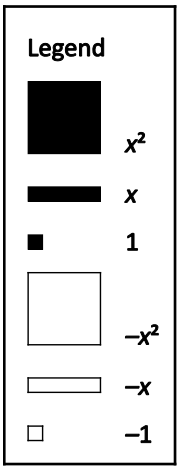
Background

Students are introduced to polynomials in Mathematics 9. They have been informally exposed to the language of algebra through their previous work with solving equations. Specific emphasis on the language of polynomials is new. To increase their familiarity and comfort with the language, the use of proper terminology should permeate mathematics classes.

Students should also be introduced to different types of polynomial expressions, and be able to distinguish between a monomial ($2x$), a binomial ($2x - 3$) and a trinomial ($x^2 + 2x - 3$). In discussing what polynomial expressions are, it is important to provide examples of

expressions that are not polynomials (e.g., $\sqrt{x}$ or $\frac{5}{n}$). Students should model polynomial expressions using algebra tiles. Concrete models provide the necessary support to model polynomial expressions symbolically.

Students have used algebra tiles in previous grades. The *x*-tiles and unit tiles were used when solving single variable linear equations involving integers. They should now be introduced to the x^2-tile. Note that the variable used is not restricted to *x*, but could be any symbol.



This will be the first time students are working with like and unlike terms. Throughout this curriculum document, shaded tiles represent positive values and white tiles represent negative values. It is very important that students are aware that x^2 and *x* are unlike terms. Using the x^2-tile and the *x*-tile to represent these terms helps students visualize and therefore understand the difference between like and unlike terms, and further understand why they cannot be combined.

Provide students with a variety of polynomial expressions and ask them to model them using the appropriate tiles. Examples should include combinations of both positive and negative terms. They should sketch a diagram that represents the polynomial. Students should also represent a polynomial expression symbolically from a concrete or pictorial representation.

It should be noted that $1x^2$ is often written as x^2 and has a coefficient of +1 whereas $–1x^2$ is often written as $-x^2$ and has a coefficient of –1. In discussing the degree of a term, it can be noted that terms with more than one variable have a degree equal to the sum of the exponents of the variables. The degree of $3x^2y^4$, for example, is 6, since 2 + 4 = 6. A variable with no exponent indicated is understood to have an exponent of 1. The degree of $5xy$ is 2 ($5xy$ can be thought of as $5x^1y^1$ and the sum of 1 + 1 is 2). The degree of a polynomial is the highest degree of any term in the polynomial. The degree of $2x^5 – 3xy^2 + 7$ is 5. This is important for future work with polynomials.

Rearranging polynomial expressions to show that some expressions are equivalent should also be explored. Students should realize that although polynomials are usually written in descending order, equivalent polynomials can be written by rearranging the terms. Stress that the signs of the terms have to remain the same (e.g., $4x – 3x^2 + 2$ is equivalent to $–3x^2 + 4x + 2$).

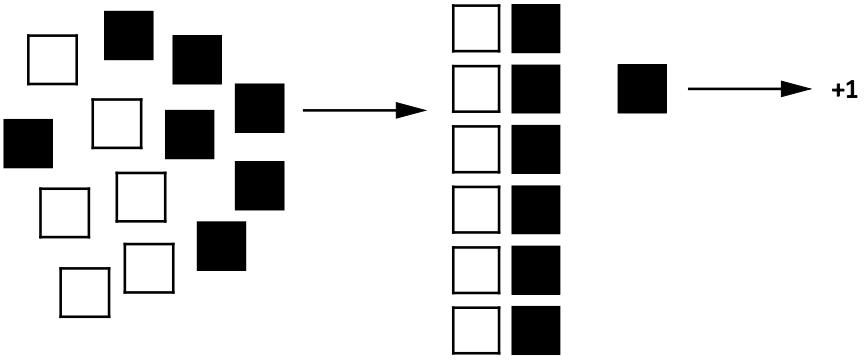
Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with integer tiles. Place the following tiles on the board or overhead and have students explain which integer the tiles represent. If necessary, remind students to make pairs of one positive and one negative to make zeros.



- Ask students to use algebra tiles to model the expression $3x - 5$.

  Change the model of $3x - 5$ if:
  - $3x$ changes to $2x$
  - -5 changes to -2
  - $3x$ is doubled
  - $3x$ changes to $-4x$
  - the entire expression is tripled

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Create a polynomial expression for each of the following descriptions. (For example, a polynomial of degree 2, with a constant of –4, would be $x^2 - 4$.)
  - A binomial with a coefficient of 4.
  - A trinomial of degree 2, with coefficients of 4 and –1.
  - A binomial with no constant term.

- Compare the polynomial in List A with the corresponding polynomial in List B, and determine if they are equivalent.

List A	List B
$3x - x^2 - 2$	$-x^2 + 3x - 2$
$7 + 2x + x^2$	$x^2 - 2x + 7$
$3x - 5$	$5 - 3x$

- Describe a real-life situation that would model the binomial expression $2x + 3$.
- Have students complete a Concept Definition Map, Frayer Model, or other graphic organizer for the concept of polynomials.

Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Use diagrams and concrete materials to demonstrate the idea of converting models to expressions.
- Create a word wall for vocabulary.

SUGGESTED LEARNING TASKS

- Provide students with algebra tiles and have them work in groups to create examples of polynomials. The polynomials should include both positive and negative terms. Polynomials should be recorded pictorially by creating a sketch of the tiles including the colours. Polynomials should also be recorded symbolically. Have students share the models and diagrams of the polynomials with other groups of students. Have students look for similarities and differences. Use examples of students' polynomials to initiate conversation involving the language of polynomials.
- Give students a description of different polynomials and ask them to create as many polynomials as they can think of to meet the description. (Sample description: Each polynomial must have four terms; the polynomial must include the coefficients 1, –2 and 4, in any order.) Students should record their list of polynomials both pictorially and symbolically. Have students compare their polynomials and identify equivalent polynomials.
- Given a polynomial, students should identify the terms, degree, variables, coefficients, and constants in each, and represent a model. Provide students with a chart similar to below and insert any polynomial expression or model and complete the remaining cells.

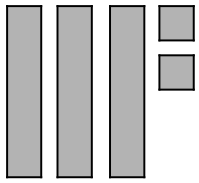
Expression	# of terms	Degree of polynomial	Variable(s)	Coefficient(s)	Constant	Model
3	1	0	none	none	3	
$x^2 + 2x - 3$	3	2	x	1, 2	–3	
$4x - 2x^2 + 3$	3	2	x	–2, 4	3	

- Students could play I Have ... Who Has to reinforce the language of polynomials. Provide them with a "loop" card as shown. A student starts and reads the "Who Has ..." part of the card aloud. A student will respond with "I have ..." answering with the correct polynomial. The student continues to read "Who Has ..." This continues until all students have read their cards.

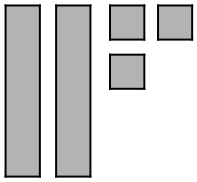
I have … $2x^2 - x + 3$	*I have …* $-3x^2 + 4x - 5$	*I have …* $-3a^2 - 6a - 5$
Who has … A polynomial with a constant term of –5	*Who has …* A polynomial with all negative coefficients	*Who has …* A polynomial with an x coefficient of –1

- As a journal prompt, tell students that Sam rearranged the polynomial $2x - 4 + 6x^2$ as $6x^2 - 2x + 4$. Ask students if she was correct. They should justify their answers using words, diagrams, pictures, models, etc.
- Ask students to match the following polynomials to the appropriate diagram (shaded represents positive).
  - _ $2x^2 + 3$
  - _ $3x + 2$
  - _ $2x + 3$

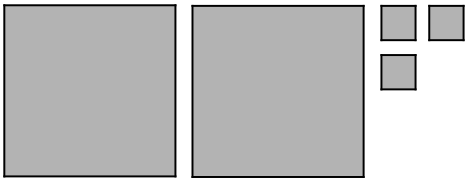
a)



b)



c)



SUGGESTED MODELS AND MANIPULATIVES ALGEBRA TILES

- algebra tiles*

*also available in *Interactive Math Tools* (Pearson n.d.)

MATHEMATICAL LANGUAGE

Teacher	Student
▪ binomial ▪ coefficient ▪ constant ▪ degree of a polynomial ▪ degree of a term ▪ equivalent ▪ expression ▪ like terms ▪ monomial ▪ polynomial ▪ term(s) ▪ trinomial ▪ unlike terms ▪ variable	▪ binomial ▪ coefficient ▪ constant ▪ degree of a polynomial ▪ degree of a term ▪ equivalent ▪ expression ▪ like terms ▪ monomial ▪ polynomial ▪ term(s) ▪ trinomial ▪ unlike terms ▪ variable

Resources

Digital

- "Algebra Tiles," *Illuminations Resources for Teaching Math* (National Council of Teachers of Mathematics 2015): http://illuminations.nctm.org/Activity.aspx?id=3482
- "Algebra Tiles [iPad App], *Apps for iPads* (Braining Camp 2013): www.brainingcamp.com/product/mobile.html
- "Polynomials," *Planning Guide* (Alberta Education, LearnAlberta.ca 2011): www.learnalberta.ca/content/mepg9/html/pg9_polynomials/index.html
- "Solving Equations [iPad App], *Apps for iPads* (Braining Camp 2013): www.brainingcamp.com/product/mobile.html
- "'Why Are There So Many Words in Math?': Planning for Content-Area Vocabulary Instruction," *Voices from the Middle*, Volume 20, Number 1, September 2012 (Smith and Angotti 2012): www.ncte.org/library/NCTEFiles/Resources/Journals/VM/0201-sep2012/VM0201Why.pdf
- Interactive Math Tools. (Pearson. n.d.): http://nsvs.ednet.ns.ca/nsps/nsps26/course/view.php?id=3875.

Print

- "Learning to Learn Vocabulary in Content Area Textbooks," *Journal of Reading*, Volume 32, No. 2 November 1988, 108–118 (Schwartz 1988)
- *Math Makes Sense 9* (Baron et al. 2009; NSSBB #: 2001644)
  - Unit 5: Polynomials
    - > Section 5.1: Modelling Polynomials
    - > Section 5.3: Adding Polynomials
    - > Section 5.4: Subtracting Polynomials
    - > Unit Problem: Algebra Patterns on a 100-Chart

- *ProGuide* (CD; Word Files; NSSBB #: 2001645)
  - > Assessment Masters
  - > Extra Practice Masters
  - > Unit Tests
- *ProGuide* (DVD; NSSBB #: 2001645)
  - > Projectable Student Book Pages
  - > Modifiable Line Masters

SCO PR06 Students will be expected to model, record, and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially, and symbolically (limited to polynomials of degree less than or equal to 2). [C, CN, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR06.01 Model addition of two given polynomial expressions, concretely and/or pictorially, and record the process symbolically.

PR06.02 Model subtraction of two given polynomial expressions, concretely and/or pictorially, and record the process symbolically.

PR06.03 Identify like terms in a given polynomial expression.

PR06.04 Apply a personal strategy for addition or subtraction of two given polynomial expressions, and record the process symbolically.

PR06.05 Identify equivalent polynomial expressions from a given set of polynomial expressions, including pictorial and symbolic representations.

PR06.06 Identify the error(s) in a given simplification of a given polynomial expression.

Scope and Sequence

Mathematics 8	**Mathematics 9**	**Mathematics 10**
–	**PR06** Students will be expected to model, record, and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially, and symbolically (limited to polynomials of degree less than or equal to 2).	**AN04** Students will be expected to demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials, and trinomials), concretely, pictorially, and symbolically. **AN05** Students will be expected to demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially, and symbolically.

Background

Students have been working with a variable when solving of equations since Mathematics 5. Algebraic reasoning and the notion of equality is a focus of the mathematics curriculum beginning at the primary level. In previous grades, students used algebra tiles or two-coloured counters to model addition and subtraction of integers. They will now model polynomial expressions with algebra tiles. Working with

concrete representations of polynomials will better prepare students to work with pictorial and symbolic representations. They will apply their understanding of addition and subtraction of integers to these operations with polynomial expressions.

In PR05 students studied equivalent polynomials, already presented in simplified form. To begin addition and subtraction, have students simplify polynomials by combining like terms concretely, pictorially, and symbolically. To simplify polynomials, have students use tools such as algebra tiles, area models, drawings or sketches, and algebraic symbols. Encourage students to record the process symbolically when using concrete materials or drawings and sketches. Simplifying polynomials should always be stressed, as this reinforces the idea that the same polynomial can be written in a variety of equivalent ways. The convention of writing polynomials with the exponents in descending order makes them easier to compare.

Students should, with the aid of algebra tiles, be able to add and subtract like terms to simplify expressions. Algebra tiles help students determine which terms can and cannot be combined. They should make the connection between combining like terms and combining polynomials. Polynomial expressions can be combined both horizontally and vertically.

Perimeter is a very useful application of addition and subtraction of polynomials. Students should recognize that, because it has linear units, perimeter can be represented by first degree polynomials.

While addition of polynomials is often straightforward, subtraction sometimes poses difficulty for students. Consideration should be given to the different representations of subtraction, including the following:

- **comparison**: comparing and finding the difference between two quantities
- **taking away** → starting with a quantity and removing a specified amount
- **finding the missing addend** → which asks the question, “What would be added to the number being subtracted to get the starting amount?”

These meanings for subtraction have been developed in previous grades in the context of number.

Teachers may need to revisit the concept of zero and the zero principle. Students have studied this concept in Mathematics 7 and Mathematics 8.

When subtracting polynomials symbolically, students can apply the properties of integers. They should be cautioned about the use of brackets. A common student error is subtracting only the first term of the second polynomial.

Students should use algebra tiles, integer properties, and the horizontal and/or vertical methods to add and subtract polynomials. They may also want to use a combination of any of these methods. Drawing on their prior knowledge of addition and subtraction of integers, students should solve addition and subtraction of polynomials questions using any strategy or materials of their choice. They should be writing number sentences and also summarizing their personal strategy to show how they found the sum or difference for each case. Have students explain how their symbolic strategies work by relating to the concrete representations.

Students should be able to explain the strategy used. Through the sharing of strategies, students will learn a variety of possible addition and subtraction strategies, and each student will adopt ones that they understand well and has made thier own. That is why these strategies are often referred to as

"personal strategies." The most appropriate strategy used may vary depending on the student and the numbers involved in the problem. Personal strategies make sense to students and are as valid as the traditional algorithm. Therefore, emphasis should be on students' algorithms rather than on the traditional algorithm. The paper-and-pencil recording of students' personal strategies should reflect their thinking and must be reliable, accurate, and efficient. It is important that students can justify how and why an algorithm works. Students should be encouraged to refine their strategies to increase their efficiency, and teachers should monitor each student's symbolic recording of the strategy to ensure that the recording is accurate, mathematically correct, organized, and efficient.

To address different learning styles and abilities, have grid paper and algebra tiles available for students who need concrete visual representation before using symbolic strategies.

Have a class discussion on the personal strategies students used to find sums and differences. Have students think about the strategies that have been discussed and have students decide which strategy is most efficient for them.

It is important that students move from the concrete to the pictorial to the symbolic. The introduction of the symbolic should be done in tandem with the concrete and pictorial representations.

It is beneficial to have students analyze solutions that contain errors. Along with providing the correct solutions, they should be able to identify incorrect solutions, including why errors might have occurred and how they can be corrected. This reinforces the importance of recording solution steps rather than only giving a final answer.

Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

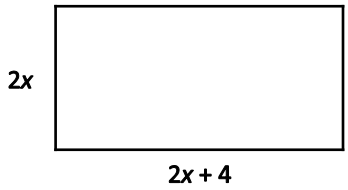
- In three different ways, model $4x^2 – 3x + 5$ with algebra tiles.
- Yvonne was asked to provide an equivalent expression for $8x+6x+2$. She said that $16x$ was an equivalent expression. Explain the mistake in Yvonne's equivalent expression.
- Write an equivalent expression for $5a + 6a + 4$ and $n^2 - 2n + 3$.

Whole-Class/Group/Individual Assessment Tasks

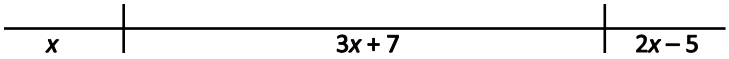
Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- A rectangular flower garden has a length of 8 cinder blocks and a width of 9 bricks. Ask students to find a simplified expression for the perimeter of the flower garden.

- Write a simplified expression for the perimeter of the rectangle.



- Write a simplified expression for the length of the string.



- The difference of two polynomials is $-2x^2 - 4x + 3$. Ask students to list three pairs of second degree polynomials that might have been subtracted to get that difference.
- Ask students to answer the following:
  - Wayne was asked to write an expression equivalent to $2x - 7 - 4x + 8$. His solution was: $2x - 4x - 7 + 8 = 2x - 1$. Identify the errors and show the correct simplification.
  - Jennifer subtracted a polynomial from $3x^2 - x - 1$. The difference required eight algebra tiles to represent it. What polynomial could she have subtracted?
- Use both the horizontal and vertical methods to simplify $(3a^2 + 2) + (-4a^2 + 2a - 7)$. Explain which method you prefer and why.
- Your friend was absent today. Your homework requires you to subtract these two polynomials: $(-3x + 5) - (4x - 3)$. Explain to your friend how to find the solution. What would you say? Write a transcript of your conversation.

Planning for Instruction

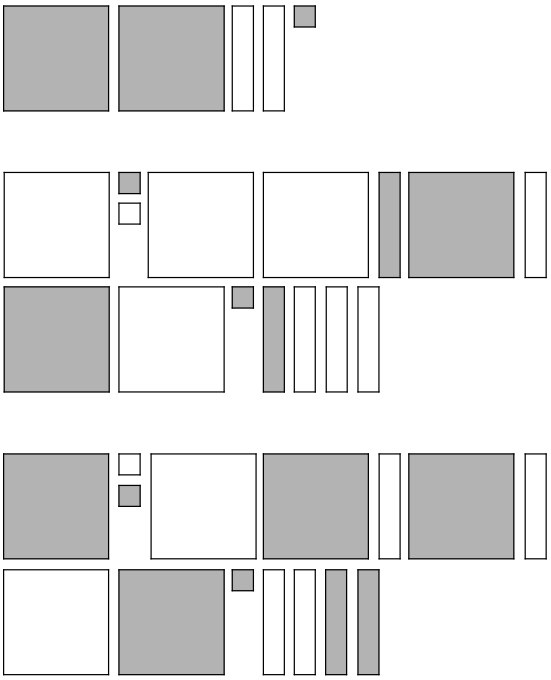
CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

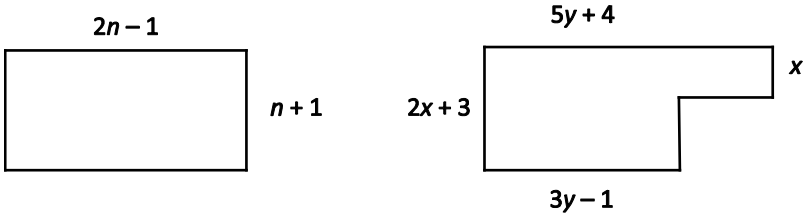
- Ensure students have opportunity to view subtraction in a variety of ways.
- Use perimeter problems as an application for addition and subtraction of polynomials.
- Take time to show students that algebra tiles may represent different variables; not just the typical models of x^2 and x. Do not stick exclusively to x variables.
- Students should practice procedures for operations with polynomials only after they have learned to make sense of operations with concrete materials.

SUGGESTED LEARNING TASKS

- Present students with models such as the following, and have them choose which ones result in the same simplified polynomial.



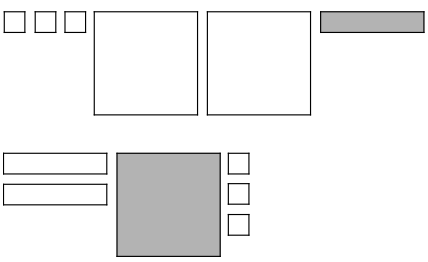
- Have students determine the perimeter of the following shapes.



- Identify the like terms: $5x^2, 3xy, -2x^2, 2x$
- Model each sum or difference concretely (algebra tiles) or pictorially and record the steps symbolically.

$$\begin{array}{l} (-2x^2 - 2x - 3) \\ \underline{+\ (-2x^2 + 3x + 1)} \\ (x^2 + 3x - 4) - (-2x^2 + 1) \end{array}$$

- Simplify using your own strategy, concretely, pictorially, or symbolically.
  - $(2x^2 - 5x) - (3x^2 + 2x)$
  - $(3m^2 - 2mn - 4) + (m^2 + 2)$
- Identify which expressions are equivalent to $-2y^2 + y - 3$.
  - $y - 3 - 2y^2$
  - $y^2 - 1 + 4y - 3y^2 - 3y - 2$
  - $-y^2 - 3$



- Circle the errors in the following work. Provide the correct solution.

  Step 1: $(2x^2 - 3x + 2) - (x^2 + x - 1)$

  Step 2: $2x^2 - 3x + 2 - x^2 + x - 1$

  Step 3: $x^2 - 2x - 1$

- Ask students to complete the magic square. Rows, columns, and diagonals must have the same sum.

		$2x^2 - 3x + 24$
	$5x^2 - 4x + 15$	$9x^2 - 2x + 3$
$8x^2 - 5x + 6$		

SUGGESTED MODELS AND MANIPULATIVES

- algebra tiles*
- grid paper

*also available in *Interactive Math Tools* (Pearson n.d.)

MATHEMATICAL LANGUAGE

Teacher	Student
▪ binomial ▪ coefficient ▪ combine ▪ constant ▪ equivalent ▪ expression ▪ like terms ▪ monomial ▪ opposite ▪ polynomial ▪ term(s) ▪ trinomial ▪ unlike terms ▪ variable ▪ zero principle	▪ binomial ▪ coefficient ▪ combine ▪ constant ▪ equivalent ▪ expression ▪ like terms ▪ monomial ▪ opposite ▪ polynomial ▪ term(s) ▪ trinomial ▪ unlike terms ▪ variable ▪ zero principle

Resources

Digital

- "Algebra Tiles," *Illuminations Resources for Teaching Math* (National Council of Teachers of Mathematics 2015): http://illuminations.nctm.org/Activity.aspx?id=3482
- "Algebra Tiles [iPad App]," *Apps for iPads* (Braining Camp 2013): www.brainingcamp.com/product/mobile.html
- "Polynomials," *Planning Guide* (Alberta Education, LearnAlberta.ca 2011): www.learnalberta.ca/content/mepg9/html/pg9_polynomials/index.html
- Interactive Math Tools. (Pearson. n.d.): http://nsvs.ednet.ns.ca/nsps/nsps26/course/view.php?id=3875.

Print

- *Big Ideas from Dr. Small: Creating a Comfort Zone for Teaching Mathematics, Grades 4–8* (Small 2009) pp. 8–9
- *Elementary and Middle School Mathematics: Teaching Developmentally*, 8th Edition (Van de Walle, Karp, and Bay-Williams 2013), pp. 481–484
- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 270–274, 639–640
- *Math Makes Sense 9* (Baron et al. 2009; NSSBB #: 2001644)
  - Unit 5: Polynomials
    - > Section 5.2: Like Terms and Unlike Terms
    - > Section 5.3: Adding Polynomials
    - > Section 5.4: Subtracting Polynomials
    - > Unit Problem: Algebra Patterns on a 100-Chart
  - *ProGuide* (CD; Word Files; NSSBB #: 2001645)

- > Assessment Masters
- > Extra Practice Masters
- > Unit Tests

– *ProGuide* (DVD; NSSBB #: 2001645)

- > Projectable Student Book Pages
- > Modifiable Line Masters

SCO PR07 Students will be expected to model, record, and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree less than or equal to 2) by monomials, concretely, pictorially, and symbolically. [C, CN, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR07.01 Model multiplication of a given polynomial expression by a given monomial, concretely or pictorially, and record the process symbolically.

PR07.02 Model division of a given polynomial expression by a given monomial, concretely or pictorially, and record the process symbolically.

PR07.03 Apply a personal strategy for multiplication and division of a given polynomial expression by a given monomial.

PR07.04 Provide examples of equivalent polynomial expressions.

PR07.05 Identify the error(s) in a given simplification of a given polynomial expression.

Scope and Sequence

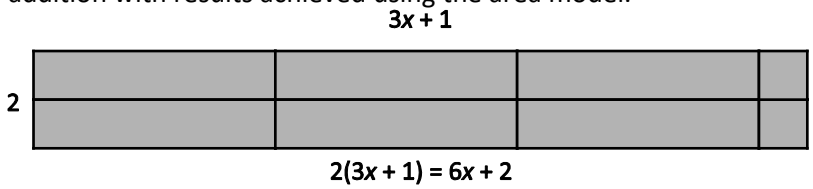
Mathematics 8	**Mathematics 9**	**Mathematics 10**
N07 Students will be expected to demonstrate an understanding of multiplication and division of integers, concretely, pictorially, and symbolically. **PR02** Students will be expected to model and solve problems, concretely, pictorially, and symbolically, where *a*, *b,* and *c* are integers, using linear equations of the form ▪ $ax = b$ ▪ $\frac{x}{a} = b,\ a \neq 0$ ▪ $ax + b = c$ ▪ $\frac{x}{a} + b = c,\ a \neq 0$ ▪ $a(x + b) = c$	**PR07** Students will be expected to model, record, and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree less than or equal to 2) by monomials, concretely, pictorially, and symbolically.	**AN04** Students will be expected to demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials, and trinomials), concretely, pictorially, and symbolically. **AN05** Students will be expected to demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially, and symbolically.

Background

When multiplying and dividing polynomial expressions, students should apply their understanding of concepts learned in previous mathematics courses, including multiplication and division of integers (Mathematics 8), the use of the distributive property (Mathematics 8), and calculation of the area of rectangles (Mathematics 6). They should be given the opportunity to connect these concepts with polynomials.

Students are expected to multiply a scalar by a polynomial, monomial by a monomial, and a monomial by a polynomial. Multiplication of a polynomial by a scalar should be developed first, with concrete materials and diagrams, and represented using repeated addition. Given 2(3x + 1), for example, students should recognize that it is the same as (3x + 1) + (3x + 1), and therefore, model the binomial twice, combine the like terms, and arrive at an answer.

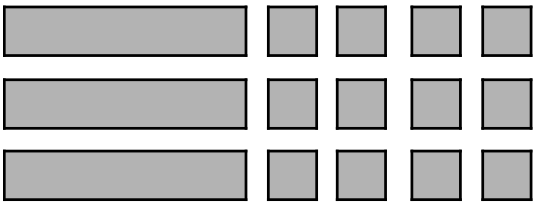
The area model should also be used so that students can relate results achieved through repeated addition with results achieved using the area model.



In Mathematics 8, students applied the distributive property to solve linear equations with integers a, b and c, a(b + c) or (a + b)c = ac + bc. This property can be demonstrated particularly well using algebra tiles. When multiplying polynomial expressions, applications should be emphasized, especially in relation to area problems. Area can be represented by second degree polynomials because it has square units.

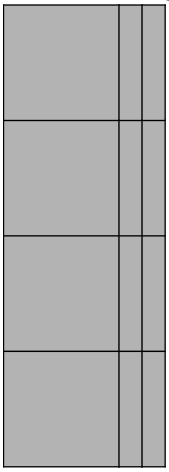
From previous work with number operations, students should be aware that division is the inverse of multiplication. This can be extended to divide polynomials by monomials. The study of division should begin with division of a monomial by a monomial, progress to a polynomial divided by a scalar, and then to division of a polynomial by any monomial.

The most commonly used symbolic method of dividing a polynomial by a monomial at this level is to divide each term of the polynomial by the monomial, and then use the exponent laws to simplify (e.g., $\frac{3x+12}{3}=\frac{3x}{3}+\frac{12}{3}$). This method can also be easily modelled using tiles, where students use the sharing model for division. They start with a collection of three *x*-tiles and 12 unit tiles, and divide them into three equal groups.



Division of a polynomial by a monomial can be visualized using area models with algebra tiles. Students should be given situations where they have a specific collection of tiles and asked to create a rectangle

with one dimension given. To model this, teachers could ask students to create a rectangle, using four x^2-tiles and eight x-tiles where 4x is one of the dimensions**.**



For this example, $x + 4$ tiles will be a part of each group, so the quotient is $x + 4$.
Because there are a variety of methods available to multiply or divide a polynomial by a monomial, students should be given the opportunity to apply their own personal strategies. They should be encouraged to use algebra tiles, area models, rules of exponents, the distributive property and repeated addition, or a combination of any of these methods, to multiply or divide polynomials. Regardless of the method used, students should be encouraged to record their work symbolically. Understanding the different approaches helps students develop flexible thinking.

Students should be encouraged to simplify polynomials. They should realize that it is often difficult to compare polynomials for equivalency until they are presented in simplified form. This is also an opportunity to highlight the value of expressing solutions in descending order.

Questions requiring error analysis can be effective tools to assess students' understanding of simplifying polynomial expressions because it requires a deeper understanding. This reinforces the idea that the process is as important as the solution.

Provide students with a variety of multiplication and division problems, such as the one below, which are not properly simplified. Ask them to identify and circle the errors in the solutions and to write the correct solution.

$$(12x^2 - 4x) \div (-2x)$$
$$= \frac{12x^2}{-2x} - \frac{4x}{-2x}$$
$$= -6x - 2$$
$$= -8x$$

Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Have students play the game "Operation Integers" using the operations of multiplication and division.

Players: Two to four.
Materials: A deck of cards (no face cards).
Description: Deal all the cards face down on the table. Black suits are positive and red suits are negative. Each player turns over two cards and decides whether to multiply or divide the two numbers on the cards. The player who has the greatest result wins all the cards that are face up.
Goal: The play continues until one person (the winner) has all the cards.
Variations:

- Use fewer cards or cards with only certain numbers.
- Turn over three or four cards instead of two cards for each player.
- The player who has the least product or quotient wins all the cards that are face up.
- Each player rolls two (or more) dice with integers on each face rather than using playing cards.
- The player with the greatest (or least) number resulting from the operations scores one point. The winner is the player with the most points.

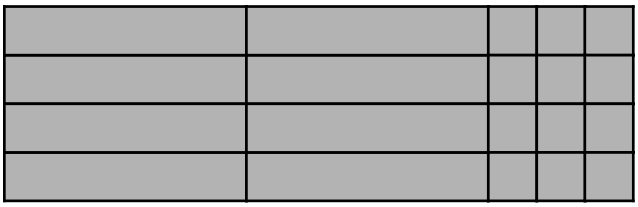
- Solve the following equations.
  - $5(b + 13) = 25$
  - $-7(n + 4) = -42$

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Demonstrate the product or quotient for each of the following using algebra tiles or diagrams and record the process symbolically.
  - $3(2x - 1)$
  - $\frac{3x^2 - 6x}{-3x}$

- Find the product or quotient using a strategy of your choice.
  - $2(x^2 + 3)$
  - $\dfrac{2x^2 + 8x - 6}{2}$
- Write the dimensions and area for the rectangle shown below. Write all the related multiplication and division equations.



- Write 2 equivalent expressions for the following:
  - $4(2x^2 + 6x)$
  - $\dfrac{3x - 9}{6}$
- Find the missing terms in following polynomials.
  - $3x$ (☐ + 4) = $6x^2$ + ☐
  - ($2x^2$ – ☐)= $x^2 - x$
- Circle the errors and correct the solution.
  - $-4m(-2 + m) = -8m + 4$
  - $\dfrac{-12y + 6}{6} = -2y$

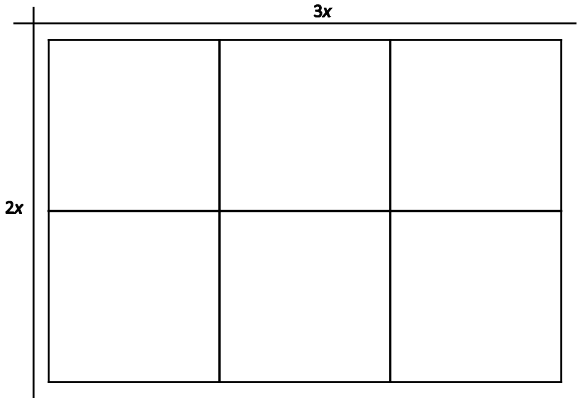
Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

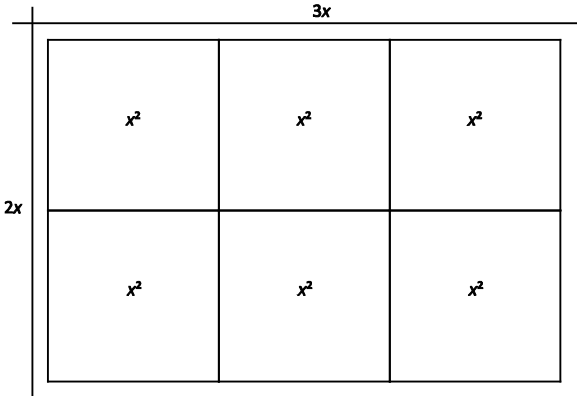
Consider the following strategies when planning daily lessons.

- Begin with multiplication and division of monomials using area models and progress toward solving algebraically. For example:

For (3*x*)(2*x*),
start by building a frame with the given dimensions.
Fill in the area with appropriate pieces.
The area of $6x^2$ is the product.



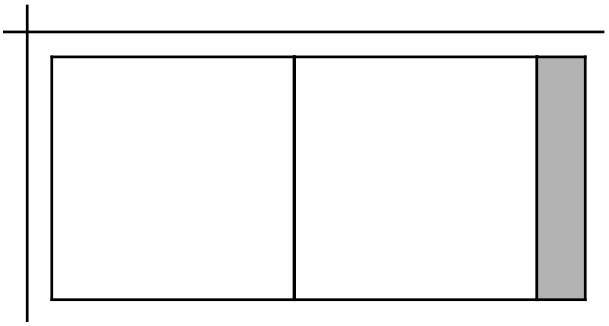
For $\frac{6x^2}{3x}$,
start by building a rectangle (dividend) using the divisor as one of the dimensions.
The other dimension is the quotient.

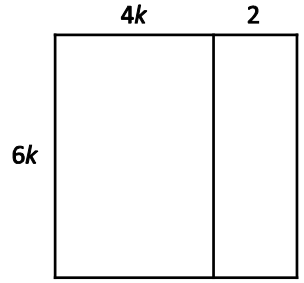


- Move toward multiplication of a polynomial by a scalar and then multiplication of a polynomial by a monomial.

SUGGESTED LEARNING TASKS

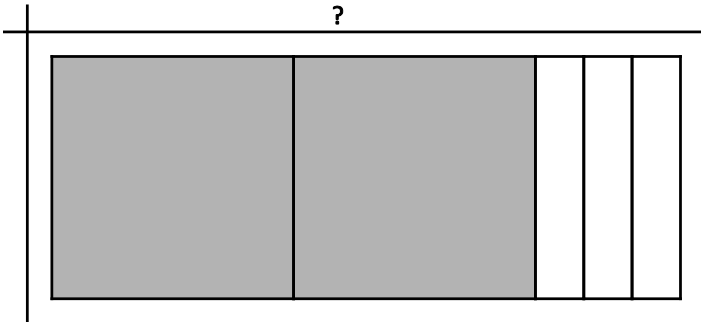
- Have students model $3(2x+1)$ with tiles.
- Ask students to model multiplication of polynomials, such as the following, using at least two different methods.
  - 2(4x2 + 3x – 2)
  - –3x(x – 4)
- Provide students with multiplication models and ask them to write a multiplication sentence for each model.



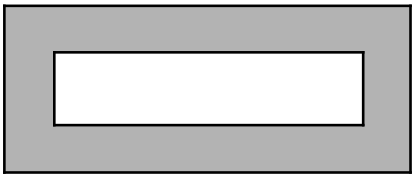


- Ask students describe two methods that could be used to multiply polynomials by monomials.

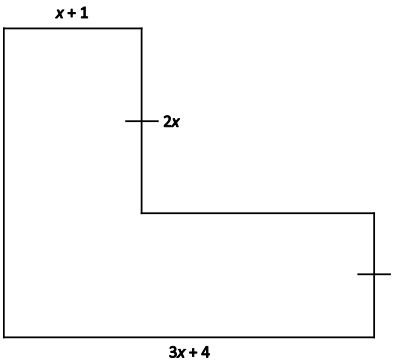
- Ask students to write a division sentence to describe the following model and then to determine the quotient.



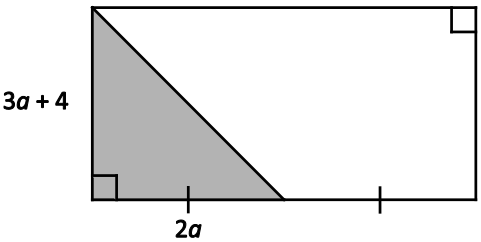
- Ask students to draw a rectangle with an area of $36a^2 + 12a$ and list as many different dimensions as possible in two minutes. After the time is up, ask students to turn to a partner to share their list. With your partner, one at a time, read an entry from the list until all entries have been exhausted. Have a class discussion about which factors are possible.
- The inside rectangle in the diagram below is a flower garden. The shaded area is a concrete walkway around it. The area of the flower garden is given by the expression $x^2 + 5x$ and the area of the large rectangle, including the walkway and the flower garden, is $2x^2 + 20x$.



  - Ask students to write an expression for the dimensions of each rectangle. Is there more than one possibility? Ask students to find the area of the walkway.
- Ask students to write a simplified expression for the area of this figure.



  - Ask students to write a simplified expression for the area of the shaded region in the figure below:



- Students could work in groups to play the Domino Game.

- Provide each group with 10 domino cards.
- One side of the card should contain a polynomial expression, while the other side contains a simplification of the polynomial expression.
- The object is to lay the dominos out such that the simplification of the polynomial expression on one card will match with the correct polynomial expression on another.
- They will eventually form a complete loop with the first card matching with the last card. A sample is shown below:

$3(2x + 5)$	$4x^2 + 20x$	$4x(x + 5)$	$-6x + 8$	$\frac{18x^2 - 24x}{-3x}$	$5x(3 - x)$

SUGGESTED MODELS AND MANIPULATIVES

- algebra tiles
- grid paper

MATHEMATICAL LANGUAGE

Teacher	Student
▪ area model ▪ binomial ▪ coefficient ▪ combine ▪ constant ▪ equivalent ▪ expression ▪ factors ▪ like terms ▪ monomial ▪ opposite ▪ polynomial ▪ product ▪ quotient ▪ scalar ▪ term(s) ▪ trinomial ▪ unlike terms ▪ variable	▪ area model ▪ binomial ▪ coefficient ▪ combine ▪ constant ▪ equivalent ▪ expression ▪ factors ▪ like terms ▪ monomial ▪ opposite ▪ polynomial ▪ product ▪ quotient ▪ scalar ▪ term(s) ▪ trinomial ▪ unlike terms ▪ variable

Resources

Digital

- "Algebra Tiles," *Illuminations Resources for Teaching Math* (National Council of Teachers of Mathematics 2015): http://illuminations.nctm.org/Activity.aspx?id=3482
- "Algebra Tiles [iPad App]," *Apps for iPads* (Braining Camp 2013): www.brainingcamp.com/product/mobile.html
- "Polynomials," *Planning Guide* (Alberta Education, LearnAlberta.ca 2011): www.learnalberta.ca/content/mepg9/html/pg9_polynomials/index.html

Print

- *Elementary and Middle School Mathematics: Teaching Developmentally*, 8th Edition (Van de Walle, Karp, and Bay-Williams 2013), pp. 484–485
- *Making Math Meaningful to Canadian Students, K–8* (Small 2009), p. 180
- *Math Makes Sense 9* (Baron et al. 2009; NSSBB #: 2001644)
  - Unit 5: Polynomials
    - > Section 5.5: Multiplying and Dividing a Polynomial by a Constant
    - > Section 5.6: Multiplying and Dividing a Polynomial by a Monomial
  - *ProGuide* (CD; Word Files; NSSBB #: 2001645)
    - > Assessment Masters
    - > Extra Practice Masters
    - > Unit Tests
  - *ProGuide* (DVD; NSSBB #: 2001645)
    - > Projectable Student Book Pages
    - > Modifiable Line Masters

Measurement (M)

GCO: Students will be expected to use direct and indirect measurement to solve problems.

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

GUIDING QUESTIONS

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Follow-up on Assessment

The evidence of learning that is gathered from student participation and work should be used to inform instruction.

GUIDING QUESTIONS

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan available at Mathematics Learning Commons: Grades 7–9: http://nsvs.ednet.ns.ca/nsps/nsps26/course/view.php?id=3875.

GUIDING QUESTIONS

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SCO M01 Students will be expected to solve problems and justify the solution strategy, using the following circle properties:

- The perpendicular from the centre of a circle to a chord bisects the chord.
- The measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc.
- The inscribed angles subtended by the same arc are congruent.
- A tangent to a circle is perpendicular to the radius at the point of tangency.

[C, CN, PS, R, T, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

M01.01 Demonstrate that

– the perpendicular from the centre of a circle to a chord bisects the chord

– the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc

– the inscribed angles subtended by the same arc are congruent

– a tangent to a circle is perpendicular to the radius at the point of tangency

M01.02 Solve a given problem involving application of one or more of the circle properties.

M01.03 Determine the measure of a given angle inscribed in a semicircle, using the circle properties.

M01.04 Explain the relationship among the centre of a circle, a chord, and the perpendicular bisector of the chord.

Scope and Sequence

Mathematics 8	Mathematics 9	Mathematics 10
M01 Students will be expected to develop and apply the Pythagorean theorem to solve problems.	**M01** Students will be expected to solve problems and justify the solution strategy, using the following circle properties: ▪ The perpendicular from the centre of a circle to a chord bisects the chord. ▪ The measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc. ▪ The inscribed angles subtended by the same arc are congruent. ▪ A tangent to a circle is perpendicular to the radius at the point of tangency.	–

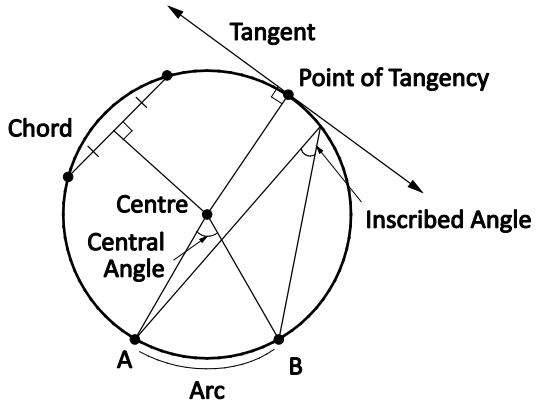
Background

Students have explored circles in Mathematics 7 in the form of radius, diameter, circumference, pi, and area.

They have developed formulas for these topics through exploration. Students are also familiar with constructing circles and central angles. While problem solving in this outcome, the Pythagorean theorem developed in Mathematics 8 will be used, and should be reviewed in context.

In Mathematics 9, students will need to develop an understanding of terms relating to circle properties. This outcome develops properties of circles and will introduce students to new terminology. Each property should be developed through a geometric exploration, which brings out the new terminology and then applies it to real life situations. Terminology includes:

- A **circle** is a set of points in a plane that are all the same distance (equidistant) from a fixed point called the centre. A circle is named for its centre.
- A **chord** is a line segment joining any two points on the circle.
- A **central angle** is an angle formed by two radii of a circle.
- An **inscribed angle** is an angle formed by two chords that share a common endpoint; that is, an angle formed by joining three points on the circle.
- An **arc** is a portion of the circumference of the circle.
- A **tangent** is a line that touches the circle at exactly one point, which is called the **point of tangency**.



Students will be exploring circle properties around chords, inscribed and central angle relationships, and tangents to circles. The treatment of these circle topics is not intended to be exhaustive, but will be determined to a significant extent by the contexts examined.

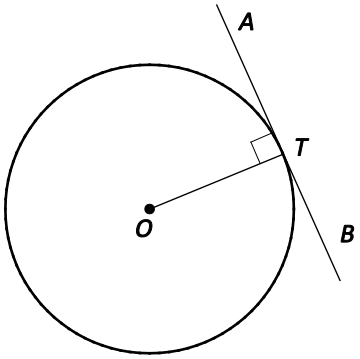
As students use circle properties to determine angle measures, it will be necessary to apply previously learned concepts. A circle may contain an isosceles triangle, for example, whose legs are radii of the circle. Students must recognize that the angle opposite the congruent sides of the isosceles triangle have equal measures. This was introduced in Mathematics 6.

Another commonly used property is that the sum of interior angles in a triangle is 180° (Mathematics 6).

The properties of a circle can be introduced in any order. By starting with the property “A tangent to a circle is perpendicular to the radius at the point of tangency,” students are introduced to only one new term. This provides the opportunity for contextual problem solving before any other properties are

developed. All properties should be developed in this manner so that students make connections with real-life situations.

- In the following diagram:
  - O is the center of the circle
  - OT is the radius
  - T is a point of tangency
  - AB is a tangent line
  - The tangent-radius property states that under the given conditions $\angle$ ATO = 90°.



Paper folding provides a good means of exploring some of the properties of circles in this outcome, such as locating the centre of a circle, determining that an inscribed angle on the diameter is a right angle, and that the perpendicular of a chord in a circle passes through the centre. (Patty paper is useful in paper folding activities.)

Locating the centre using diameters:

- Draw a large circle on a piece of paper.
- Fold the circle to form a diameter and mark endpoints A and B.
- Fold the circle again using a different mirror line mark the end points C and D.
- The point of intersection of these two diameters is the centre of the circle.

An inscribed angle on the diameter is a right angle:

- Draw a large circle on a piece of paper.
- Fold the circle to form a diameter and mark endpoints A and B.
- Mark a point C on the circumference. Fold to form chord AC.
- Fold to form chord BC.
- Measure angle C. What do you notice?

The perpendicular of a chord pass through the centre:

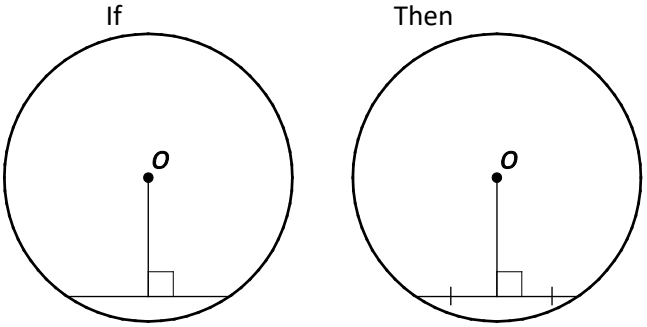
- Draw a large circle on a piece of paper.
- Draw two chords on the circle that are not parallel.
- Use folding to find the perpendicular bisector of each chord.
- The point of intersection of the two perpendicular bisectors is the centre of the circle.

Students should come to realize that if any two of the following three conditions are in place, then the third condition is true for a given line and a given chord in a circle:

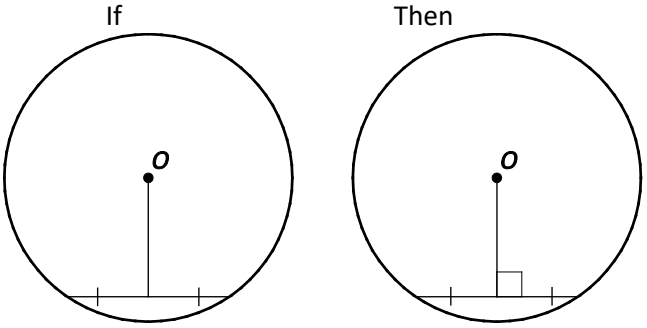
- the line bisects the chord
- the line passes through the centre of the circle
- the line is perpendicular to the chord

Illustrate the properties of a circle using the following diagrams:

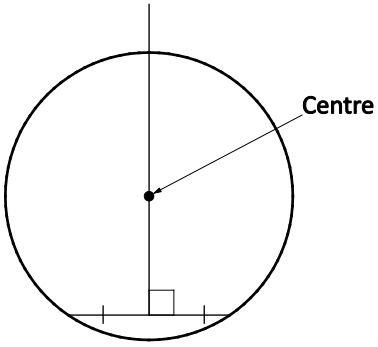
- Property 1: A line from the centre of the circle that is perpendicular to a chord will bisect the chord.



- Property 2: A line from the centre that bisects a chord is perpendicular to the chord.



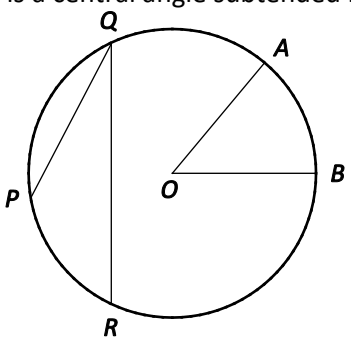
- Property 3: If a line is a perpendicular bisector of a chord, then the line passes through the centre of the circle.



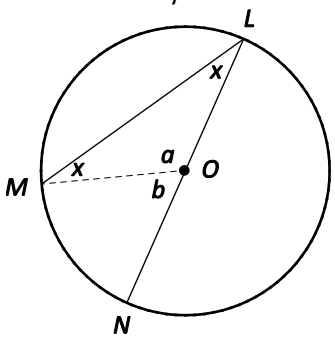
Students should also discover relationships between central and inscribed angles. Circle geometry is very visual, and students should be encouraged to draw diagrams. Some students may have difficulty identifying the arc that subtends an inscribed or central angle. They may benefit from using different colours to outline and label different lines that make angles. Reinforce the idea that an angle subtended by an arc is an angle that has common endpoints with the arc.

$\angle$ PQR is an inscribed angle subtended by arc PR

$\angle$ AOB is a central angle subtended by arc AB



Students should discover the relationship between inscribed and central angles that are subtended on the same arc. One way to demonstrate the relationship is indicated below.



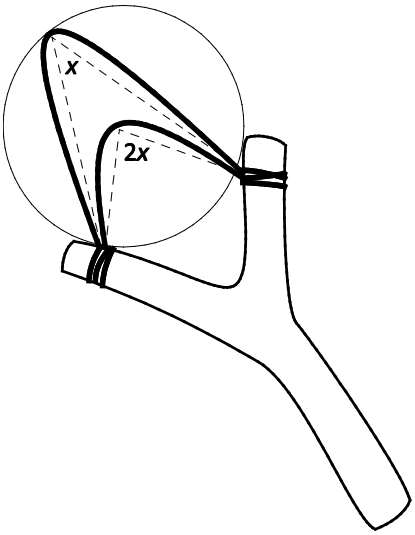
Notice $a + 2x = 180°$

Also, $a + b = 180°$

Therefore $b = 2x$

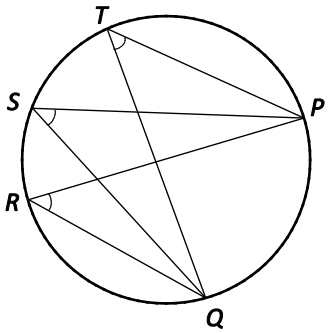
Since *b* represents a central angle and *x* represents an inscribed angle, students should conclude that inscribed angles are equal to half the measure of the central angle subtended by the same arc.

A common error occurs when students double the measure of the central angle to determine the inscribed angle. The use of diagrams is a good visual tool to show the impossibility of an inscribed angle being larger than a central angle subtending the same arc. Students could think about the act of drawing back a slingshot and measuring the angle that is formed by the elastic. The further the slingshot is pulled back the more acute (smaller) the angle becomes. This mental exercise will reinforce the notion that the inscribed angle is smaller than the central angle subtended on the same arc.



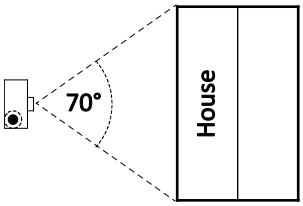
Alternatively, if students understand that the diameter is a central angle measuring 180°, they should conclude that an inscribed angle is half of the central angle subtended by the same arc and, since the central angle is 180°, the inscribed angle must be 90°.

Students should also have an opportunity to discover that inscribed angles subtended by the same arc are equal.

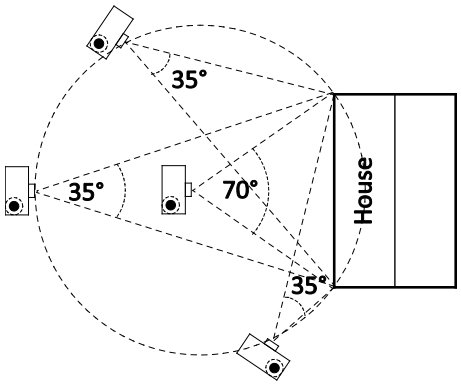


Work through the following example with students to help them develop the relationship between angles in a circle:

- Jackie works for a realtor photographing houses that are for sale. She photographed a house two months ago using a camera lens that has a 70° field of view. She has returned to the house to update the photo, but she has forgotten her lens. Today she only has a telephoto lens with a 35° field of view. From what location(s) could Jackie photograph the house with the telephoto lens, so that the entire house still fills the width of the picture? Explain your choices.



A possible solution is shown here. This also illustrates that inscribed angles subtended by the same arc are congruent.



Once all properties have been developed, students can solve problems involving a combination of properties. The use of technology is encouraged. Dynamic geometry software packages can help students explore the relationships.

Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with bull's eye compasses. Have them explore drawing circles. Remind them to label the length of the radius and the diameter of each circle.

- A student performed the following steps using the Pythagorean Theorem. **Circle** the step where the student made an error and write the corrected solution (including all steps) to the right of the student's work. For example: If ***a* = 4** and ***b* = 6**

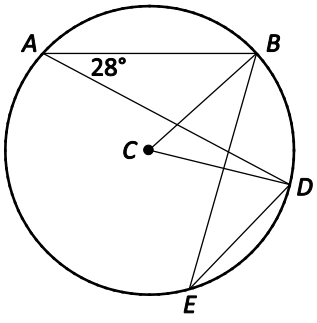
$$4^2 + 6^2 = c^2$$
$$8 + 12 = c^2$$
$$20 = c^2$$
$$\sqrt{20} = \sqrt{c^2}$$
$$4.47 = c$$

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

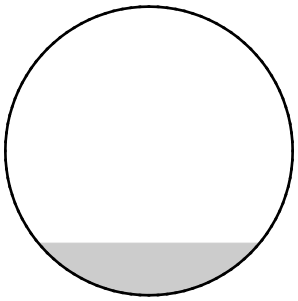
Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- You have just purchased a new umbrella to put in the centre of your wooden circular picnic table. You want to place the umbrella in the centre of the table, but the hole is not cut. Explain how you would figure out where to cut the hole for your new umbrella.

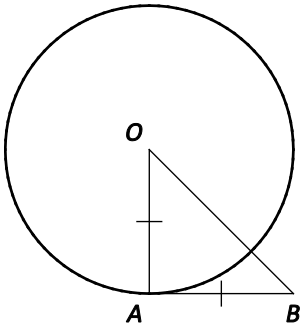
- Find $\angle$ BCD and $\angle$ BED.



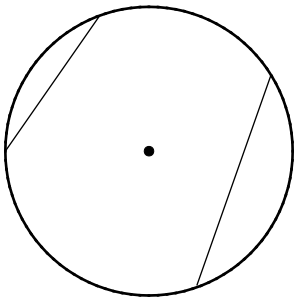
- The diagram represents the water level in a pipe. The surface of the water from one side of the pipe to the other measures 30 mm and the inner diameter of the pipe 44 mm. What is the depth of the water?



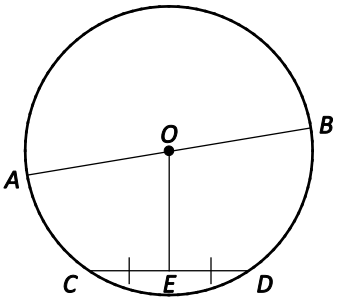
- If OA = BA, and BA is tangent to the circle at A, determine the measure of $\angle$ ABO.



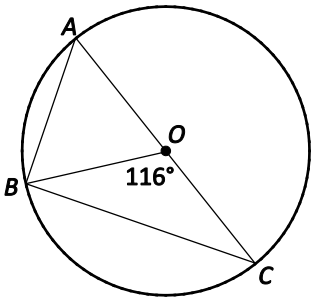
- Mike has a rock tied to the end of a 5 m rope and is swinging it over his head to form a circle with him at the center. The rock comes free of the rope and flies along a tangent from the circle until it hits the side of a building that is 14 m away from Mike. How far along the tangent did the rock travel? Determine the answer to the nearest meter.
- Ask students to explain how they could locate the center of a circle if they were given any two chords in the circle that are not parallel.



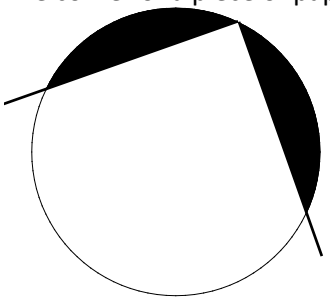
- In the following circle with center O, the diameter is 40 cm, and chord CD is 34 cm. What is the length of *OE*?



- In the following circle with centre O, $\angle$ BOC = 116°. What is the measure, in degrees, of $\angle$ ABO and $\angle$ BCO?

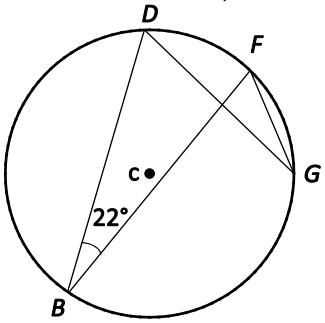


- The corner of a piece of paper is a 90° angle and is placed on a circle as shown.

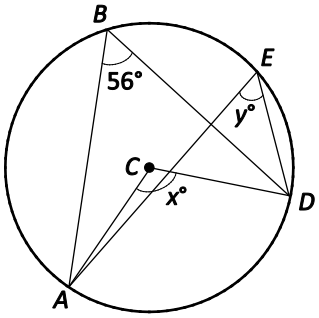


  - Why is *AB* the diameter?
  - How can the corner of the paper be used to find the centre of the circle?

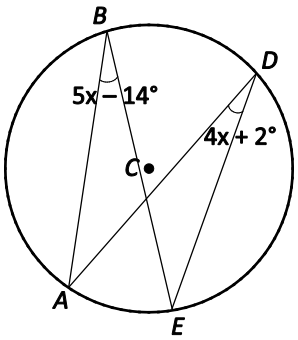
- In the circle below, what is the measure of $\angle$ DGF?



- What are the measures of *x* and *y*?



- What is the value of *x*? the measure of $\angle$ ABE?



- A city is building a pedestrian tunnel under a street using a large culvert. The culvert has a diameter of 5 meters. The city is going to fill the bottom of the culvert with concrete to create a surface for walking. Regulations state that there must be 4.2 m of space between the top of the culvert and the walking surface.
  - How deep must the city pour the concrete in the bottom of the culvert?
  - How wide will the walking surface be when it is completed?

Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

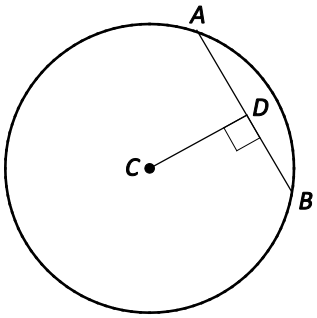
Consider the following strategies when planning daily lessons.

- Provide students with a handout of circles with labelled centres to explore circle properties.
  - Ask students to draw two non-parallel chords in the same circle. Using the triangle from a geometry set have them draw a line perpendicular to each chord passing through the centre, and then measure each part of the divided chords. This exploration should lead to establishing that the perpendicular bisector of a chord will pass through the centre of the circle and conversely that the line from the centre of the circle that meets the chord at a right angle, will also bisect the chord.
  - Provide opportunities for students to draw and measure central and inscribed angles subtended by the same arc and draw conclusions from their answers.
  - Ask students to place a point outside of one of the circles and ask them to draw the two possible tangents to the circle. From the point where each tangent touches the circle (point of tangency), ask students to draw a line to the centre of the circle. Students should then measure the angle formed by the tangent and the radius. What do the students notice about these measurements?
  - Ask students to draw a diameter on one of the circles. They should then draw and measure an inscribed angle subtended by the semi-circle.

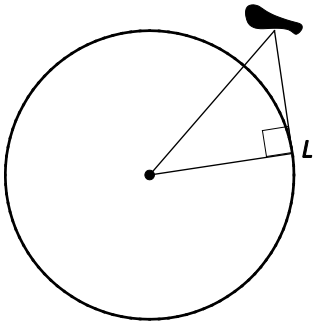
SUGGESTED LEARNING TASKS

- Challenge students with the following problem: A surveillance camera is taping people coming through the entrance of the school. While reviewing the tape, school administrators realized that the camera was broken. When shopping for a new one, the cameras available have a field of view of 40° compared to the broken one that had a field of view of 80°. Where should they position the new camera to cover the same area?
- Provide students with an arc and ask them to find the radius of the circle from which the arc was taken (could be extended to a variety of arcs).

- Have students respond to the following problems:
  - The radius of the circle to the right measures 6 cm. If the distance between the centre and the chord (CD) is 4 cm, what is the length of the chord AB?



  - The radius of the earth is 6400 km. If a bird is 1500 m from the ground, how far is it from Leslie standing at point L?



- Ask students to complete the following paper folding activity to develop the relationship between the perpendicular from the centre of the circle and a chord.
  - Construct a large circle on tracing paper and draw two different chords.
  - Construct the perpendicular bisector of each chord.
  - Label the point inside the circle where the two perpendicular bisectors intersect.
  - What do you notice about the point of intersection of the two perpendicular bisectors?

SUGGESTED MODELS AND MANIPULATIVES

- circular objects to trace circles
- string
- tracing (patty) paper
- bull's-eye compass
- The Geometer's Sketchpad
- circle template (See Digital Resources)

MATHEMATICAL LANGUAGE

Teacher	Student
▪ arc ▪ area ▪ bisect ▪ centre ▪ central angle ▪ chord ▪ circle ▪ circumference ▪ congruent ▪ diameter ▪ equidistant ▪ inscribed angle ▪ line segment ▪ perpendicular ▪ perpendicular bisector ▪ pi ▪ point of tangency ▪ radii ▪ radius ▪ subtend ▪ subtended ▪ tangent	▪ arc ▪ area ▪ bisect ▪ centre ▪ central angle ▪ chord ▪ circle ▪ circumference ▪ congruent ▪ diameter ▪ equidistant ▪ inscribed angle ▪ line segment ▪ perpendicular ▪ perpendicular bisector ▪ pi ▪ point of tangency ▪ radii ▪ radius ▪ subtend ▪ subtended ▪ tangent

Resources

Digital

- "Circle Template," *Illuminations: Resources for Teaching Math* (National Council of Teachers of Mathematics 2015) http://illuminations.nctm.org/uploadedFiles/Content/Lessons/Resources/9-12/FoldingCircle-CircleTemplate.pdf
- "Exploring Circle Geometry Properties: Use It," *Math Interactives* (Alberta Education, LearnAlberta.ca 2015): www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.SHAP&ID2=AB.MATH.JR.SHAP.CIRC&lesson=html/object_interactives/circles/use_it.html
- "Folding Circles: Exploring Circle Theorems through Paper Folding," *Illuminations: Resources for Teaching Math* (National Council of Teachers of Mathematics 2015): http://illuminations.nctm.org/Lesson.aspx?id=3777
- "Point-Circle," LearnAlberta.ca (Alberta Education, LearnAlberta.ca 2015): www.learnalberta.ca/content/meda/html/pointcircle/index.html

Print

- *Math Makes Sense 9* (Baron et al. 2009; NSSBB #: 2001644)
  - Unit 8: Circle Geometry

    - > Section 8.1: Properties of Tangents to a Circle
    - > Section 8.2: Properties of Chords in a Circle
    - > Technology: Verifying the Tangent and Chord Properties
    - > Game: Seven Counters
    - > Section 8.3: Properties of Angles in a Circle
    - > Technology: Verifying the Angle Properties
    - > Unit Problem: Circle Designs
  - – *ProGuide* (CD; Word Files; NSSBB #: 2001645)
    - > Assessment Masters
    - > Extra Practice Masters
    - > Unit Tests
  - – *ProGuide* (DVD; NSSBB #: 2001645)
    - > Projectable Student Book Pages
    - > Modifiable Line Masters
- *Patty Paper Geometry* (Serra 2011) , pp. 103–119
- *Developing Thinking in Geometry* (Johnston-Wilder and Mason 2006), pp. 41–45
- *Geometry: Seeing, Doing, Understanding,* Third Edition (Jacobs 2003), pp. 484–485, 491–492, 497–499, 504–505

Geometry (G)

GCO: Students will be expected to describe the characteristics of 3-D objects and 2-D shapes and analyze the relationships among them.

GCO: Students will be expected to describe and analyze position and motion of objects and shapes.

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

GUIDING QUESTIONS

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Follow-up on Assessment

The evidence of learning that is gathered from student participation and work should be used to inform instruction.

GUIDING QUESTIONS

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan available at Mathematics Learning Commons: Grades 7–9: http://nsvs.ednet.ns.ca/nsps/nsps26/course/view.php?id=3875.

GUIDING QUESTIONS

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SCO G01 Students will be expected to determine the surface area of composite 3-D objects to solve problems. [C, CN, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

G01.01 Determine the area of overlap in a given composite 3-D object, and explain the effect on determining the surface area (limited to right cylinders, right rectangular prisms, and right triangular prisms).

G01.02 Determine the surface area of a given composite 3-D object (limited to right cylinders, right rectangular prisms, and right triangular prisms).

G01.03 Solve a given problem involving surface area.

Scope and Sequence

Mathematics 8	**Mathematics 9**	**Mathematics 10**
M03 Students will be expected to determine the surface area of right rectangular prisms, right triangular prisms, and right cylinders to solve problems. **G01** Students will be expected to draw and interpret top, front, and side views of 3-D objects composed of right rectangular prisms.	**G01** Students will be expected to determine the surface area of composite 3-D objects to solve problems.	**M01** Students will be expected to solve problems that involve linear measurement, using SI and imperial units of measure, estimation strategies, and measurement strategies. **M03** Students will be expected to solve problems, using SI and imperial units, that involve the surface area and volume of 3-D objects, including right cones, right cylinders, right prisms, right pyramids, and spheres.

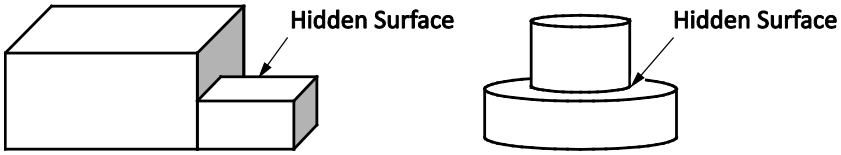
Background

In Mathematics 8, students have had experience calculating surface areas of right rectangular prisms, right triangular prisms, and right cylinders. The focus was on gaining a conceptual understanding of surface area through the use of nets rather than on the use of formulae. In Mathematics 9, this will be extended to composite objects.

To calculate the surface area of a 3-D object, students should be provided with nets of various 3-D objects, or should be given polydron pieces to build right rectangular and triangular prisms, and take them apart to create nets. With some experience they will no longer need the net, but will learn to visualize it, and the relationship between a 2-D net and its 3-D object.

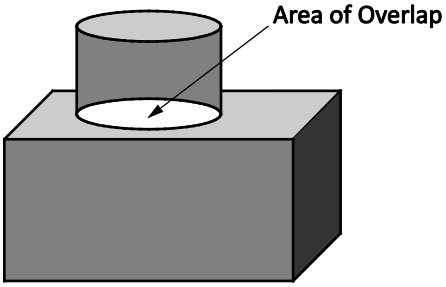
When solids are combined into composite 3-D objects, students can use concrete items to determine what surface is hidden. Students should explore how the surface area changes when right cylinders, right rectangular prisms, and right triangular prisms are arranged collectively, causing some of the faces to be covered as a result of contact with other objects. When objects are combined, there will be an area of overlap.

Students should consider how such a shape is made from its component parts, determine the surface area of each part, and remove the area of overlapping both surfaces. Alternatively, they could determine the area of each exposed surface and add to find the total area.



In general, when determining the surface area of composite geometric figures, all sides, except overlap, are included. In contextual situations, however, sides other than the overlap may also need to be excluded.

The composite figure below is composed of a right cylinder and a right rectangular prism.



Ask students to identify the area of overlap. They should conclude the area of overlap is circular. Students can determine the surface area of the composite figure by calculating:

$\text{Surface Area}_{\text{prism}} + \text{Surface Area}_{\text{cylinder}} - 2\text{Area}_{\text{circle}}$

Ask students to now consider the object as the base for a patio post that sits on the ground. Ask them if they would paint the bottom of the rectangular prism. They should reason that when painting this object the surface touching the ground would not be painted and therefore would not be included in the surface area calculation.

Discuss with students other examples where it is important to keep the context in mind. To determine how much paint is needed to paint a flat-bottom dresser, for example, the area of the bottom would be omitted because it would not be painted. Similarly, when icing a cake, the bottom of the cake would not be covered with icing.

Assessment, Teaching, and Learning

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

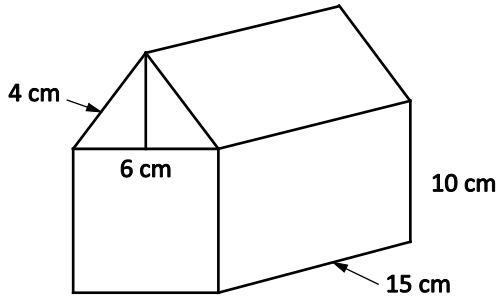
- Tell students that Mandy has 1 m^2 of paper to wrap a gift box 27 cm long, 25 cm wide, and 14 cm high. Does she have enough paper?

- Have students sketch the net and calculate the surface area of a right cylinder that has a diameter of 3.3 m and is 4.7 m long.

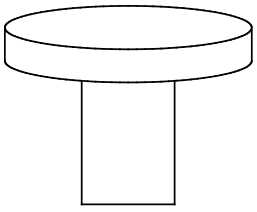
WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

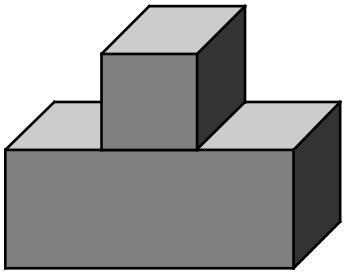
- In the scale model of a steel barn has below 1 cm represents 0.5 m. Calculate the amount of steel needed to cover the actual barn.



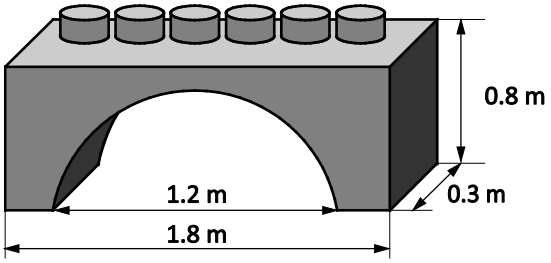
- A table has been built out of two cylinders. The diameter of the large cylinder is 40 cm and the diameter of the small cylinder is 15cm. The height of the larger cylinder is 10cm and the height of the small cylinder is 50cm. Determine the surface area of the table. Do not include the overlap.



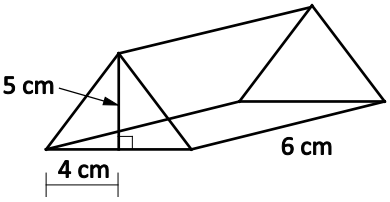
- The surface area of this composite figure was calculated incorrectly to be 582 cm^2. The figure on the top is a cube with sides of 5 cm. The large prism on the bottom has a length of 12 cm, a width of 6 cm, and a height of 8 cm. Determine the error in the calculation and give the correct solution.



- A set for a school theatre production uses giant multi linking cubes. Each cylindrical connector is 0.20 m in diameter and 0.15 m high. Ask students to calculate the total area that must be painted.



- Calculate the surface area of the composite figure:



Planning for Instruction

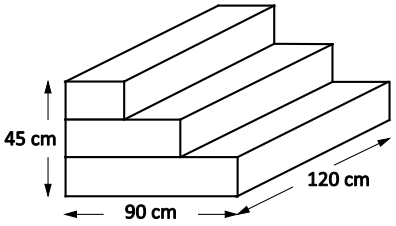
CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

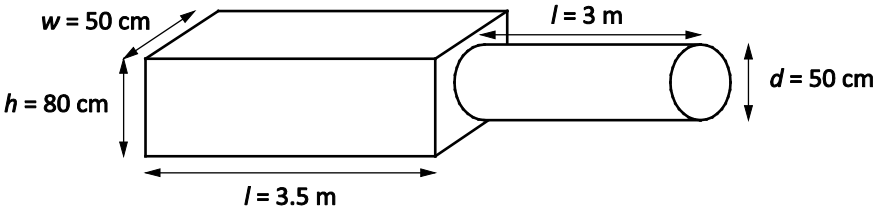
- Begin by having students sum surface areas that have already been calculated, to establish an understanding of the meaning of surface area before focusing on calculations.
- Use concrete materials to promote the understanding of which surfaces overlap and which are exposed when objects are combined into composite 3-D objects.
- Explore objects that may not appear to be composite. Students may require assistance in separating these objects into familiar pieces.
- Encourage the use of estimation skills in calculations and conversion of units.
- Have students brainstorm situations that necessitate determining surface area as well as ones that require adding or removing some area calculation such as icing a wedding cake, building a garage over a permanent floor, wrapping a birthday gift or package to mail.
- Have students find the error(s) in a surface area calculation.
- As an extension, have students find one dimension, given the other dimensions, as well as the surface area.

SUGGESTED LEARNING TASKS

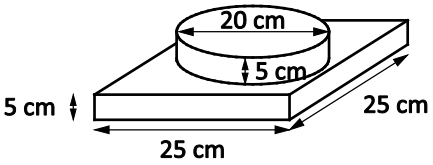
- Distribute sets of geometric solids to groups of students. Ask students to create composite shapes from the solids. Ask them to determine the surface area of the composite figure and explain how they found the surface area. An extension to this may involve giving students the amount of wrapping paper equal to their calculated surface area. Ask students to wrap their object with the paper to see if their calculation was accurate. Note: If sets of geometric solids are not available some advance planning might be necessary. Teachers could ask students to bring in boxes, cans, tins, paper towel rolls, etc., from home for this activity.
- Build two different objects using 12 multi linking cubes. Determine the surface area of each object. How can symmetry help determine the surface area more efficiently? Slide the two objects together and determine which surfaces overlap. How does the overlap affect the total surface area of the new composite figure?
- Estimated surface area coverage is provided on paint cans. Determine the surface area of a set of concrete steps to determine how much paint will be needed to cover them. Assume all steps are the same depth and height.



- A play structure has been designed by combining a prism with a cylinder. How much fabric would be required to cover the entire surface?



- Ask students to calculate the total surface area of a composite figure made up of 42 centimeter cubes with 12 overlaps.
- Todd makes a two-layer cake. He puts strawberry jam between the layers instead of icing. He plans to cover the outside of the cake with icing. Ask students to describe how he can calculate the area that needs icing.



SUGGESTED MODELS AND MANIPULATIVES

- multi linking cubes
- geometric solids
- grid paper

- boxes and tins of various shapes and sizes
- paper towel rolls
- Polydron pieces

MATHEMATICAL LANGUAGE

Teacher	Student
▪ 2-D shape ▪ 3-D object ▪ area ▪ base ▪ composite object ▪ face ▪ overlap ▪ right cylinder ▪ right rectangular prism ▪ right triangular prism	▪ 2-D shape ▪ 3-D object ▪ area ▪ base ▪ composite object ▪ face ▪ overlap ▪ right cylinder ▪ right rectangular prism ▪ right triangular prism

Resources

Digital

- "Interactives: Geometry 3D Shapes," *Annenberg Learner* (Annenberg Foundation 2014): www.learner.org/interactives/geometry/index.html

Print

- *Developing Thinking in Geometry* (Johnston-Wilder and Mason 2006), pp. 98–99,
- *Geometry: Seeing, Doing, Understanding,* Third Edition (Jacobs 2003), pp. 634–635, 654–655
- *Math Makes Sense 9* (Baron et al. 2009; NSSBB #: 2001644)
  - Unit 1: Square Roots and Surface Area
    - > Section 1.3: Surface Area of Objects Made from Right Rectangular Prisms
    - > Section 1.4: Surface Area of Other Composite Objects
    - > Unit Problem: Design of a Play Structure
  - *ProGuide* (CD; Word Files; NSSBB #: 2001645)
    - > Assessment Masters
    - > Extra Practice Masters
    - > Unit Tests
  - *ProGuide* (DVD; NSSBB #: 2001645)
    - > Projectable Student Book Pages
    - > Modifiable Line Masters

SCO G02 Students will be expected to demonstrate an understanding of similarity of polygons. [C, CN, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

G02.01 Determine if the polygons in a given presorted set are similar, and explain the reasoning.
G02.02 Model and draw a polygon similar to a given polygon, and explain why the two are similar.
G02.03 Solve a given problem using the properties of similar polygons.

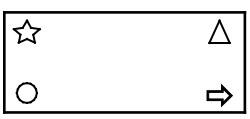
Scope and Sequence

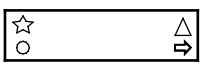
Mathematics 8	**Mathematics 9**	**Mathematics 10**
–	**G02** Students will be expected to demonstrate an understanding of similarity of polygons.	**M04** Students will be expected to develop and apply the primary trigonometric ratios (sine, cosine, tangent) to solve problems that involve right triangles.

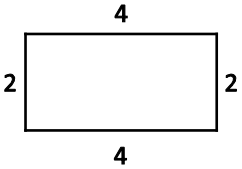
Background

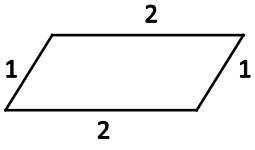
Students are introduced to the concept of similarity in Mathematics 9. Through investigation, they should recognize that polygons that have corresponding angles that are congruent and corresponding sides that are proportional are similar.

When shapes are similar, corresponding angles are congruent. This alone, however, is not enough to determine similarity. For example, a square and a long, thin rectangle both have four right angles, but they are not similar shapes. To determine similarity, side lengths must also be considered. The ratio of corresponding side lengths will be the same if the polygons are similar. To determine similarity, students can compare the side lengths to determine if they are proportional. This proportional relationship is called scale factor. One could use the following examples to demonstrate the importance of satisfying both conditions for polygons. Below, the two rectangles on the left, have congruent corresponding angles without being similar. Similarly, the rectangle and the parallelogram on the right, have proportional corresponding sides without being similar. Therefore both conditions must exist in order for triangles to be similar.









When first constructing similar shapes, students can draw a shape on a square grid, and then copy the shape onto another grid with larger or smaller squares. Rulers and protractors can also be used to draw

similar polygons. The use of technology (e.g., overhead projectors, printers, photocopiers, design software) can enhance the study of similarity. Students can use dynamic geometry software to draw a shape and then reduce or enlarge it.

Students should be given a wide variety of problem-solving situations that involve similarity. Encourage students to use a variety of strategies, such as observation, measurement, proportional reasoning, and scale factor, to determine unknown measures. Accuracy of measurement, proper labelling, and correct notation are important.

When using the properties of similar figures to find unknown measures, emphasize the relationship between similarity and scale factor.

The connection between proportional reasoning and similarity is very important. Similar figures provide a visual representation of proportions, and proportional thinking enhances the understanding of similarity. It is through using ratios of sides within triangles that the trigonometric ratios will be developed in Mathematics 10.

Students should be exposed to a variety of situations, including pairs of similar figures that vary in orientation. The properties of similar polygons can be used to find the measures of missing sides and angles. This topic lends itself well to real-life situations, such as estimating heights of buildings or distances that are normally difficult to measure directly, such as estimating the distance across a pond.

This outcome should be done in conjunction with Outcome G03.

Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

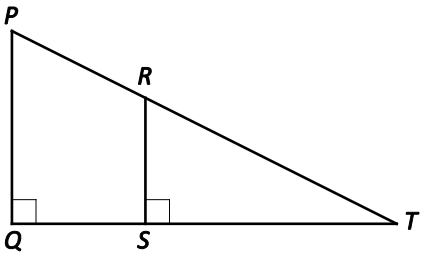
Tasks such as the following could be used to determine students' prior knowledge.

- Find the missing number in each proportion.
  - 1:3 = 5:__
  - 2:__ = 3:7.5
  - $\frac{3}{\square} = \frac{9}{24}$
  - $\frac{64}{12} = \frac{\square}{9}$
  -

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- One triangle has two 50° angles. Another triangle has a 50° angle and an 80° angle. Are the triangles similar? Explain your thinking.
- A software program offers these preset paper sizes for printing:
  - A4 (210 mm by 297 mm)
  - A5 (148 mm by 210 mm)
  - B5 (182 mm by 257 mm)
  - Use scale factors to determine if the paper sizes are similar.
- Answer the questions that follow based on the diagram given:

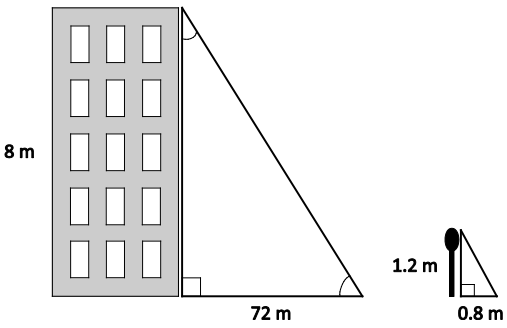


  - Which triangles are similar?
  - Measure the sides and determine the ratios of:

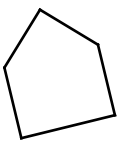
$$\frac{PQ}{RS}=\frac{QT}{ST}\ \frac{PQ}{PT}=\frac{RS}{RT}\ \frac{QT}{PT}=\frac{ST}{RT}$$

What do you notice about the values?

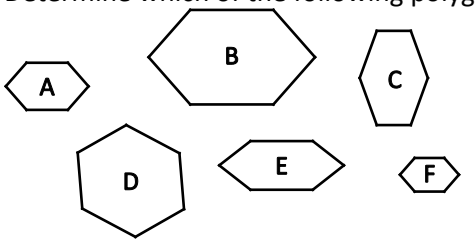
- A building casts a shadow 72 metres long. At the same time, a parking meter that is 1.2 metres tall casts a shadow that is 0.8 metres long. Ask students to determine the height of the building.



- Sketch a polygon that is similar to the given polygon below. Explain the criteria you have used to show that they are similar.



- Determine which of the following polygons are similar. Justify your answer.



- A photograph measuring 12.5 cm by 17.5 cm needs to be enlarged by a factor of one and a half. What will be the new dimensions of the photograph? Draw a diagram of both photographs to support your reasoning.

Planning for Instruction

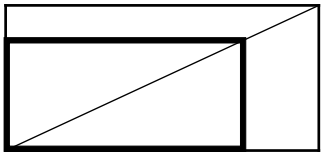
CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Support students' understanding of comparison of corresponding angles with concrete materials.
- Promote reasoning strategies by having students prove that similar polygons have equal corresponding angles and proportional sides. Polygons of differing levels of difficulty can be presented to extend student's thinking.
- Facilitate constructions with technology by using the tools for enlarging and reducing figures.
- Emphasize the use of proportional thinking for problems involving similar polygons.

SUGGESTED LEARNING TASKS

- Test rectangles for similarity by placing two rectangles on top of each other with the smaller one fitting into a corner of the larger one. If the diagonal of the larger rectangle is also the diagonal of the smaller rectangle, the rectangles are similar. Note: This test is only approximate and is only as precise as our measurement. This could be an opening for a discussion about accuracy in measurement.



- Provide students with a set of polygons (e.g., all 4 sided polygons or all triangles). Have students sort them by identifying those that are similar. Similar polygons could then be traced and colour coded. Have students provide a justification for why the polygons are similar.
- Cut out of card stock, various regular and irregular polygons that include one or more matching similar polygons. Place in a container and have students draw one or more shape, and then find the student or student with the matching similar polygon or polygons.

SUGGESTED MODELS AND MANIPULATIVES

- grid paper
- Power Polygons
- rulers
- protractor
- geo-strips

MATHEMATICAL LANGUAGE

Teacher	Student
▪ corresponding angles ▪ corresponding sides ▪ enlarge/enlargement ▪ orientation ▪ proportional ▪ ratio ▪ reduce/reduction ▪ scale factor	▪ corresponding angles ▪ corresponding sides ▪ enlarge/enlargement ▪ orientation ▪ proportional ▪ ratio ▪ reduce/reduction ▪ scale factor

Resources

Print

- *Developing Thinking in Geometry* (Johnston-Wilder and Mason 2006), pp. 28–32
- *Elementary and Middle School Mathematics: Teaching Developmentally*, 8th Edition (Van de Walle, Karp, and Bay-Williams 2013), pp. 361–363, 404
- *Geometry: Seeing, Doing, Understanding,* Third Edition (Jacobs 2003), pp. 385–387
- *Making Math Meaningful to Canadian Students, K–8* (Small 2008), pp. 319–321
- *Math Makes Sense 9* (Baron et al. 2009; NSSBB #: 2001644)
  - Unit 7: Similarity and Transformations
    - > Section 7.3: Similar Polygons
    - > Section 7.4: Similar Triangles
  - *ProGuide* (CD; Word Files; NSSBB #: 2001645)
    - > Assessment Masters
    - > Extra Practice Masters
    - > Unit Tests
  - *ProGuide* (DVD; NSSBB #: 2001645)
    - > Projectable Student Book Pages
    - > Modifiable Line Masters

SCO G03 Students will be expected to draw and interpret scale diagrams of 2-D shapes. [CN, R, T, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

G03.01 Identify an example of a scale diagram in print and electronic media.
G03.02 Draw a diagram to scale that represents an enlargement or a reduction of a given 2-D shape.
G03.03 Determine the scale factor for a given diagram drawn to scale.
G03.04 Determine if a given diagram is proportional to the original 2-D shape, and if it is, state the scale factor.
G03.05 Solve a given problem that involves the properties of similar triangles.

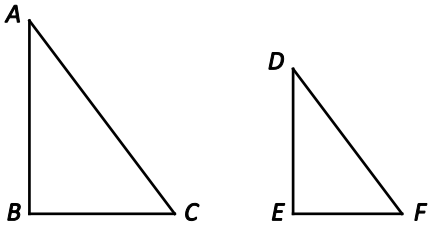
Scope and Sequence

Mathematics 8	**Mathematics 9**	**Mathematics 10**
–	**G03** Students will be expected to draw and interpret scale diagrams of 2-D shapes.	**M03** Students will be expected to solve problems, using SI and imperial units, that involve the surface area and volume of 3-D objects, including right cones, right cylinders, right prisms, right pyramids, and spheres.

Background

Students were introduced to angles and determining angle measures in Mathematics 6. Investigations will involve measuring segments and angles, where accuracy of measurement is important.

A scale factor is a comparison between the actual size of an object and the size of its image, therefore a scale diagram is a drawing that is similar to the actual figure. Scale diagrams can be either an enlargement or a reduction of the actual object, depending on the context. If the scale factor is bigger than one, this will result in an enlarged image, whereas if the scale factor is less than one, the result is a reduced image. Students have an intuitive sense of shapes that are enlargements or reductions of each other. Students have experienced maps and pictures that have been drawn to scale, as well as images produced by photocopiers and computer software.



Students should understand the relationships between the corresponding sides of similar triangles. That is, if $\triangle ABC$ is similar to $\triangle DEF$, then the following ratios are equivalent: $\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$.

Students will be responsible for the correct use of language, symbols, and conventions. There should be opportunities to use dynamic geometry software.

Students should be provided with an opportunity to explore real world examples of scale diagrams. Through these investigations, students should come to understand the concept of a scale factor, be able to determine scale factor, and use it to create enlargements and reductions. Enlargements and reductions are also known as dilatations.

Students should also be aware of the effect of the magnitude of a scale factor (i.e., What happens when a scale factor is greater than 1? less than 1?). A common student error is interchanging the numerator and denominator while calculating the scale factor. Understanding that for an enlargement the scale factor is greater than 1, and for a reduction the scale factor is less than 1, should help students avoid making that mistake. A ratio of 2:1 means the new figure is an enlargement to twice the size of the original. Likewise, a ratio of 1: 3 means that the new figure is a reduction to $\frac{1}{3}$ of the original, or the original is three times the size of the new figure. Have students work with scale factors as fractions, decimals, and percents.

It would be useful to study the scales used in town and provincial maps. Students could be asked to find the actual distance, using the scale, or to convert the scale provided in one form to a different form. For example:

- If the scale is given as the ratio 1: 500 000, how many kilometres does 7.5 cm represent?

  To solve this problem students might recognize that they need to multiply 500 000 by 7.5 cm and do this calculation directly, or they might set up the proportion $\frac{1\ cm}{500\,000\ km} = \frac{7.5\ cm}{x}$. They should initially determine that x = 3 750 000. Since 7.5 is in cm, then 3 750 000 is also in cm, and when converted equals 37.5 km.

  Unit analysis could be introduced here as a way to verify that units in a conversion are correct. To convert 3 750 000 cm to kilometres:

  $$3\,750\,000\ cm \times \frac{1\ km}{100\,000\ cm}$$

  $$\frac{1\ km}{100\,000\ cm} = 1$$

Students should recognize that when a numerical value is multiplied by one, the value remains the same. This is the basis of unit conversion. Students will work with unit analysis in Mathematics 10.

Reading, interpreting, and constructing scale diagrams provides a good introduction to similarity. This outcome should be done in conjunction with outcome G02.

Assessment, Teaching, and Learning

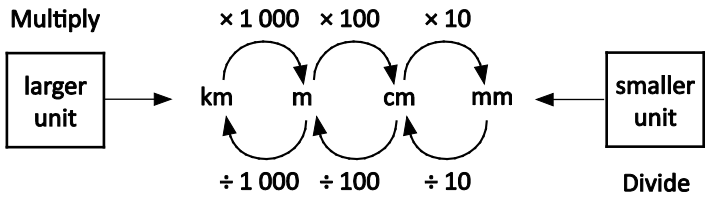
Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with the following charts to illustrate that each place value is 10 times the place value to its right. Provide students with conversion questions.

1 000	100	10	1	0.1	0.01	0.001
thousands	hundreds	tens	ones	tenths	hundredths	thousandths
kilo	hecto	deka	basic unit	deci	centi	millie

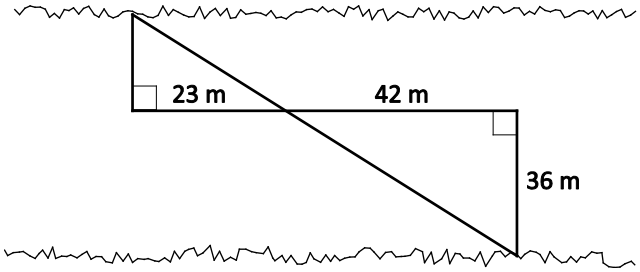


- As Yasmine's puppy grows, she can feed her more food. She feeds her puppy 1 cup of protein mixed with half a cup of formula. If she increases the protein to 1.5 cups, how much formula must she mix it with?
- Play this "Fish Simulator" applet (Drexel University 2015) to explore ratios: http://mathforum.org/escotpow/puzzles/fish/applet.html

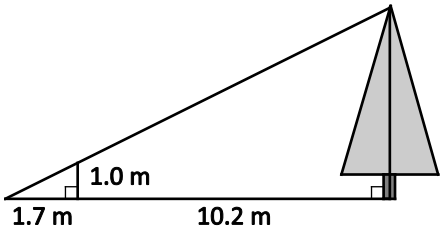
WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

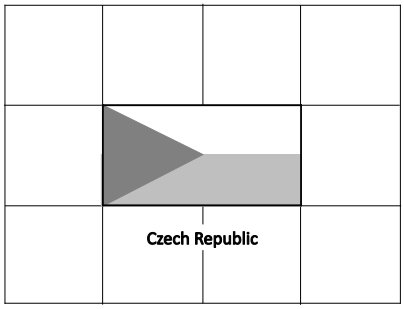
- Are the two triangles in the diagram below similar? Justify. If enough information is available, find the width of the river. If there is not enough information, what other information is needed?



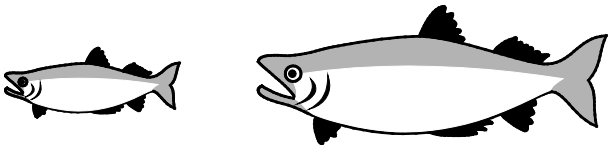
- Are the two triangles similar in the figure below? Justify. If enough information is available, find the height of the tree.



- A billboard picture is 8 m wide and 12 m long and needs to be recreated as a sign that is only 1 m wide. Explain how to determine the length of the sign.
- Draw an enlargement of the flag of the Czech Republic using a scale factor of 3.



- For the second image, is the scale factor equal to 1? greater than 1? less than 1? Explain how you know.



- Explain how you could determine if Figure B is an accurate enlargement of Figure A.

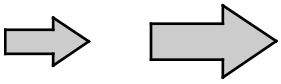


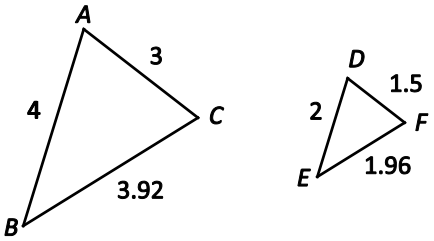
Figure A **Figure B**

Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

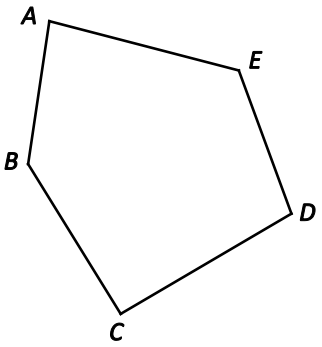
Consider the following strategies when planning daily lessons.

- Give students two triangles and have them determine if they are similar. If they are, state the scale factor.



- Provide students with pictures of original and scale diagrams and have them determine the scale factor. For example, take a picture of a group of students and then measure the height of one of them. Have students determine the scale factor, and then use it to determine the heights of the others in the picture.

- Provide students with a 2-D shape on graph paper and ask them to come up with a procedure to either reduce or enlarge the diagram. For example, give students the shape below and ask them to redraw the shape to be 3 times larger.



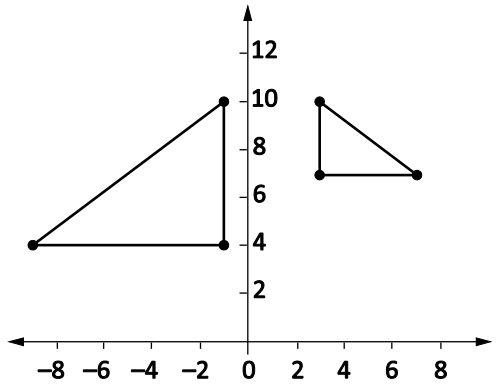
SUGGESTED LEARNING TASKS

- Use a table, such as below, to reinforce as a useful way to reinforce the use of scale factor.

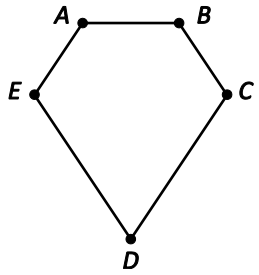
Size of Scale Diagram	**Scale Factor (scale size: original size)**	**Size of Original Diagram**
160 cm	?	40 cm
?	2: 1 000 000	150 km
3m		54 m

- Given the following list of points to form two triangles, have students determine if they are similar.
-

$\triangle ABC$ with A(3,7) B(7,7) and C(3,10)
$\triangle DEF$ with D(-1,4) E(-9,4) and C(-1,10)



- Have students create an enlargement or reduction of the following shape (various shapes could be used).



- Bring to class a scaled image of a landmark and use the scale to determine the actual size.
- Provide students with a table that requires them to record missing measurements or scale factor.

Scale Factor	Original Length	Image Length
4	6 cm	
0.6		30 cm
25%	160 m	
	18 km	6 m

- Ask students to answer questions such as the following:
  - The flying distance from Halifax to Montreal is 791 km. If this distance on a map is 5 cm, what is the scale factor?
  - Some viruses measure 0.0001 mm in diameter. An artist's diagram of a virus shows the diameter as 5 mm. What is the scale factor used?
- Provide students with a simple diagram on a grid and ask them to reduce or enlarge the diagram using a given scale factor.
- A baseball coach wants to have a diagram of a baseball diamond that is similar to a real baseball diamond. A real baseball diamond is a square with side lengths of 27.4m. Have students draw a model using a ratio of 1:500.
- Provide students with the coordinates of the vertices of an object and the coordinates of the vertices of the image. Ask them to plot the two, and then determine the type of dilatation and the scale factor.
- Students could design a logo that includes geometric shapes. Once they have created a design, they should:
  - decide on the dimensions of an enlargement of the logo that would fit on a banner or billboard

- determine the scale factor
- create a business card using the logo by repeating the process for a reduction

SUGGESTED MODELS AND MANIPULATIVES

- blueprints
- floorplans
- GeoGebra
- Geometer's sketchpad
- graph paper
- maps

MATHEMATICAL LANGUAGE

Teacher	Student
▪ dilatation ▪ enlarge/enlargement ▪ proportional ▪ ratio ▪ reduce/reduction ▪ scale factor	▪ enlarge/enlargement ▪ proportional ▪ ratio ▪ reduce/reduction ▪ scale factor

Resources

Digital

- "Fish Simulator [applet]," *The Math Forum @ Drexel* (Drexel University 2015): http://mathforum.org/escotpow/puzzles/fish/applet.html (Explore ratios)
- GeoGebra [Software] (International GeoGebra Institute 2015): www.geogebra.org/cms/en
- The Geometer's Sketchpad [Software] (Key Curriculum Press 2013; NSSBB #: 50474, 50475, 51453)

Print

- *Developing Thinking in Geometry* (Johnston-Wilder and Mason 2006), pp. 158–159
- *Elementary and Middle School Mathematics: Teaching Developmentally*, 8th Edition (Van de Walle, Karp, and Bay-Williams 2013), pp. 364–372
- *Geometry: Seeing, Doing, Understanding,* Third Edition (Jacobs 2003), pp. 414–415
- *Making Math Meaningful to Canadian Students, K–8* (Small 2008), p. 375
- *Math Makes Sense 9* (Baron et al. 2009; NSSBB #: 2001644)
  - Unit 7: Similarity and Transformations
    - > Section 7.1: Scale Diagrams and Enlargements
    - > Section 7.2: Scale Diagrams and Reductions
    - > Technology: Drawing Scale Diagrams
    - > Unit Problem: Designing a Flag
  - *ProGuide* (CD; Word Files; NSSBB #: 2001645)
    - > Assessment Masters
    - > Extra Practice Masters
    - > Unit Tests

- *ProGuide* (DVD; NSSBB #: 2001645)
  - > Projectable Student Book Pages
  - > Modifiable Line Masters

SCO G04 Students will be expected to demonstrate an understanding of line and rotation symmetry. [C, CN, PS, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

G04.01 Classify a given set of 2-D shapes or designs according to the number of lines of symmetry.
G04.02 Complete a 2-D shape or design, given one half of the shape or design and a line of symmetry.
G04.03 Determine if a given 2-D shape or design has rotation symmetry about the point at its centre, and if it does, state the order and angle of rotation.
G04.04 Rotate a given 2-D shape about a vertex, and draw the resulting image.
G04.05 Identify the type of symmetry that arises from a given transformation on a Cartesian plane.
G04.06 Complete, concretely or pictorially, a given transformation of a 2-D shape on a Cartesian plane, record the coordinates, and describe the type of symmetry that results.
G04.07 Identify and describe the types of symmetry created in a given piece of artwork.
G04.08 Determine whether or not two given 2-D shapes on a Cartesian plane are related by either rotation or line symmetry.
G04.09 Draw, on a Cartesian plane, the translation image of a given shape using a given translation rule such as R2, U3, or →→, ↑↑↑; label each vertex and its corresponding ordered pair; and describe why the translation does not result in line or rotation symmetry.
G04.10 Create or provide a piece of artwork that demonstrates line and rotation symmetry, and identify the line(s) of symmetry and the order and angle of rotation.

Scope and Sequence

Mathematics 8	**Mathematics 9**	**Mathematics 10**
G01 Students will be expected to determine the surface area of composite 3-D objects to solve problems. **G02** Students will be expected to demonstrate an understanding of the congruence of polygons under a transformation.	**G04** Students will be expected to demonstrate an understanding of line and rotation symmetry.	–

Background

Students will work with two types of symmetry in 2-D geometry: reflective and rotational. When a 2-D shape is divided along a line, and the opposite sides are mirror images of each other, the shapes have reflective, or line symmetry. It is possible a 2-D shape may have more than one line of reflective symmetry. Rotational symmetry refers to the number of times a 2-D shape coincides with the image of

itself when it is rotated a full rotation. Students will examine both line and rotation symmetry in artwork, as well as transformations on the Cartesian plane. They have had previous experience with lines of symmetry in Mathematics 4.

Students have also worked with the Cartesian plane. A 2-D shape has line symmetry if one half of the shape is a reflection of the other half. The reflection occurs across a line. The line of symmetry, or line of reflection, can be horizontal, vertical, or oblique, and may or may not be part of the diagram itself. To help explain line symmetry, students need to see examples and non-examples.

Another approach is to have students fold a sheet of paper in half and cut out a shape of their choosing. When they open the paper, the fold line will be a line of symmetry. Another way is to use transparent mirrors. If the shape is symmetrical where a Mira has been placed, the image of one half of the shape will fall right on top of the other half of the shape.

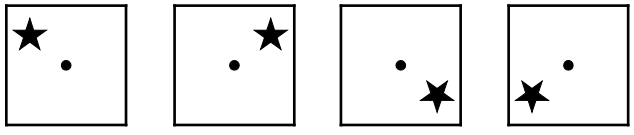
Students should be given an opportunity to investigate the number of lines of symmetry that exist in various 2-D shapes. These shapes could include polygons, letters, pictures, logos, etc. In examining shapes to be classified based on the number of lines of symmetry, it is important to include shapes that are asymmetrical as well. Students should conclude that the number of lines of symmetry in a regular polygon is equal to the number of vertices.

Students can complete shapes and designs with line symmetry using square tiles or pattern blocks, folded paper, a Mira, grid paper, or technology such as a drawing program or dynamic geometry software. Relating a line of symmetry to a line of reflection should enable students to complete a figure, describe the completed shape, and describe the reflection. This should include working with and without a coordinate plane.

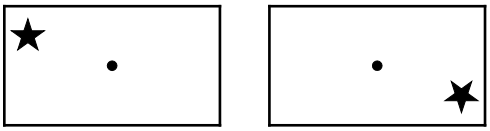
Students have worked with rotations of 2-D shapes in earlier grades (Mathematics 5, 6, and 7). The concept of rotation symmetry, however, is new to them.

A 2-D shape has rotation symmetry if, when turned around its centre point, it coincides with itself at least once before it has completed a full rotation. One way to test for this is to trace the shape, and then turn the tracing over the original shape around a pencil point to see whether it fits over itself. For example, a rectangle fits over itself twice—once after a half turn, and once again after a complete turn. Students should use grid paper to investigate the rotation symmetry of various objects.

Another way to test this is as follows: Ask each student to use a square from the pattern blocks, mark the top left corner and trace around it to make a square on a sheet of paper. With the square block placed inside its picture as illustrated below, it is rotated 90^{o} clockwise, with the centre of rotation being the centre of the square (intersection point of its two diagonals), until it perfectly matches its picture again. Students should notice that the marked vertex is at the next corner (top right). They then repeat this rotation. Because the square can appear in four identical positions during one complete 360^{o} rotation (see below), it is said to have rotational symmetry of order 4.



Similarly, students can show that a rectangle has rotational symmetry of order two. This can be repeated with all of the pattern blocks.

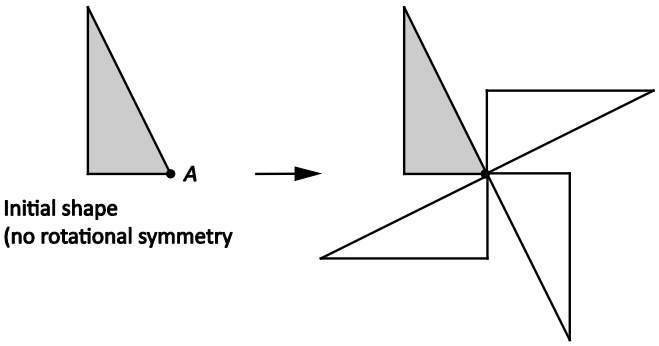


Students should determine the order of rotation and the angle of rotation. For example, a square has rotation symmetry of order 4, an equilateral triangle has rotation symmetry of order 3, and the order of rotation symmetry for a circle is infinite. A shape with order 2 symmetry has an angle of rotation of 180°, one with order 3 symmetry has an angle of rotation of 120°, and a shape with order 4 symmetry has a 90° angle of rotation. The number of degrees refers to the smallest angle through which the shape must be rotated to coincide with itself. Students should be able to express the angle of rotation in both degrees and fractions of a turn (e.g., 90° angle of rotation is a one-quarter turn).

Students can determine the angle of rotation by dividing the number of degrees in a circle by the order of rotation. Make students aware that, because every shape always coincides with itself after a 360° rotation, rotation symmetry of order 1 is not identified.

Students should be able to rotate objects in both a clockwise and counter-clockwise direction. They will need to use various types of grid paper / dot paper for these activities. It may be useful to provide tracing paper for students who have difficulty visualizing a rotation.

Students should identify rotational symmetry and order of rotation for a new shape formed by rotating an initial shape about a vertex. The result of a number of rotations about vertex A below, for example, is a new figure with rotation symmetry of order 4.



Students should be given an opportunity to explore symmetry in the context of everyday experiences. They should explore different types of art, such as paintings, jewelry, quilts, tiles, murals, and cultural artwork. Students could analyze pictures, logos, flags, signs, playing cards, kaleidoscopes, etc. They may use computer software to experiment with symmetry of designs or photos.

In small groups, students could search for company logos that contain various geometric shapes. They should copy them and draw in any lines of symmetry. They should then discuss the following:

- Are logos that contain lines of symmetry more pleasing to the eye than those without lines of symmetry?
- Are logos that contain lines of symmetry easier to remember than those without lines of symmetry?

This topic also provides a good opportunity to collaborate with the Art teacher on a cross-curricular project or activity.

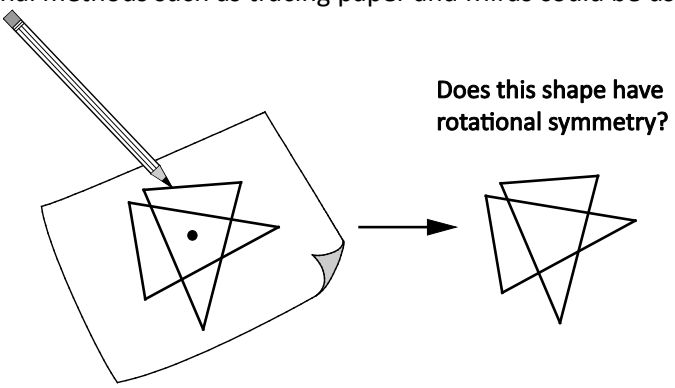
Students will also explore line and rotation symmetry for transformations on the Cartesian plane. They have previously worked with transformations in Mathematics 6, 7, and 8. In Mathematics 7, they worked with translations, rotations, and reflections of a 2-D shape in all four quadrants of a Cartesian plane.

Students should be able to identify symmetries in the combined shape of an object and its resulting image. They should be provided with examples of transformations that have already been graphed, as well as be expected to perform the transformations themselves. Students should be encouraged to use conventions when labelling axes, vertices, coordinates, etc. Accuracy in the drawings is important.

Students should discover that when a figure is reflected, the combined shape will have line symmetry. It may or may not have rotation symmetry. When a figure is rotated, the combined shape may have rotation symmetry. It may or may not have line symmetry. The symmetry of a combined shape created when a figure is translated depends on the type of translation and also on the original shape's symmetry.

The intent of this outcome is to apply transformations to create new shapes. Those shapes are then examined to determine if symmetry exists.

Traditional methods such as tracing paper and Miras could be used.



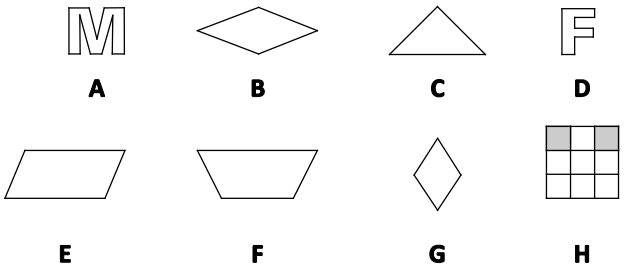
Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Have students draw examples of triangles with symmetry and triangles without symmetry.
- Provide students with the following labelled 2-D shapes.

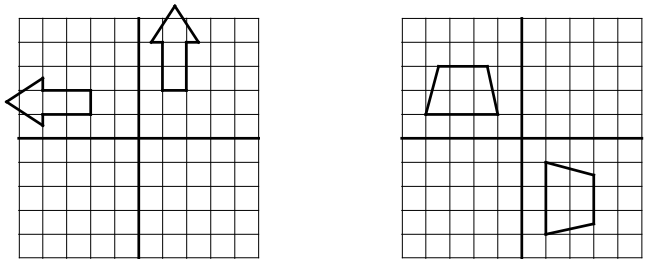


Ask students to circle all the symmetrical shapes. Then instruct them to draw all the lines of symmetry on the symmetrical shapes. Finally, have them sort the shapes by the number of lines of symmetry in each shape: no lines of symmetry, one line of symmetry, more than 1 line of symmetry. Ask students to show the lines of symmetry.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

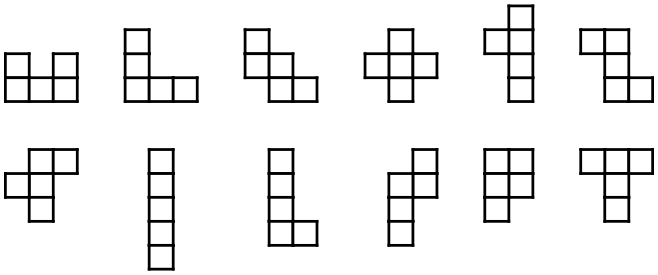
- Determine whether or not these pairs of shapes are related by line or rotational symmetry.



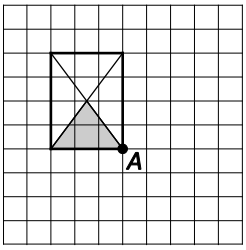
- Give the number of lines of symmetry in the given figure below, and determine the order and angle of rotation.



- Determine which objects below have rotation symmetry. If they have rotation symmetry, identify the center of rotation and then state the order and the angle of rotation.



- A shape has rotation symmetry of order infinity. What is the shape and what does this mean?
- Use point A as the center of rotation and create a design that has an order of rotation equal to 4.



Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

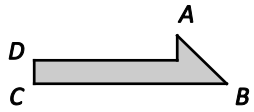
- Provide students with (or have them bring in) various 2-D shapes and classify them according to the number of lines of symmetry, and identify the order of rotation.
- Provide students with (or have them bring in) artwork or wallpaper designs to identify lines of symmetry or the order and angle of rotation symmetry. This would be a great cross-curricular activity with art classes. The designs produced can make interesting wall hangings for the classroom.
- Ask students to draw a shape on graph paper and to cut it along a line of symmetry. Have students exchange their drawings with another student who will complete the 2-D shape. Students should approach this by counting the spaces from the vertices to the line of symmetry in order to place each of the mirrored vertices and complete the shape.
- Ensure that students are exposed to a variety of shapes when looking at translations of 2-D shapes on the Cartesian plane, in order to recognize that translations often do not result in line or rotational symmetry.
- The works of M.C. Escher would make an interesting research project. Another avenue to research could be Islamic Art, which is often geometry-based and expresses the logic and order inherent in the Islamic vision of the universe.
- Wallpaper is a good source of designs that utilize transformational geometry and Escher-like transformations. If there is a wallpaper store close by, teachers can request old wallpaper books from discontinued designs. Students can look at the designs to find evidence of translations, reflections, and rotations, and record the transformations they observe. Many wallpaper designs incorporate multiple transformations. Fabrics/textiles from various cultures may also be explored.

SUGGESTED LEARNING TASKS

- On a Cartesian plane,
  - sketch a quadrilateral
  - label and record the coordinates of its vertices
  - translate the quadrilateral [3R, 2U]
  - label and record the coordinates of the corresponding vertices of the image
  - determine whether the shapes are related by either rotational or line symmetry and describe why
- Draw the following 2-D shapes: triangles (scalene, isosceles, and equilateral), quadrilaterals (square, rectangle, parallelogram, trapezoid, rhombus) and other regular polygons (pentagon, hexagon,

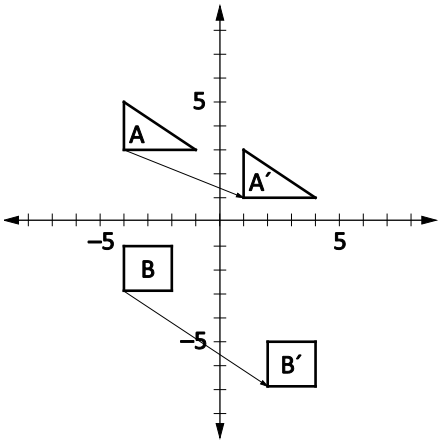
heptagon, octagon). Classify according to the number of lines of symmetry. Determine whether the shape has rotational symmetry and if it does, state the order and angle of rotation.

- Give students a 2-D shape from the previous activity, and have them rotate the shape about a vertex and draw the resulting image.
- Ask students to use a geoboard to create a shape with one rubber band. Students can use other bands to make as many lines of symmetry as possible. As a class, students can classify each shape according to the number of lines of symmetry.
- Ask students to explain how the number of lines of symmetry of a regular polygon relates to the number of sides it has.
- Ask students to respond to the following journal prompt: Any rectangle has only two lines of symmetry. Do you agree or disagree with this statement? Explain. Use drawings to support your argument.
- Students can sketch, or use a geoboard, to create the following shape image here. This shape represents half of another shape. Ask students to create the final shape by constructing the missing half, given each of the following cases:
  - Line of symmetry is $\overline{CB}$
  - Line of symmetry is CD
  - Line of symmetry is $\overline{AB}$

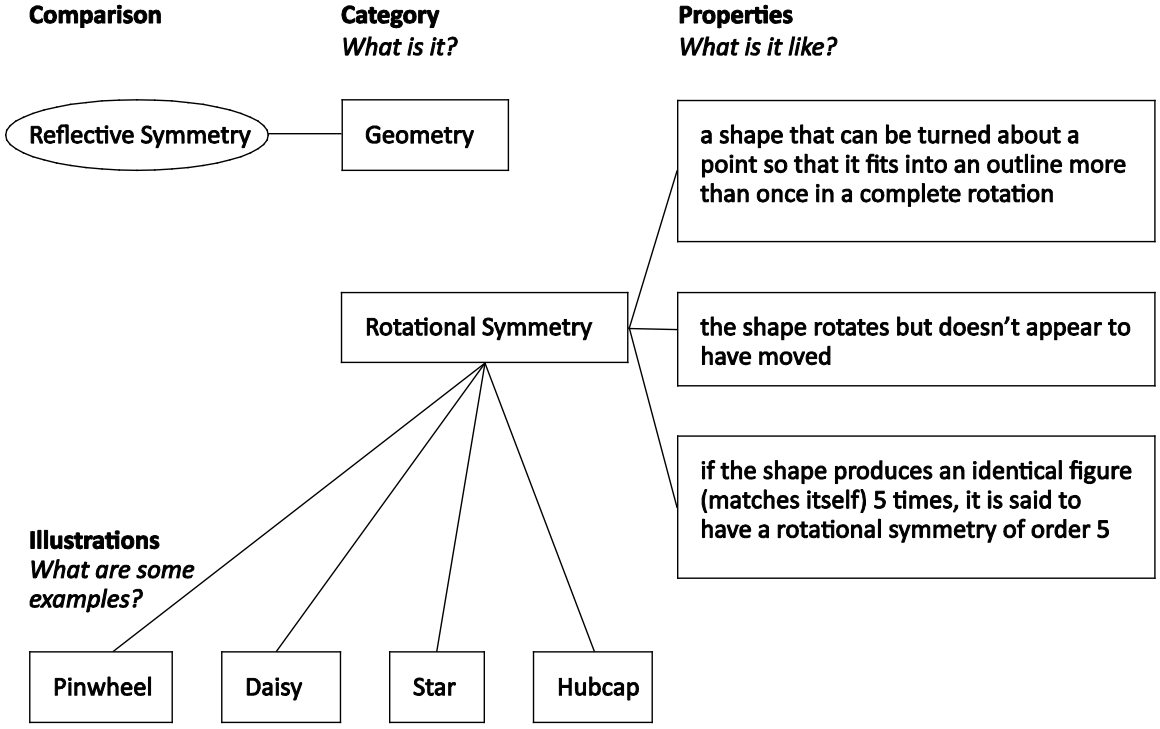


- For this activity, students need to use either isometric or rectangular dot grid paper. Students should draw a horizontal, vertical, or oblique line through several dots. Have them make a design completely on one side of the drawn line that touches the line in some way. The task is to make a mirror image of their design on the other side of the line. Students can exchange designs and make the mirror image of a classmate's design. When finished, they can use a mirror to check their work. You can also challenge them to make designs with more than one line of symmetry.
- Ask students to find a photograph or drawing of each item.
  - a 2-D shape with line and rotation symmetry
  - a 2-D shape with line symmetry but without rotation symmetry
  - a 2-D shape with rotation symmetry but without line symmetry
- Ask students to find examples of both line symmetry and rotational symmetry in artwork. Have them print various samples and determine:
  - the number of lines of symmetry
  - the order of rotation
  - the angle of rotation
- Ask students to use a digital camera to take photographs of their faces. Instruct them to look directly at the camera and avoid tilting their heads. Using a drawing program, such as Paint Shop Pro or Adobe Photoshop, students can then follow the steps outlined below:
  - Crop the right side of the face
  - Copy the remaining left side
  - Paste a mirror image of the left side in the right-hand position.
  - This will create a perfectly symmetrical face that can be compared with the original picture.
  - Repeat the procedure to produce a mirror image of the right side of the face.

  - Print the three photos.
  - Compare the "symmetrical" photos with the original.
- Ask students to identify the type of transformation presented in each case below.



  - Ask them to determine if the object and image for each situation are related by line symmetry and/or rotation symmetry.
  - Ask students to complete several assigned transformations and analyze their own drawings for symmetry.
- Ask students to answer the following: Some regular shapes, such as an equilateral triangle, a square, or a regular hexagon, appear to show line symmetry when they are translated in one direction. Do you agree or disagree with this statement? Give examples to support your argument. Discuss your answer with a partner.
- Have students complete concept definition maps for rotational and reflective symmetry. An example for rotational symmetry is below:



- Create a grid on the floor with masking tape. Use rope or coloured tape to place the axes. Student A student chooses a spot. Student B directs Student B to translate the position. This activity could progress to three or more students on the grid holding an elastic to form a 2-D shape. A different student directs them (as vertices) to "walk through" various transformations.

SUGGESTED MODELS AND MANIPULATIVES

- artwork
- Cartesian plane
- digital camera
- geoboard
- Mira

MATHEMATICAL LANGUAGE

Teacher	Student
▪ angle of rotation ▪ line symmetry ▪ order of rotation ▪ rotational symmetry	▪ angle of rotation ▪ line symmetry ▪ order of rotation ▪ rotational symmetry

Resources

Digital

- GeoGebra [Software] (International GeoGebra Institute 2015): www.geogebra.org/cms/en
- The Geometer's Sketchpad [Software] (Key Curriculum Press 2013; NSSBB #: 50474, 50475, 51453)

Print

- *Math Makes Sense 9* (Baron et al. 2009; NSSBB #: 2001644)
  - Unit 7: Similarity and Transformations
    - > Section 7.5: Reflections and Line Symmetry
    - > Game: Make: Your Own Kaleidoscope
    - > Section 7.6: Rotations and Rotational Symmetry
    - > Section 7.7: Identifying Types of Symmetry on the Cartesian Plane
    - > Unit Problem: Designing a Flag
  - *ProGuide* (CD; Word Files; NSSBB #: 2001645)
    - > Assessment Masters
    - > Extra Practice Masters
    - > Unit Tests
  - *ProGuide* (DVD; NSSBB #: 2001645)
    - > Projectable Student Book Pages

> Modifiable Line Masters

- *Developing Thinking in Geometry* (Johnston-Wilder and Mason 2006), pp. 27–30
- *Elementary and Middle School Mathematics: Teaching Developmentally*, 8th Edition (Van de Walle, Karp, and Bay-Williams 2013), pp. 420–421
- *Geometry: Seeing, Doing, Understanding,* Third Edition (Jacobs 2003), pp. 319–320
- *Making Math Meaningful to Canadian Students, K*–8 (Small 2008), pp. 298–303

Statistics and Probability (SP)

GCO: Students will be expected to collect, display, and analyze data to solve problems.

GCO: Students will be expected to use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Assessment Strategies

Assessment for learning can and should happen every day as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts should be used for assessing all students—as a class, in groups, and individually.

GUIDING QUESTIONS

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Follow-up on Assessment

The evidence of learning that is gathered from student participation and work should be used to inform instruction.

GUIDING QUESTIONS

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Long-term Planning

- Yearly plan available at Mathematics Learning Commons: Grades 7–9: http://nsvs.ednet.ns.ca/nsps/nsps26/course/view.php?id=3875.

GUIDING QUESTIONS

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote learning of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SCO SP01 Students will be expected to describe the effect on the collection of data of bias, use of language, ethics, cost, time and timing, privacy, and cultural sensitivity. [C, CN, R, T]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

SP01.01 Analyze a given case study of data collection; and identify potential problems related to bias, use of language, ethics, cost, time and timing, privacy, or cultural sensitivity.

SP01.02 Provide examples to illustrate how bias, use of language, ethics, cost, time and timing, privacy, or cultural sensitivity may influence data.

Scope and Sequence

Mathematics 8	**Mathematics 9**	**Mathematics 10**
SP01 Students will be expected to critique ways in which data is presented.	**SP01** Students will be expected to describe the effect on the collection of data of bias, use of language, ethics, cost, time and timing, privacy, and cultural sensitivity.	–

Background

In Mathematics 8, students learned a variety of ways in which to present data (bar graphs, line graphs, circle graphs, pictographs), and to evaluate the strengths and limitations of different presentations of data. Terms such as discrete and continuous data, accuracy, choice of intervals, and trends were reinforced. Students learned how to justify their conclusions and identify inconsistent and misrepresented data.

In Mathematics 9, students will continue to develop data analysis and focus on factors that affect the collection of data.

There are many factors within the data collection process that have the potential to influence the results. Students should consider factors such as the method used, the reliability and usefulness of data, and the ability to make generalizations about the population from a sample. To critically analyze data collection, students must have an understanding of the factors that might lead to problems in the data collection process.

A good way to approach this would be for students to analyze survey questions showing only one problem. The following situation, for example, illustrates how timing can affect data collection:

- Free samples of sunscreen are sent to every home in the fall and winter. A mail reply card asks people if they would use the product again.

When preparing to collect data, appropriate questions and a representative sample of the whole population are important.

Students should consider the following:

- appropriate questions are clearly written, easy to answer, and effective in generating the desired data
- multiple choice questions are useful for identifying respondents' preferences
- questions should be ordered appropriately

Students should analyze how the phrasing of questions might affect the data collected. For a given case study, they should ask questions such as:

- Does asking this question collect the required information?
- Does the question make one response sound right and another one wrong (i.e., does it have bias)?
- Is the question respectful?

Remind students that when wording survey questions, factors that may influence the responses should be considered.

Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- What graph would you use to represent the data below? Explain your choice.
  - the cost of cell phones over the past 20 years
  - prices of different brands of jeans
  - the favourite sports of boys and girls
  - the percentage of favorite pizza toppings of grade 9s

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- For the following, identify the sources of bias and suggest ways to remove it.
  - At a soccer game, a survey was given and the results showed that when asked to give their favourite sport, 85% of the youth responded it was soccer.
  - Do you think that small dogs make good pets even though they are yappy?
- Create a survey question with bias about the use of technology among teenagers.
- Your friend is unclear what the term "bias" means. Develop an example to help explain the term.

Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Use graphs from various sources (newspapers, magazines, online, etc.), computer spreadsheet applications (e.g., Microsoft Excel), and websites such as Statistics Canada (www40.statcan.gc.ca/z01/cs0002-eng.htm).

SUGGESTED LEARNING TASKS

- Ask students to bring in a variety of surveys and discuss the following:
  - **Bias:** How many people were surveyed? What was the age range of the population?
  - **Use of language:** Is the question clear, and not leading to a particular response?
  - **Ethics:** Will the results be used for appropriate purposes originally given?
  - **Cost:** Does the cost of the study outweigh the benefits?
  - **Time:** Is the timing of, and the time required to do the survey, appropriate?
  - **Privacy:** Are the results confidential?
  - **Cultural sensitivity:** Are the questions offensive to a particular culture?
- Have students, in groups, develop survey questions. For example:
  - Since students have many after school activities, do you think teachers should assign less homework?
  - Do you agree that the price of concert tickets is too high?

  Ask students to identify the potential problems associated with the questions asked.
- Compile some data examples (good/bad) from math texts from previous years for analysis and discussion.
- Investigate misleading information (e.g., the endangered tree octopus, the house hippo) and discuss the value of critical thinking skills.
- Students could be given a case study, such as the following, and asked to determine the factor(s) that might affect data collection. Ask students to rewrite the scenario without any bias.
  - A marketing agency wants to determine how Canadians spend their clothing dollars. Jody wrote this question to determine how much is spent on imported clothing: What does your closet contain more of? A. less expensive, foreign made clothes or B. high-quality, made-in Canada clothes.
  - What specific information is Jody trying to obtain?
  - Rewrite the question to avoid bias and sensitivity issues.
- Students could be asked to develop their own survey question that involves factor(s) that affect data collection. They could then identify the factor(s) involved and rewrite the question to collect accurate data.

SUGGESTED MODELS AND MANIPULATIVES

- computer spreadsheet applications (e.g., Microsoft Excel, Google docs)
- graphs from various sources (newspapers, magazines, etc.)

- computer, tablet
- websites for data sources (e.g., Statistics Canada: www.statcan.gc.ca)
- websites for interactives (e.g., Shodor: www.shodor.org\interactive)

MATHEMATICAL LANGUAGE

Teacher	Student
▪ bias ▪ bar graph ▪ circle graph ▪ continuous ▪ discrete ▪ distort ▪ double bar graph ▪ double line graph ▪ interval ▪ line graph ▪ misrepresentation ▪ pictograph ▪ trend	▪ bias ▪ bar graph ▪ circle graph ▪ continuous ▪ discrete ▪ distort ▪ double bar graph ▪ double line graph ▪ interval ▪ line graph ▪ misrepresentation ▪ pictograph ▪ trend

Resources

Digital

- "Data Analysis and Probability, [manipulatives]," *National Library of Virtual Manipulatives* (Utah State University 2015): http://nlvm.usu.edu/en/nav/topic_t_5.html
- "Tables by Subject," *Statistics Canada* (Government of Canada 2015): www40.statcan.gc.ca/z01/cs0002-eng.htm
- *Interactivate,* (Shodor 2015): www.shodor.org\interactive
- *Statistics Canada* (Government of Canada 2015): www.statcan.gc.ca

Print

- *Math Makes Sense 9* (Baron et al. 2009; NSSBB #: 2001644)
  - Unit 9: Probability and Statistics
    - > Game: Cube Master
    - > Section 9.2: Potential Problems with Collecting Data
    - > Technology: Using *Census at School* (Currently, link is not working.)
  - *ProGuide* (CD; Word Files; NSSBB #: 2001645)
    - > Assessment Masters
    - > Extra Practice Masters
    - > Unit Tests
  - *ProGuide* (DVD; NSSBB #: 2001645)
    - > Projectable Student Book Pages

> Modifiable Line Masters

- *Elementary and Middle School Mathematics: Teaching Developmentally*, 8th Edition (Van de Walle, Karp, and Bay-Williams 2013), pp. 434–448
- *Making Math Meaningful to Canadian Students, K–8* (Small 2008), pp. 525–532

SCO SP02 Students will be expected to select and defend the choice of using either a population or a sample of a population to answer a question. [C, CN, PS, R]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

SP02.01 Identify whether a given situation represents the use of a sample or a population.
SP02.02 Provide an example of a situation in which a population may be used to answer a question, and justify the choice.
SP02.03 Provide an example of a question where a limitation precludes the use of a population, and describe the limitation.
SP02.04 Identify and critique a given example in which a generalization from a sample of a population may or may not be valid for the population.
SP02.05 Provide an example to demonstrate the significance of sample size in interpreting data.

Scope and Sequence

Mathematics 8	**Mathematics 9**	**Mathematics 10**
SP01 Students will be expected to critique ways in which data is presented.	**SP02** Students will be expected to select and defend the choice of using either a population or a sample of a population to answer a question.	–

Background

To analyze whether a given situation represents a **sample** or a **population**, students must clearly understand these terms in the context of data collection and analysis. Some students may not understand the difference between a population and a sample. Reinforce the meaning of each term using a familiar example. Stress the importance of the context to help determine the population or the sample. Students may think that the term population only refers to a group of people. In fact, the term population refers to a complete group of anything, such as people, objects, animals, businesses, etc.

Since it is often impractical to gather information about entire populations, sampling is a common statistical technique. Any group of individuals (or units) selected from the population would be referred to as a sample. For example, in a federal election a sample could be taken of 100 individuals chosen from each province or territory.

When a sample is representative of the population, the data collected from the sample leads to valid conclusions. Students will need to understand issues with respect to sampling strategies and **sample size** in order to properly draw inferences from sample data.

Examining different types of sampling would be beneficial to students when they are developing a project plan, and have to consider the choice of a data collection method. Students should also be able to determine when it is best to use a population versus a sample when limitations are present. Suppose students want to conduct a survey to find out where to go on a class trip. It would be appropriate to ask everyone in the class. With a large population, however, it is impractical to survey everyone, so students need to use a representative sample group. Suppose they want to conduct a survey to determine if people in their community support year-round schooling. They would have to carefully consider whom to ask and how many people to ask. There are many factors that affect the feasibility of using the entire population.

Students should be encouraged to carefully consider any generalizations made from a sample to a population, as sometimes they may not be **valid**. For example, students could consider the following scenario:

- All grade 9 students in the province were surveyed to determine the start time for the school day. Ninety percent of the students in urban HRM wanted school to start at 7:50 a.m., as they wanted to finish up early.

  This sample might not represent the majority of students outside urban HRM because they would be considering different factors, such as length of time spent on bus travel to school, when completing their survey. On the other hand, students should realize that if proper sampling techniques were used, the survey results would be valid.

Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Collect graphs from newspapers or other sources and show them to students.
  - What are some advantages for using the graph that was chosen?
  - What are some disadvantages for using the graph that was chosen?
  - What other graphs could have been used?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Identify the population for each situation given below, and indicate if you would survey the population or just a sample.
  - Vito's restaurant would like to know which lunch menu item its customers would prefer.
  - A phone service provider would like to know how many of its customers would like to have caller id as a feature.

  - Health Canada would like to find out reasons why some Canadians chose not to get the flu vaccine.
- For the following scenarios, what problems can you see with the generalizations that were made?
  - At the school cafeteria, employees conducted a survey about what snacks would be offered at break time during the school day. The cafeteria worker handed out a survey to every fourth person who came through the line on a particular day and gathered the data from these. It was concluded from this that students would like to see more granola bars offered during the breaks.
  - Student council surveyed students about how best to spend the activities budget for the coming year. It randomly surveyed students at a soccer game. Student council concluded more money should be spent on athletic teams.
- Students could be asked to explain the factors that would determine using a sample rather than a population in the following scenarios:
  - Is there a need in Nova Scotia for a mass vaccination for the flu virus?
  - Is there a need to check each light bulb coming off an assembly line for defects?
  - Is there a need to survey all people in an electoral district before an election to predict the winner?

Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Consider the following two situations. In the first situation, a television survey is done to find the fraction of the public that watches a particular television program on a particular night and to determine if more women than men watch it. In the second situation, a quality engineer must estimate what percentage of bottles rolling off an assembly line is defective. In both these situations, information is gathered about a large group of people or things. The expense of contacting every person or inspecting every bottle, and the time it would take, is formidable so information is gathered about only part of the group (a sample) in order to draw conclusions about the whole group (the population).
- Ensure students recognize that choosing a representative sample from a large and varied population can be a complex task. It is important to be clear about what population is to be described and exactly what is to be measured. Areas of concern:
  - How can a sample be chosen so that it is truly representative of the population?
  - If a sample from a population differs from another sample from the same population, how confident can you be about predicting the true population percentage?
  - Does the size of the sample make a difference?

SUGGESTED LEARNING TASKS

- Identify a population for a specific situation (e.g., a cell phone carrier wants to know which brand and model of phone the students are currently using) and state whether the whole population or a sample of the population should be sampled. Explain your reasoning.
- Ask students to identify whether a sample or a population is used in each of the following situations:

  - All residents of a town are asked where a new school for the town should be located.
  - One out of every 100 tablets is tested for defects.
  - A student from each class in a school is questioned about removing chocolate milk from the lunch menu.
- Students could be asked to explain why data collection should include the entire population for situations such as the following:
  - Jet engines, produced by a factory, should be tested before use.
  - A government official is elected.
  - Determine whether or not a school should have a uniform for students.
- Each student is given a cut-out of a fish and is asked to label the fish as male or female. Ask students to determine the percentage of male and female in the populations. Place all fish in a fishbowl and depending on class size draw a:
  - small sample
  - medium sample
  - large sample
  - Ask students to calculate the percentage of male/female in the sample and compare this value to the experimental results of the population.

SUGGESTED MODELS AND MANIPULATIVES

- computer and Internet access

MATHEMATICAL LANGUAGE

Teacher	Student
▪ census ▪ inference ▪ population ▪ random ▪ sample ▪ sample size ▪ validity	▪ census ▪ inference ▪ population ▪ random ▪ sample ▪ sample size ▪ validity

Resources

Digital

- "Data Analysis and Probability, [manipulatives]," *National Library of Virtual Manipulatives* (Utah State University 2015): http://nlvm.usu.edu/en/nav/topic_t_5.html

Print

- *Math Makes Sense 9* (Baron et al. 2009; NSSBB #: 2001644)
  - Unit 9: Probability and Statistics

    - > Section 9.3: Using Samples and Population to Collect Data
      (The Nova Scotia Mathematics 9 curriculum requires students to provide an example to demonstrate the significance of sample size in interpreting data [PI 9SP02.05].)
    - > Section 9.4: Selecting a Sample
  - *ProGuide* (CD; Word Files; NSSBB #: 2001645)
    - > Assessment Masters
    - > Extra Practice Masters
    - > Unit Tests
  - *ProGuide* (DVD; NSSBB #: 2001645)
    - > Projectable Student Book Pages
    - > Modifiable Line Masters
- *Elementary and Middle School Mathematics: Teaching Developmentally*, 8th Edition (Van de Walle, Karp, and Bay-Williams 2013), pp. 434–448
- *Making Math Meaningful to Canadian Students, K–8* (Small 2008), pp. 525–532

SCO SP03 Students will be expected to develop and implement a project plan for the collection, display, and analysis of data by
- formulating a question for investigation
- choosing a data collection method that includes social considerations
- selecting a population or a sample
- collecting the data
- displaying the collected data in an appropriate manner
- drawing conclusions to answer the question

[C, PS, R, T, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

SP03.01 Create a rubric to assess a project that includes the assessment of
- a question for investigation
- the choice of a data collection method that includes social considerations
- the selection of a population or a sample and the justification for the selection
- the display of collected data
- the conclusions to answer the question

SP03.02 Develop a project plan that describes
- a question for investigation
- the method of data collection that includes social considerations
- the method for selecting a population or a sample
- the methods for display and analysis of data

SP03.03 Complete the project according to the plan, draw conclusions, and communicate findings to an audience.

SP03.04 Self-assess the completed project by applying the rubric.

Scope and Sequence

Mathematics 8	Mathematics 9	Mathematics 10
SP01 Students will be expected to critique ways in which data is presented.	**SP03** Students will be expected to develop and implement a project plan for the collection, display, and analysis of data by ▪ formulating a question for investigation ▪ choosing a data collection method that includes social considerations ▪ selecting a population or a sample ▪ collecting the data ▪ displaying the collected data in an appropriate manner ▪ drawing conclusions to answer the question	–

Background

In previous grades, students have collected, displayed, and interpreted data represented in various tables and graphs. In Mathematics 9, students will plan and carry out a data project to answer a question, and individually or as a group, design a rubric to assess the project. The project will include formulating an appropriate question, collecting data from a sample or population, displaying the data, and drawing conclusions..

Proper planning should identify potential problems with questions or data collection methods. Problem solving should permeate the whole process, as students decide on interesting topics, formulate questions, plan the collection of data, implement plans, and analyze results. This outcome should incorporate many of the other Statistics and Probability outcomes. This outcome is to be assessed based on the development and implementation of an individualized or group project.

The following are useful guidelines for project-based learning:

- Students may in groups or work independently.
- Allow students a choice on the topic and methods of presentation.
- Plan the project with drafts and timeline benchmarks.
- Familiarize students with the assessment plan.
- Begin stages of the project at various times of the year.

In the mathematics curriculum, this is the first exposure students have had developing rubrics. A rubric should be developed prior to the beginning of the project to clarify exactly what is expected and how the project will be assessed. A rubric should include the criteria that will be assessed and a description of each level of performance. Rubric development could be completed as a class activity. A sample rubric is shown below.

Criteria	Level 1	Level 2	Level 3	Level 4
The question for investigation				
The choice of data collection method that includes social considerations				
The selection of a population or a sample and justification of the choice				
The presentation of collected data				
Conclusions to answer the question				

Students can do a written or oral report that includes information about the project plan and conclusions. The report should also include

- the question(s) asked in the survey
- appropriate display of the data
- valid conclusions based on the data

Assessment, Teaching, and Learning

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

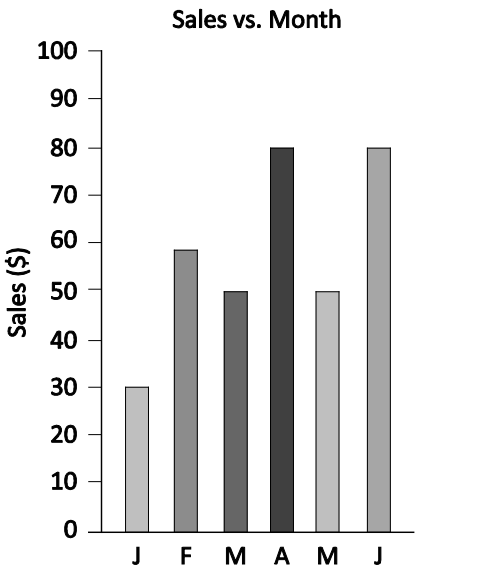
- Answer **True** or **False** for each statement below:
  - The purpose of a graph is to display data in a way that is easier to interpret.
  - All graphs do not require a legend.
  - All graphs must have a vertical and a horizontal scale.
  - All graphs must have a title.
  - A circle graph is always the best way to represent data.
- For the following sets of graphs, indicate which of the necessary components match ✓ or do not match "×". Then determine if the graphs represent the same data.

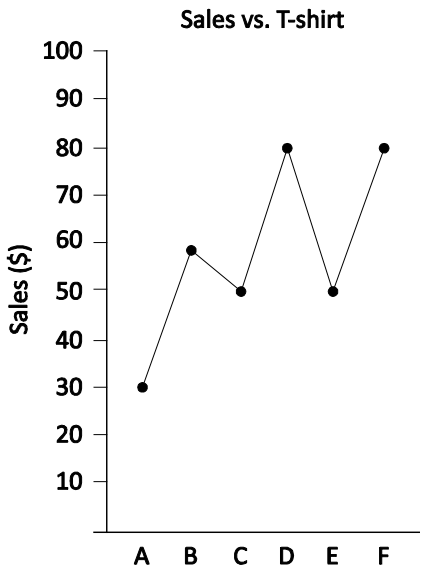
___ Titles

___ Scaling of X axis

___ Scaling of Y axis

___ Sales ($)





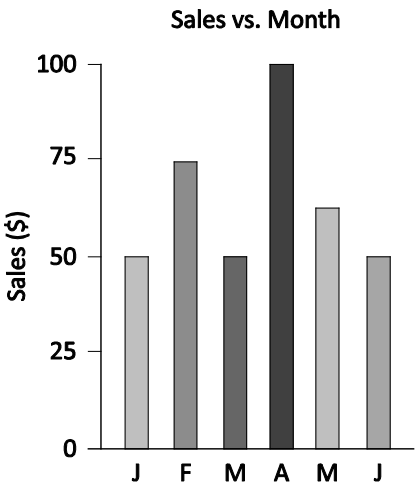
Do these graphs represent the same data? **YES** **NO**

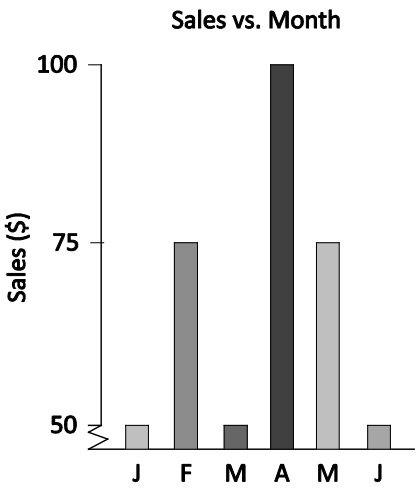
___ Titles

___ Scaling of X axis

___ Scaling of Y axis

___ Sales ($)





Do these graphs represent the same data? **YES** **NO**

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

The following represents a list of ideas for use in the development of a statistics project. Each can be shaped by students to better reflect their interests.

- Find out how much time is spent per week on each subject area when doing homework. Does this change from grade 7 to grade 9?
- Do students in your school text their friends more or less often than they speak to them in person?
- Find out what type of transportation students in your school use to get to school. Does it differ by grade level? Does it differ with the time of year?
- Find out the most popular types of after-school activities of students in your school. Does it differ by grade level?
- Find out what the favourite cereals of students are in your class or school. Students could also compare adult versus student cereal consumption. Compare this with sales volume at the local supermarket to determine how the class compares to the rest of the community.
- Find out the favourite kind of jeans for people in your age group. Use the result of the survey to write a recommendation to a local store regarding their ordering of the types of jeans. A comparison could also be made for various age groups.
- Ask the student council or community council to suggest issues they would like investigated. Use this as a source for project work.
- Collect data to look for a relationship between average grade on students' last report cards and time spent (1) watching television; (2) time spent on homework, and (3) shoe size.
- Conduct a survey to find out information related to: students' favourite NHL team; students' favourite musical instrument; students' favourite snack food; effects of social media or technology on sleep patterns; bullying; interests; online vs. in-store shopping; etc.
- Survey or interview grade 9 students to determine preferred part-time jobs and the amount of money typically earned. They may wish to include jobs such as babysitting, lawn mowing, and paper routes.

Planning for Instruction

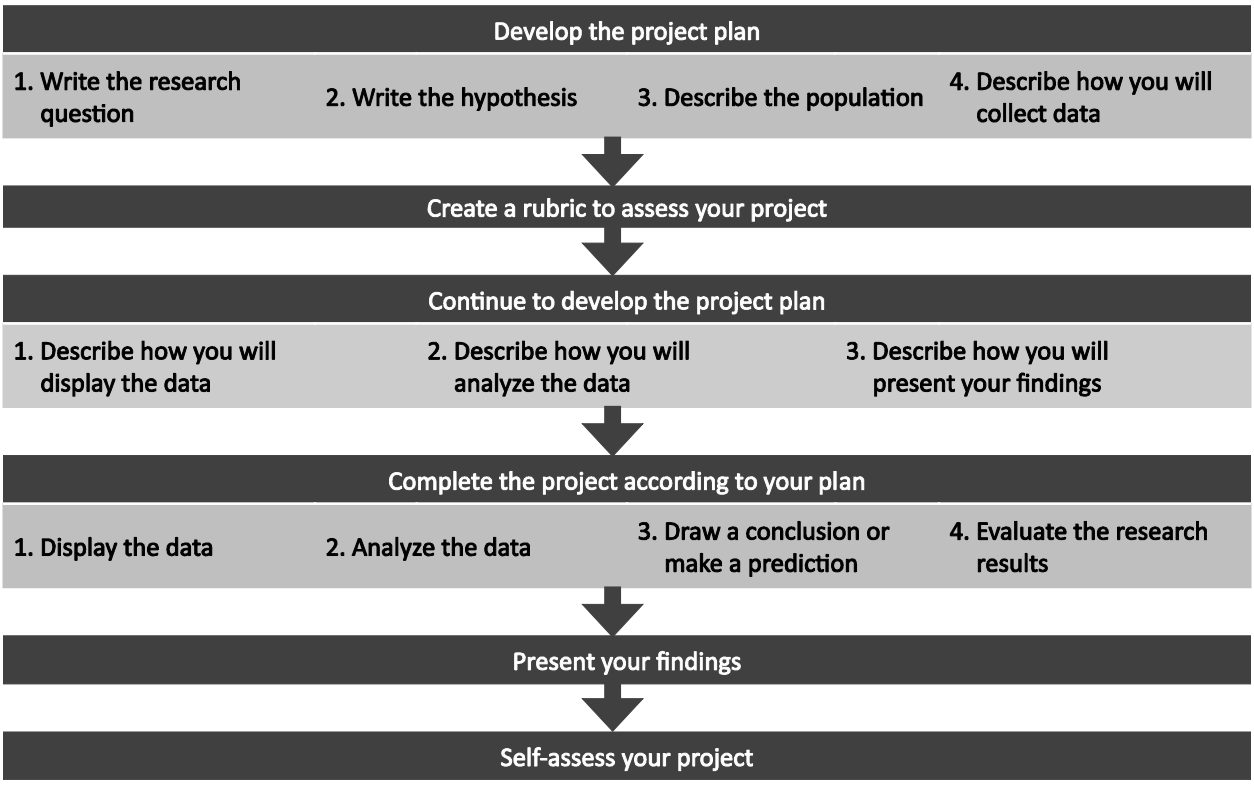
CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Create a plan with the various components of a data management project: the question that will be investigated, methods of data collection, sampling procedures, data collection, data displays, and conclusions.
- Complete statistics projects in pairs or small groups. Students could assess their projects in pairs using the rubric created for this project.
- Consider using technology to create data displays.

SUGGESTED LEARNING TASKS

- Explore and critique rubrics that have been developed so students are exposed to models.
- Ask students to brainstorm possible questions, ideas, and/or issues that could be investigated. They could organize their thoughts with mind maps.
- Ask students to create an organizer, such as the flow chart below, to help organize the research project and carry out the plan.



SUGGESTED MODELS AND MANIPULATIVES

- spreadsheets

MATHEMATICAL LANGUAGE

Teacher	Student
▪ population ▪ rubric ▪ sample ▪ sample size ▪ sampling technique	▪ population ▪ rubric ▪ sample ▪ sample size ▪ sampling technique

Resources

Print

- *Math Makes Sense 9* (Baron et al. 2009; NSSBB #: 2001644)
  - Unit 8: Circle Geometry
  - Start Where You Are: How Do I Best Learn Math
  - Unit 9: Probability and Statistics
    - > Technology: Using Census at Schools (Currently, link is not working.)
    - > Technology: Using Spreadsheets and Graphs to Display Data
    - > Section 9.5: Designing a Project Plan
  - *ProGuide* (CD; Word Files; NSSBB #: 2001645)
    - > Assessment Masters
    - > Extra Practice Masters
    - > Unit Tests
  - *ProGuide* (DVD; NSSBB #: 2001645)
    - > Projectable Student Book Pages
    - > Modifiable Line Masters
- *Elementary and Middle School Mathematics: Teaching Developmentally*, 8th Edition (Van de Walle, Karp, and Bay-Williams 2013), pp. 434–448
- *Making Math Meaningful to Canadian Students, K–8* (Small 2008), pp. 525–532

Digital

- "Data Analysis and Probability, [manipulatives]," *National Library of Virtual Manipulatives* (Utah State University 2015): http://nlvm.usu.edu/en/nav/topic_t_5.html

SCO SP04 Students will be expected to demonstrate an understanding of the role of probability in society. [C, CN, R, T]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

SP04.01 Provide an example from print and electronic media where probability is used.
SP04.02 Identify the assumptions associated with a given probability, and explain the limitations of each assumption.
SP04.03 Explain how a single probability can be used to support opposing positions.
SP04.04 Explain, using examples, how decisions may be based on a combination of theoretical probability, experimental probability, and subjective judgment.

Scope and Sequence

Mathematics 8	**Mathematics 9**	**Mathematics 10**
SP02 Students will be expected to solve problems involving the probability of independent events.	**SP04** Students will be expected to demonstrate an understanding of the role of probability in society.	–

Background

In previous grades students have explored the difference between **theoretical** and **experimental probability**, and how to be able to express probabilities for single and independent events in fractions, percents, and decimals.

In Mathematics 9, the **focus** of study is for students to understand the role that probability plays in society by looking at the probability of events occurring, and by examining decisions that are based on those predictions. Students should be exposed to a variety of examples from daily life in which probability is used. For examples,

- insurance premiums that are established based on the historical data of a certain gender, age group, or region making claims
- warranty periods based on probable lifespan of a product
- number of units manufactured based on probable units that will sell
- projecting of winners of an election from past data
- determining probability of experiencing the side effects of a drug
- airline companies developing a schedule of flights and crew, and setting fares based on the probability of demand at different times of year
- weather forecasts and the probability of precipitation and other weather systems

Students should be focused on familiar situations. Discussions about how predictions are made (a mix of theoretical and experimental probability and subjective judgment) should be a focus. Students will quickly realize that experimental probability is the one most widely used when making predictions. The types of **assumptions** made when making these predictions should also be addressed.

Calculations of probability are based on assumptions. Students should be encouraged to identify and examine assumptions to help them determine whether the calculated probability is meaningful when making a decision.

Students should engage in evaluating situations that lend themselves to reasonably accurate predictions, those that are questionable, and those for which the unknowns are not quantifiable. Injury as a result of road accidents with/without seatbelts is a good example for safe prediction.

Health professionals predicting that people of lower socioeconomic status will have more health problems, is a more questionable situation.

There are many situations where the unknowns are too great to make probabilistic arguments. Attempting to find the probability of someone having the same name, age, and birthdate as yourself, for example, involves too many unknowns to make an accurate prediction. Teachers could discuss, with students:

- What are the reasons for the uncertainty?
- What are the important questions to ask about a situation in order to reduce it to probabilistic form?

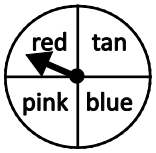
Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Jamila and Brooke are playing a game in which Jamila rolls a pair of dice and finds the sum of the numbers rolled. If the sum is six or eight, then Brooke gets three points. If the sum is not six or eight, then Jamila receives one point. Is this game fair? Give reasons for your answer.
- Have students compare the theoretical and experimental probability for spinning pink on the spinner below and rolling a composite number on a dice.





WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Access "Climate" (Government of Canada 2015) http://climate.weather.gc.ca/data_index_e.html and search for data about your hometown to make predictions for the current month (precipitation amounts, mean temperature, etc.). Discuss any assumptions you may have had when making these predictions and explain the limitations of these assumptions.
- For your school determine the probable number of students who will go on to post-secondary education next year. Think about various ways to determine this probability. For example, use school data such as a request for transcripts from previous years, or approach the Awards Committee to see how many scholarships were obtained.
- Wayne thinks that a good way to model the performance of a baseball player who gets a hit 1 time in 4 at bats is to use a spinner with 4 sections. What assumptions is he making? Are his assumptions valid?

Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Give students the opportunity to explore the media to find examples of predictions based on probability in everyday life.
- Give students the opportunity to explore decision-making based on probability. They should use a sample to determine the probability of an event, use the results and subjective judgment to make predictions and explain the reasonableness of the predictions, based on any assumptions that they made. If at all possible, the reasonableness of the predictions should be tested.
- As a class, look for examples in the media where probability is used to support or reject a position.

SUGGESTED LEARNING TASKS

- Use the sampling from outcome SP03 to make predictions about the general population by
  - identifying the assumptions made and the limitations of these assumptions
  - discussing how much reliance there was on theoretical probability, experimental probability, and subjective judgment to make that prediction
- Ask students to provide a report on examples of where probability is used in print and electronic media.
- Ask students to think of a TV game show where players consider probabilities when deciding how to proceed, and then explain the extent to which probability is involved.
- Ask students to look through print media and the Internet to find examples of cases such as the following:
  - a situation where decisions affecting your community were made that might have been based on probabilities
  - a situation where a medical organization might make a decision based on probabilities

  Ask them to describe how probabilities were involved.
- Ask students to find an article that includes probabilities and discuss the possible opposing viewpoints.
- Provide pairs of students with 2 cards containing a misleading probability statement. Ask their partner to explain the limitations of the statement.

1. I've tossed a fair quarter 3 times and got heads. It is more likely to be tails than heads if I toss it again.	**2.** The Rovers Team play against the Shooters Team. The Rovers can win, lose, or draw, so the probability that the Rovers will win is $\frac{1}{3}$.
3. There are 3 red beads and 5 blue beads in a bag. I pick a bead at random. The probability that it is red is $\frac{3}{5}$	**4.** I roll 2 dice and add the results. The probability of getting a total of 6 is $\frac{1}{12}$ because there are 12 different possibilities and 6 is one of them.
5. It is less likely to throw a 6 than a 3 with a dice.	**6.** Tomorrow it will either rain or it will not rain, so the probability that it will rain is 0.5.
7. Mr. Brown has to have a major operation. 90% of the people who have this operation make a complete recovery. There is a 90% chance that Mr. Brown will make a complete recovery if he has this operation.	**8.** If 6 fair dice are thrown at the same time, I am less likely to get 1,1,1,1,1,1 than 1,2,3,4,5,6.

- Students could be asked to give a written or oral report on scenarios such as the following:
  - Jolene's mother has an important presentation to make in the morning at a conference 200 km away. She has an evening meeting at work tonight. The weather network has reported a 50% probability of snow in the morning. The company she works for would pay for her hotel. What are the probabilities Jolene's mother has to consider when deciding whether to make the drive tonight or in the morning? Which probability do you think would have the most impact on her decision? Explain.
  - What probabilities might a government consider when deciding whether to turn a two-lane highway into a four-lane highway?
- Many insurance companies charge drivers under the age of 25 higher insurance premiums based on the probability of accidents. Ask students to find an article about car insurance costs based on the probability of collision and answer the following questions.
  - In the article, what are the assumptions associated with each probability? Explain.
  - In your opinion, is there a bias against young drivers?
  - "Discussions about car insurance costs are based on a combination of experimental probability, theoretical probability, and subjective judgement." Do you agree or disagree with this statement? Explain.
- Odette states that she has a 1 in 2 chance of getting heads when she flips a coin. Claude states that a particular coin is unfair based on the fact that, when flipped 50 times, it came up heads 40 of the 50 times. Ingrid feels that even if there is an equal chance of getting heads, heads will appear more often because she feels it is her lucky choice. Ask students to categorize whether or not the three individual's decisions are based on subjective, experimental, or theoretical probabilities, and describe how each can play a part in decision making.

SUGGESTED MODELS AND MANIPULATIVES

- dice
- spinners

MATHEMATICAL LANGUAGE

Teacher	Student
▪ assumptions ▪ experimental probability ▪ fair ▪ judgment ▪ subjective ▪ theoretical probability	▪ assumptions ▪ experimental probability ▪ fair ▪ judgment ▪ subjective ▪ theoretical probability

Resources

Digital

- "Climate," *Government of Canada* (Government of Canada 2015): http://climate.weather.gc.ca/data_index_e.html

Print

- *Math Makes Sense 9* (Baron et al. 2009; NSSBB #: 2001644)
  - Unit 9: Probability and Statistics
    - > Section 9.1: Probability in Society
  - *ProGuide* (CD; Word Files; NSSBB #: 2001645)
    - > Assessment Masters
    - > Extra Practice Masters
    - > Unit Tests
  - *ProGuide* (DVD; NSSBB #: 2001645)
    - > Projectable Student Book Pages
    - > Modifiable Line Masters
- *Elementary and Middle School Mathematics: Teaching Developmentally*, 8th Edition (Van de Walle, Karp, and Bay-Williams 2013), pp. 454–469
- *Making Math Meaningful to Canadian Students, K–8* (Small 2008), pp. 548–560

References

4 Numbers. 2015. “24 [Game].” *4 Numbers.* www.4nums.com/apps.

Alberta Education, LearnAlberta.ca. 2011. “Polynomials,” *Planning Guide.* www.learnalberta.ca/content/mepg9/html/pg9_polynomials/index.html.

———. 2012. “Single Variable Linear Inequalities.” *Planning Guide.* www.learnalberta.ca/content/mepg9/html/pg9_singlevariablelinearinequalities/index.html.

———. 2012. *Mathematics Planning Guide, Grade 9: Working with Linear Equations, Patterns and Relations (Variables and Equations)*. Edmonton, AB: Alberta Education. (Available at www.learnalberta.ca/content/mepg9/html/pg9_linearequations/pdf/pg9_linearequations.pdf.)

———. 2015. “Exploring Circle Geometry Properties: Use It.” *Math Interactives.* www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.SHAP&ID2=AB.MATH.JR.SHAP.CIRC&lesson=html/object_interactives/circles/use_it.html.

———. 2015. “Exploring Laws of Exponents: Use It.” *Math Interactives*. www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.NUMB&ID2=AB.MATH.JR.NUMB.EXPO&lesson=html/object_interactives/exponent_laws/use_it.html.

———. 2015. “Exploring Laws of Exponents: Explore It,” *Math Interactives*. www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.NUMB&ID2=AB.MATH.JR.NUMB.EXPO&lesson=html/object_interactives/exponent_laws/explore_it.html.

———. 2015. “Exploring Order of Operations: Use It,” *Math Interactives.* www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.NUMB&ID2=AB.MATH.JR.NUMB.INTE&lesson=html/object_interactives/order_of_operations/use_it.html.

———. 2015. “Point-Circle,” *LearnAlberta.ca.* www.learnalberta.ca/content/meda/html/pointcircle/index.html.

American Association for the Advancement of Science [AAAS-Benchmarks]. 1993. *Benchmark for Science Literacy*. New York, NY: Oxford University Press.

Annenberg Foundation. 2014. “Interactives: Geometry 3D Shapes.” *Annenberg Learner*. www.learner.org/interactives/geometry/index.html.

Armstrong, T. 1999. *Seven Kinds of Smart: Identifying and Developing Your Many Intelligences*. New York, NY: Plume.

Baron, Lorraine, Trevor Brown, Garry Davis, Sharon Jeroski, Susan Ludwig, Sandra Glanville Maurer, Kanwal Neel, Robert Sidley, Shannon Sookochoff, David Sufrin, David Van Bergeyk, Jerrold Wiebe. 2009. *Math Makes Sense 9*. Toronto, ON: Pearson Education Canada. (NSSBB #: 2001644)

———. 2009. *Math Makes Sense 9 ProGuide*. Toronto, ON: Pearson Education Canada. (NSSBB #: 2001645)

Black, Paul, and Dylan Wiliam. 1998. "Inside the Black Box: Raising Standards through Classroom Assessment." *Phi Delta Kappan* 80, No. 2 (October 1998), 139–144, 146–148.

Braining Camp. 2013. *Apps for iPads*. www.brainingcamp.com/product/mobile.html.

British Columbia Ministry of Education. 2000. *The Primary Program: A Framework for Teaching*. Victoria, BC: Province of British Columbia.

Caine, Renate Numella, and Geoffrey Caine. 1991. *Making Connections: Teaching and the Human Brain*. Reston, VA: Association for Supervision and Curriculum Development.

Davies, Anne. 2000. *Making Classroom Assessment Work*. Courtenay, BC: Classroom Connections International, Inc.

Drexel University. 2015. "Fish Simulator [applet]." *The Math Forum @ Drexel*. http://mathforum.org/escotpow/puzzles/fish/applet.html.

Frankenstein, Marilyn. 1995. "Equity in Mathematics Education: Class in the World outside the Class." *New Directions for Equity in Mathematics Education*. Cambridge, MA: Cambridge University Press.

GeoGebra. 2013. GeoGebra. International GeoGebra Institute. Computer Software. www.geogebra.org/cms/en.

Government of Canada. 2015. "Climate." Government of Canada. http://climate.weather.gc.ca/data_index_e.html.

———. 2015. "Tables by Subject." *Statistics Canada*. www40.statcan.gc.ca/z01/cs0002-eng.htm.

———. 2015. *Statistics Canada*. www.statcan.gc.ca.

Gutstein, Eric. 2003. "Teaching and Learning Mathematics for Social Justice in an Urban, Latino School." *Journal for Research in Mathematics Education* 34, No. 1. Reston, VA: National Council of Teachers of Mathematics.

Hawes, Kathy. 2006. "Using Error Analysis to Teach Equation Solving," *Mathematics Teaching in the Middle School,* Volume 12, Issue 5, January 2007, 238–242. Reston, VA: National Council of Teachers of Mathematics.

Herzig, Abbe. 2005. "Connecting Research to Teaching: Goals for Achieving Diversity in Mathematics Classrooms." *Mathematics Teacher*, Volume 99, No. 4. Reston, VA: National Council of Teachers of Mathematics.

Hope, Jack A., Larry Leutzinger, Barbara Reys, and Robert Reys. 1988. *Mental Math in the Primary Grades*. Palo Alto, CA: Dale Seymour Publications.

Hume, Karen. 2011. *Tuned Out: Engaging the 21st Century Learner*. Don Mills, ON: Pearson Education Canada.

Jacobs, Harold R. 2003. *Geometry: Seeing, Doing, Understanding*, Third Edition. New York, NY: W. H. Freeman & Company.

Johnston-Wilder, Sue, and John Mason. 2006. *Developing Thinking in Geometry.* London, UK: Sage Publications.

Ladson-Billings, Gloria. 1997. "It Doesn't Add Up: African American Students' Mathematics Achievement." *Journal for Research in Mathematics Education* 28, No. 6. Reston, VA: National Council of Teachers of Mathematics.

Manitoba Education. 2014. *Grades 9–12 Mathematics: Manitoba Curriculum Framework of Outcomes*. Winnipeg, MB: Government of Manitoba.

National Council of Teachers of Mathematics. 2000. *Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

———. 2005. "Computation, Calculators, and Common Sense: A Position of the National Council of Teachers of Mathematics" (position paper, May 2005). Reston, VA: National Council of Teachers of Mathematics.

———. 2015. "Algebra Tiles." *Illuminations : Resources for Teaching Math*. http://illuminations.nctm.org/Activity.aspx?id=3482.

———. 2015. "Folding Circles: Exploring Circle Theorems through Paper Folding." *Illuminations: Resources for Teaching Math.* http://illuminations.nctm.org/Lesson.aspx?id=3777.

———. 2015. "Circle Template." *Illuminations: Resources for Teaching Math*. http://illuminations.nctm.org/uploadedFiles/Content/Lessons/Resources/9-12/FoldingCircle-CircleTemplate.pdf.

———. 2015. "Order of Operations Bingo," *Illuminations: Resources for Teaching Math*. http://illuminations.nctm.org/Lesson.aspx?id=2583.

New Brunswick Department of Education. 2010. *Mathematics Grade 9 Curriculum*. Fredericton, NB: New Brunswick Department of Education.

Newfoundland and Labrador Department of Education. 2014. *Mathematics: Grade 9 Curriculum Guide*. St. John's, NF: Government of Newfoundland and Labrador.

Nova Scotia Department of Education. 2010. *Gifted Education and Talent Development*. Halifax, NS: Province of Nova Scotia.

OECD Centre for Educational Research and Innovation. 2006. *Formative Assessment: Improving Learning in Secondary Classrooms*. Paris, France: Organization for Economic Co-operation and Development (OECD) Publishing.

Pearson. n.d. *Interactive Math Tools*. Pearson Canada Inc. http://nsvs.ednet.ns.ca/nsps/nsps26/course/view.php?id=3875.

Rubenstein, Rheta N. 2001. "Mental Mathematics beyond the Middle School: Why? What? How?" *Mathematics Teacher*, September 2001, Vol. 94, No. 6. Reston, VA: National Council of Teachers of Mathematics.

Schwartz, Robert M. 1988. "Learning to Learn Vocabulary in Content Area Textbooks," *Journal of Reading,* Volume 32, No. 2, November 1988, 108–118. Hoboken, NJ: Wiley.

Serra, Michael. 2011. *Patty Paper Geometry*. San Francisco, CA: Playing It Smart.

Shaw, J. M., and Cliatt, M. F. P. 1989. "Developing Measurement Sense." In P.R. Trafton (Ed.), *New Directions for Elementary School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Shodor. 2015. "Algebra Four." *Interactivate*. www.shodor.org/interactivate/activities/AlgebraFour.

Small, Marian. 2009. *Making Math Meaningful to Canadian Students, K–8*. Toronto, ON: Nelson Education Ltd.

———. 2013. *Making Math Meaningful to Canadian Students, K–8*, Second Edition. Toronto, ON: Nelson Education Ltd.

———. 2009. *Big Ideas from Dr. Small: Creating a Comfort Zone for Teaching Mathematics, Grades 4–8*. Toronto, ON: Nelson Canada.

———. 2010. *Big Ideas from Dr. Small: Creating a Comfort Zone for Teaching Mathematics, Grades 9–12*. Toronto, ON: Nelson Canada.

Smith, Anthony T., and Robin L. Angotti. 2012. "'Why Are There So Many Words in Math?': Planning for Content-Area Vocabulary Instruction," *Voices from the Middle*, Volume 20, Number 1, September 2012. Urbana, IL: National Council of Teachers of English. (Available at www.ncte.org/library/NCTEFiles/Resources/Journals/VM/0201-sep2012/VM0201Why.pdf.)

Softschools.com. 2015. "Ordering Rational Numbers." *SoftSchools.com*. www.softschools.com/math/rational_numbers/ordering_rational_numbers.

Steen, L.A. (ed.). 1990. *On the Shoulders of Giants: New Approaches to Numeracy*. Washington, DC: National Research Council.

Tate, William F. 1995. "Returning to the Root: A Culturally Relevant Approach to Mathematics Pedagogy." *Theory into Practice* 34, Issue 3. Florence, KY: Taylor & Francis.

Utah State University. 2015. "Algebra Balance Scale [Unnamed]." National Library of Virtual Manipulatives. http://nlvm.usu.edu/en/nav/frames_asid_324_g_4_t_2.html?open=instructions&from=category_g_4_t_2.html.

———. 2015. "Data Analysis and Probability [manipulatives]." *National Library of Virtual Manipulatives.* http://nlvm.usu.edu/en/nav/topic_t_5.html.

Van de Walle, John A., and LouAnn H. Lovin. 2006. *Teaching Student-Centered Mathematics, Grades 5–8,* Volume Three. Boston, MA: Pearson Education, Inc.

Van de Walle, John A., Karen S. Karp, and Jennifer M. Bay-Williams. 2013. *Elementary and Middle School Mathematics: Teaching Developmentally*, 8th Edition. Boston, MA: Pearson Education, Inc.

Western and Northern Canadian Protocol (WNCP) for Collaboration in Education. 2006. *The Common Curriculum Framework for K–9 Mathematics*. Edmonton, AB: Western and Northern Canadian Protocol (WNCP) for Collaboration in Education.