Mathematics at Work 11

Guide

NOVA SCOTIA
NOUVELLE-ÉCOSSE

2014

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Mathematics at Work 12

Prepared by the Department of Education and Early Childhood Development

This is the most recent version of the current curriculum materials as used by teachers in Nova Scotia.

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July 2014

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CURRICULUM GUIDE

NOVA SCOTIA

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Contents

Introduction

Background and Rationale

Mathematics curriculum is shaped by a vision that fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in society. It is essential that the mathematics curriculum reflect current research in mathematics instruction. To achieve this goal, Western and Northern Canadian Protocol's (WNCP) *The Common Curriculum Framework for Grades 10–12 Mathematics* (2008) has been adopted as the basis for the new mathematics curriculum in Nova Scotia.

The Common Curriculum Framework was developed by the seven ministries of education (Alberta, British Columbia, Manitoba, Northwest Territories, Nunavut, Saskatchewan, and Yukon Territory) in collaboration with teachers, administrators, parents, business representatives, post-secondary educators, and others. The framework identifies beliefs about mathematics, general and specific student outcomes, and performance indicators agreed upon by the seven jurisdictions. The outcomes and performance indicators have been adapted for Nova Scotia. This document is based on both national and international research by the WNCP and the National Council of Teachers of Mathematics (NCTM).

There is an emphasis in the Nova Scotia curriculum on particular key concepts at each grade that will result in greater depth of understanding and, ultimately, stronger student achievement. There is also a greater emphasis on number sense and operations concepts in the early grades to ensure students develop a solid foundation in numeracy.

Purpose

This document provides sets of outcomes and performance indicators to be used as a mandated common base for defining mathematics curriculum expectations. This common base should result in consistent student outcomes in mathematics within the province of Nova Scotia. It should also enable easier transfer for students moving within the province or from any jurisdiction that has adopted the WNCP framework. This document is intended to clearly communicate to all education partners across the province the high expectations for students' mathematical learning.

Program Design and Components

Pathways

The *Common Curriculum Framework for Grades 10–12 Mathematics* (Western and Northern Canadian Protocol 2008), on which the Nova Scotia Mathematics 10–12 curriculum is based, includes pathways and topics rather than strands as in *The Common Curriculum Framework for K–9 Mathematics* (Western and Northern Canadian Protocol 2006). In Nova Scotia, four pathways are available: Mathematics Essentials, Mathematics at Work, Mathematics, and Pre-calculus.

Each topic area requires that students develop a conceptual knowledge base and skill set that will be useful to whatever pathway they have chosen. The topics covered within a pathway are meant to build upon previous knowledge and to progress from simple to more complex conceptual understandings.

Goals of Pathways

The goals of all four pathways are to provide prerequisite attitudes, knowledge, skills, and understandings for specific post-secondary programs or direct entry into the work force. All four pathways provide students with mathematical understandings and critical-thinking skills. It is the choice of topics through which those understandings and skills are developed that varies among pathways. When choosing a pathway, students should consider their interests, both current and future. Students, parents, and educators are encouraged to research the admission requirements for post-secondary programs of study as they vary by institution and by year.

Design of Pathways

Each pathway is designed to provide students with the mathematical understandings, rigour, and critical-thinking skills that have been identified for specific post-secondary programs of study and for direct entry into the work force.

The content of the Mathematics Essentials courses was designed in Nova Scotia to fill a specific need for Nova Scotia students. The content of each of the Mathematics at Work, Mathematics, and Pre-calculus pathways has been based on the *Western and Northern Canadian Protocol (WNCP) Consultation with Post-Secondary Institutions, Business and Industry Regarding Their Requirements for High School Mathematics: Final Report on Findings* (Alberta Education 2006) and on consultations with mathematics teachers.

Mathematics Essentials (Graduation)

This pathway is designed to provide students with the development of the skills and understandings required in the workplace, as well as those required for everyday life at home and in the community. Students will become better equipped to deal with mathematics in the real world and will become more confident in their mathematical abilities.

MATHEMATICS AT WORK (GRADUATION)

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into some college programs and for direct entry into the work force. Topics include financial mathematics, algebra, geometry, measurement, number, and statistics and probability.

MATHEMATICS (ACADEMIC)

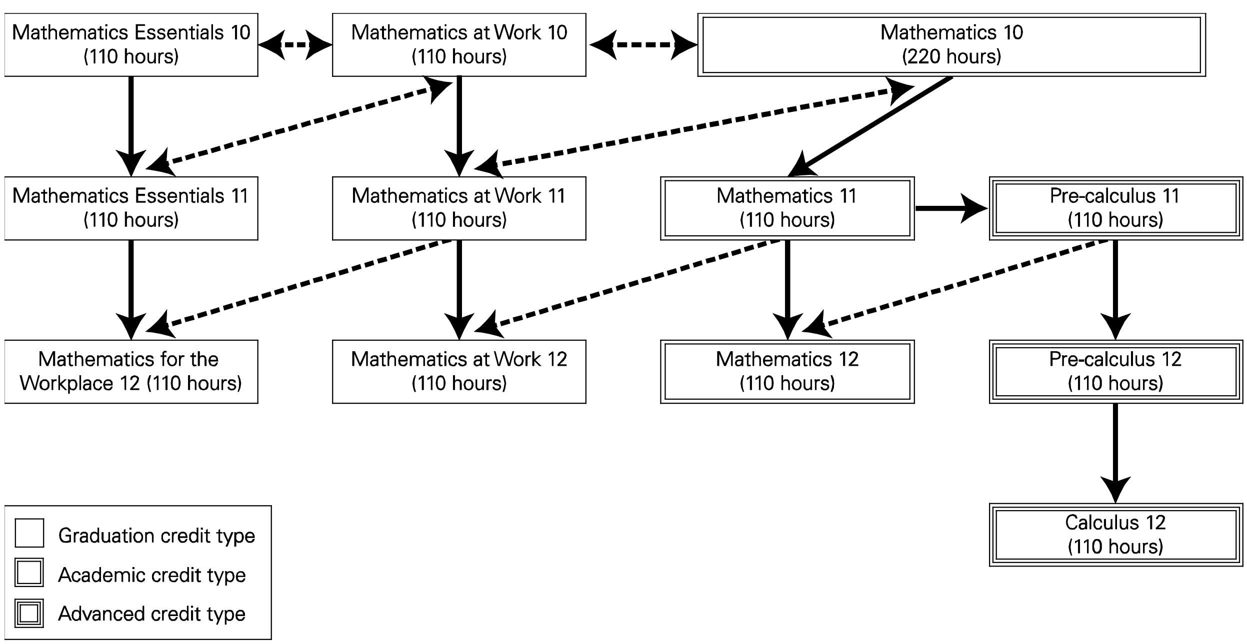
This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for post-secondary studies in programs that require an academic or Pre-calculus mathematics credit. Topics include financial mathematics, geometry, measurement, number, logical reasoning, relations and functions, and statistics and probability. **Note:** After completion of Mathematics 11, students have the choice of an academic or Pre-calculus pathway.

PRE-CALCULUS (ADVANCED)

This pathway is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into post-secondary programs that require the study of theoretical calculus. Topics include algebra and number, measurement, relations and functions, trigonometry, and permutations, combinations, and binomial theorem.

Pathways and Courses

The graphic below summarizes the pathways and courses offered.



Instructional Focus

Each pathway in senior high mathematics pathways is arranged by topics. Students should be engaged in making connections among concepts both within and across topics to make mathematical learning experiences meaningful.

Teachers should consider the following points when planning for instruction and assessment.

- The mathematical processes that are identified with the outcome are intended to help teachers select effective pedagogical approaches for the teaching and learning of the outcome.
- All seven mathematical processes must be integrated throughout teaching and learning approaches and should support the intent of the outcomes.
- Wherever possible, meaningful contexts should be used in examples, problems, and projects.
- Instruction should flow from simple to complex and from concrete to abstract.
- The assessment plan for the course should be a balance of assessment for learning and assessment of learning.

The focus of student learning should be on developing a conceptual and procedural understanding of mathematics. Students' conceptual understanding and procedural understanding must be directly related.

Assessment

Ongoing assessment for learning is essential to effective teaching and learning. Research has shown that assessment for learning (formative assessment) practices produce significant and often substantial learning gains, close achievement gaps, and build students' ability to learn new skills (Black & Wiliam 1998; OECD 2006). Student involvement in assessment promotes learning. Timely and effective teacher feedback and student self-assessment allow students to reflect on and articulate their understanding of mathematical concepts and ideas.

Assessment in the classroom includes

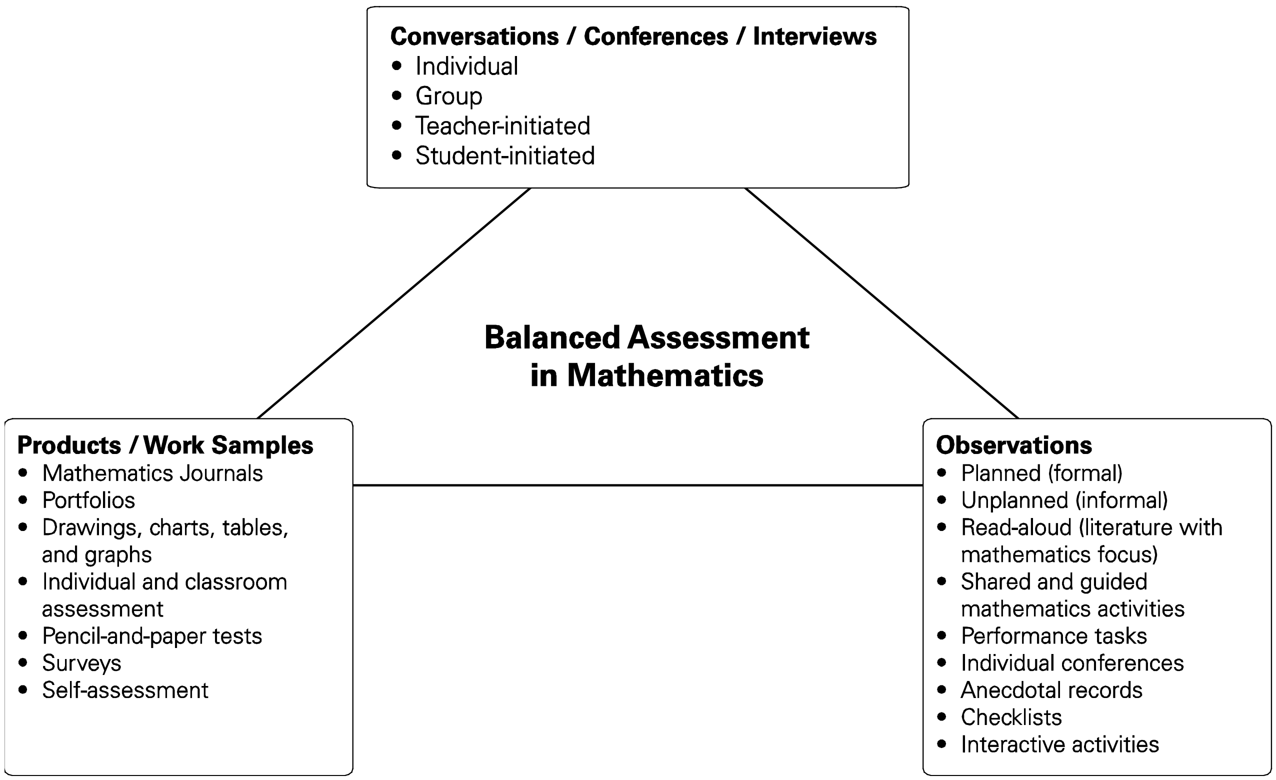
- providing clear goals, targets, and learning outcomes
- using exemplars, rubrics, and models to help clarify outcomes and identify important features of the work
- monitoring progress towards outcomes and providing feedback as necessary
- encouraging self-assessment
- fostering a classroom environment where conversations about learning take place, where students can check their thinking and performance and develop a deeper understanding of their learning

(Davies 2000)

Assessment for learning practices act as the scaffolding for learning, which only then can be measured through assessment of learning (summative assessment). Assessment of learning tracks student progress, informs instructional programming, and aids in decision making. Both forms of assessment are necessary to guide teaching, stimulate learning, and produce achievement gains.

Assessment of student learning should

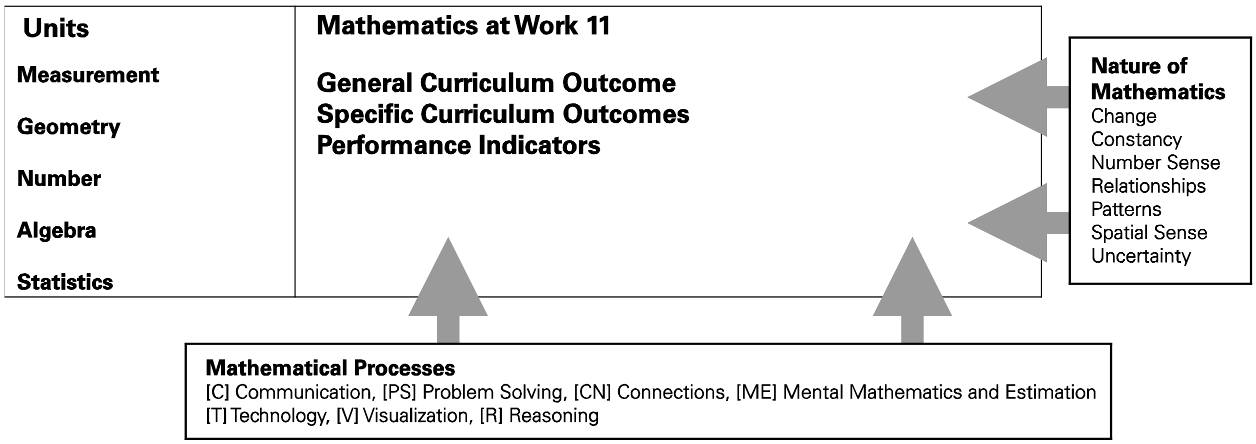
- align with curriculum outcomes
- clearly define criteria for success
- make explicit the expectations for students' performance
- use a wide variety of assessment strategies and tools
- yield useful information to inform instruction



Outcomes

Conceptual Framework for Mathematics 10–12

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



(Adapted with permission from Western and Northern Canadian Protocol, *The Common Curriculum Framework for K–9 Mathematics*, p. 5. All rights reserved.)

Structure of the Mathematics at Work 11 Curriculum

Units

Mathematics at Work 11 comprises five units:

- Measurement (M) (20–25 hours)
- Geometry (G) (25–30 hours)
- Number (N) (15–20 hours)
- Algebra (A) (15–20 hours)
- Statistics (S) (10 hours)

Outcomes and Performance Indicators

The Nova Scotia curriculum is stated in terms of general curriculum outcomes, specific curriculum outcomes, and performance indicators.

General Curriculum Outcomes (GCOs)

General curriculum outcomes are overarching statements about what students are expected to learn in each strand/sub-strand. The GCO for each strand/sub-strand is the same throughout the pathway.

Measurement (M)

Students will be expected to develop spatial sense through direct and indirect measurement.

Geometry (G)

Students will be expected to develop spatial sense.

Number (N)

Students will be expected to develop number sense and critical-thinking skills.

Algebra (A)

Students will be expected to develop algebraic reasoning.

Statistics (S)

Students will be expected to develop statistical reasoning.

Specific Curriculum Outcomes (SCOs) and Performance Indicators

Specific curriculum outcomes are statements that identify specific concepts and related skills underpinned by the understanding and knowledge attained by students as expected for a given grade.

Performance indicators are samples of how students may demonstrate their performance of the goals of a specific curriculum outcome. The range of samples provided is meant to reflect the scope of the SCO. In the SCOs, the word **including** indicates that any ensuing items *must* be addressed to fully achieve the learning outcome. The phrase **such as** indicates that the ensuing items are provided for clarification only and are *not* requirements that must be addressed to fully achieve the learning outcome. The word **and** used in an outcome indicates that both ideas must be addressed to achieve the learning outcome, although not necessarily at the same time or in the same question.

MEASUREMENT (M)

M01 Students will be expected to solve problems that involve SI and imperial units in surface area measurements and verify the solutions.

Performance Indicators

M01.01 Explain, using examples, the difference between volume and surface area.
M01.02 Explain, using examples, including nets, the relationship between area and surface area.
M01.03 Explain how a referent can be used to estimate surface area.
M01.04 Estimate the surface area of a 3-D object.
M01.05 Illustrate, using examples, the effect of dimensional changes on surface area.
M01.06 Solve a contextual problem that involves the surface area of 3-D objects, including spheres, and that requires the manipulation of formulas.

M02 Students will be expected to solve problems that involve SI and imperial units in volume and capacity measurements.

Performance Indicators

M02.01 Explain, using examples, the difference between volume and capacity.
M02.02 Identify and compare referents for volume and capacity measurements in SI and imperial units.
M02.03 Estimate the volume or capacity of a 3-D object or container, using a referent.
M02.04 Identify a situation where a given SI or imperial volume unit would be used.
M02.05 Solve problems that involve the volume of 3-D objects and composite 3-D objects in a variety of contexts.
M02.06 Solve a problem that involves the capacity of containers.
M02.07 Write a given volume expressed as another unit in the same measurement system.
M02.08 Write a given capacity expressed as another unit in the same measurement system.
M02.09 Determine the volume of prisms, cones, cylinders, pyramids, spheres, and composite 3-D objects using a variety of measuring tools such as rulers, tape measures, calipers, and micrometers.
M02.10 Determine the capacity of prisms, cones, pyramids, spheres, and cylinders, using a variety of measuring tools and methods, such as graduated cylinders, measuring cups, measuring spoons, and displacement.
M02.11 Describe the relationship between the volumes of
- cones and cylinders with the same base and height
- pyramids and prisms with the same base and height

M02.12 Illustrate, using examples, the effect of dimensional changes on volume.
M02.13 Solve a contextual problem that involves the volume of a 3-D object, including composite 3-D objects, or the capacity of a container.
M02.14 Solve a contextual problem that involves the volume of a 3-D object and requires the manipulation of formulas.

Geometry (G)

G01 Students will be expected to solve problems that involve two and three right triangles.

Performance Indicators

G01.01 Identify all of the right triangles in a given illustration for a context.
G01.02 Determine if a solution to a problem that involves two or three right triangles is reasonable.
G01.03 Sketch a representation of a given description of a problem in a 2-D or 3-D context.
G01.04 Solve a contextual problem that involves angles of elevation or angles of depression.
G01.05 Solve a contextual problem that involves two or three right triangles, using the primary trigonometric ratios.

G02 Students will be expected to solve problems that involve scale.

Performance Indicators

G02.01 Describe contexts in which a scale representation is used.
G02.02 Determine, using proportional reasoning, the dimensions of an object from a given scale drawing or model.
G02.03 Construct a model of a 3-D object, given the scale.
G02.04 Draw, with and without technology, a scale diagram of a given object.
G02.05 Solve a contextual problem that involves scale.

G03 Students will be expected to model and draw 3-D objects and their views.

Performance Indicators

G03.01 Draw a 2-D representation of a given 3-D object.
G03.02 Draw, using isometric dot paper, a given 3-D object.
G03.03 Draw to scale top, front, and side views of a given 3-D object.
G03.04 Construct a model of a 3-D object, given the top, front, and side views.
G03.05 Draw a 3-D object, given the top, front, and side views.
G03.06 Determine if given views of a 3-D object represent a given object, and explain the reasoning.
G03.07 Identify the point of perspective of a given one-point perspective drawing of a 3-D object.
G03.08 Draw a one-point perspective view of a given 3-D object.

G04 Students will be expected to draw and describe exploded views, component parts, and scale diagrams of simple 3-D objects.

Performance Indicators

(It is intended that the simple 3-D objects come from contexts such as flat-packed furniture or sewing patterns.)

G04.01 Draw the components of a given exploded diagram, and explain their relationship to the original 3-D object.
G04.02 Sketch an exploded view of a 3-D object to represent the components.
G04.03 Draw to scale the components of a 3-D object.
G04.04 Sketch a 2-D representation of a 3-D object, given its exploded view.

NUMBER (N)

N01 Students will be expected to analyze puzzles and games that involve numerical reasoning, using problem-solving strategies.

Performance Indicators

(It is intended that this outcome be integrated throughout the course by using puzzles and games such as cribbage, magic squares, and Kakuro.)

N01.01 Determine, explain, and verify a strategy to solve a puzzle or to win a game; for example,

- guess and check
- look for a pattern
- make a systematic list
- draw or model
- eliminate possibilities
- simplify the original problem
- work backward
- develop alternative approaches

N01.02 Identify and correct errors in a solution to a puzzle or in a strategy for winning a game.

N01.03 Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.

N02 Students will be expected to solve problems that involve personal budgets.

Performance Indicators

N02.01 Identify income and expenses that should be included in a personal budget.

N02.02 Explain considerations that must be made when developing a budget (e.g., prioritizing, recurring and unexpected expenses).

N02.03 Create a personal budget based on given income and expense data.

N02.04 Collect income and expense data and create a budget.

N02.05 Modify a budget to achieve a set of personal goals.

N02.06 Investigate and analyze, with or without technology, “what if …” questions related to personal budgets.

N03 Students will be expected to demonstrate an understanding of compound interest.

Performance Indicators

N03.01 Solve a problem that involves simple interest, given three of the four values in the formula $I = Prt$.

N03.02 Compare simple and compound interest and explain their relationship.

N03.03 Solve, using a formula, a contextual problem that involves compound interest.

N03.04 Explain, using examples, the effect of different compounding periods on calculations of compound interest.

N03.05 Estimate, using the Rule of 72, the time required for a given investment to double in value.

N04 Students will be expected to demonstrate an understanding of financial institution services used to access and manage finances.

Performance Indicators

N04.01 Describe the type of banking services available from various financial institutions, such as online services.
N04.02 Describe the types of accounts available at various financial institutions.
N04.03 Identify the type of account that best meets the needs for a given set of criteria.
N04.04 Identify and explain various automated teller machine (ATM) service charges.
N04.05 Describe the advantages and disadvantages of online banking.
N04.06 Describe the advantages and disadvantages of debit card purchases.
N04.07 Describe ways that ensure the security of personal and financial information (e.g., passwords, encryption, protection of personal identification number [PIN] and other personal identity information).

N05 Students will be expected to demonstrate an understanding of credit options, including credit cards and loans.

Performance Indicators

N05.01 Compare advantages and disadvantages of different types of credit options, including bank and store credit cards, personal loans, lines of credit, and overdraft.
N05.02 Make informed decisions and plans related to the use of credit, such as service charges, interest, payday loans, and sales promotions, and explain the reasoning.
N05.03 Describe strategies to use credit effectively, such as negotiating interest rates, planning payment timelines, reducing accumulated debt, and timing purchases.
N05.04 Compare credit card options from various companies and financial institutions.
N05.05 Solve a contextual problem that involves credit cards or loans.
N05.06 Solve a contextual problem that involves credit linked to sales promotions.

ALGEBRA (A)

A01 Students will be expected to solve problems that require the manipulation and application of formulas related to

- volume and capacity
- surface area
- slope and rate of change
- simple interest
- finance charges

Performance Indicators

A01.01 Solve a contextual problem involving the application of a formula that does not require manipulation.
A01.02 Solve a contextual problem involving the application of a formula that requires manipulation.
A01.03 Explain and verify why different forms of the same formula are equivalent.
A01.04 Describe, using examples, how a given formula is used in a trade or an occupation.
A01.05 Create and solve a contextual problem that involves a formula.
A01.06 Identify and correct errors in a solution to a problem that involves a formula.

A02 Students will be expected to demonstrate an understanding of slope
- as rise over run
- as rate of change
- by solving problems

Performance Indicators

A02.01 Describe contexts that involve slope (e.g., ramps, roofs, road grade, flow rates within a tube skateboard parks, ski hills).

A02.02 Explain, using diagrams, the difference between two given slopes (e.g., a 3:1 and a 1:3 roof pitch), and describe the implications.

A02.03 Describe the conditions under which a slope will be either 0 or undefined.

A02.04 Explain, using examples and illustrations, slope as rise over run.

A02.05 Verify that the slope of an object, such as a ramp or a roof, is constant.

A02.06 Explain, using illustrations, the relationship between slope and angle of elevation (e.g., for a ramp with a slope of 7:100, the angle of elevation is approximately 4°).

A02.07 Explain the implications, such as safety and functionality, of different slopes in a given context.

A02.08 Explain, using examples and illustrations, slope as rate of change.

A02.09 Solve a contextual problem that involves slope or rate of change.

A03 Students will be expected to solve problems by applying proportional reasoning and unit analysis.

Performance Indicators

A03.01 Explain the process of unit analysis used to solve a problem (e.g., given kmh and time in hours, determine how many kilometres; given revolutions per minute, determine the number of seconds per revolution).

A03.02 Solve a problem, using unit analysis.

A03.03 Explain, using an example, how unit analysis and proportional reasoning are related (e.g., to change kmh to km/min., multiply by 1 h/60 min. because hours and minutes are proportional [constant relationship]).

A03.04 Solve a problem within and between systems using proportions or tables (e.g., km to m or kmh to ft./sec.).

STATISTICS (S)

S01 Students will be expected to solve problems that involve creating and interpreting graphs, including bar graphs, histograms, line graphs, and circle graphs.

Performance Indicators

S01.01 Determine the possible graphs that can be used to represent a given data set and explain the advantages and disadvantages of each.

S01.02 Create, with and without technology, a graph to represent a given data set.

S01.03 Describe the trends in the graph of a given data set.

S01.04 Interpolate and extrapolate values from a given graph.

S01.05 Explain, using examples, how the same graph can be used to justify more than one conclusion.

S01.06 Explain, using examples, how different graphic representations of the same data set can be used to emphasize a point of view.

S01.07 Solve a contextual problem that involves the interpretation of a graph.

Mathematical Processes

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and encourage lifelong learning in mathematics.

Students are expected to
- communicate in order to learn and express their understanding of mathematics (Communication [C])
- connect mathematical ideas to other concepts in mathematics, to everyday experiences, and to other disciplines (Connections [CN])
- demonstrate fluency with mental mathematics and estimation (Mental Mathematics and Estimation [ME])
- develop and apply new mathematical knowledge through problem solving (Problem Solving [PS])
- develop mathematical reasoning (Reasoning [R])
- select and use technologies as tools for learning and solving problems (Technology [T])
- develop visualization skills to assist in processing information, making connections, and solving problems (Visualization [V])

The Nova Scotia curriculum incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning. The key to these process standards is presented in a box, as shown below, with each specific outcome within the units.

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Communication [C]

Students need opportunities to read about, represent, view, write about, listen to, and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics. Communication is important in clarifying, reinforcing, and modifying ideas, knowledge, attitudes, and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics.

Students also need to communicate their learning using mathematical terminology. Communication can help students make connections between and among the different representational modes—contextual, concrete, pictorial, linguistic/verbal, written and symbolic—of mathematical ideas. Students must communicate *daily* about their mathematics learning. This enables them to reflect, to validate, and to clarify their thinking and provides teachers with insight into students' interpretations of mathematical meanings and ideas.

Emerging technologies enable students to engage in communication beyond the traditional classroom to gather data and share mathematical ideas.

Problem Solving [PS]

Problem solving is one of the key processes and foundations within the field of mathematics. Learning through problem solving should be the focus of mathematics at all grade levels. Students develop a true understanding of mathematical concepts and procedures when they solve problems in meaningful contexts.

When students encounter new situations and respond to questions of the type, How would you ... ? or How could you ... ?, the problem-solving approach is being modeled. Students develop their own problem-solving strategies by listening to, discussing, and trying different strategies.

In order for an activity to be problem-solving based, it must ask students to determine a way to get from what is known to what is sought. If students have already been given ways to solve the problem, it is not a problem, but practice. Students should not know the answer immediately. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement. Students will be engaged if the problems relate to their lives, cultures, interests, families, or current events.

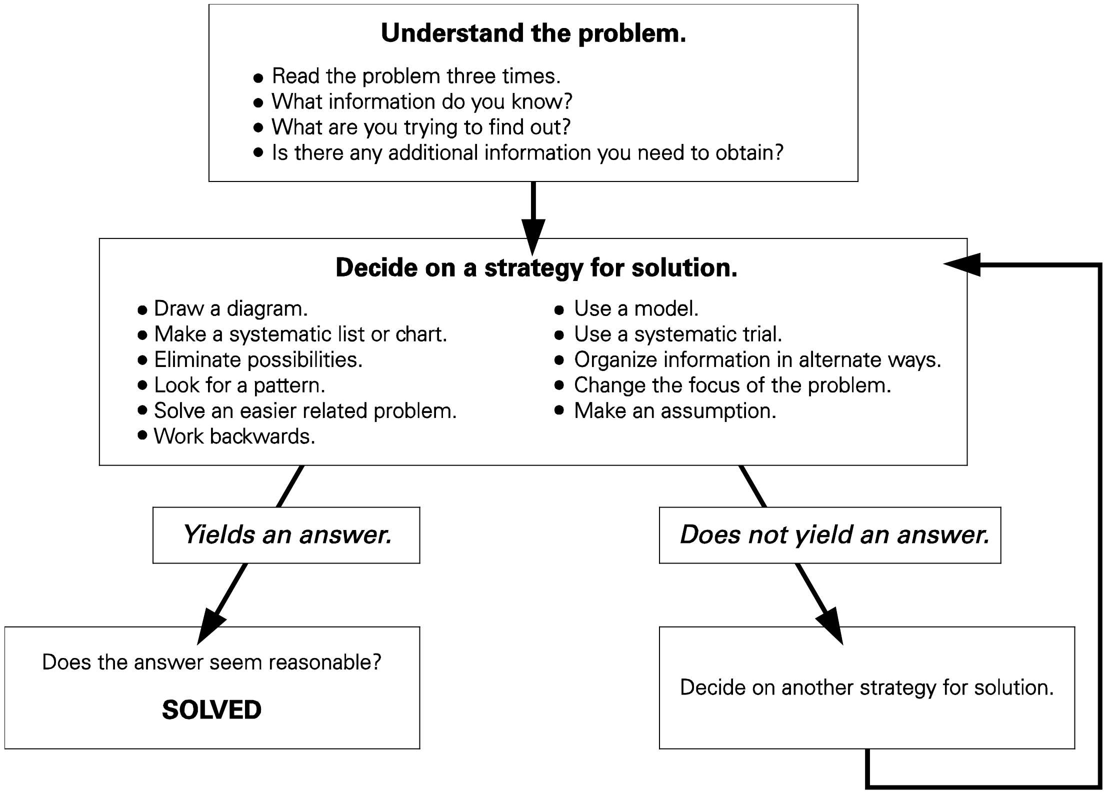
Both conceptual understanding and student engagement are fundamental in molding students' willingness to persevere in future problem-solving tasks. Problems are not just simple computations embedded in a story, nor are they contrived. They are tasks that are rich and open-ended, so there may be more than one way of arriving at a solution or there may be multiple answers. Good problems should allow for every student in the class to demonstrate their knowledge, skill, or understanding. Problem solving can vary from being an individual task to a class (or beyond) undertaking.

In a mathematics class, there are two distinct types of problem solving: solving contextual problems outside of mathematics and solving mathematical problems. Finding the maximum profit given manufacturing constraints is an example of a contextual problem, while seeking and developing a general formula to solve a quadratic equation is an example of a mathematical problem.

Problem solving can also be considered in terms of engaging students in both inductive- and deductive-reasoning strategies. As students make sense of the problem, they will be creating conjectures and looking for patterns that they may be able to generalize. This part of the problem-solving process often involves inductive reasoning. As students use approaches to solving the problem, they often move into mathematical reasoning that is deductive in nature. It is crucial that students be encouraged to engage in both types of reasoning and be given the opportunity to consider the approaches and strategies used by others in solving similar problems.

Problem solving is a powerful teaching tool that fosters multiple, creative, and innovative solutions. Creating an environment where students openly look for, and engage in finding, a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive mathematical risk-takers.

A possible flow chart to share with students is as follows:



Connections [CN]

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to one another or to real-world phenomena, students can begin to view mathematics as useful, relevant, and integrated. Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged. The brain is constantly looking for and making connections.

"Because the learner is constantly searching for connections on many levels, educators need to *orchestrate the experiences* from which learners extract understanding. … *Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching*." (Caine and Caine 1991, 5).

Mathematics should be viewed as an integrated whole rather than as the study of separate strands or units. Connections must also be made between and among the different representational modes—contextual, concrete, pictorial, linguistic/verbal, and symbolic. The process of making connections, in turn, facilitates learning. Concepts and skills should also be connected to everyday situations and other curricular areas. For example, when developing literacy skills students learn to make text-to-world, text-to-text, and text-to-self connections. Students can also make connections to make mathematics come alive through math-to-world, math-to-math, and math-to-self connections.

Mental Mathematics and Estimation [ME]

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external aids.

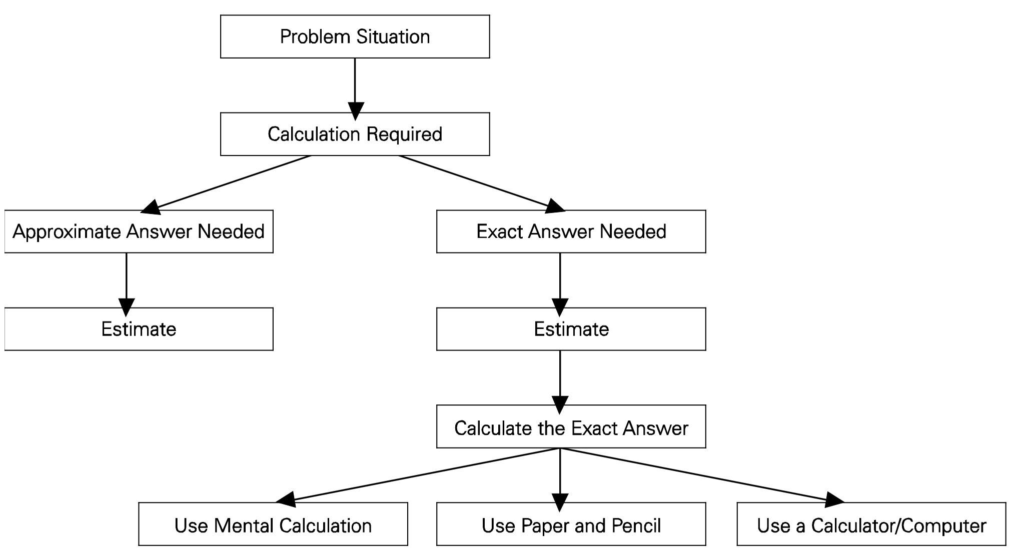
Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy and flexibility.

"Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental math." (National Council of Teachers of Mathematics, May 2005).

Students proficient with mental mathematics "become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers, and are more able to use multiple approaches to problem solving." (Rubenstein 2001) Mental mathematics "provides a cornerstone for all estimation processes, offering a variety of alternative algorithms and nonstandard techniques for finding answers." (Hope 1988, v)

Estimation is a strategy for determining approximate values or quantities, usually by referring to benchmarks or using referents, or for determining the reasonableness of calculated values. Students need to know how, when, and what strategy to use when estimating. Estimation is used to make mathematical judgments and develop useful, efficient strategies for dealing with situations in daily life. When estimating, students need to learn which strategy to use and how to use it.

Students need to develop both mental mathematics and estimation skills through context and not in isolation so they are able to apply them to solve problems. Whenever a problem requires a calculation, students should follow the decision-making process as illustrated below.



The skill of estimation requires a sound knowledge of mental mathematics. Both are necessary to many everyday experiences, and students should be provided with frequent opportunities to practise these skills.

Technology [T]

Technology can be effectively used to contribute to and support the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures, and solve problems.

Calculators, computers, and other technologies can be used to

- explore and represent mathematical relationships and patterns in a variety of ways
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- increase the focus on conceptual understanding by decreasing the time spent on repetitive procedures
- reinforce the learning of foundational concepts
- develop personal procedures for mathematical operations
- simulate situations
- develop number and spatial sense
- generate and test inductive conjectures

Technology contributes to a learning environment in which the curiosity of students can lead to rich mathematical discoveries at all grade levels. The use of technology should not replace mathematical understanding. Instead, technology should be used as one of a variety of approaches and tools for creating mathematical understanding.

Visualization [V]

Visualization "involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world." (Armstrong 1993, 10). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them. Visual images and visual reasoning are important components of number, spatial, and measurement sense. Number visualization occurs when students create mental representations of numbers. Being able to create, interpret and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and spatial reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure, when to estimate and to know several estimation strategies. (Shaw and Cliatt 1989, 150)

Visualization is fostered through the use of concrete materials, technology, and a variety of visual representations. It is through visualization that abstract concepts can be understood by the student. Visualization is a foundation to the development of abstract understanding, confidence, and fluency.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. Questions that challenge students to think, analyze, and synthesize help them develop an understanding of mathematics. All students need to be challenged to answer questions such as, Why do you believe that's true/correct? or What would happen if ...

Mathematical experiences provide opportunities for students to engage in inductive and deductive reasoning. Students use inductive reasoning when they explore and record results, analyze observations, make generalizations from patterns, and test these generalizations. Students use deductive reasoning when they reach new conclusions based upon the application of what is already known or assumed to be true.

Mathematics reasoning involves informal thinking, conjecturing, and validating—these help students understand that mathematics makes sense. Students are encouraged to justify, in a variety of ways, their solutions, thinking processes, and hypotheses. In fact, good reasoning is as important as finding correct answers. The thinking skills developed by focusing on reasoning can be used in daily life in a wide variety of contexts and disciplines.

Nature of Mathematics

Mathematics is one way of trying to understand, interpret, and describe our world. There are a number of components that define the nature of mathematics, and these are woven throughout this document. These components include change, constancy, number sense, relationships, patterns, spatial sense, and uncertainty.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics. Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as

- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen 1990, 184).

Students need to learn that new concepts of mathematics as well as changes to previously learned concepts arise from a need to describe and understand something new. Integers, decimals, fractions, irrational numbers, and complex numbers emerge as students engage in exploring new situations that cannot be effectively described or analyzed using whole numbers.

Students best experience change to their understanding of mathematical concepts as a result of mathematical play.

Constancy

Different aspects of constancy are described by the terms **stability**, **conservation**, **equilibrium**, **steady state** and **symmetry** (AAAS–Benchmarks 1993, 270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include the following:

- The area of a rectangular region is the same regardless of the methods used to determine the solution.
- The sum of the interior angles of any triangle is 180°.
- The theoretical probability of flipping a coin and getting heads is 0.5.
- Lines with constant slope.

Number Sense

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy. (British Columbia Ministry of Education, 2000, 146) Continuing to foster number sense is fundamental to growth of mathematical understanding.

A true sense of number goes well beyond the skills of simply counting, memorizing facts, and the situational rote use of algorithms. Students with strong number sense are able to judge the reasonableness of a solution, describe relationships between different types of numbers, compare quantities, and work with different representations of the same number to develop a deeper conceptual understanding of mathematics.

Number sense develops when students connect numbers to real-life experiences and use benchmarks and referents. This results in students who are computationally fluent, flexible with numbers, and have intuition about numbers. The evolving number sense typically comes as a by-product of learning rather than through direct instruction. However, number sense can be developed by providing rich mathematical tasks that allow students to make connections.

Relationships

Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects, variables, and concepts. The search for possible relationships involves collecting and analyzing data, analyzing patterns and describing possible relationships visually, symbolically, orally or in written form.

Patterns

Mathematics is about recognizing, describing, and working with numerical and non-numerical patterns. Patterns exist in all of the mathematical topics, and it is through the study of patterns that students can make strong connections between concepts in the same and different topics.

Working with patterns also enables students to make connections beyond mathematics. The ability to analyze patterns contributes to how students understand their environment. Patterns may be represented in concrete, visual, auditory, or symbolic form. Students should develop fluency in moving from one representation to another.

Students need to learn to recognize, extend, create, and apply mathematical patterns. This understanding of patterns allows students to make predictions and justify their reasoning when solving problems. Learning to work with patterns helps develop students' algebraic thinking, which is foundational for working with more abstract mathematics.

Spatial Sense

Spatial sense involves the representation and manipulation of 3-D objects and 2-D shapes. It enables students to reason and interpret among 3-D and 2-D representations.

Spatial sense is developed through a variety of experiences with visual and concrete models. It offers a way to interpret and reflect on the physical environment and its 3-D or 2-D representations.

Some problems involve attaching numerals and appropriate units (measurement) to dimensions of objects. Spatial sense allows students to make predictions about the results of changing these dimensions.

Spatial sense is also critical in students' understanding of the relationship between the equations and graphs of functions and, ultimately, in understanding how both equations and graphs can be used to represent physical situations.

Uncertainty

In mathematics, interpretations of data and the predictions made from data may lack certainty. Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty. The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation. Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately. This language must be used effectively and correctly to convey valuable messages.

Curriculum Document Format

This guide presents the mathematics curriculum so that a teacher may readily view the scope of the outcomes that students are expected to achieve during that year. Teachers are encouraged, however, to examine what comes before and what follows, to better understand how students' learning at a particular grade level is part of a bigger picture of concept and skill development.

The order of presentation in no way assumes or prescribes a preferred order of presentation in the classroom, but simply lays out the specific curriculum outcomes in relation to the overarching general curriculum outcomes.

When a specific curriculum outcome is introduced, it is followed by the mathematical processes and performance indicators for that outcome. A scope and sequence is then provided, which relates the SCO to previous and next grade SCOs. Also for each SCO, there are background information, assessment strategies, suggested instructional strategies, and suggested models and manipulatives, mathematical vocabulary, and resource notes. For each section, the guiding questions should be used to help with unit and lesson preparation.

SCO

Mathematical Processes

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections
[ME] Mental Mathematics and Estimation		
[T] Technology	**[V]** Visualization	**[R]** Reasoning

Performance Indicators

Describes observable indicators of whether students have met the specific outcome.

Scope and Sequence

Previous grade or course SCOs	Current course SCO	Following grade or course SCOs

Background

Describes the "big ideas" to be learned and how they relate to work in previous grade and work in subsequent courses.

Assessment, Teaching, and Learning

Assessment Strategies

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Sample tasks that can be used to determine students' prior knowledge.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Some suggestions for specific activities and questions that can be used for both instruction and assessment.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcome and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Suggestions for general approaches and strategies suggested for teaching this outcome.

Guiding Question

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

SUGGESTED MODELS AND MANIPULATIVES

MATHEMATICAL VOCABULARY

Resources/Notes

Contexts for Learning and Teaching

Beliefs about Students and Mathematics Learning

“Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.” (National Council of Teachers of Mathematics 2000, 20).

- The Nova Scotia mathematics curriculum is based upon several key assumptions or beliefs about mathematics learning that have grown out of research and practice. These beliefs include the following:
- Mathematics learning is an active and constructive process.
- Learning is most effective when standards of expectation are made clear with ongoing assessment and feedback.
- Learners are individuals who bring a wide range of prior knowledge and experiences and who learn via various styles and at different rates.
- Leaning is most effective when standards of expectation are made clear with ongoing assessment and feedback.

Students are curious, active learners with individual interests, abilities, and needs. They come to classrooms with varying knowledge, life experiences, and backgrounds. A key component in successfully developing numeracy is making connections to these backgrounds and experiences.

Students construct their understanding of mathematics by developing meaning based on a variety of learning experiences. This meaning is best constructed when learners encounter mathematical experiences that proceed from simple to complex and from the concrete to the abstract. The use of manipulatives, visuals, and a variety of pedagogical approaches can address the diversity of learning styles and developmental stages of students. At all levels of understanding, students benefit from working with a variety of materials, tools, and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions also provide essential links among concrete, pictorial, and symbolic representations of mathematics. The learning environment should value, respect, and address all students’ experiences and ways of thinking so that students are comfortable taking intellectual risks, asking questions, and posing conjectures. Students need to explore mathematics through solving problems in order to continue developing personal strategies and mathematical literacy. It is important to realize that it is acceptable to solve problems in different ways and that solutions may vary depending upon how the problem is understood.

Goals of Mathematics Education

The main goals of mathematics education are to prepare students to
- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- make connections between mathematics and its applications
- become mathematically literate adults, using mathematics to contribute to society
- commit themselves to lifelong learning

Students who have met these goals will

- gain understanding and appreciation of the contributions of mathematics as a science, a philosophy, and an art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity about mathematics and situations involving mathematics

In order to assist students in attaining these goals, teachers are encouraged to develop a classroom atmosphere that fosters conceptual understanding. Students should be encouraged to

- take risks
- think and reflect independently
- share and communicate mathematical understanding
- solve problems in individual and group projects
- pursue greater understanding of mathematics
- appreciate the value of mathematics throughout history

Opportunities for Success

A positive attitude has a profound effect on learning. Environments that create a sense of belonging, encourage risk taking, and provide opportunities for student success help develop and maintain positive attitudes and self-confidence. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations, and engage in reflective practices.

To experience success, students must be taught to set achievable goals and assess their progress as they work toward these goals. Striving toward success and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting the setting and assessing of personal goals.

Engaging All Learners

"No matter how engagement is defined or which dimension is considered, research confirms this truism of education: *The more engaged you are, the more you will learn."* (Hume 2011, 6)

Student engagement is at the core of learning. Engagement in learning occurs when students are provided with opportunities to become more invested in their learning. This is critical for teachers to take into account when planning and implementing instruction. Effective instruction engages, embraces, and supports all learners through a range of learning experiences that are both age and developmentally appropriate.

This curriculum is designed to provide learning opportunities that are equitable, accessible, and inclusive of the many facets of diversity represented in today's classrooms. When teachers know their students as individual learners and as individual people, their students are more likely to be motivated to learn, persist in challenging situations, and apply reflective practices.

Supportive Learning Environments

A supportive and positive learning environment has a profound effect on students' learning. Students need to feel physically, socially, emotionally, and culturally safe in order to take risks with their learning. In classrooms where students feel a sense of belonging, see their teachers' passion for learning and teaching, are encouraged to actively participate, and are challenged appropriately, they are more likely to be successful.

Teachers recognize that not all students progress at the same pace nor are they equally positioned in terms of their prior knowledge of particular concepts, skills, and learning outcomes. Teachers are able to create more equitable access to learning when

- instruction and assessment are flexible and offer multiple means of representation
- students have options to engage in learning through multiple ways
- students can express their knowledge, skills, and understanding in multiple ways

(Hall, Meyer, and Rose 2012)

In a supportive learning environment, teachers plan learning experiences that support *each* student's ability to achieve curriculum outcomes. Teachers use a variety of effective instructional approaches that help students to succeed, such as

- providing a range of learning opportunities that build on individual strengths and prior knowledge
- providing all students with equitable access to appropriate learning strategies, resources, and technology
- involving students in the creation of criteria for assessment and evaluation
- engaging and challenging students through inquiry-based practices
- verbalizing their own thinking to model comprehension strategies and new learning
- balancing individual, small-group, and whole-class learning experiences
- scaffolding instruction and assignments as needed and giving frequent and meaningful descriptive feedback throughout the learning process
- integrating "blended learning" opportunities by including an online environment that extends learning beyond the physical classroom
- encouraging students to take time and to persevere, when appropriate, in order to achieve a particular learning outcome

Multiple Ways of Learning

"Advances in neuroscience and education research over the past 40 years have reshaped our understanding of the learning brain. One of the clearest and most important revelations stemming from brain research is that there is no such thing as a 'regular student.'" (Hall, Meyer, and Rose 2012, 2) Teachers who know their students well are aware of students' individual learning differences and use this understanding to inform instruction and assessment decisions.

The ways in which students make sense of and demonstrate learning vary widely. Individual students tend to have a natural inclination toward one or a few learning styles. Teachers are often able to detect learning strengths and styles through observation and through conversation with students. Teachers can also get a sense of learning styles through an awareness of students' personal interests and talents. Instruction and assessment practices that are designed to account for multiple learning styles create greater opportunities for all students to succeed.

While multiple learning styles are addressed in the classroom, the three most commonly identified are:

- auditory (such as listening to teacher-modelled think-aloud strategies or participating in peer discussion)
- kinesthetic (such as examining artifacts or problem-solving using tools or manipulatives)
- visual (such as reading print and visual texts or viewing video clips)

For additional information, refer to *Frames of Mind: The Theory of Multiple Intelligences* (Gardner 2007) and *How to Differentiate Instruction in Mixed-Ability Classrooms* (Tomlinson 2001).

A Gender-Inclusive Curriculum and Classroom

It is important that the curriculum and classroom climate respect the experiences and values of all students and that learning resources and instructional practices are not gender-biased. Teachers promote gender equity and inclusion in their classrooms when they

- articulate equally high expectations for all students
- provide equal opportunity for input and response from all students
- model gender-fair language, inclusive practices, and respectful listening in their interactions with students
- identify and openly address societal biases with respect to gender and sexual identity

Valuing Diversity: Teaching with Cultural Proficiency

"Instruction that is embedded in socially meaningful contexts, and tasks that are meaningful and relevant to the lives of students, will engage students in high-level problem-solving and reasoning and enhance students' engagement (Frankenstein 1995; Gutstein 2003; Ladson-Billings 1997; Tate 1995)." (Herzig 2005)

Teachers appreciate that students have diverse life and cultural experiences and that individual students bring different prior knowledge to their learning. Teachers can build upon their knowledge of their students as individuals, value their prior experiences, and respond by using a variety of culturally proficient instruction and assessment practices in order to make learning more engaging, relevant, and accessible for all students. For additional information, refer to *Racial Equity Policy* (Nova Scotia Department of Education 2002) and *Racial Equity / Cultural Proficiency Framework* (Nova Scotia Department of Education 2011).

Students with Language, Communication, and Learning Challenges

Today's classrooms include students who have diverse language backgrounds, abilities, levels of development, and learning challenges. By observing and interacting with students and by conversing with students and/or their families, teachers gain deeper insights into the student as a learner. Teachers can use this awareness to identify and respond to areas where students may need additional support to achieve their learning goals. For students who are experiencing difficulties, it is important that teachers distinguish between those students for whom curriculum content is challenging and those for whom language-based factors are at the root of apparent academic difficulties. Students who are learning English as an additional language may require individual support, particularly in language-based subject areas, while they become more proficient in their English language skills. Teachers understand that many students who appear to be disengaged may be experiencing difficult life or family circumstances, mental health challenges, or low self-esteem, resulting in a loss of confidence that affects their engagement in learning. A caring, supportive teacher demonstrates belief in the students' abilities to learn and uses the students' strengths to create small successes that help nurture engagement in learning and provide a sense of hope.

STUDENTS WHO DEMONSTRATE EXCEPTIONAL TALENTS AND GIFTEDNESS

Modern conceptions of giftedness recognize diversity, multiple forms of giftedness, and inclusivity. Some talents are easily observable in the classroom because they are already well developed and students have opportunities to express them in the curricular and extracurricular activities commonly offered in schools. Other talents only develop if students are exposed to many and various domains and hands-on experiences. Twenty-first century learning supports the thinking that most students are more engaged when learning activities are problem-centred, inquiry-based, and open-ended. Talented and gifted students usually thrive when such learning activities are present. Learning experiences may be enriched by offering a range of activities and resources that require increased cognitive demand and higher-level thinking with different degrees of complexity and abstraction. Teachers can provide further challenges and enhance learning by adjusting the pace of instruction and the breadth and depth of concepts being explored. For additional information, refer to *Gifted Education and Talent Development* (Nova Scotia Department of Education 2010).

Connections across the Curriculum

The teacher should take advantage of the various opportunities available to integrate mathematics and other subjects. This integration not only serves to show students how mathematics is used in daily life, but it helps strengthen the students' understanding of mathematical concepts and provides them with opportunities to practise mathematical skills. There are many possibilities for integrating mathematics in business education, career education, literacy, music, physical education, science, social studies, technology education, and visual arts.

Measurement
20–25 hours

GCO: Students will be expected to develop spatial sense through direct and indirect measurement.

SCO M01 Students will be expected to solve problems that involve SI and imperial units in surface area measurements and verify the solutions. [C, CN, ME, PS, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

M01.01 Explain, using examples, the difference between volume and surface area.
M01.02 Explain, using examples, including nets, the relationship between area and surface area.
M01.03 Explain how a referent can be used to estimate surface area.
M01.04 Estimate the surface area of a 3-D object.
M01.05 Illustrate, using examples, the effect of dimensional changes on surface area.
M01.06 Solve a contextual problem that involves the surface area of 3-D objects, including spheres, and that requires the manipulation of formulas.

Scope and Sequence

Mathematics at Work 10	Mathematics at Work 11	Mathematics at Work 12
M01 Students will be expected to demonstrate an understanding of the International System of Units (SI) by ▪ describing the relationships of the units for length, area, volume, capacity, mass, and temperature ▪ applying strategies to convert SI units to imperial units **M02** Students will be expected to demonstrate an understanding of the imperial system by ▪ describing the relationships of the units for length, area, volume, capacity, mass, and temperature ▪ comparing the American and British imperial units for capacity ▪ applying strategies to convert imperial units to SI units **M03** Students will be expected to solve and verify problems that involve SI and imperial linear measurements, including decimal and fractional measurements.	**M01** Students will be expected to solve problems that involve SI and imperial units in surface area measurements and verify the solutions.	**M01** Students will be expected to demonstrate an understanding of the limitations of measuring instruments including, precision, accuracy, uncertainty, and tolerance, and solve problems.

Mathematics at Work 10	Mathematics at Work 11	Mathematics at Work 12
M04 Students will be expected to solve problems that involve SI and imperial area measurements of regular, composite, and irregular 2-D shapes and 3-D objects, including decimal and fractional measurements, and verify the solutions. **A01** Students will be expected to solve problems that require the manipulation and application of formulas related to perimeter, area, the Pythagorean theorem, primary trigonometric ratios, and income.		

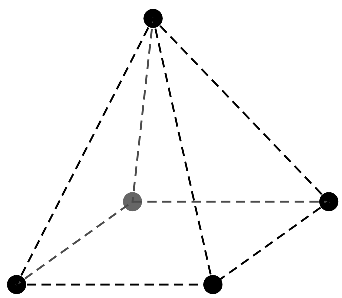
Background

In Mathematics 7, students developed an understanding of 2-D objects and calculated the area of triangles, circles, and parallelograms (7M02). In Mathematics 8, they developed a conceptual understanding of the surface area of 3-D objects, using nets, and then calculated the surface area of right rectangular prisms, right triangular prisms, and right cylinders (8M03, 8M04). In Mathematics 9, students extended their knowledge of surface area to calculating the surface area of composite 3-D objects (9M02). In Mathematics at Work 10, work with surface area was reviewed and extended to include cones (10M04). SI and imperial units were used in estimations and calculations. A review of unit conversions may be necessary.

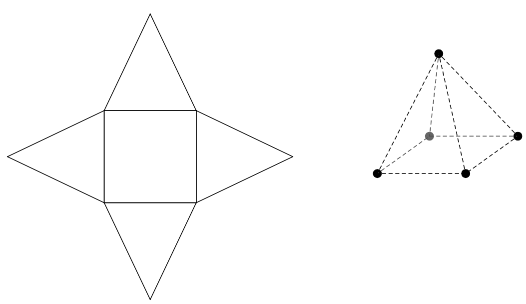
This outcome reinforces previous work completed on finding the surface area of rectangular prisms, triangular prisms, cylinders, and cones. Students will explore, estimate, and calculate the surface area of pyramids and spheres using referents, net diagrams and formulas.

Students have previously investigated the relationship between area and surface area, but a review may be necessary. They have worked with area as the amount of 2-D space that a shape covers, and surface area as the sum of the areas of all faces or surfaces of a solid, measured in square units. Students should be aware that they will need to identify an object’s net and use the appropriate formulas to determine the area of each face. In the beginning, focus should be on right prisms and cylinders. Students have already worked with these 3-D shapes. They may need to be reminded that prisms consist of two parallel congruent faces called bases. A prism is named for its bases.

Students will work with objects composed of rectangles, circles, and triangles. A review of these area formulas may be necessary. They will be introduced to pyramids and examine the relationship between area and surface area of this figure. An example of a net diagram for a square pyramid is below. They should recognize that the surface area is the sum of the areas of the base and four triangular faces.



Students worked with volume in Mathematics 8 (8M02) where they investigated the connection between the area of the base of a given 3-D object and the formula for the volume of the object (8M04). It is important to have discussions about the difference between volume and surface area of 3-D objects, and have students gain further understanding of the distinction between the concepts of surface area and volume through relevant problem-solving activities. To support a better understanding of these concepts, they need to be encouraged to draw diagrams and nets to help them visualize 3-D objects.



The exploration with nets should lead into generalizing formulas and using symbolic representations to calculate surface area. Students have previously used formulas to calculate the surface area of right prisms, right cylinders, and cones. They will now extend this to include calculating the surface area of right pyramids with rectangular bases and spheres.

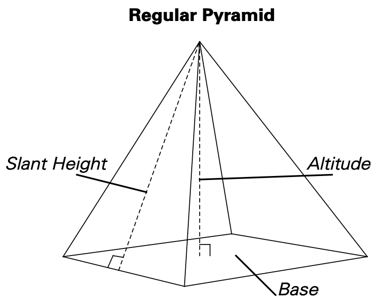
Begin with a review of calculating the surface area of a prism and a cylinder. Calculating the surface area of a prism using its net allows for easy identification of congruent faces, which sometimes avoids the necessity of having to find the area of each face individually. Students may conclude that the surface area of a rectangular prism can be calculated using the formula $SA_{\text{prism}} = 2lw + 2lh + 2wh$. Some students may never use this formula, but continue to find the sum of the areas of all the faces.

Next, revisit the cylinder. The net of a cylinder consists of two circles and a rectangle. The curved surface opens up to form a rectangle. A good way to demonstrate this is to unpeel the label on a can to show it is a rectangle.

$SA_{cylinder}$ = 2 × (area of circle) + (area of rectangle)

$$SA = 2\pi r^2 + 2\pi rh$$

A pyramid is a 3-D figure with a polygon base. The shape of the base determines the name of the pyramid. Students should work with square-based pyramids and be able to recognize the difference between the height of a right pyramid and its slant height. The height refers to the perpendicular height from the apex to the base and the slant height is the altitude of the triangular face.



The surface area of the pyramid is the total area of all five sides: the square base and the four triangles. In addition to prisms and pyramids, students also worked with the surface area of cones in Mathematics at Work 10. Revisit the formula.

$SA_{cone} = \pi r^2 + \pi rs$, where r is the radius of the base and s is the slant height of the cone. In some cases, students will have to apply the Pythagorean theorem to determine the slant height. Finally, students will investigate the surface area of a sphere. They will investigate the connection between the area of a circle and the surface area of a sphere. Students should reach the conclusion that the surface area of a sphere is the area of four circles and is calculated using $4\pi r^2$. The effect of dimensional changes on surface area will also be explored.

Students need to be able to use referents. A referent is an object or idea that can be used to help estimate a measurement. From the earliest introduction to metric units, students have had experience relating non-standard and standard units of measurement. They have previously used referents to estimate the dimensions of a 2-D shape or 3-D object. Students will use a referent to determine approximate dimensions of objects from which they can estimate the surface area. While referents are given as suggestions, students should be encouraged to select their own personal referents. Give students an opportunity to estimate measurements of various items using their referents. Encourage students to analyze their answers to determine if they are reasonable.

This outcome provides a good opportunity for students to make connections between mathematics and real-world situations. Surface area can be used to determine such things as the amount of

- vinyl siding for a new house
- paint needed to cover walls and ceiling
- materials for a cylindrical punching bag
- shingles for a roof

In some situations, surface area will be calculated when all measurements are given, requiring no formula manipulation. Alternatively, students may be given the surface area and asked to find another measurement. In such cases, be selective with the missing dimensions students are asked to find. Avoid cases involving factoring. For a cylinder, for example, provide the surface area and radius and ask students to determine the height.

Students will investigate how increasing or decreasing the dimension(s) of an object changes the surface area. Students can be given several objects to calculate the surface area of and then asked to calculate the new surface area once one of the dimensions is doubled, tripled, or halved. Have students discover why doubling a dimension does not double the surface area.

As an extension, students could explore the effect on surface area of changing all dimensions by the same factor.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- Soma estimates that the top of her school desk is 60 cm^2. Does her estimate seem reasonable? Explain.

- Given various shapes, estimate the area of the shapes using square centimetres and square inches. Overlay the shapes on centimetres and inch grid papers to determine the reasonableness of your estimate.

- A jet is flying at a height of 28 000 ft. How high is this in metres?

- Identify objects in the room that would be approximately
  - 1 cm long
  - 2 m long
  - 3 ft. long

- Compare and contrast the terms **perimeter** and **area**.

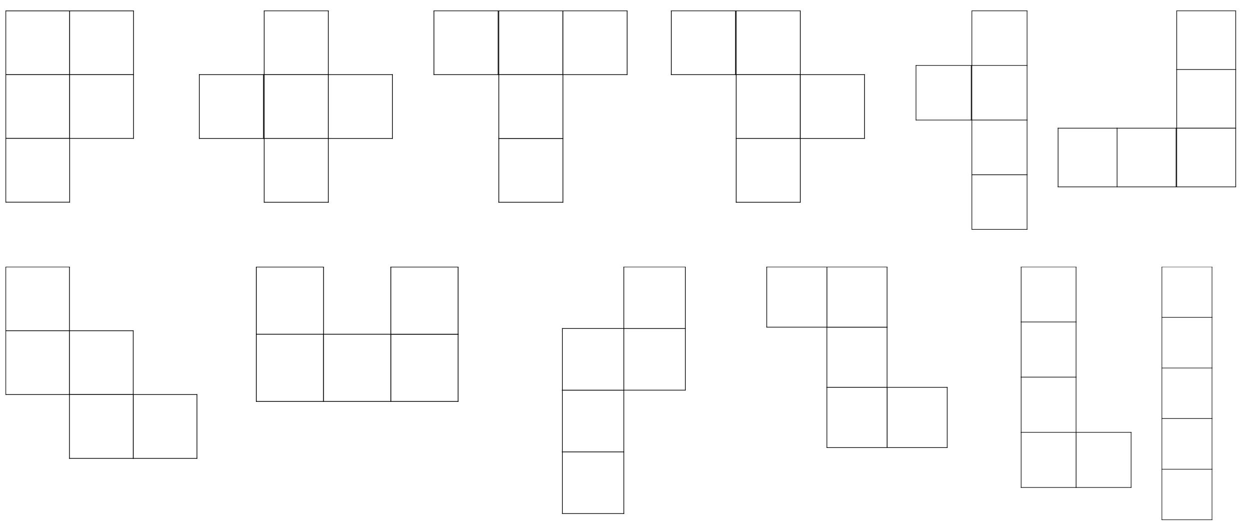
- Convert the following:
  (a) 25 dam = ____ cm
  (b) 130 cm = ____ m
  (c) 3.25 km = ____ cm
  (d) 4 ft. = ____ in.
  (e) 3 mi. = ____ yd.
  (f) 42 cm = ____ in.
  (g) 45 km/h = ____ mph
  (h) 26.2 km = ____ mi.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

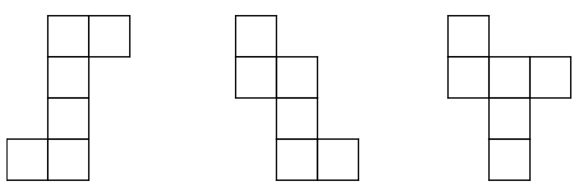
Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- If a cone has no top (like a drinking cup or ice cream cone), how would you find the surface area of the cone? Explain.

- A pentomino is a shape made of five identical squares connected along their sides. There are 12 different pentominoes that can be made from five squares. Determine which of the 12 patterns can be folded to form open cubes.



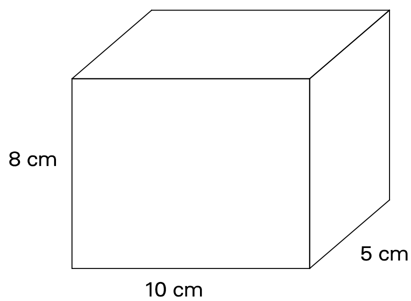
- A hexomino is a shape made of six identical squares connected along their sides. There are 35 different hexominoes that can be made from six squares. Below are three different hexominoes.

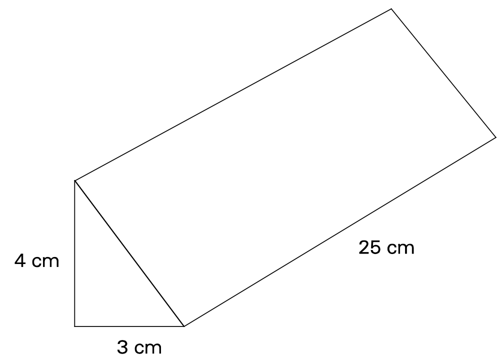


- Create at least three more hexominoes. Using visualization, decide if any of the created hexominoes can be folded to form a closed cube.

- Use SI and imperial referents to estimate the surface area of objects inside or outside the classroom (e.g., teacher’s desk, nearby building, tube slide, basketball). Compare your estimates with those of your classmates.

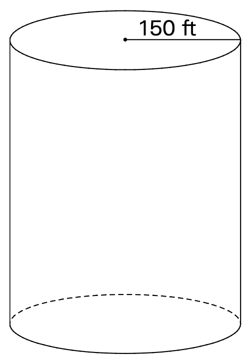
- The owners of a salt factory are trying to choose a box to hold their new brand of salt. They want a box that uses the least amount of cardboard. They are choosing between a rectangular prism and triangular prism as shown. Which box should they choose? Explain.



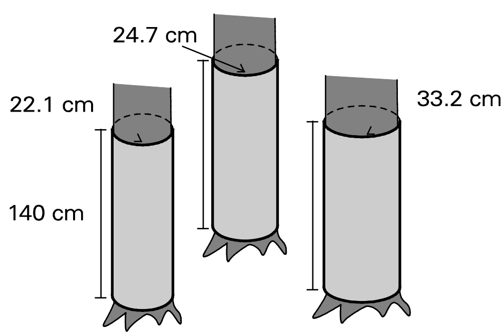


- What fraction of 1 m^2 is a sticky note that is 10 cm × 10 cm?

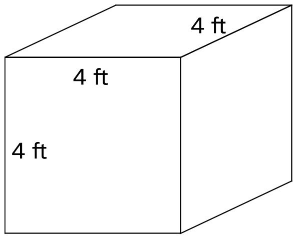
- You are painting a cylindrical oil tank with a radius of 150 ft. If one gallon of paint covers 350 $ft.^2$ and you have 16 gallons of paint, how far up the tank can you paint?



- Brad is purchasing burlap to protect his three apple trees against the cold winter weather. He will wrap the burlap around the bottom 140 cm of each tree trunk. The trees are 22.1 cm, 24.7 cm, and 33.2 cm in circumference. How much burlap will he need?

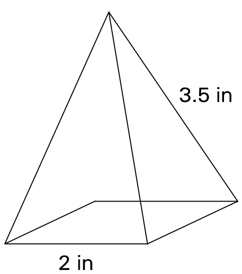


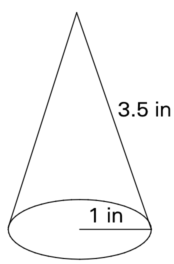
- Luisa builds a shipping crate out of quarter-inch plywood. The crate is a cube with a side dimension of 4 ft.?



(a) What is the surface area of the crate?
(b) She buys plywood in standard sheet sizes of 4 ft. × 8 ft. How many sheets of plywood does she need to build one shipping crate?
(c) She builds a second crate that is one-half the height, but has the same length and width. How many sheets of plywood will she need to build the smaller shipping crate? Explain.

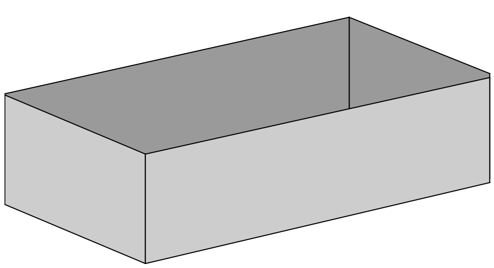
- A toy block manufacturer needs to cover its wooden blocks with a non-toxic paint. One block is a right square pyramid with a base of 2 in. and a slant height of $3\frac{1}{2}$ in. Another is a right cone that has a slant height of $3\frac{1}{2}$ in. and a radius of 1 in.





Answer the following:
(a) Which block requires more paint?
(b) When the blocks rest on their bases, which block is taller? How do you know?

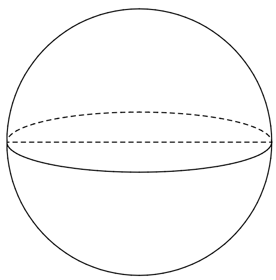
- You are building a child's toy box. The toy box is in the shape of a rectangular prism with no top. It is to be 1 m long and 0.5 m wide. The sides of the toy box will be 0.3 m high. Determine the surface area of the toy box, in square metres.



- Emily is placing a gazing ball in one of her customer's gardens. The ball has a diameter of 2 ft. and will be covered with reflective crystals. One jar of these crystals covers 10 ft.^{2}.
  (a) Estimate the surface area to decide whether one jar of the crystals will cover the ball.
  (b) Calculate the surface area, to the nearest square foot.
  (c) Was your estimate reasonable? Explain.

- A basketball has a diameter of 24.8 cm.
  (a) How much leather is required to cover this ball?
  (b) If one square metre of leather costs $28, how much will it cost to cover the ball?

- The surface area of an official five-pin bowling ball is approximately 459.96 cm^2. Determine the diameter of the bowling ball.



- Answer the following using a sphere with a radius of 15 cm.
  (a) Predict how much the surface area increases if the radius changes by a factor of 2.
  (b) Calculate the change in the surface area, to the nearest square centimetre.
  (c) How accurate was your prediction?

- Suppose that all of the dimensions of a square-based prism are doubled. How does that affect its surface area?

- If the diameter of a sphere triples, what happens to its surface area?

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should be used to inform instruction.

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?

- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?
- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Suggested Learning Tasks

Consider the following sample instructional strategies when planning lessons.

- To visualize nets, provide students with an opportunity to deconstruct rectangular prism and cylindrical-shaped objects such as tissue boxes and paper towel rolls.

- In groups of two, students play a memory matching game where they must match net diagrams to their corresponding shapes. The images will be on cards turned facedown and students will receive a point when they turn two up that display a shape and its matching net. Include multiple copies of each net. Composite objects could also be included.

- Students should be given an opportunity to estimate measurements of various items using their referents.
  - They may use their waist height as a referent for one metre. If they determine the height of the seat of a chair to be approximately half of their waist height, then the seat of the chair is about 0.5 metres high.
  - If students are determining the length of a room, they could count the tiles on the floor knowing that the length of a standard tile is one foot.

- Opportunities should be provided to explore common referents in both SI and imperial units. Including:

thickness of a dime	≈ 1 mm
width of a paper clip	≈ 1 cm
distance from a door knob to the floor	≈ 1 m
thickness of a hockey puck	≈ 1 in.
length of a standard floor tile	≈ 1 ft.
distance from the tip of the nose to the outstretched fingers	≈ 1 yd.

- Students should relate 1 kilometre to a distance between two well-known points in their own communities.

- Area referents should be used to estimate the surface area of an object. Some common referents for area measurement include the following:

area of a floor tile	≈ 1 ft.^{2}
area of a postage stamp	≈ 1 in.^{2}
area of a fingernail	≈ 1 cm^2
area of an exterior house door	≈ 2 m^2
area of an exercise notebook	≈ 93.5 in.^{2} or 600 cm^2
area of an ice rink surface	≈ 1500 m^2 or 17 000 ft.^{2}
area of a sheet of gyproc or plywood	≈ 32 ft.^{2} or 3 m^2
area of a square of shingles (3 packs)	≈ 100 ft.^{2}

- The following task investigates the connection between the area of a circle and the surface area of a sphere. Students will need a sphere-shaped object that has a removable coating, such as an orange or a covered ball, a caliper or ruler, and a compass.
  - Measure and record the circumference of the sphere-shaped object.
  - Calculate the radius by dividing the circumference by 2π.
  - Use a compass to draw 6 circles with radius equal to the radius of the sphere-shaped object.
  - Arrange cut-up pieces of the removed sphere coating onto the circles. Completely fill one circle before moving to the next.
  - Continue filling the circles until all of the coating has been used.
  - Use the filled circles and the formula for the area of a circle to estimate the surface area of the sphere.

  Students should reach the conclusion that the surface area of a sphere is the area of four circles and is calculated using $4\pi r^2$.

- The effect of dimensional changes can be explored through a task that requires students to select an answer and provide a justification for their answer, such as the following task.

> **What's the surface area?**
>
> *The radius of a sphere is doubled.*
>
> Choose the statement that describes how the surface area of the new sphere compares to the surface area of the original sphere.
>
> A. Same size.
> B. Doubles in size.
> C. Quadruples in size.
> D. Not enough information to compare. Explain your reasoning.

- Ask students to consider if doubling the radius of a cylinder doubles its' surface area. They should explain their reasoning.

SUGGESTED MODELS AND MANIPULATIVES

- calipers
- compass
- metre and yard sticks
- rulers and measuring tapes
- string

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- altitude
- apex
- cone
- cylinder
- diameter
- net
- radius
- rectangular-based prism
- referent
- right pyramid
- slant height
- sphere
- surface area
- triangular-based prism

Resources/Notes

Internet

- Professional Learning, K–12, Newfoundland and Labrador (Professional Learning NL 2013)
  www.k12pl.nl.ca

Print

Math at Work 11 (Etienne et al. 2012)
- Chapter 1: Surface Area
  - Sections 1.1, 1.2, 1.3 and 1.4
  - Skill Check
  - Test Yourself
  - Chapter Project
  - Games and Puzzles
- Chapter 2: Drawing and Design
  - Section 2.1
  - Chapter Project
- Chapter 3: Volume and Capacity
  - Section 3.1
  - Skill Check
  - Test Yourself
  - Chapter Project

Notes

SCO M02 Students will be expected to solve problems that involve SI and imperial units in volume and capacity measurements. [C, CN, ME, PS, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

M02.01 Explain, using examples, the difference between volume and capacity.
M02.02 Identify and compare referents for volume and capacity measurements in SI and imperial units.
M02.03 Estimate the volume or capacity of a 3-D object or container, using a referent.
M02.04 Identify a situation where a given SI or imperial volume unit would be used.
M02.05 Solve problems that involve the volume of 3-D objects and composite 3-D objects in a variety of contexts.
M02.06 Solve a problem that involves the capacity of containers.
M02.07 Write a given volume expressed as another unit in the same measurement system.
M02.08 Write a given capacity expressed as another unit in the same measurement system.
M02.09 Determine the volume of prisms, cones, cylinders, pyramids, spheres, and composite 3-D objects using a variety of measuring tools such as rulers, tape measures, calipers, and micrometers.
M02.10 Determine the capacity of prisms, cones, pyramids, spheres, and cylinders, using a variety of measuring tools and methods, such as graduated cylinders, measuring cups, measuring spoons, and displacement.
M02.11 Describe the relationship between the volumes of
- cones and cylinders with the same base and height
- pyramids and prisms with the same base and height

M02.12 Illustrate, using examples, the effect of dimensional changes on volume.
M02.13 Solve a contextual problem that involves the volume of a 3-D object, including composite 3-D objects, or the capacity of a container.
M02.14 Solve a contextual problem that involves the volume of a 3-D object and requires the manipulation of formulas.

Scope and Sequence

Mathematics at Work 10	Mathematics at Work 11	Mathematics at Work 12
M03 Students will be expected to solve and verify problems that involve SI and imperial linear measurements, including decimal and fractional measurements. **M04** Students will be expected to solve problems that involve SI and imperial area measurements of regular, composite, and irregular 2-D shapes and 3-D objects, including decimal and fractional measurements, and verify the solutions. **A01** Students will be expected to solve problems that require the manipulation and application of formulas related to perimeter, area, the Pythagorean theorem, primary trigonometric ratios, and income.	**M02** Students will be expected to solve problems that involve SI and imperial units in volume and capacity measurements.	**M01** Students will be expected to demonstrate an understanding of the limitations of measuring instruments including precision, accuracy, uncertainty, and tolerance, and solve problems.

Background

In Mathematics 7, students worked with areas of triangles, circles, and parallelograms (7M02). In Mathematics 8, they developed and applied formulas for determining the volume of right rectangular prisms, right triangular prisms, and right cylinders (8M04). In Mathematics at Work 10, students were introduced to strategies to convert within and between the imperial and SI measurement systems (10M01, 10M02). Additionally, they approximated linear measurements using referents (10M03). Work with volume will now be extended to include right pyramids, right cones, spheres, and composite 3-D objects. Although students worked with volume in Mathematics 8 (8M02), it will be necessary to revisit the concept here. Work should begin with rectangular prisms, triangular prisms, and cylinders. They will use the formula, V = (area of base × height), developed in Mathematics 8 to determine the volume of these 3-D objects.

Students will also work with capacity, with a focus on understanding how it differs from volume. Capacity is sometimes used interchangeably with volume. Since the two are not the same, it is important to emphasize the difference between them. Volume is the measure of the space an object occupies. Capacity is the amount a 3-D object can hold. A milk container, for example, could have a capacity of 250 mL of milk and a volume of 300 cubic centimetres. All objects have volume.

An example such as the following may highlight this difference.

- A brick has volume because it occupies space. A box has volume because it occupies space, and it also has capacity because it can contain another material within it.

Students should understand that hollow objects have volume and capacity, but solid objects only have volume. Volume is measured in units such as cubic metres (m^3) or cubic yards ($yd.^3$), reflecting the fact that it is measured in three dimensions. Measures of capacity includes units such as litres (L) or gallons (gal.), cups (c.), tablespoons (Tbsp.) and teaspoon (tsp.).

From their earliest introduction to linear measurements, students have used personal referents to estimate lengths that would normally be measured in SI or imperial units. The use of referents is now extended to estimate volume and capacity. Students should be encouraged to use a referent that represents approximately one unit of measurement. Consider the following examples:

Item	Approximate Volume / Capacity
sugar cube	1 cm^3
large bottle of water	1 L
large can of paint	1 gal.

Students will extend their use of referents to estimating the volume and capacity of 3-D objects. Emphasis should be placed on hands-on activities using manipulatives and a variety of measuring tools such as graduated cylinders, calipers, and micrometers. Problems should involve a mixture of SI and imperial units and include job-related contexts, where possible. The intent is that students measure the dimensions of a variety of 3-D objects and substitute those values into the appropriate volume formula. It is strongly recommended that teachers develop volume formulas to promote student understanding of these formulas and minimize formula memorization.

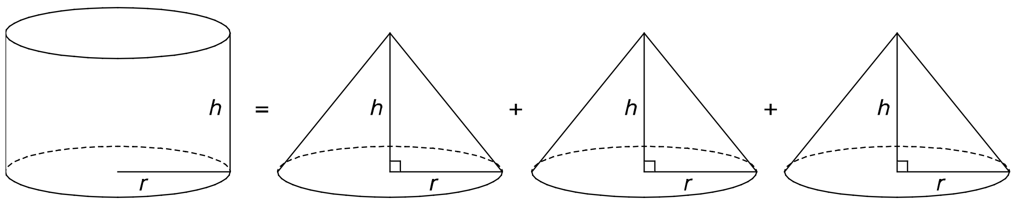
Students may be interested in exploring the “handful”; a nonstandard unit for measuring capacity that is related to the body. They could estimate and then measure how many handfuls of peas, beans or corn it takes to fill a margarine container. Students should compare their results using handfuls as a unit of measurement.

As students investigate dimensional changes, the expectation is that, initially, only one dimension would be changed and the effect on the volume discussed. This would then be extended to two dimensions, and then three. The same scale factor should be used when changing dimensions.

Students should know that to convert centimetres to metres they divide by 100. A common error is to also divide cubic centimetres by 100 to obtain cubic metres. To avoid this, ask students by what they would divide 1 000 000 cm^3 to convert it to cubic metres. Show that the correct answer is 1 000 000. Similarly, a common error occurs when converting cubic feet to cubic yards. Students should be reminded to divide by three for each dimension (i.e., divide by 27). Where possible, converting the linear measurements prior to calculating volume is recommended.

When calculating volume, the focus should be on right 3-D objects with regular bases such as squares, rectangles, right triangles, and circles and students should then progress to composite 3-D objects. Students should be given the opportunity to explore the relationships between the volumes of cones and cylinders with the same base and height, and between pyramids and prisms with the same base and height. Use a substance such as rice or water to develop the 1:3 ratio.

The volume formula for a cylinder ($V = \pi r^2 h$) was developed in Mathematics 8. To explore the volume of a cone with the same base and height as a cylinder, students can fill three congruent cones, all having the same height and radius as a cylinder, with water. They pour the water from the cones into the cylinder.



Students will see that the water from the cones fills the cylinder entirely. This means it takes the volume of three cones to equal one cylinder when the height and radius are the same.

The volume formula for a cone is therefore $V = \frac{\pi r^2 h}{3}$.

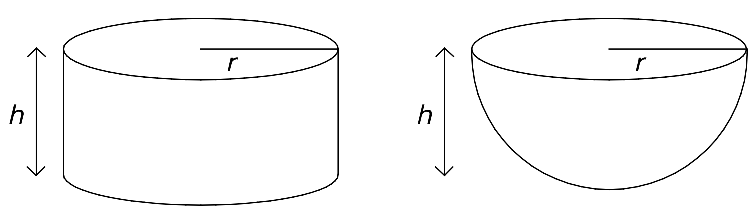
The volume of a rectangular prism was also developed in Mathematics 8 using the formula V = (area of base) × height. Students will continue to use this formula for any right prism. The volume of a pyramid is found by calculating one third of the volume of its related prism. A rectangular pyramid, for example, can be filled with rice, and students can investigate how many times it takes to fill a rectangular prism with the same base length, base width, and height.

The result of these investigations should be the development of the related formulas.

Three Dimensional Object	**Formula**
Cylinder or Prism	V = Area of Base × Height
Cone or Pyramid	$V = \frac{\text{Area of Base} \times \text{Height}}{3}$

The volume formula of a sphere could be introduced using the following task. The cylinder and hemisphere must have the same height and radius.

- Cut a small spherical object in half, creating a hemisphere.
- Use a cylinder having the same radius and height as the hemisphere.
- Fill the hemisphere completely with rice. Then pour the rice into the cylinder. It will fill two-thirds of the cylinder.



Students have already worked with the volume of a cylinder, $V = \pi r^2 h$. From the task, they should see that the volume of the hemisphere is $V = \frac{2}{3}(\pi r^2 h)$. The height is the same as the radius of the hemisphere, resulting in $V = \frac{2}{3}\pi r^3$. Since the volume of a sphere is twice the volume of a hemisphere, it is $V = \frac{4}{3}\pi r^3$.

The composite 3-D objects students work with would normally consist of two objects, or three objects where two of them are congruent.

Initially, students should find the volume by substituting values for the dimensions without having to manipulate the formulas and later progress to problems where the manipulation of formulas is required.

In all cases, an effort should be made to contextualize problems using trades and occupational examples. Students should be given the opportunity to investigate other trades and occupations that use volume and capacity formulas. Many construction problems, for example, involve knowing how to use the volume formulas of common geometric shapes.

- How much soil needs to be moved? How many truckloads will actually be hauled? How many trucks are needed on site?
- How much concrete has to be ordered for the pour?
- How much does something weigh? What weight will it take to lift it?
- How much water will be in that pool? How long will it take to fill it?

Expose students to incorrect solutions to problems and challenge them to find and correct the error(s). Common errors occur in the substitution of values for variables, in the rearrangement of the formula, in the order of operations, and in unit conversions within the problem.

It is sometimes more practical to use certain units of volume or capacity. While we may talk about millilitres of milk in a small carton, for example, it is more practical to talk about litres of milk in a container on a transport truck. It is also sometimes necessary to convert volume to capacity.

Engine displacements may need to be converted from cubic centimetres (sometimes referred to as cc's) to litres. When calculating the amount of cement needed, cubic inches or cubic feet is converted into cubic yards and perhaps then into cubic metres.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- Cory is measuring a fishing "haul-up" line for turbot nets. He uses his two outstretched arms as his fathom referent (1 fathom = 6 feet). If he measures 125 fathoms, how many feet and inches has he measured?

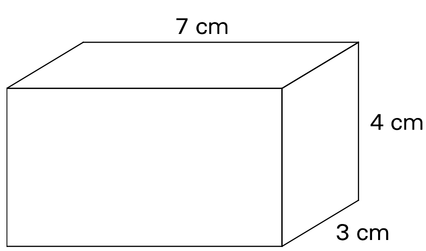
- Convert the following:
  (a) ____ g = 150 mg
  (b) 60 L = ____ mL
  (c) ____ g = 0.68 k
  (d) $\frac{1}{4}$ cup = ____ Tbsp.

- When you convert a measurement from a larger unit to a smaller unit, do you expect the number of units to increase or decrease? Why?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

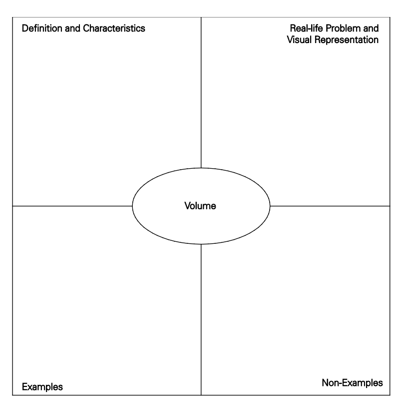
Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- The large block of cheese shown below is being cut into 1 cm cubes. How many cubes can be cut from the block. Using centimetre cubes, construct an open container with a capacity equal to the volume of the cheese block.



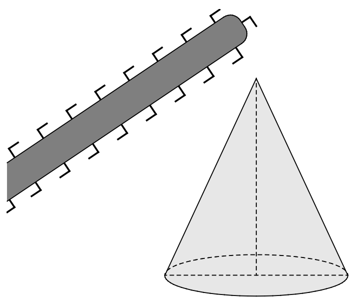
- Convert the following:
  (a) 3 000 000 cm^3 = ____ m^3
  (b) 1 $ft.^3$ = ____ $in.^3$
  (c) 4 L = ____ mL
  (d) 872 mL = ____ L
  (e) 8 $yd.^3$ = ____ $ft.^3$

- Using a template of the Frayer Model, complete the sections individually or as a group to consolidate your understanding of volume. Perform this same task for capacity.



- A cube has a volume of 27 cm^3. If the side length of the cube is doubled, will the volume of the cube also be doubled? Explain.

- A conveyor belt drops gravel into a cone shaped pile. If the radius of the base of the pile is 7 ft., and the pile is 12 ft. high, what is the volume of the gravel in the pile? If the gravel costs \$0.66/$ft.^3$, how much is the pile of gravel worth?

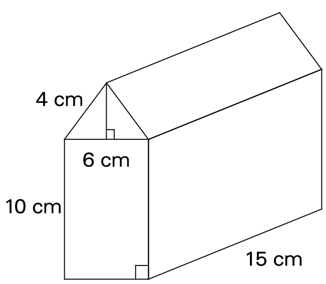


- Sam says that tripling the radius of a sphere will also triple its volume. Lydia disagrees with Sam. She thinks the volume will become much larger. Who is correct and why?

- A certain cube has a surface area of 96 cm^2. What is the volume of the cube?

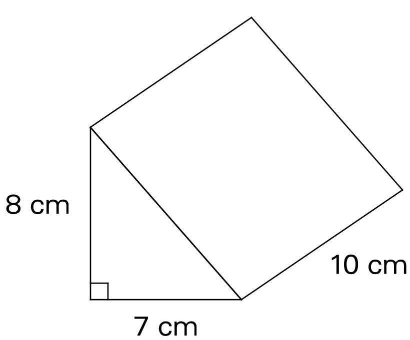
- The Canola Oil Company is designing cans for its oil. Their cans hold 1 L, which is equivalent to 1000 cm^3. The area of the base of their can is 80 cm^2. How tall is the can?

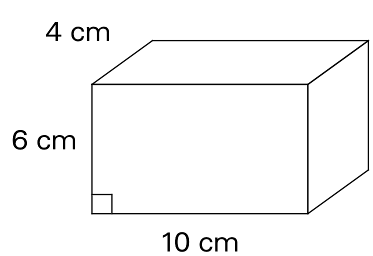
- Find the volume of the following object:



- Josh's car will hold 45 L of gas. If gas costs \$1.27/L, how much does it cost to fill his gas tank?

- Each of the following pieces of cheese costs \$5. Which is the better deal? Why?





- A school is having a fundraiser by selling popcorn, and the students are making their own containers to save on expenses. If they have sheets of cardboard for the sides with dimensions 27 cm × 43 cm, would the volume be greater if the sheets were folded to make cylindrical containers with a height of 27 cm or with a height of 43 cm? Justify your answer mathematically.

- A hot air balloon has a spherical shape with a diameter of 4 m. If 30 additional cubic metres of air are pumped into the balloon, what will be the new diameter? Round off the answer to one decimal place.

- A sphere that is 12 cm in diameter fits exactly into a cube. Find the volume of the cube.

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should be used to inform instruction.

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?
- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

SUGGESTED LEARNING TASKS

Consider the following sample instructional strategies when planning lessons.

- Discuss with students when they would use the concepts of volume and capacity in everyday life. (Responses might include filling a tank with gas, a measuring cup with water, a rectangular flower pot with soil, etc.)

- Show students a hollow unit cube for overhead base-ten blocks. Pour water into a 1 mL measuring spoon to verify that this amount is one millilitre. This connects volume to capacity (i.e., 1 cm^3 occupies the same space as 1 mL of water).

- Provide nets of various sized cubes to students. Ask students to measure the side length of the cube and to find the volume of the cube using the formula. Students should then cut out and fold their nets to create a cube (leave the top open). They can then fill the cube with rice or sand and measure the capacity by pouring the rice or sand into a graduated cylinder. Students should recognize that the volume in cubic centimetres should be equal to the capacity in mL. (This task can be done using any of the regular 3-D objects.)

- Give each student, or group of students, 36 centimetre cubes. Ask the students to make a rectangular prism using all 36 cubes. Ask students to find the volume and surface areas of their prisms. As a class, compare the different shapes of the prisms, the surface areas, and the volumes. Ask students to look for relationships between the shapes of the prisms, the surface areas, and the volumes. (**Note:** This task can be optimized by starting with a perfect cubed number of centimetre cubes, for example, 64 or 125 cubes.)

- Ask students to make a cylinder by rolling a 8.5" × 11" sheet of paper lengthwise. Make another cylinder by rolling another 8.5" × 11" sheet of paper widthwise. Will the two cylinders have the same capacity?

- Another way to find the volume of a sphere is to use the displacement method. This uses the amount of liquid displaced by an object to determine its volume. Provide pairs of students with a cup, a graduated cylinder, a pan, and several spherical objects to measure (e.g., a tennis ball, a golf ball). To do this task, students will
  - place the cup on a pan and fill it completely with water
  - slowly drop their object into the cup and let the excess water pour into the pan
  - pour the displaced water from the pan into the graduated cylinder to see how much water was displaced.

SUGGESTED MODELS AND MANIPULATIVES

- 3-D solids
- base-ten blocks
- calipers
- metre and yard sticks
- nets of various prisms, pyramids, cylinders, and cones
- relational geo-solids
- rice
- rulers and measuring tapes
- string

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- capacity
- cone
- cube
- cylinder
- diameter
- dimensional change
- displacement
- hemisphere
- net
- new units of measure
  - cubic foot (ft.^{3})
  - cubic inch (in.^{3})
  - cubic centimeter (cm^3)
  - cubic metre (m^3)
- prism
- pyramid
- radius
- rectangular prism

- referent
- slant height
- sphere
- square-based pyramid
- surface area
- triangular prism
- volume

Resources/Notes

Internet

- Professional Learning, K–12, Newfoundland and Labrador (Professional Learning NL 2013) www.k12pl.nl.ca

Print

Math at Work 11 (Etienne et al. 2012)

- Chapter 3: Volume and Capacity
  - Sections 3.1, 3.2, 3.3, and 3.4
  - Skill Check
  - Test Yourself
  - Chapter Project

Notes

Geometry
25–30 hours

GCO: Students will be expected to develop spatial sense.

SCO G01 Students will be expected to solve problems that involve two and three right angles. [CN, PS, T, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

G01.01 Identify all of the right triangles in a given illustration for a context.
G01.02 Determine if a solution to a problem that involves two or three right triangles is reasonable.
G01.03 Sketch a representation of a given description of a problem in a 2-D or 3-D context.
G01.04 Solve a contextual problem that involves angles of elevation or angles of depression.
G01.05 Solve a contextual problem that involves two or three right triangles, using the primary trigonometric ratios.

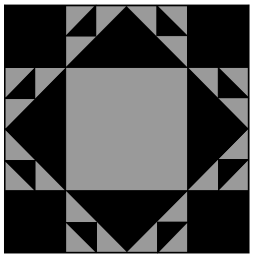
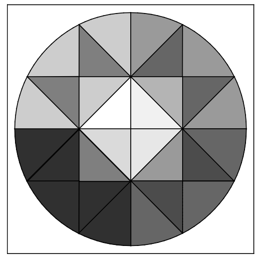
Scope and Sequence

Mathematics at Work 10	Mathematics at Work 11	Mathematics at Work 12
G02 Students will be expected to demonstrate an understanding of the Pythagorean theorem by identifying situations that involve right triangles, verifying the formula, applying the formula, and solving problems. **G04** Students will be expected to demonstrate an understanding of primary trigonometric ratios (sine, cosine, tangent) by applying similarity to right triangles, generalizing patterns from similar right triangles, applying the primary trigonometric ratios, and solving problems.	**G01** Students will be expected to solve problems that involve two and three right triangles.	**G01** Students will be expected to solve problems by using the sine law and cosine law, excluding the ambiguous case. **G02** Students will be expected to solve problems that involve triangles, quadrilaterals, and regular polygons.

Background

Students are familiar with the Pythagorean theorem. They were introduced to it in Mathematics 8 (8M01), used it in problem-solving situations in Mathematics 9 (9N06, 9M01, 9M02) and used real-world examples to promote the importance and relevance of the theorem in Mathematics at Work 10 (10G02, 10A01). They were also introduced to the three primary trigonometric ratios (10G04). Students solved right triangles and contextual problems using the primary trigonometric ratios but they were limited to working with only one right triangle. This outcome extends their learning to solving problems with two or three right triangles. In this unit, students will also work with angles of elevation and depression.

Identification of right triangles is an important starting point when solving problems involving trigonometry. Providing students with diagrams, such as the ones below, and discussing what information is needed to determine if the triangles are right triangles is a possible teaching strategy.

When solving problems involving trigonometry, sketching the scenario helps students determine the answer. Students sometimes have difficulty correctly identifying the opposite and adjacent sides in relation to the reference angle. They should be exposed to right triangles with reference angles in various locations so that they recognize that the opposite and adjacent sides are relevant to the reference angle.

A review of the primary trigonometric ratios and their applications to right triangles will be necessary. A trigonometric ratio is a ratio of the measures of two sides of a right triangle. The three primary trigonometric ratios are tangent, sine and cosine.

The short form for the tangent ratio of angle *A* is tan *A*. It is defined as

$$\tan A = \frac{\text{length of side opposite } \angle A}{\text{length of side adjacent to } \angle A}.$$

The short form for the sine ratio of angle *A* is sin *A*. It is defined as

$$\sin A = \frac{\text{length of side opposite } \angle A}{\text{length of hypotenuse}}.$$

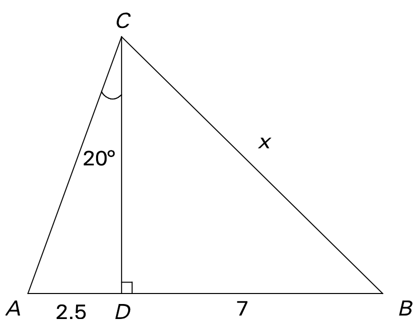
The short form for the cosine ratio of angle *A* is cos *A*. It is defined as

$$\cos A = \frac{\text{length of side adjacent to } \angle A}{\text{length of hypotenuse}}.$$

Students apply the sine, cosine, and tangent ratios to determine missing sides and acute angles in right triangles. This requires the use of the inverse of the ratio. It is expected that students will use a calculator to find trigonometric ratios and angle measurements of given ratios. They will have to be reminded of the need to work in degree mode. In Mathematics 9, students solved equations of the form $a = \frac{b}{c}$ (9PR03). A review of this may be necessary prior to solving equations such as, $\tan 30° = \frac{x}{10}$; $\tan 30° = \frac{5}{x}$ and $\tan\theta = \frac{5}{12}$.

$\tan 30° = \frac{x}{10}$ $10 (\tan 30°) = \frac{x}{10} (10)$ $5.7735 = x$	$\tan 30° = \frac{5}{x}$ $x (\tan 30°) = \frac{5}{x} (x)$ $x (\tan 30°) = 5$ $x = \frac{5}{\tan 30°}$ $x = 8.6603$	$\tan\theta = \frac{5}{12}$ $\angle\theta = \tan^{-1}(\frac{5}{12})$ $\angle\theta = 22.6°$

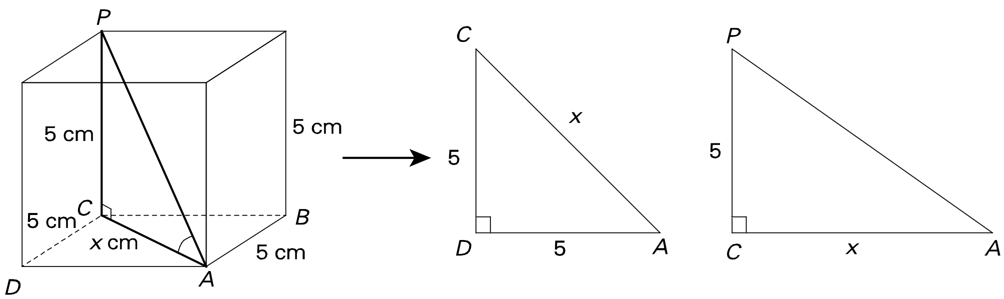
Students will extend their knowledge of solving contextual problems involving one right triangle to solving problems with two and three right triangles. They should practise determining missing side lengths and angles from given diagrams before solving contextual problems. Trigonometry can be used to solve for lengths or angle measures in a sequence of triangles by using the solution of one triangle to provide information needed to solve the next triangle. An effective method of solving trigonometric problems involving more than one right triangle is to separate the triangles and label them. Students could work with a partner to explore this. Ask them, for example, to determine the length of *CB* in the diagram below.



To calculate the length of *CB*, more than one triangle will be used. Students should recognize that ΔACD will be used before ΔBCD.

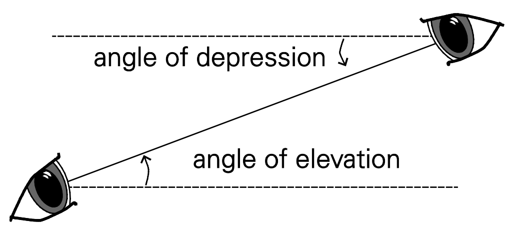
When solving problems involving more than one right triangle, expose students to three-dimensional problems. When solving a three-dimensional problem, it is important for students to be able to visualize right triangles within the diagram. Students can then redraw the right triangles in two dimensions and use the appropriate trigonometric ratio and/or apply the Pythagorean theorem to solve.

For example, given the following diagram, determine the value of theta.

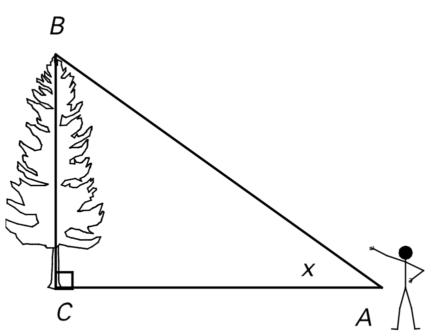


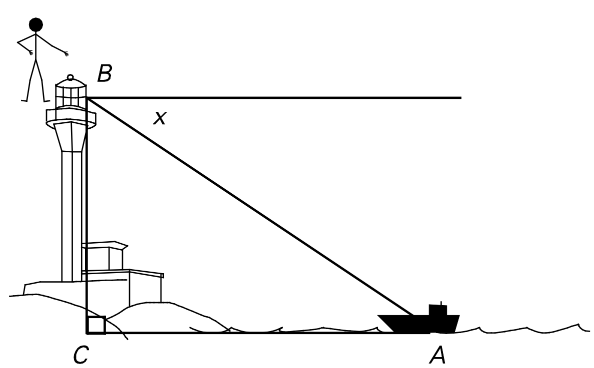
Using a 2-D diagram, the value of *x* can be determined in Δ*ACD* using the Pythagorean theorem. The tangent ratio can then be applied to find the measure of θ in Δ*APC*.

When students solved contextual problems using trigonometric ratios in Mathematics at Work 10, the terms **angle of elevation** and **angle of depression** may have been introduced to them. These angles are always measured relative to the horizontal. The line of sight is an invisible line from one person or object to another person or object. Some applications of trigonometry involve an angle of elevation or an angle of depression. An angle of elevation is the angle formed by the horizontal and a line of sight looking upward from the horizontal (or when a person is looking at an object above them). An angle of depression is the angle formed by the horizontal and a line of sight looking downward from the horizontal (or when a person is looking at an object below them). As the diagram below shows, the angle of elevation and the angle of depression along the same line of sight are equal.

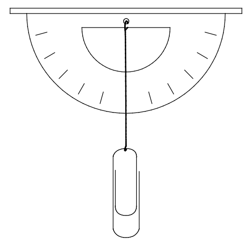


Have students solve contextual problems that include angles of elevation and depression, limited to one right triangle. Providing students with visuals of real-world situations and indicating where the angle of elevation or depression is located should give students an understanding of these terms.





In situations where it is inconvenient or impossible to make measurements directly, a clinometer or hypsometer can be used. A hypsometer, used by forestry technicians, is an instrument that uses changes in atmospheric pressure to determine land elevations. A clinometer (a protractor-like device used to measure angles), shown below can be used to gather data for the indirect measurement of the height of an object or angle of elevation or depression.

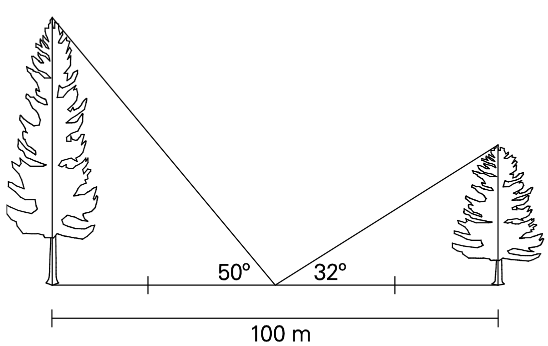


Directions to build a clinometer can be found on page 85 of the student text for Mathematics 10 (Foundations and Pre-calculus Mathematics 10).

Students can measure the angle between the horizontal and the line of sight to the top of the object. Measuring the horizontal distance from the observer to the object should provide the data needed to calculate the height of the object using trigonometry. To link this to the workplace, invite a surveyor to visit the class and describe the requirements of their job, show the tools used on the job, and discuss and demonstrate how they would perform a similar task.

Students should always be encouraged to use the properties of a triangle to check the reasonableness of their solutions. The smallest angle, for example, is located opposite the side with the shortest length, the sum of any two side lengths of a triangle is greater than the length of the third side, and the sum of the angles in a triangle is 180°. Students should also check if their solution is reasonable within the context of the problem. They should be exposed to problems such as the following:

- Two trees are 100 m apart. From a point on the ground halfway between the trees, the angle of elevation to the top of the shorter tree is 32° and the angle of elevation to of the top of the taller tree is 50°. What is the height of each tree?



This involves using the tangent ratio to find side lengths. Common student errors, such as using cosine rather than tangent or incorrectly identifying the opposite and adjacent sides, could lead to answers resulting in the shorter tree having a longer length than the taller tree. In such cases, by asking themselves questions such as Does this make sense? or Is this possible?, students should realize they have made an error.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

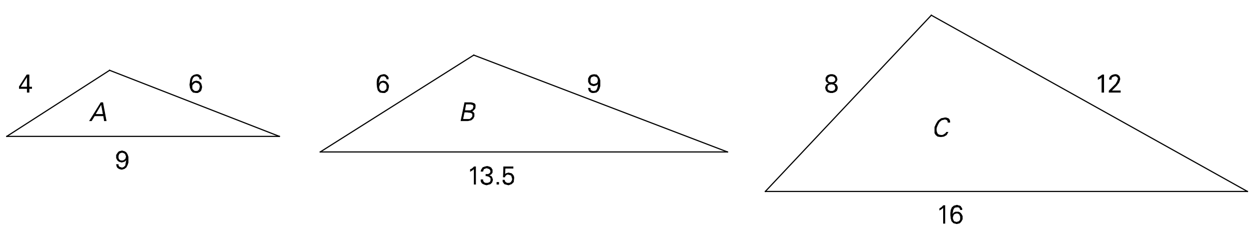
Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- Determine whether each triangle with sides of given lengths is a right triangle.
  (a) 9 cm, 12 cm, 15 cm
  (b) 16 mm, 29 mm, 18 mm

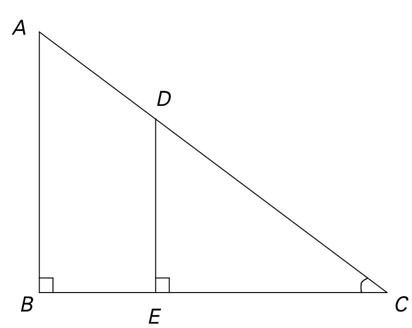
- Anja is building a garage on a floor that measures 18 feet × 24 feet. Anja measures the length of the diagonal to be 29.5 feet. Are the angles at the corners of the garage right angles? Explain.

- Identify situations where trigonometric ratios are used to determine measurement of angles and lengths indirectly (e.g., determining the height of a flagpole or a tree from the ground).

- Determine if either triangle B or C is similar to triangle A.



- Answer the questions that follow based on the diagram given.
  (a) Which triangles are similar?
  (b) Measure the sides and determine the ratios of
    (i) $\frac{AB}{DE}, \frac{AC}{DC}$
    (ii) $\frac{AB}{BC}, \frac{DE}{EC}$
    (iii) $\frac{BC}{EC}, \frac{AC}{DC}$
    What do you notice about the values?



  (c) If AB = 9 cm, DE = 6 cm, and EC = 8, what is the length of BC?

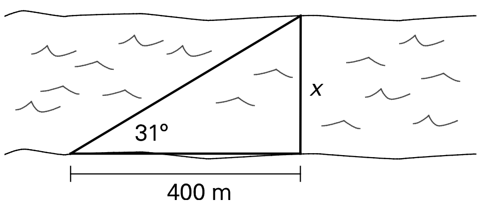
- A ramp 4 m long is being built to reach a loading dock that is 1.5 m in height. What is the measure of the angle between the ramp and the ground?

- If $\sin a = \frac{3}{5}$, what is $\tan a$? Using this information, draw a triangle.

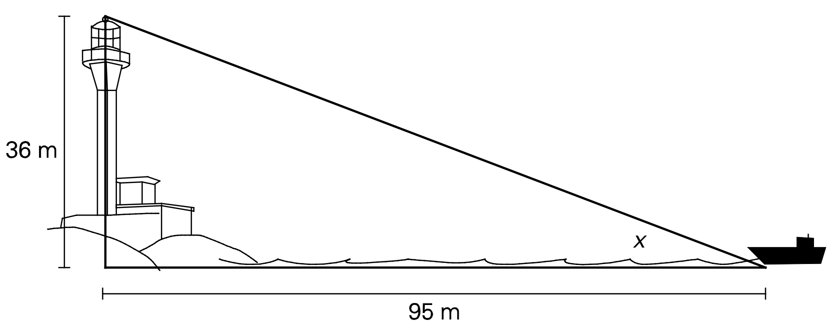
WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- A surveyor wants to determine the width of a river for a proposed bridge. The distance from the surveyor to the proposed bridge site is 400 m. The surveyor measures a 31° angle to the bridge site across the river. What is the width of the river, to the nearest metre?



- A small boat is 95 m from the base of a lighthouse that has a height of 36 m above sea level. Calculate the angle from the boat to the top of the lighthouse. Round off the answer to one decimal place.



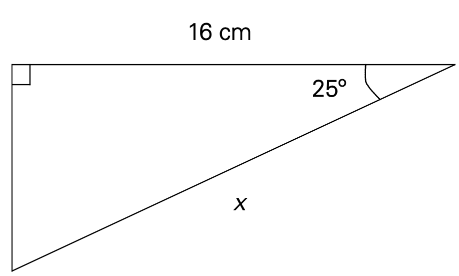
- Identify the error(s) in the following calculations. Determine the correct solution and explain how the person doing this question could have easily discovered that there was an error.

Solve for x:

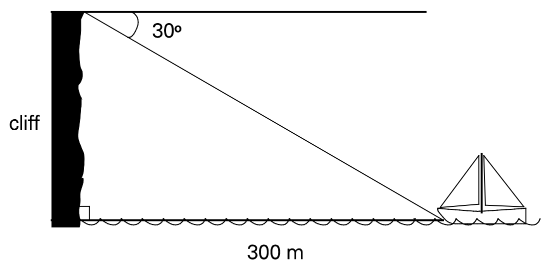
$$\sin 25° = \frac{16}{x}$$

$x = 16(\sin 25°)$

$x = 6.8$ cm



- The angle of depression from the top of a cliff to a sailboat below is 30°. If the sailboat is 300 m from the cliff, what is the height of the cliff?

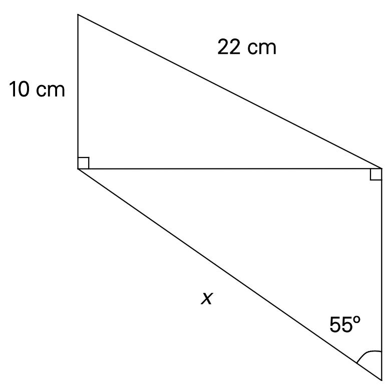


- Two poles on horizontal ground are 60 m apart. The shorter pole is 3 m high. The angle of depression from the top of the longer pole to the top of the shorter pole is 20°. Sketch a diagram to represent the situation.

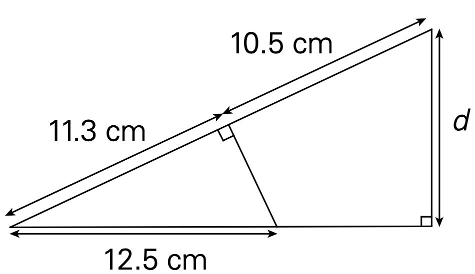
- A man who is 2 m tall stands 30 m away from a tree. The angle of elevation to the top of the tree from his eyes is 28°. Sketch a diagram and determine the height of the tree.

- From a height of 50 m in his fire tower near the lake, a ranger observes the beginnings of two fires. One fire is due west at an angle of depression of 9°. The other fire is due east at an angle of depression of 7°. What is the distance between the two fires to the nearest tenth of a metre?

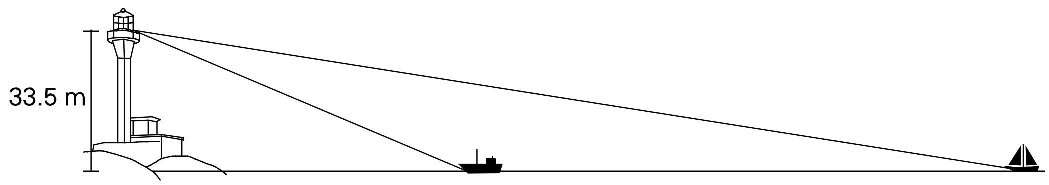
- Determine the value of *x* in the following diagram.



- Calculate the length *d* shown in the diagram.

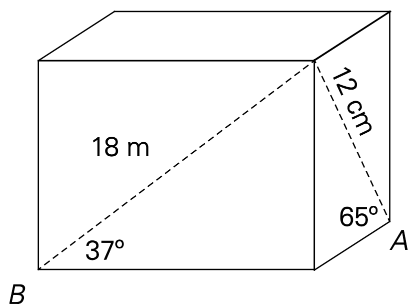


- A tourist at the Cape Forchu Lighthouse looks out and sees two boats in his line of site, a fishing boat at an angle of depression of 23° and a sailboat at an angle of depression of 9°. If the tourist is 33.5 m above the water, determine how far apart the two vessels are.

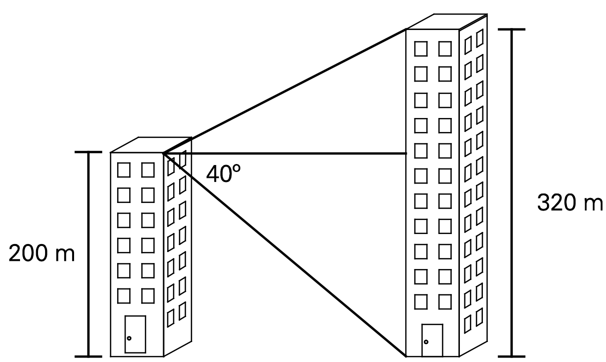


- A lighthouse is located at the top of a cliff. From a point 150 m offshore, the angle of elevation to the foot of the lighthouse is 25° and the angle of elevation to the top of the lighthouse is 31°. Determine the height of the lighthouse to the nearest tenth of a metre.

- Determine the shortest distance between *A* and *B* in the rectangular prism shown.

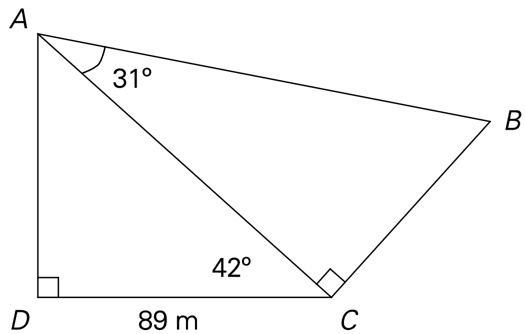


- Roger is on top of the smaller building and views the base of the other building at an angle of depression of 40°.

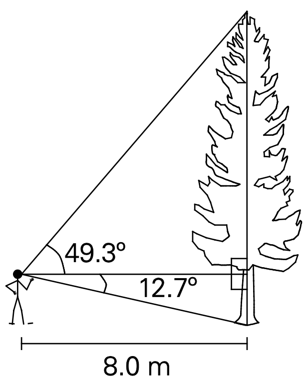


(a) What is the distance between the two buildings?
(b) At what angle of elevation does Roger view the top floor of the other building?

- Find the length of *AB* to the nearest tenth of a metre.



- A forest technician wishes to measure the height of a tree using a hypsometer. At eye level, he finds the horizontal distance to the tree is 8.0 m, the angle of elevation to the top of the tree is 49.3° and the angle of depression to the base of the tree is 12.7°. According to these measurements, what is the approximate height of the tree?



FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should be used to inform instruction.

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?

- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?
- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

SUGGESTED LEARNING TASKS

Consider the following sample instructional strategies when planning lessons.

- Remind students that a diagram showing all of the given information should be the first step in the solution of any trigonometric problem. Their sketches need not be accurate, but reasonable representations of the given situation can help them decide on a strategy.

- Students should be encouraged to verify their work. If trigonometry was used to find the lengths of the missing sides, the Pythagorean theorem can be used to verify the results. When verifying angle measurements, encourage students to ensure that the sum of the angles totals 180°. Students should also check the reasonableness of their answers by ensuring that the smallest angle, for example, is opposite the shortest side.

- Set up stations at which groups of students find the height of various objects using trigonometry. They can measure the angle of elevation using a clinometer. Some objects may include the height of a basketball net, the height of a clock on the wall, the height of a gymnasium wall, and the height of a door.

- Ask students to describe a work or recreational situation where trigonometry could be used to find a length or distance. Ask them to explain what information is needed to make the calculation and to describe how they could gather this information. They should also provide a method for the calculation.

- Choose a problem that involves two or three right triangles. Put each step for solving the problem on a card and distribute the cards to small groups of students. Ask students to decide on a logical sequence in which to place the cards. They should justify their reasons for the sequence. A blank card could also be included in the set. Students would have to provide the missing card to complete the steps required to solve the problem.

- Students should judge the reasonableness of their answers. Consider asking questions such as the ones that follow:
  (a) Given that the tangent is smaller than 1, should the opposite side be longer or shorter than the adjacent side?
  (b) What can be said about the angles of a right triangle when the tangent ratio is larger than one or less than one?

- A common mistake when working with angle of elevation is incorrectly transferring information to the corresponding right triangle. To help students avoid this, remind learners that the angle of elevation is always formed with the horizontal and never with the vertical.

- Students could research a real-life application of right triangle trigonometry and then create a poster that illustrates this application.

- Use clinometers and have students go outside to find the height of tall buildings or trees using trigonometric ratios.

- Invite a tradesperson in to discuss and demonstrate how skills in trigonometry are used in their work.

Suggested Models and Manipulatives

- clinometers (these can be constructed using cardboard, a straw, string, and a weight)
- metre and yard sticks
- rulers and measuring tapes

Mathematical Vocabulary

Students need to be comfortable using the following vocabulary.

- adjacent side
- angle of depression
- angle of elevation
- cosine θ; cos θ
- hypotenuse
- opposite side
- sine θ; sin θ
- tangent θ; tan θ
- theta (θ)

Resources/Notes

Internet

- *Professional Learning K–12*, Newfoundland and Labrador, “Trigonometry” (Professional Learning NL 2013)
  www.k12pl.nl.ca/curr/10-12/math/math2202/classroomclips/trig.html
  In the Trigonometry clip, students use a clinometer to measure the angle of elevation of various objects. They then use the tangent ratio to determine the heights of the objects.

- *Illuminations: Resources for Teaching Math* (National Council of Teachers of Mathematics 2013)
  - *Building Height*
    http://illuminations.nctm.org/LessonDetail.aspx?id=L764
  - *Construction of Clinometer*
    http://illuminations.nctm.org/Lessons/BuildingHeight/BuildingHeight-OH.pdf

Print

Math at Work 11 (Etienne et al. 2012)

- Chapter 7: Right Triangles and Trigonometry
  - Sections 7.1, 7.2, and 7.3
  - Skill Check
  - Test Yourself
  - Chapter Project

Software

- Geometer's Sketchpad (Key Curriculum 2013)

Notes

SCO G02 Students will be expected to solve problems that involve scale. [PS, R, T, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

G02.01 Describe contexts in which a scale representation is used.
G02.02 Determine, using proportional reasoning, the dimensions of an object from a given scale drawing or model.
G02.03 Construct a model of a 3-D object, given the scale.
G02.04 Draw, with and without technology, a scale diagram of a given object.
G02.05 Solve a contextual problem that involves scale.

Scope and Sequence

Mathematics at Work 10	Mathematics at Work 11	Mathematics at Work 12
G03 Students will be expected to demonstrate an understanding of similarity of convex polygons, including regular and irregular polygons.	**G02** Students will be expected to solve problems that involve scale.	**G03** Students will be expected to demonstrate an understanding of transformations on a 2-D shape or a 3-D object, including translations, rotations, reflections, and dilations.

Background

A scale is a comparison between the actual size of an object and the size of its image, therefore a scale diagram would be a drawing that is similar to the actual figure. Scale diagrams can be either an enlargement or a reduction of the actual object depending on the context. If the scale factor is bigger than one, this will result in an enlargement; whereas if the scale factor is between zero and one, it is a reduction.

Students have used models and diagrams to investigate ratios. They wrote and compared equivalent ratios and set up proportions to solve problems (8N05). In Mathematics 9, students were introduced to drawing and interpreting scale diagrams representing enlargements and reductions (9G03). This outcome reviews these concepts and extends them to include 3-D modelling. The basic modelling done here will be further expanded upon in outcome G03. In Mathematics at Work 10, students used proportions and proportional reasoning to solve problems that require conversions.

Students have an intuitive sense of shapes that are enlargements or reductions of each other. Students have experienced maps and pictures that have been drawn to scale, and images produced by photocopiers and computer software. Although students have already been exposed to scale concepts and proportions, a review of these key concepts is important. Students may be familiar with scale statements, but they may not understand exactly what they mean. Examples of real-world applications, such as maps, sewing patterns, car models, and construction blueprints, would be a great way to introduce the concept and capture student interest. Students with experience using scale diagrams for activities such as building models of boats, cars, or planes, sewing, and orienteering can share their insight on the importance of using a scale.

When a ratio is used to represent an enlargement or reduction, the format of the ratio is New:Original. A ratio of 2:1 means the new shape is an enlargement to twice the size of the original. Likewise, a ratio of 1:3 means that the new shape is a reduction to $\frac{1}{3}$ of the size of the original, or the original is three times the size of the new shape.

The use of computer software can allow for a great deal of flexibility in the investigation of enlargement and reduction. Other methods, such as using graph paper or a protractor and ruler, can also be used to draw scale diagrams.

Students will determine dimensions for real-world situations using their knowledge of scale diagrams and proportions. Including examples that revolve around student interest, such as cars, video game animations, motorcycles, and sport surfaces, will help engage learners.

Using scale, students will determine the dimensions of a reduced or enlarged object. Setting up and solving proportions is crucial to student success. Given a basic 3-D object, such as a cubic box, students will construct a scale model. Working in groups, they can choose their media (e.g., linking cubes, cardboard, straws and pipe cleaners, or modelling clay) and use an appropriate scale to produce their model.

Students must also be made aware of the importance of units in scale diagrams. A review of converting different units in both the metric and imperial systems may be necessary. It is also important to emphasize the meaning of units so students better understand the concept of scale and can gain a visual appreciation of the object that is being reduced or enlarged. When using a scale, the units do not have to be the same, however the units must be stated. If the units are not stated, they must be the same scale.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

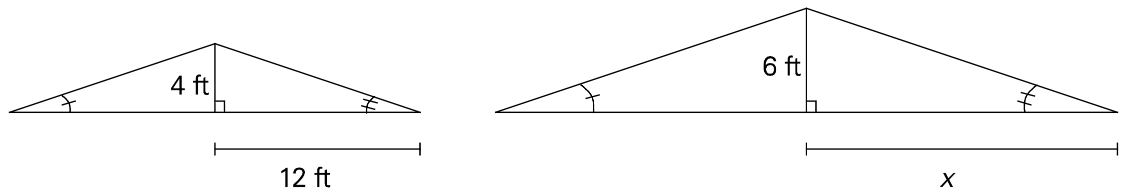
Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

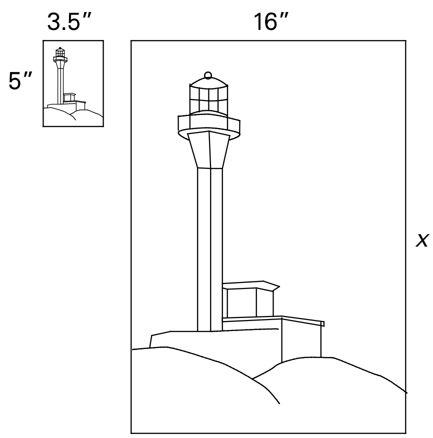
ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- Determine the value of the variable in the proportion 5:x = 40:56.
- Omar wants to build a roof truss that is "4 on 12" (see diagram below). If the roof truss height changes to six feet, how wide will the new roof truss be?



- You are asked to create a poster to advertise a field trip to see the Cape Forchu Lighthouse in Yarmouth. The owners want you to enlarge a photo of the lighthouse that is 3.5 inches wide and 5 inches long. Determine how long the poster will be if the enlargement is to be 16 inches wide.

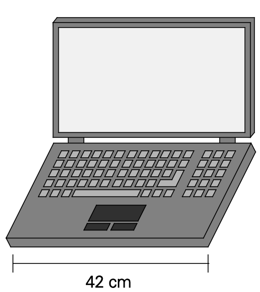


- Convert the following:
  (a) 130 cm = ____ m
  (b) 3.25 km = ____ cm
  (c) 42 cm = ____ in.
  (d) 26.2 km = ____ mi.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

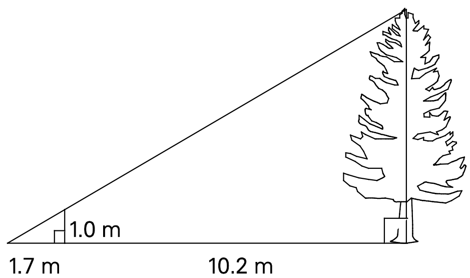
Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- An actual laptop has a width of 42 cm. Determine the scale factor used for the image of the laptop.

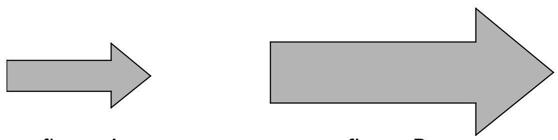


- A driving distance is 600 km. The distance shown on a map is 4 cm. What is the scale factor? Express the answer as a fraction.

- Use similar triangles to find the height of the tree.



- Explain how you could determine if Figure B is an accurate enlargement of Figure A.



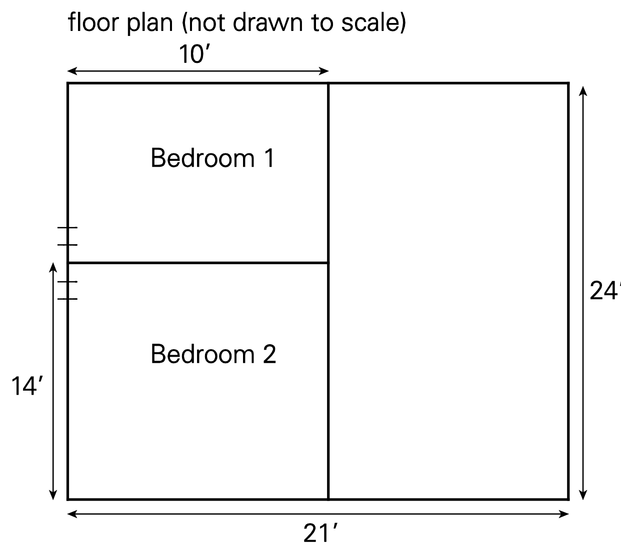
- A model of a sailboat is 9 inches long and the actual length of the boat is 32 feet.
  (a) Convert 32 feet to inches.
  (b) On the model, the mast is 6 inches high. What is the height of the mast of the actual boat?

- Rashem is drawing a scale diagram of his house. He will let 1 inch represent 3 feet. If his house measures 30 feet by 42 feet, how big will his diagram be?

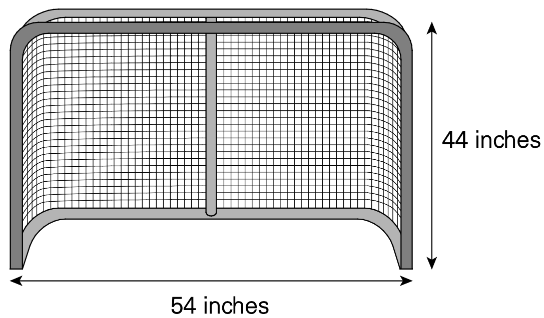
- Write each as a ratio whose first term is one.
  (a) 2 inches to 3 yards
  (b) 5 cm to 1 m

- Given the sketch of a floor plan below, complete the following activities.
  (a) Find the scale factor that will allow the largest possible view of the floor plan to fit on a sheet of $8\frac{1}{2}$ inch × 11 inch paper.
  (b) Draw a scale floor plan using these measurements.
  (c) Create a model of the floor plan using a media of choice.

  (The diagram is not drawn to scale. Students should note that the bedroom doors are 30 inches wide and 6 inches from the corner of the room.)



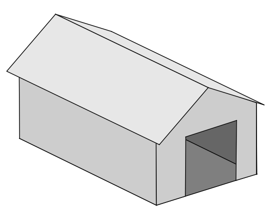
- This hockey net has dimensions as shown in the diagram. Use a scale of 1 inch to 20 inches to create a scale drawing of the front of the net.



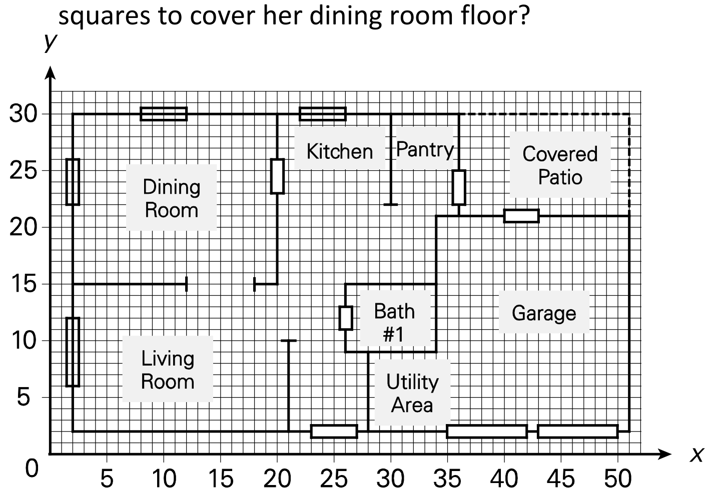
- Locate and print a copy of a building of a familiar community building from Google Maps. Using measurements of the actual building and their building on Google Maps, determine how much the measurements have been reduced. Using Google SketchUp (SketchUp 2014), create a scale.

- A scale for a mini car toy collection is 1:67. The dimensions of the toy car are 34 × 25 × 15 cm. Determine the actual dimensions of the vehicle.

- Gideon has a doghouse that has a length of 5 feet and a width of 3 feet. His neighbour Leah likes the design of Gideon's doghouse, but has a smaller dog and wants her doghouse to have a length of 4 feet.
  (a) What scale factor would Raquel use to find the dimensions of her doghouse?
  (b) What would the width of Raquel's doghouse be?



- Mrs. Lewis intends to have the dining room floor redone with wood flooring that comes in 1-foot wide squares. The scale is 1 unit = 1 ft. A 12-pack of squares sells for $38.40, and single squares can be purchased for $3.70 each. Determine the cost, in dollars, for Mrs. Lewis to buy just enough squares to cover her dining room floor?



Follow-up on Assessment

The evidence of learning that is gathered from student participation and work should be used to inform instruction.

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?
- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Suggested Learning Tasks

Consider the following sample instructional strategies when planning lessons.

- With the students, brainstorm a list of real-world examples where scales are used. Examples of real-world applications, such as maps, classroom fire exit plans, sewing patterns, car models, and construction blueprints, would be an effective way to introduce the concept of scale and capture students' interest.

- Have students generate a list of jobs and hobbies that require the use of scales.

- Students can be given a 2-D shape on graph paper and then asked to come up with a procedure to either reduce or enlarge the shape.

- Have students bring in some models such as small versions of cars, trucks, airplanes, or action figures. Ask them to do some research to determine the scale factor that such models represent.

- Students will be given a basic 3-D object, such as a cubic box, and asked to construct a scale model.
  (a) Working in groups, students can choose their media (e.g., linking cubes, cardboard, straws and pipe cleaners, or modelling clay) and use an appropriate scale to produce their model.
  (b) The dimensions of the original cubic box could be provided or students could be required to measure the box to determine the dimensions.

(c) Based on their model or directly from the 3-D object, students should construct 2-D scale diagrams with or without technology. (This is a good lead-in to Outcome G03.) Visually, students' 2-D representations may look quite different depending on how they view the 3-D object. However, if they use the scale properly, the drawings will be correct. A class discussion regarding the difference in appearance of their drawings would be helpful.

- An appropriate task would involve students designing a floor plan of their dream bedroom, kitchen, or a room of their choice. They will have to consider what to include in their drawing, the scale they will use, and the measurements needed. Students must also understand that it is important for the measurements to be realistic. If a blueprint is using a scale of 1 in. = 1 ft., for example, the doorway should be exactly 3 in. wide on the drawing so that in real life it would be 3 ft. wide.

- Consider asking the following questions to guide students with their design.
  - How are the dimensions recorded on the diagram?
  - How are the doors, windows, closets, and walls represented?
  - Is the scale indicated?
  - Should a key be included to identify the symbols used in the drawing?
  - Will furniture be included?

  Completed floor plans can be posted around the classroom for students to see other examples.

SUGGESTED MODELS AND MANIPULATIVES

- 3-D shapes
- cardboard
- cube-a-links
- graph paper
- modelling clay
- pipe cleaners
- straws
- unifix cubes

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- 2-D representation
- 3-D representation
- proportion
- ratio
- scale
- scale drawing
- similar figure

Resources/Notes

Internet

- *Google Maps* (Google 2014)
  https://maps.google.ca/

- *Illuminations: Resources for Teaching Math* (National Council of Teachers of Mathematics 2013)
  http://illuminations.nctm.org

Print

Math at Work 11 (Etienne et al. 2012)

- Chapter 2: Drawing and Design
  - Sections 2.1 and 2.2
  - Skill Check
  - Chapter Project

Software

- Geometer's Sketchpad (Key Curriculum 2014)
- Google SketchUp (SketchUp 2014)

Video

- *The Futures Channel*, "Designing Toy Cars" (approximately 3 min.) (The Futures Channel 2014)
  http://thefutureschannel.com/videogallery/designing-toy-cars
  As a professional toy car designer for Mattel's Hot Wheels, Larry Wood uses basic mathematics concepts such as fractions, measurement and scale to create accurate replicas of the coolest cars on the road.

Notes

SCO G03 Students will be expected to model and draw 3-D objects and their views. [CN, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

G03.01 Draw a 2-D representation of a given 3-D object.
G03.02 Draw, using isometric dot paper, a given 3-D object.
G03.03 Draw to scale top, front, and side views of a given 3-D object.
G03.04 Construct a model of a 3-D object, given the top, front, and side views.
G03.05 Draw a 3-D object, given the top, front, and side views.
G03.06 Determine if given views of a 3-D object represent a given object, and explain the reasoning.
G03.07 Identify the point of perspective of a given one-point perspective drawing of a 3-D object.
G03.08 Draw a one-point perspective view of a given 3-D object.

Scope and Sequence

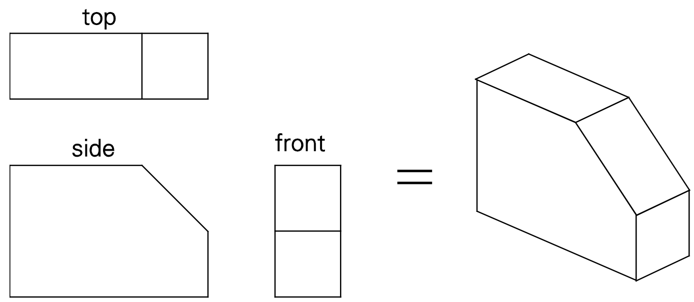
Mathematics at Work 10	Mathematics at Work 11	Mathematics at Work 12
–	**G03** Students will be expected to model and draw 3-D objects and their views.	**G03** Students will be expected to demonstrate an understanding of transformations on a 2-D shape or a 3-D object, including translations, rotations, reflections, and dilations.

Background

Observing and learning to represent 2-D shapes and 3-D objects in various positions by drawing and construction helps students develop spatial sense. Students' mathematical experience with 3-D is often derived from 2-D pictures. It is important that students are able to interpret information from 2-D pictures of the world, as well as to represent real-world information in 2-D. Students will explore multiple ways to draw representations of 3-D objects. They will draw a variety of 3-D objects and then discuss their representations.

In Mathematics 8, students drew and interpreted top, front, and side views of 3-D objects composed of right rectangular prisms. As they continue to explore the various views, the use of concrete models, such as linking cubes, will be beneficial. When building 3-D objects, each view of an object provides some of the information required to build the object. Through exploration, students should learn that several views are required to accurately build a unique 3-D object. Some objects can be completely represented with fewer than six views because they are symmetrical, or have sides that are equal in shape and measure.

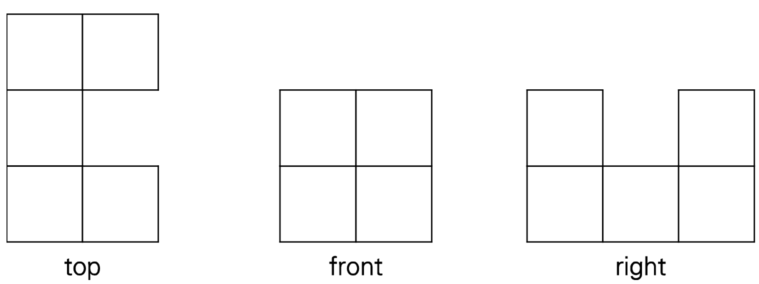
Students will apply their knowledge of scale factor and 3-D structure to determine whether given views represent a 3-D object. Students may be given the scale or they might have to determine the scale by making corresponding measurements on both the object and diagram. Measurements on the scale diagram should correspond to measurements taken of the 3-D object with the correct use of the scale factor. If the object is to be represented by the views, hidden irregularities must be shown.



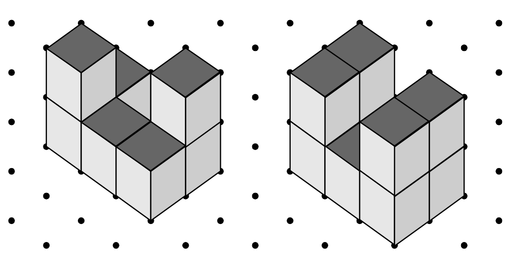
Though views are useful in representing 3-D objects, they are limited. If an object is hollow, for example, it may be difficult to see its thickness or the positions of materials inside.

Students will be given a variety of 3-D objects to construct scale orthographic diagrams. Orthographic drawings are 2-D views of 3-D objects. These drawings include front, top, and side views of the object. Assembling furniture is an example of a real-world situation where 2-D images of component parts are put together to form a 3-D object. Students should be encouraged to take precise measurements of the object and choose an appropriate scale.

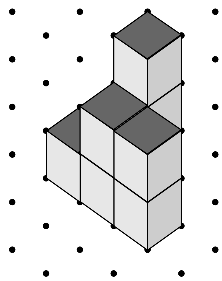
Students can be given a series of 2-D views of a 3-D object, such as in the example below, and be asked to construct, using cubes, a building that adheres to the plans. Such plans are often referred to as orthographic plans or views.



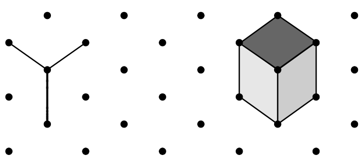
This outcome will also be explored using isometric drawings. An isometric drawing is a representation of a 3-D object where the same scale is used to draw the same object height, width, and depth. Lines that are parallel in reality are shown to be parallel in the drawing. Isometric drawings combine the depth of an object with an undistorted view of the object's dimensions. This makes it easy to measure and visualize the assembly of the component parts. The orthographic plans shown above are satisfied by each of the following isometric drawings.



Students can discover that when they are given only one view of an isometric drawing, they often cannot see all of the cubes because some are hidden. Students can be given an isometric drawing, such as the one shown below, and be asked to create a building from it. Generally, not all students make the same structure, and they understand that one drawing can lead to more than one 3-D object. They can again explore the maximum, minimum, and variety of structures that can support a given drawing.

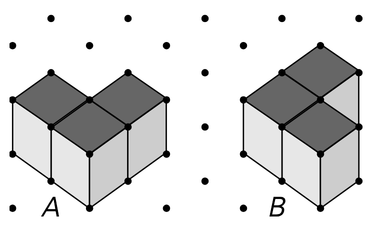


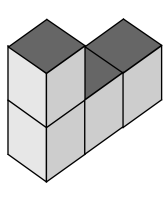
Demonstrate an isometric drawing with a simple object, such as a cube.



It is interesting to note that the angle made is 120° but in context of a drawing of the cube above, it will appear as 90°. This is an example of **perceptual constancy** in spatial visualization. Isometric drawings are always done from a front-right, front-left, back-right or back-left view, not by looking straight on.

After successively drawing a single cube, students should try to draw a series of cubes linked together, and then extend to more complex shapes using isometric dot paper.





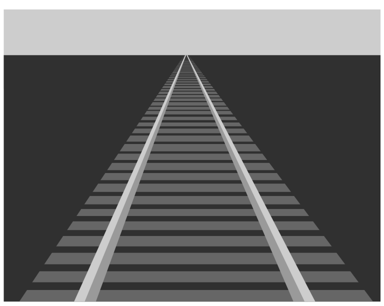
Discuss with students how views and isometric drawings are different. They should realize that each view shows only one face of the structure. Point out that an isometric drawing can show two faces, such as the front and the side, as well as part of the top structure. An isometric drawing shows the structure in space, whereas a view shows it flat.

Students should gain an understanding of perspective drawing versus isometric drawing and identify situations where one or the other is more appropriate. Both give a representation of a 3-D object. However, in isometric drawings, because parallel lines do not converge in the distance as they do in perspective drawings and in real life, they do not look as realistic.

Perspective drawings allow objects to appear proportionally smaller with distance while isometric drawings keep the same scale for height, width, and depth.

In a one-point perspective drawing, an object is drawn as it really looks from a single point. It is used to create the impression of depth and space. Lines that are parallel in reality appear to converge at a vanishing point, and objects in the foreground are represented as larger than those in the background. The vanishing point is the point at which the object appears to disappear into the distance. It is drawn on the page to appear to be behind the object.

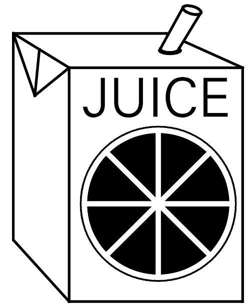
Any perspective representation of a scene that includes parallel lines has one or more vanishing points in the perspective drawing. A one-point perspective drawing means that the drawing has a single vanishing point, usually (though not necessarily) directly opposite the viewer’s eye and usually (though not necessarily) on the horizon line. All lines parallel with the viewer’s line of sight recede to the horizon towards this vanishing point. This is the standard “receding railroad tracks” phenomenon, as shown in the picture below. One vanishing point is typically used for roads, railway tracks, hallways, or buildings viewed so that the front is directly facing the viewer. Any objects that are made up of lines either directly parallel with the viewer's line of sight or directly perpendicular (the railroad slats) can be represented with one-point perspective.



When the point of perspective is above an object, the object appears to go upward into the distance; when the point of perspective is below an object, the object appears to go downward into the distance; and the same phenomenon happens when the point of perspective is to the left or right of an object. If the artist wishes the point of perspective to be below and to the left of the object, for example, the vanishing point is drawn on the page below and to the left of the object and appears to be behind the object in the distance.

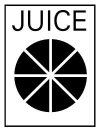
Students should be given examples of one-point perspective drawing from which they can determine the point of perspective.

Students should also be able to construct a one-point perspective drawing given a 3-D object. Key concepts include the horizon line that is at the eye level of the observer and the vanishing point located on the horizon line. All horizontal lines should be parallel to the horizon line. All vertical lines need to be perpendicular to the horizon.

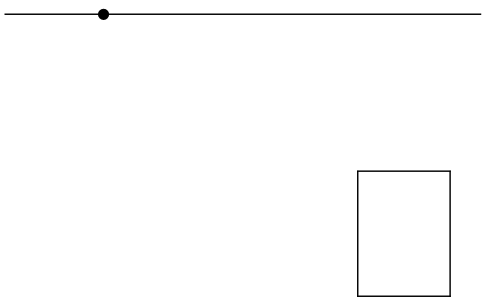


To draw a one-point perspective of a juice box, for example, students would follow the steps outlined below.

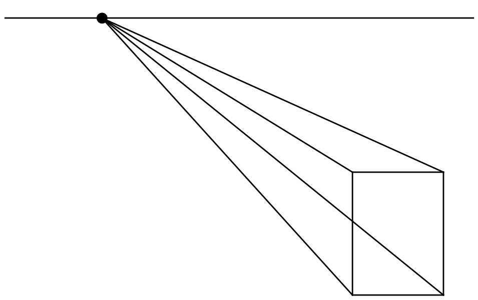
Step 1: Draw the front view.



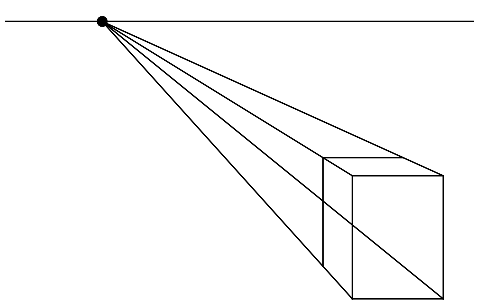
Step 2: Decide the perspective the viewer will have. If students want the perspective to be above and to the left, they select a vanishing point above and to the left.



Step 3: Draw a line from the vanishing point to each of the corners of the object.



Step 4: Draw a horizontal line between the two top lines that end at the vanishing point, and from the left end of this line, draw a vertical line.



The viewer can now also see the top and left side of the juice box.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in assessing students' prior knowledge.

- Using a supply of interlocking cubes, build a shape using a specified number of cubes.
  (a) Make a sketch of the views of the top, front, and side views on square dot paper.
  (b) Exchange models and views to verify each other's work.

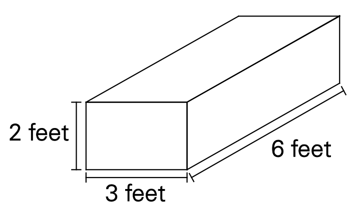
- A classmate insists that you need all six views of an object to create a physical model. Is this correct? Explain why or why not.

- Sketch and label the top, front, right side, and left side views of your school.

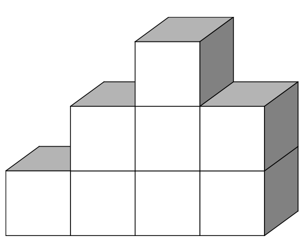
WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

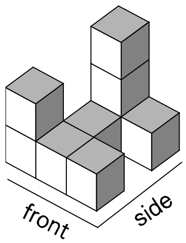
- State the dimensions for the front view, side view, and top view of the rectangular prism shown.

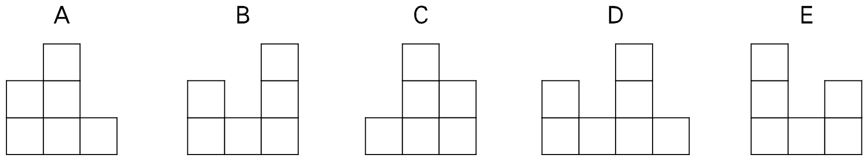


- Using blank paper, draw the top, front, and side views of this object.



- Examine this picture drawn from its right-front corner. Which one of A–E is the right orthographic view?



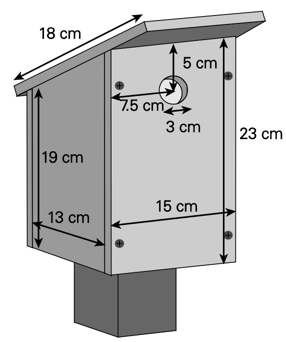


- In groups, measure a piece of furniture in the classroom. Using your measurements and an appropriate scale, draw the top, front, and side views. Draw a 2-D representation of your chosen piece of furniture.

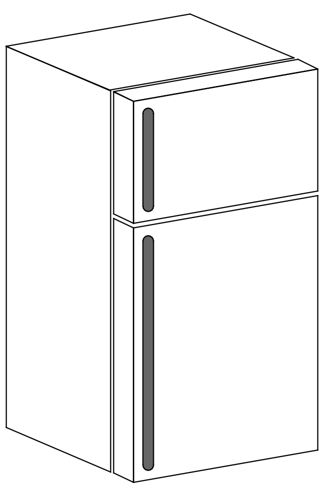
- Make a rectangular prism out of linking cubes. Draw the top, front, and side views. How are the views alike?

- Construct two different objects that have the same front, top, and side views.

- Sue built the birdhouse pictured here. The height in the front measures 23 cm and the height in the rear measures 19 cm. The width of the side panel is 13 cm and the width of the front panel is 15 cm. The roof has a length of 18 cm and a width of 15 cm. The bird hole has a diameter of 3 cm and is centred 5 cm from the top and 7.5 cm from each side.
  (a) Determine the size of the base.
  (b) Draw the component parts of the diagram using a scale of 1:5.

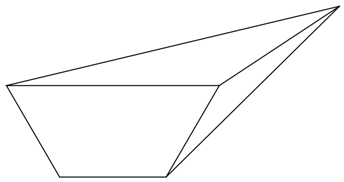


- Cody is designing a flyer for a local furniture store. He wants to include a perspective view of a refrigerator. Sketch what his drawing could look like.



- Sketch a triangle on a piece of paper. Locate a vanishing point that is below left of the triangle. Create a one-point perspective drawing of the triangle.

- Identify the point of perspective for this drawing.



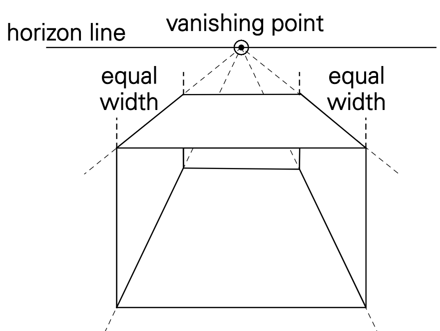
- Draw a one-point perspective view of a milk carton.

- In your journal respond to what you learned about one-point perspective drawings.

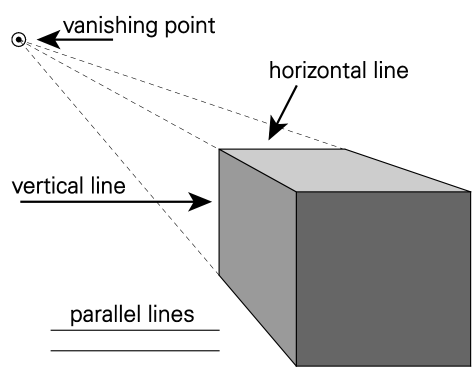
Three new things I learned: 1. 2. 3. Two things I am still struggling with: 1. 2. One thing that will help me tomorrow: 1.

- Identify the point of perspective in the following drawings. Explain how you decided on the perspective.

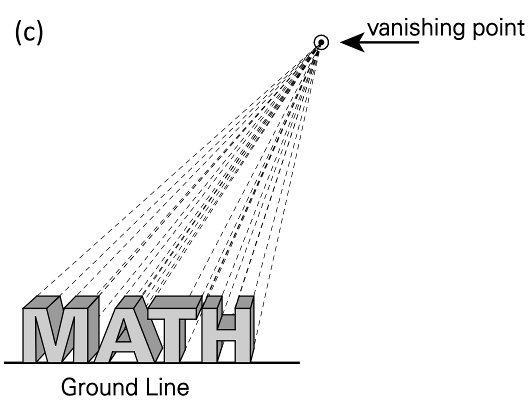
(a)



(b)



(c)



FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should be used to inform instruction.

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?
- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

SUGGESTED LEARNING TASKS

Consider the following sample instructional strategies when planning lessons.

- This material is new to the curriculum, and teachers may need to experiment with various types of questions before presenting concepts in the classroom. Collaborate with technology and arts education teachers for potential cross-curricular activities and resources.

- Research various resources that explain and demonstrate one-point perspective and isometric drawings. The Mr. D‘N’T website (Quarry 2009) at www.mr-d-n-t.co.uk/index.htm is a good starting point.

- Using isometric dot paper, have students draw a variety of 3-D objects, such as tissue boxes, teacher desks, and bookcases, and then discuss their representations. Discussion should include such questions as,
  (a) Was a scale factor used?
  (b) Does the diagram show all parts of the object?
  (c) Which object was most difficult to draw?

- A view can be drawn to scale or it can be an approximation. Discuss with students when an approximation might provide enough information and when a scale diagram might be needed.

- Ask a student to construct an original 3-D object using linking cubes. (Keep hidden from class.)
  (a) Draw front, back, side, and top views of the object.
  (b) Ask the class to re-construct the object using linking cubes based on the views.

- Ask students to complete the following tasks:
  (a) Create a figure using three geo-blocks.
  (b) Draw and label the front view, top view, and side view of the figure.
  (c) Have a partner build the figure and compare it to the original.
  (d) Use the sketches from your partner to build a figure that has the same top view, but has different side and/or front views. Draw a perspective drawing of it on grid paper.

- Provide students with various orthographic diagrams and ask them to construct a scale model of the 3-D object. Students can use the following guidelines to help construct their model.
  - Determine the amount of space and materials available to work with.
  - Determine the measurements of the actual object.
  - Use an appropriate scale.
  - Convert actual measurements to scale measurements.

- A view can be drawn to scale or it can be an approximation. Discuss with students when an approximation might provide enough information and when a scale diagram might be needed.

- Use linking cubes as the basic building blocks for 3-D objects, as they are very versatile. With the front of the object facing the students, have them turn it 90° clockwise and sketch the face of the object. Now have them turn it another 90° clockwise and sketch it again. Have them turn it one more time 90° clockwise and produce the third sketch. Have them continue rotating at 90° intervals until the sketch looks identical to the one already drawn.

- When drawing perspectives of 3-D objects, have students use a ruler to ensure that they create the most accurate drawing.

- Discuss how one-point perspective drawings can be used to represent products in advertisements or scenes in paintings.

- Provide pictures of a variety of drawings that demonstrate a one-point perspective (e.g., DaVinci's Last Supper). Ask students to identify the point of perspective in each picture. Be sure to include pictures with points of perspective that are not centred.

Suggested Models and Manipulatives

- geo-blocks
- graph paper
- isometric dot paper
- linking cubes
- pictures
- polydrons
- rulers
- unifix cubes
- various 3-D shapes

Mathematical Vocabulary

Students need to be comfortable using the following vocabulary.

- isometric drawing
- one-point perspective drawing
- orthographic drawing
- perceptual constancy
- point of perspective
- vanishing point

Resources/Notes

Internet

- *Mr. D'N'T: Secondary Education and Design* (Quarry 2009)
  www.mr-d-n-t.co.uk/index.htm
  A demonstration and explanation of isometric drawings, as well as activities.

- *Illuminations, Resource for Teaching Mathematics* (National Council of Teachers of Mathematics 2013).
  – *Isometric Drawing Tool*
    http://illuminations.nctm.org/ActivityDetail.aspx?ID=4182
    An interactive isometric drawing tool.
  – *Balancing Act*
    http://illuminations.nctm.org/LessonDetail.aspx?ID=1576
    Using cubes and isometric drawings.

Print

Math at Work 11 (Etienne et al. 2012)
- Chapter 2: Drawing and Design
  – Sections 2.2 and 2.3
  – Skill Check
  – Test Yourself
  – Chapter Project
  – Games and Puzzles

Software

- Google SketchUp (SketchUp 2014)

Notes

SCO G04 Students will be expected to draw and describe exploded views, component parts, and scale diagrams of simple 3-D objects. [CN, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

(It is intended that the simple 3-D objects come from contexts such as flat-packed furniture or sewing patterns.)

G03.01 Draw the components of a given exploded diagram, and explain their relationship to the original 3-D object.
G04.02 Sketch an exploded view of a 3-D object to represent the components.
G04.03 Draw to scale the components of a 3-D object.
G04.04 Sketch a 2-D representation of a 3-D object, given its exploded view.

Scope and Sequence

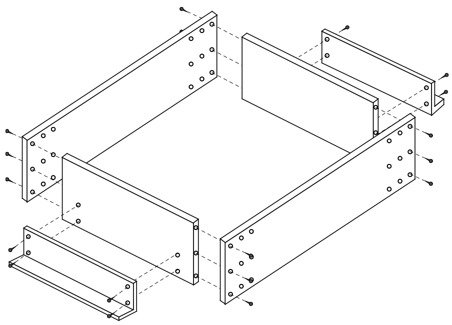
Mathematics at Work 10	Mathematics at Work 11	Mathematics at Work 12
M04 Students will be expected to solve problems that involve SI and imperial area measurements of regular, composite, and irregular 2-D shapes and 3-D objects, including decimal and fractional measurements, and verify the solutions.	**G04** Students will be expected to draw and describe exploded views, component parts, and scale diagrams of simple 3-D objects.	**G03** Students will be expected to demonstrate an understanding of transformations on a 2-D shape or a 3-D object, including translations, rotations, reflections, and dilations.

Background

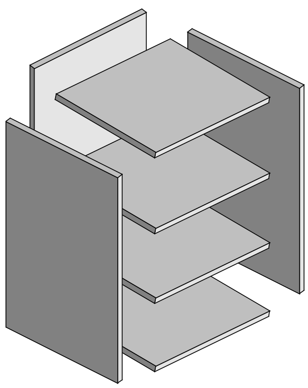
Many careers require the ability to read and interpret different types of diagrams. As consumers, students may encounter situations where knowledge of diagrams may be beneficial (e.g., assembling furniture and ordering parts online).

Students should be aware that an **exploded diagram** can be either a perspective or isometric drawing. In an exploded diagram, the components that make up an object are shown separated, but in their relative positions. This allows the viewer to see where all the parts go and how they fit together. An exploded diagram used in a parts diagram or instruction manual would be a perspective drawing, whereas an isometric exploded diagram would be useful when constructing the object.

Examples of various exploded diagrams can be shown or distributed to students when introducing the concept. Students could also find their own examples on the Internet. Students could be asked what they noticed about the diagrams and when they would be useful. This could lead to a discussion of the advantages and disadvantages of each type of diagram.



Students should be able to draw the components of a given exploded, simple 3-D object. In addition, they should be able to identify the relationship between the individual components and the original 3-D object. Students could be given diagrams like the ones below, for example, and asked to draw and label all components (i.e., front, back, side, shelf).



Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in assessing students' prior knowledge.

- Solve a problem that involves the conversion of units within or between SI and imperial systems.

- Construct various polygons on grid paper, and then copy the same polygon onto larger or smaller grid paper to create a similar figure.

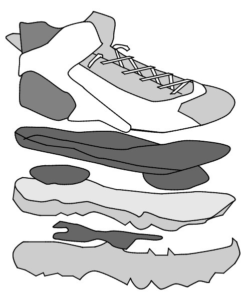
- Justify, using mental mathematics, the reasonableness of a solution to a conversion problem.

- Design a logo for a new application (play park, skateboard park, etc.) incorporating 2-D and composite shapes.

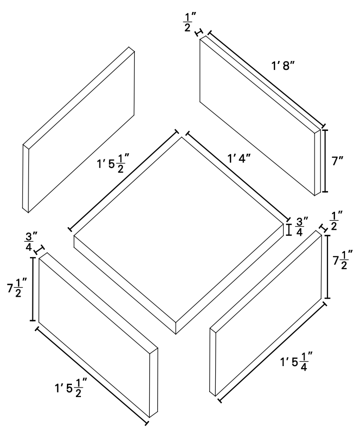
WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

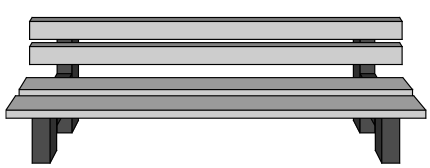
- Identify the item shown in the exploded view diagram. How many parts are shown in the diagram?



- Given the exploded view of a drawer, draw the individual components. Explain what each component represents with respect to the drawer.



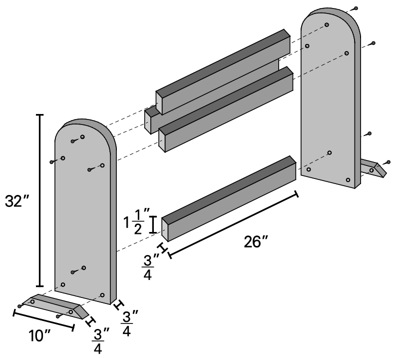
- Measure a wooden seat, such as the one below. (Similar seats could be found around the school.)



  (a) Sketch an exploded view of the seat.
  (b) Draw to scale the components of the seat.

- Create the instruction pamphlet for an object of your choice. Your pamphlet needs to include the exploded view and the written directions for assembling the object. Have the object available for hands-on demonstration or testing.

- Sketch a 2-D representation of a 3-D object with the exploded view given. The exploded view of a quilt rack is shown here:



FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should be used to inform instruction.

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?
- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

SUGGESTED LEARNING TASKS

Consider the following sample instructional strategies when planning lessons.

- Ensure that students understand how exploded view drawings are used to help put together the different components of an object to create a whole object.

- Use polydron shapes to help demonstrate the parts to an exploded view.

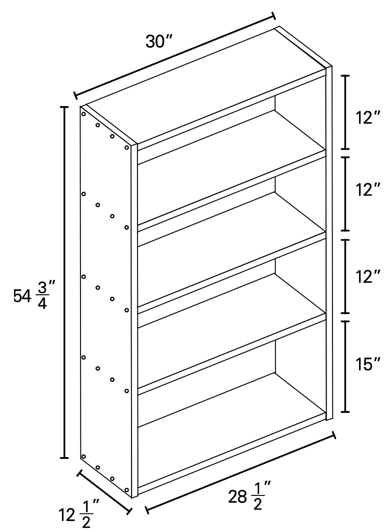
- Examples of various exploded diagrams should be shown or distributed to students when introducing the concept. Students could be asked what they noticed about the diagrams and when they would be useful. This could lead to a discussion of the advantages and disadvantages of each type of diagram.

- Have students sketch exploded views of 3-D objects found in the classroom or at home. They should keep in mind that all parts of the object should be shown. Drawing the components isometrically can be helpful. Draw dashed lines to show the relationship between the parts, or how the parts could be assembled together to form the 3-D object. Students should start with simple objects and progress towards more complicated shapes.

- Do as many hands-on activities as possible. Work both from diagram to object, and from object to diagram.

- Collaborate with technology, arts education, and other teachers for potential cross-curricular activities and resources.

- Provide problems where a sketch of a 3-D object with measurements is provided, and have students draw its components to scale. One such example would be a bookcase as shown below.



- Have students sketch a 2-D representation of a 3-D object given an exploded view. They could be asked to sketch a diagram that shows how the pieces of an exploded diagram would fit together.

SUGGESTED MODELS AND MANIPULATIVES

- 3-D objects
- blue prints
- polydrons
- various exploded view diagrams

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- exploded view diagram

Resources/Notes

Internet

- *Mr. D'N'T*: Secondary Education and Design (Quarry 2009)
  www.mr-d-n-t.co.uk/index.htm
  Examples of exploded views.

Print

Math at Work 11 (Etienne et al. 2012)
- Chapter 2: Drawing and Design
  - Section 2.3
  - Skill Check
  - Test Yourself
  - Chapter Project
  - Games and Puzzles

Notes

Number
15–20 hours

GCO: Students will be expected to develop number sense and critical-thinking skills.

SCO N01 Students will be expected to analyze puzzles and games that involve numerical reasoning, using problem-solving strategies. [C, CN, PS, R]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

(It is intended that this outcome be integrated throughout the course by using puzzles and games such as cribbage, magic squares, and Kakuro.)

N01.01 Determine, explain, and verify a strategy to solve a puzzle or to win a game; for example,
- guess and check
- look for a pattern
- make a systematic list
- draw or model
- eliminate possibilities
- simplify the original problem
- work backward
- develop alternative approaches

N01.02 Identify and correct errors in a solution to a puzzle or in a strategy for winning a game.

N01.03 Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.

Scope and Sequence

Mathematics at Work 10	Mathematics at Work 11	Mathematics at Work 12
G01 Students will be expected to analyze puzzles and games that involve spatial reasoning, using problem-solving strategies.	**N01** Students will be expected to analyze puzzles and games that involve numerical reasoning, using problem-solving strategies.	**N01** Students will be expected to analyze puzzles and games that involve logical reasoning, using problem-solving strategies.

Background

It is intended that this outcome not be taught in isolation but that it be integrated throughout the course.

Mathematics at Work 10 focuses on spatial reasoning to solve puzzles and play games. In Mathematics at Work 11 the focus shifts to numerical reasoning to solve puzzles and play games. However, it is not enough for students to only do the puzzle or play the game. They must be given a variety of opportunities to analyze the puzzles they solve and the games they play. The goal is to develop their problem-solving abilities using a variety of strategies and to be able to apply these skills to other contexts in mathematics. In Mathematics at Work 12 the focus will shift to logical reasoning.

Students need time to play and enjoy each game before analysis begins. They can then discuss the game, determine the winning strategies, and explain these strategies by demonstration, orally, or in writing.

A variety of puzzles and games, such as board games, online puzzles and games, and pencil-and-paper games should be used. It would be beneficial to share with other teachers games and puzzles that promote the intent of this outcome.

Problem-solving strategies will vary depending on the puzzle or game. Some students will use strategies such as working backwards, looking for a pattern, using guess and check, or by eliminating possibilities, while others will plot their moves by trying to anticipate their opponents' moves. As students play games and analyze strategies, they explore mathematical ideas and compare different strategies for efficiency.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

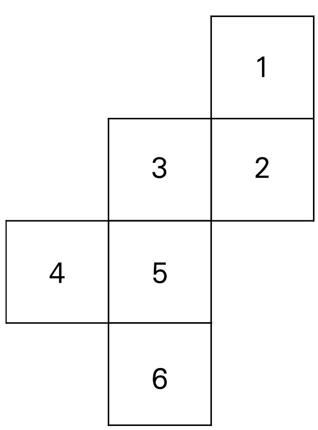
Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- Recall games that you have played that require strategies involving numerical reasoning.

- If the numbered net shown below is folded to form a cube, what is the product of the numbers on the four faces sharing an edge with the face number 1?



WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- Magic 15 is a game for two players. Begin with the numbers 1 to 9. Players take turns selecting a number, with each number used only once. The winner is the first player to have exactly three numbers that total 15.

- In Roll Six, players roll six dice and use five of the numbers together with any of the four operations to make the sixth number. Points are scored for successful equations. (As an alternate or extension task, this game can be played using the Order of Operations.)

- Create a game using the rules of an existing game but using different materials **or** use the idea for an existing game and change the rules. For example, the game Snakes and Ladders can be modified to Operation Snakes and Ladders. The board can be used with two dice. On each turn, to determine the number of spaces to move, the player has the option of multiplying, dividing, adding or subtracting the two numbers, with a maximum answer of twenty.

- Using a puzzle or game of your choice, answer the following questions:
  - What problem-solving strategies did you try? Which worked well and which did not?
  - Explain the rules of the game in your own words. Show your rules to another student. Do they agree with your explanation? Can other people find loopholes in your rules?
  - What did you do when you got stuck? Explain through words or diagrams the strategies you tried in solving the puzzle or playing the game.
  - What general advice would you give to other students trying to solve the puzzle or play the game?

- Create a variation of a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
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Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
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- What teaching strategies and resources should be used?
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Suggested Learning Tasks

Consider the following sample instructional strategies when planning lessons.

- The following are some tips for using games in the mathematics class.
  - Keep the number of players from two to four, so that turns come around quickly.
  - Communicate to students the purpose of the game.
  - Engage students in post-game discussions.
  - Use games for specific purposes.

- As students play a game or solve a puzzle, it is important to pose questions and engage students in discussions about the strategies they are using. Ask probing questions and listen to their responses. Record the different strategies and use these strategies to begin a class discussion. Possible discussion starters include the following:
  - Thumbs up if you liked the game, thumbs sideways if it was okay, and thumbs down if you did not like it. What did you like about it? Why?
  - What did you notice while playing the game?
  - Did you make any choices while playing?
  - Did anyone figure out a way to quickly find a solution?

- Students will benefit from solving puzzles and playing games if they take time to reflect on their experiences. Ask them to choose one of the puzzles or games they have worked on and write about it. Students may find it easier to record their thoughts if they talk about what they are thinking as they work through a puzzle, while a partner takes notes.

- Choose a variety of games or puzzles for students to play online, with pencil and paper, or using models that require a variety of strategies to solve.

- Try games in advance as instructions to games are not always clear.

- Have students look for patterns and then develop strategies to fit these patterns. Numerical examples of this would be in Magic Squares and Sudoku.

- Set up stations with one or two games at each centre. Circulate among the groups and assess students' understanding. Puzzles and games involving numerical reasoning could include the following:
  - Cribbage
  - Kakuro
  - Magic Square
  - Monopoly
  - Yahtzee
  - Game of Life
  - Sudoku

- Students can work through the puzzles or games individually or with a partner. At regular intervals, have students rotate to the next station. They could record their progress in a table such as the one shown below.

Puzzle	**Solved?**	**Strategy**	**Comments/hints**

- Have students develop a game for classmates to play and/or use a known game. Students can then change a rule or parameter and explain how it affects the outcome of the game. The following guiding questions could be used to help students evaluate their games.
  - Can the game be completed in a short time?
  - Is there an element of chance built in?
  - Are there strategies that can be developed to improve the likelihood of winning?

- Have students switch partners periodically to provide opportunities for new strategies to be shared.

- Find a game online and critique the quality of it. Several websites are listed in the Internet section of this outcome.

- As students work through the games and puzzles, keep a checklist of the games and puzzles each student is working on. Students can work in groups where each member has been exposed to a different game or puzzle. Ask the group to do the following:
  - Explain the rules of the game in your own words to the other group members, and give a brief demonstration of how the game is played.
  - What advice would you give to other students trying to solve the puzzle or play the game?
  - What did you do when you got stuck?

- Invite students to bring in their own games and puzzles involving numerical reasoning. Students can introduce this game/puzzle by providing information such as the following:
  - What is the puzzle or game, and where did you find it?
  - Describe the puzzle or game. Why did you select it?
  - Describe the objective of the game and the rules of play.
  - What strategy would you use to solve the puzzle or play the game?

- This outcome provides tremendous opportunity to differentiate for the needs of students. Prepare an extension question ready for a group that finishes before others. Have one or two extra clues ready. These can be used to allow the inclusion of an extra group member, to give help to a group that is stuck, or to assist checking when a group thinks they have finished the task. While observing the task, focus on questions such as,
  - How did your group get started?
  - How did the group "join" the clues together?
  - Were there any challenges?
  - Which clue did you find most helpful?
  - What might you do differently next time?

- Have students keep a games and puzzles journal that they write in every "Games and Puzzles Day." They should reflect on the strategies they used to solve the puzzle or win the game. The following is an example of how the journal could be set up.

Games Journal			
Date	Game	Win or Lose **or** Score	Explain your strategies

- *Pieces of the Puzzle*
  Create cards that display pieces of each puzzle. Students are each given a clue card, and they must find their groups based on which puzzle name their card displays. It is important to have each card titled with the puzzle name to allow students to form their groups easily. The student must hold on to their clue card and is responsible for communicating the content to the group. Each student's role is to work with the group to solve the puzzle using the clues. It is important for groups to report on their problem-solving processes, as well as confirming the correctness of the solution.

  A sample puzzle is shown below. In this case, students have to find the mystery number. It may be helpful if members of the group were provided with a hundreds chart.

PETER'S NUMBER The number is a multiple of four.	PETER'S NUMBER The number is in the bottom half of the hundreds chart.
PETER'S NUMBER One of the digits is not double the other digit.	PETER'S NUMBER The sum of the digits is an odd number.
PETER'S NUMBER The number is a multiple of six.	PETER'S NUMBER The number is even.

SUGGESTED MODELS AND MANIPULATIVES

- dice
- puzzles
- various games

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- special reasoning
- strategy
- systematic list
- visualization

Resources/Notes

Equipment/Materials

- 24 Game: (Double Digits)—Add, Subtract, Multiply, and Divide (Orwell, n.d.; NSSBB #: 17232)
- 24 Game: Factors and Multiples Game (Orwell, n.d.; NSSBB #: 17310)
- 24 Game: Fractions and Decimals (Orwell, n.d.; NSSBB #: 23955))
- 24 Game: Integers (Orwell, n.d.; NSSBB #: 23956)

Internet

- There are numerous games and puzzles available on the Internet. What follows are just a few suggestions of spatial games and puzzles, available for free online. Many can also be acted out, completed on paper, or solved using models.

  Note: It is expected that the teacher confirm the validity of the site prior to directing students to it.

- MathIsFun.com (MathIsFun.com 2013)
  - Tic-Tac-Toe
    www.mathsisfun.com/games/tic-tac-toe.html
  - Dots and Boxes Game
    www.mathsisfun.com/games/dots-and-boxes.html
  - Four in a Line
    www.mathsisfun.com/games/connect4.html

- *Calculation Nation* (National of Teachers of Mathematics 2013)
  http://calculationnation.nctm.org

- *Nrich: Enriching Mathematics* (University of Cambridge 2014)
  http://nrich.maths.org

- *Coolmath-Games.com*, “Coolmath’s B-Cubed” (Coolmath.com 2014)
  www.coolmath-games.com/0-b-cubed/index.html

- Numerous numerical puzzles are available at:
  - Humorous Implications, "Numerical Puzzles" (Declarative Languages and Artificial Intelligence 2014)
    http://dtai.cs.kuleuven.be/projects/ALP/newsletter/archive_93_96/humour/index- num.html
  - Samgine, "Number Puzzles and Math Puzzles" (Samgine 2011)
    http://samgine.com/free/number-puzzles
  - *PBS Kids*, "123 Games" (PBS Kids 2013)
    http://pbskids.org/games/numbers/html
  - *Fibonicci*, "Number Sequence Test—Medium" (Fibonicci 2011)
    http://www.fibonicci.com/numeracy/number-sequences-test/medium
  - MindJolt Games (MindJolt Games 2013)
    http://www.mindjolt.com
  - CombinationLock.com (OriginalGames.com 2014)
    www.combinationlock.com

- *Jefferson Lab, "*The Nim Number Game" (Jefferson Lab 2014)
  http://education.jlab.org/nim/index.html
  Nim is an ancient game that can be played online, on paper, or using sticks. The winner is the player to not pick up the last stick (Common Nim) or to pick up the last stick (Straight Nim).

Print

Math at Work 11 (Etienne et al. 2012)

- Chapter 1: Surface Area
  - Sections 1.2 and 1.3 Puzzlers
  - Games and Puzzles
- Chapter 2: Drawing and Design
  - Section 2.3 Puzzlers
  - Games and Puzzles
- Chapter 3: Volume and Capacity
  - Section 3.4 Puzzlers
  - Games and Puzzles
- Chapter 4: Interpreting Graphs
  - Section 4.1 Puzzler
  - Games and Puzzles
- Chapter 5: Banking and Budgeting
  - Section 5.2 Puzzler
  - Games and Puzzles
- Chapter 6: Slope
  - Section 6.1 Puzzler
  - Games and Puzzles
- Chapter 7: Right Triangles and Trigonometry
  - Sections 7.1 and 7.2 Puzzlers
  - Games and Puzzles

Notes

SCO N02 Students will be expected to solve problems that involve personal budgets. [CN, PS, R, T]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N02.01 Identify income and expenses that should be included in a personal budget.
N02.02 Explain considerations that must be made when developing a budget (e.g., prioritizing, recurring and unexpected expenses).
N02.03 Create a personal budget based on given income and expense data.
N02.04 Collect income and expense data and create a budget.
N02.05 Modify a budget to achieve a set of personal goals.
N02.06 Investigate and analyze, with or without technology, “what if …” questions related to personal budgets.

Scope and Sequence

Mathematics at Work 10	**Mathematics at Work 11**	**Mathematics at Work 12**
N02 Students will be expected to demonstrate an understanding of income to calculate gross pay and net pay, including wages, salary, contracts, commissions, and piecework.	**N02** Students will be expected to solve problems that involve personal budgets.	**N02** Students will be expected to solve problems that involve the acquisition of a vehicle by buying, leasing, or leasing to buy.

Background

In Mathematics at Work 10 students were expected to demonstrate an understanding of income (e.g., net and gross pay) and deductions (e.g., CPP, EI, income tax, health plans, and union dues) including wages, salary, contracts, commissions, and piecework.

Income, deduction, and expense data will now be used by students to prepare budgets. Budgets are useful for

- determining how money is being made and spent
- preventing the accumulation of large amounts of debt
- planning for future purchases or emergencies

Students are expected to create a budget based on data that are provided, as well as data they collect themselves. At the start of the unit, students could be asked to record their personal expenses and income for one week in preparation for creating their own budget later in the unit. It is important to also have an example available for students to use if they have no income or if they prefer not to use their own income and expenses. Once completed, students should modify their budget to achieve immediate personal goals (such as purchasing a car, and making car and insurance payments) and analyze changes to their budget necessary to achieve future goals.

To create a personal budget, students should identify

- net pay
- income from other sources (e.g., investments, tax credits, and rental properties)
- fixed expenses (e.g., rent, car payments, and telephone/Internet bills)
- variable expenses (e.g., heat, electricity, dining out, and repairs)

It may be useful for students to first identify general categories of expenses (e.g., housing, transportation, insurance, education, food, pets, personal, entertainment, loans, taxes, savings and investments, recreation, miscellaneous) and then give specific expenses under each category.

When developing personal budgets, students need to prioritize expenses to first ensure that the necessities of life are met. Ideally, some money will remain for investing and unexpected expenses (e.g., auto repair, health care, gifts, appliance replacement). Some expenses, such as property tax, are billed annually or semi-annually but can be paid monthly. Students need to be cognizant of this additional expense when making a monthly budget. Students must understand that, for some expenses, the amount budgeted and the amount actually spent may not match. It is suggested that students be provided with the data needed to complete a monthly budget, based on stated assumptions about income levels and expenses. Additional assumptions that should be stated in assignments include owning or renting housing, transportation options such as owning or leasing a car, or using public transit and taxis. Assignments should reflect diversity of lifestyles, values, and economic realities for students and their families.

Once their personal budgets are completed, students should identify some personal goals, the monetary cost of each goal, and a timeline for achieving each goal. Students should then determine how to modify their budget to attain these goals. For example, students may select an educational goal (such as attending a trade program at a local college), the purchase of a new vehicle, or living on their own. Students would then determine what budget changes are necessary to fund their goals. These changes may involve a combination of increasing income and decreasing or eliminating expenses. Students should be helped to understand that a realistic and achievable goal can sometimes be derailed by unexpected and uncontrollable events. This can be investigated by posing “what if” scenarios and determine if previously identified goals are still attainable. Spreadsheets are an excellent tool for investigating the solutions to these scenarios. For example, students could be faced with loss of income, major health expenses, vehicle or property damage, or unexpected family costs.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- Determine the tax on a purchase of $120. (Tax rate is 15%.)
- Estimate the sum of $43.52, $24.31, and $57.48. Explain your strategy.
- Mentally determine 3 × $8.50 and explain your strategy.
- Mentally determine 25% of $440 and explain your strategy.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- Make a list of variable and fixed expenses that should be included in your personal budget. Based on a weekly income of $130, prepare a personal budget where you will save at least $20 per week.

- Explore options for when you finish high school (such as post-secondary studies or employment). List the sources of income (e.g., part-time jobs, scholarships, savings, loans) and possible expenses that you would incur (e.g., tuition, transportation, rent). Then, using the collected data, develop a personal budget.

- Using the completed budget and a goal provided by your teacher, determine if the goal is attainable/realistic.

- Michelle is a high school student living at home. She has a part-time job. Michelle's budget for a month is in the following table.

Monthly Net Income	**Budget**
Job	$475
Monthly expenses	**Fixed expenses**
Savings	$45
Cell phone	$45
Transportation (bus fare)	$40
Variable expenses	
Clothing	$250
Entertainment	$50
Personal items	$55
School expenses	$35
Other (gifts)	$50

(a) Calculate her total expenses.
(b) How much more are her planned expenses than her income?
(c) Suggest ways she can alter her spending to keep her planned expenses less than or equal to her income.

- It has been suggested that families have a reserve equal to three to six times the family's monthly income for unexpected events such as job loss. List the advantages and disadvantages of this suggestion. This could be extended to ask how realistic this is for some families (single parent; only one parent working; low-income earners; people on social assistance). For situations in which this is not feasible, what other strategies could be employed to prepare for unexpected events?

- Use a chart similar to the following as a guide to list all the expenses you think you might incur living on your own or with one or more roommates (this can be used as a group task).

Expense	**Amount ($)**
Getting started costs One-time costs, such as hook-up fees for phone, cable/Internet, purchase of furniture, dishes, appliances.	
Rent or Mortgage	
Utilities Electricity, heat	
Food Staples, such as flour, spices, condiments, beans; regular groceries. Home-cooked meals are cheaper and usually healthier than restaurant food.	
Transportation Public transit, bicycle, or car. If you have a car, you will need to budget for insurance, gas, maintenance, and parking.	
Medical/Dental Medical plan payments and/or costs such as glasses, contacts, prescriptions, and dental care not covered by Nova Scotia Health Insurance or by a medical plan.	
Clothing Consider clothes required for work and seasonal clothes, such as boots and a winter coat.	
Miscellaneous This may include laundry, entertainment, toiletries, and cleaning supplies. Also consider purchasing gifts for birthdays and holidays.	
Other This includes anything else that is not included in other categories, such as loan payments, vacations, memberships, or workshop fees.	
TOTAL OF ALL ESTIMATED COSTS	

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?
- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

SUGGESTED LEARNING TASKS

Consider the following sample instructional strategies when planning lessons.

- Spreadsheets, such as Excel, could be effectively used in situations such as calculating and revising budgets as well as to keep track of expenses and income.

- Provide students with a budget and a goal that is currently unattainable. Ask them to decide which expenses to reduce or eliminate in order to make the goal attainable.

- After completing a budget from given data, students may be provided with income data and some basic expenses and asked to add at least three expenses of their own. Using these data, students could create a second budget. Eventually, students will be expected to collect their own income and expense data and create a budget. This data may be collected from sources such as telephone and electricity bills, credit card statements, catalogues, and advertising flyers. Using the data collected at the beginning of the unit, and data from other sources, students can create their personal budget.

Sample Monthly Family Budget (2 incomes)

Income ($)	Expected Expenses ($)	Actual Expenses ($)	Difference ($)
Net Pay (1): 2300	Mortgage: 700	700	0
Net Pay (2): 1500	Property tax: 120	120	0
Rental Income: 500	Groceries: 600	650	–50
	Child care: 800	800	0
	Dining out: 250	200	50
	Heat and light: 200	230	–30
	Telephone: 30	70	–40
	Mobile phone: 50	30	20
	Cable/Internet: 120	120	0
	Car payment: 400	400	0
	Gas: 300	350	–50
	Insurance (vehicle): 200	200	0
	Insurance (house): 50	50	0
	Clothing: 150	120	30
	Personal care: 50	70	–20
	Investments: 120	120	0

Sample Monthly Family Budget (1 income)

Income ($)	Expected Expenses ($)	Actual Expenses ($)	Difference ($)
Net Pay (1): 680	Rent: 600	600	0
	Property tax: 0	NA	NA
	Groceries: 300	325	–25
	Child care: 400	400	0
	Dining out: 20	25	–5
	Heat and light: 150	180	–30
	Telephone: 35	70	–40
	Mobile phone: 0	NA	NA
	Cable/Internet: 0	NA	NA
	Car payment: 0	NA	NA
	Public transit: 75	75	0
	Gas: 0	NA	NA
	Insurance (vehicle): 0	NA	NA
	Insurance (tenant): 30	NA	NA
	Clothing: 40	50	–10
	Personal care: 30	10	20
	Investments: 0	NA	NA

- Present students with various scenarios—living on their own and working, living as a single parent with an infant or school-aged child, going to school and working part-time, working full-time, living as a two-income family with two children, among other possible scenarios. Have them develop their own budgets that include all of their expenses or complete a budget such as the one found in Chapter 9 of *Money and Youth* (Canadian Foundation for Economic Education 2013) at http://moneyandyouth.cfee.org/en/resources/pdf/moneyyouth_chap9.pdf.

  Note: You may have students who are living in the scenarios listed above. This could also be an opportunity for students to interview someone living in these various circumstances to get a true picture of expenses, especially hidden ones.

- Additional questions and activities can be found in the textbook and teachers guide used in Mathematics 10, *Financial Mathematics* (Etienne and Kalwarowsky 2013)

Mathematical Vocabulary

Students need to be comfortable using the following vocabulary.

- fixed expenses
- variable expenses

Resources/Notes

Internet

Use some of the free online resources available on financial literacy such as

- *Get Smarter About Money.ca* (Investor Education Fund 2013)
  www.getsmarteraboutmoney.ca

- *Literacy at Every Level*, "Skills for Life, Unit 6—Managing My Money." (Quebec Literacy Working Group 2013)
  www.qlwg.ca/index.php
  A free download, Managing my Money.

- *RBC Royal Bank*, "Create a Budget Calculator—Manage your Debt Effectively." (Royal Bank of Canada 2013)
  www.rbcroyalbank.com/products/personalloans/budget/budget-calculator.html
  A resource provided by RBC to explore the financial implications of "What if?" situations such as job loss, loss of roommate, and illness.

- *Starting Point: Teaching Entry Level Geoscience*, "How to use Excel" (Science Education Resource Centre 2013)
  http://serc.carleton.edu/introgeo/mathstatmodels/xlhowto.html
  Part of Carleton College in Minnesota, this site has links to several useful tutorials for using the Excel spreadsheet program.

- *The City: A Financial Life Skills Resource* (Financial Consumer Agency of Canada 2013)
  www.fcac-acfc.gc.ca/eng/education/index-eng.asp

Print

Math at Work 11 (Etienne et al. 2012)
- Chapter 5: Banking and Budgeting
  - Section 5.2
  - Skill Check
  - Test Yourself
  - Chapter Project

Software

- Spreadsheets

Notes

SCO N03 Students will be expected to demonstrate an understanding of compound interest. [CN, ME, PS, T]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N03.01 Solve a problem that involves simple interest, given three of the four values in the formula $I = Prt$.
N03.02 Compare simple and compound interest and explain their relationship.
N03.03 Solve, using a formula, a contextual problem that involves compound interest.
N03.04 Explain, using examples, the effect of different compounding periods on calculations of compound interest.
N03.05 Estimate, using the Rule of 72, the time required for a given investment to double in value.

Scope and Sequence

Mathematics at Work 10	Mathematics at Work 11	Mathematics at Work 12
–	**N03** Students will be expected to demonstrate an understanding of compound interest.	**N02** Students will be expected to solve problems that involve the acquisition of a vehicle by buying, leasing, or leasing to buy.

Background

This will be students' first exposure to simple and compound interest. In this unit, the initial focus is on simple interest and its use in problem solving. This is followed by a comparison of simple and compound interest with a focus on the relationship between the two. Students will then work with compound interest, considering the effect of different compounding periods, and use compound interest in problem-solving situations. Finally, they will **estimate** the doubling time for an investment using the Rule of 72.

New terminology will include the following:

- **Principal:** The original amount invested or borrowed.
- **Interest:** Money earned on an investment or a fee paid for borrowing money, usually expressed as a percentage.
- **Interest rate:** The percentage charged, usually stated as a per year rate.
- **Simple interest:** Interest calculated as a percentage of the principal.
- **Term:** The time in years for an investment or loan.
- **Compound interest:** The interest paid on the principal and its accumulated interest.
- **Compounding period:** The time between calculation of interest, also called the interest period.

Students are introduced to the concept of simple interest and the use of the simple interest formula, $I = Prt$. P represents the principal, r represents the rate of interest, and t represents the term of the investment. To calculate final or future value use the formula $FV = P + I$. FV represents the final or future value. (The formula is sometimes written as $A = P + I$ where A represents the final amount.)

Simpler problems will require that students substitute the values for the principal, rate (as a decimal), and time (in years) into the formula to obtain the interest earned. Other problems may require formula rearrangement, either before or after values are substituted. When given the time in months, students are expected to change it into years before substituting into the interest formula.

Compound interest is much more common than simple interest. To understand compound interest, however, it is important to have an understanding of simple interest first.

The relationship between simple and compound interest should be investigated using investment vehicles such as savings accounts or guaranteed investment certificates (GICs). Initially, students should use an iterative process to compare the effects of simple and compound interest. The tables below illustrate the effects of simple and compound interest on the value of $1000 in a savings account earning annual interest of 5%.

Simple Interest

Year	Principal	Amount of Annual Interest	Total Amount at the End of the Year
1	$1000	1000 × 0.05 = 50	$1050
2	$1000	1000 × 0.05 = 50	$1100
3	$1000	1000 × 0.05 = 50	$1150
4	$1000	1000 × 0.05 = 50	$1200

Compound Interest

Year	Principal plus Interest	Amount of Annual Interest	Total Amount at the End of the Year
1	$1000	1000 × 0.05 = 50	$1050
2	$1050	1050 × 0.05 = 52.50	$1102.50
3	$1102.50	1102.50 × 0.05 = 55.13	$1157.63
4	$1157.63	1157.63 × 0.05 = 57.88	$1215.51

To highlight the effect of compound interest in the short term, larger values for the initial amount of money in the savings account can be used. Alternatively, a longer time period can be used.

Once students have explored the iterative process, they can be introduced to the appropriate compound interest formulas and use them to calculate the value of an investment after a specific amount of time. In Mathematics 9, students evaluated powers with integral bases and whole number exponents (9N01). They are now expected to evaluate powers with decimals in both the base and the exponent. This is easily calculated using the y^x function on a calculator. In Mathematics at Work 11, it is not an expectation that students rearrange the formula in compound-interest problems.

To calculate the value of an investment with compound interest over a long term, students will use the following formula:

- The final amount for compound interest can be calculated using the formula, $FV = P\,(1 + i)^n$.
  FV represents final amount or future value,
  P represents the principal,
  i represents the interest earned per compounding period (or annual interest rate divided by the

number of compounding periods per year), and
n represents the total number of compounding periods of the investment.

For an annual interest rate of 7.2% compounded quarterly for 2 years,
$i = \frac{0.072}{4} = 0.018$ and $n = 2 \times 4 = 8$.

- Students who have difficulty determining the value of i and the value of n may prefer to use the formula $FV = P(1 + \frac{r}{n})^{nt}$.
  V represents the final or future value,
  P represents the principal,
  r represents the annual interest rate,
  n represents the number of compounding periods in a year, and
  t represents the term of the investment in years.

Students should compare the value of an investment when interest is compounded annually, semi-annually, quarterly, monthly, and daily. Students should understand that the more often interest is compounded, the greater the value of their investment as length of time invested increases.

In the study of compound interest, have students investigate how long it takes for an amount to double if it is invested at different rates. This should lead to the "Rule of 72." This is a method that is used to estimate the time it takes for an investment compounded annually to double in value. Dividing 72 by the annual interest rate expressed as a percentage, will give you the number of years it would take to double the value. For example, an investment with an interest rate of 3% per annum will double in $\frac{72}{3} = 24$ years.

Students can use the Rule of 72 to quickly approximate doubling time for an investment. This provides an opportunity to use mental mathematics. Students can use the Rule of 72, for example, to estimate how long it will take for an investment to double at an interest rate of 1.95% compounded annually. They can round the interest rate to 2% and use the formula to determine that half of 72 is 36. They should conclude that an investment purchased at a rate of 1.95% would take over 36 years to double.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- The price of a pair of shoes is $54. Calculate the final price after HST of 15% is added.

- Evaluate the following expressions by substituting the values given.
  (a) $5x^3 + 6xy$, if $x = 2$, $y = 3$
  (b) $(2x)^2$, if $x = 4$

- Leah solved the following equation. Check for any errors. If any were made, indicate where and make the necessary changes to correct them.
  $\frac{1}{3}(x - 2) = 5(x + 6)$
  $3(x - 2) = 5(x + 6)$
  $3x - 6 = 5x + 30$
  $3x - 6 + 6 = 5x + 30 + 6$
  $3x - 5x = 5x - 5x + 36$
  $-2x = 36$
  $\frac{-2x}{2} = \frac{36}{-2}$
  $x = -18$

- Solve and verify.
  (a) $3(x + 2) = 12$
  (b) $\frac{-5}{x} = -2$
  (c) $\frac{x}{2} - 3 = 1\frac{1}{6}$
  (d) $\frac{m}{3} - \frac{3m}{4} = 10$
  (e) $\frac{1}{2}k - 5 = 4 - k$

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- Determine how much simple interest is earned in each case.
  - $2500 at 3%/year for 5 years
  - $800 at 2.5%/year for 2 years
  - $1000 at 6%/year for 6 months

- Determine how much compound interest is earned in each case.
  - $5000 at 3%/year for 10 years
  - $350 at 1.5%/year for 2 years
  - $1250 at 2.25%/year for 3 years

- A \$1500 investment earns simple interest at an annual rate of 2.5%. Determine the number of years it will take to earn \$270 in interest.

- Meika is 25. She would like to retire at age 50. She decided to invest an inheritance of \$60,000 at 2.5% compounded annually. How much will she have at age 50?

- Jane, who is 18 years old, won \$1,000,000 in a lottery. She invested \$500,000 of her winnings into a bank account that pays 3.25% per year for her retirement, when she turns 65, in 47 years. How much money will there be in her account when she turns 65?

- Calculate the yearly interest rate on a \$500 loan that was borrowed for six months if the total amount of interest paid is \$35. Use the formula, I = Prt.

  Interest *I* = Principal Amount *P* • Interest Rate *r* • time *t*

- A house is bought today for \$335,000. If the value of real estate increases by 7% every year, what is the value of the house after six months?

- For each of the following, determine the future value and the total interest earned.
  - \$200 invested for 9 years at 1.75% compounded monthly
  - \$750 invested for 12 years at 2.4% compounded quarterly

- Which is the better investment? Explain.
  - 2.5% compounded annually
  - 2.5% compounded quarterly

- A couple decides to set aside \$5000 in a savings account for a second honeymoon trip. It is compounded quarterly at 1.75%. Find the amount of money they will have in four years.

- *Good Deal Financial* is offering a 4.4% interest rate, compounded monthly, on amounts in savings accounts over \$10,000. Set up a chart or spreadsheet and determine how much interest you would have earned after one year for a \$12,000 investment. Use the formula to calculate the interest for 10 years.

- Ron and Ann have just won \$100,000 in a lottery. They have a young daughter who will be two in August. Ron suggests that they buy a GIC at 2.3% with the \$100,000 over a 16-year period. If they cash in the GIC after 16 years, will they have enough interest to pay for their daughter's first year at university? First-year tuition is expected to grow 175% from the present \$3500 in 16 years.

- Suzette and Raymon each invest \$5000 for six years at a rate of 3.2% per annum. Suzette chooses simple interest as she would like access to the interest each year, and Raymon chooses compound interest. How much interest did each person earn over the six years?

- Rajesh has $1000 to invest. He checks out five different banks, each having an annual interest rate of 1.5%. However, each bank uses a different compounding period. Complete the following table to help Rajesh decide which bank he should choose.

Bank	Compounding Period	Interest Rate	Number of Compounding Periods	Amount of Money After One Year
1	annually			
2	semi-annually			
3	quarterly			
4	monthly			
5	daily			

- Use the Rule of 72 and mental math to estimate how long it would take an investment to double at an interest rate of 3.1% compounded annually.

Follow-up on Assessment

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?
- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Suggested Learning Tasks

Consider the following sample instructional strategies when planning lessons.

- Students should be given plenty of practice determining which formula to use in which situation, and then in calculating the answers.

- Pablo earns $60 in interest. Jerika earns $70 interest. Ask students to discuss some possible reasons why Jerika earns more.

- Ask students to explain why the amount of interest earned from the end of year 1 to the end of year 2 is much less than the amount of interest earned from the end of year 29 to the end of year 30.

Year	Principal	Total Amount at End of Year
1	$1000	$A = \$1000(1.08)^1 = \$1080$
2	$1080	$A = \$1000(1.08)^2 = \$1166.40$
29	$8627.11	$A = \$1000(1.08)^{29} = \$9317.27$
30	$9317.27	$A = \$1000(1.08)^{30} = \$10,062.66$

- As a journal entry, ask students to respond to the following: Two GICs have the same principal, the same rate per annum, and the same term. One is compounded semi-annually and the other is compounded daily. Which do you think would earn more interest? How could you confirm your thinking?

- Have students compare the current rates for borrowing using a credit card and borrowing using a line of credit. Ask them to find out how interest is calculated for each.

- Students could check the current interest rates for savings and borrowing and then solve problems using these rates. One example might be to have students investigate whether it would be profitable to borrow a large sum of money and invest it for 10 years. There could be a variety of repayment plans that students could choose from.

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- compound interest
- compounding period
- GIC (Guaranteed Investment Certificate)
- interest
- interest rate
- principal
- Rule of 72
- simple interest
- term

Resources/Notes

Print

Math at Work 11 (Etienne et al. 2012)
- Chapter 5: Banking and Budgeting
  - Sections 5.3 and 5.4
  - Skill Check
  - Test Yourself

Notes

SCO N04 Students will be expected to demonstrate an understanding of financial institution services used to access and manage finances. [C, CN, R, T]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N04.01 Describe the type of banking services available from various financial institutions, such as online services.
N04.02 Describe the types of accounts available at various financial institutions.
N04.03 Identify the type of account that best meets the needs for a given set of criteria.
N04.04 Identify and explain various automated teller machine (ATM) service charges.
N04.05 Describe the advantages and disadvantages of online banking.
N04.06 Describe the advantages and disadvantages of debit card purchases.
N04.07 Describe ways that ensure the security of personal and financial information (e.g., passwords, encryption, protection of personal identification number [PIN] and other personal identity information).

Scope and Sequence

Mathematics at Work 10	Mathematics at Work 11	Mathematics at Work 12
N02 Students will be expected to demonstrate an understanding of income to calculate gross pay and net pay, including wages, salary, contracts, commissions, and piecework.	**N04** Students will be expected to demonstrate an understanding of financial institution services used to access and manage finances.	–

Background

Students in grade 11 will have various levels of experiences with financial institutions. This outcome provides an opportunity for students to become aware of the services offered by financial institutions, to become familiar with the terminology used, to learn how to access and manage finances, and to critically consider the best options for them depending upon their situation.

Financial literacy involves being able to understand money and financial issues, how to “read and write” in the language of dollars and cents. It includes having money knowledge and money skills, and being able to apply them. Financial literacy also means being able to make informed decisions about money and planning for the future.

Bank accounts, credit cards, and debt have become a reality in today’s society. It is important for students to be aware of the financial realities they will face throughout their adult lives. The ability to make informed financial decisions, ranging from daily spending and budgeting to choices involving

insurance or saving for post-secondary education, home ownership, and retirement, is essential to the well-being of society.
The focus of this outcome is on the various services provided by financial institutions, such as banks and credit unions. Students will investigate and describe the types of accounts available at financial institutions in order to select an account that best meets their needs. The focus will be on three of the most commonly used services available from financial institutions: Automated Teller Machine (ATM), online banking, and debit card purchases. Students will examine ways to protect their personal and financial information.

Financial institutions provide a wide variety of services, including various types of savings and chequing accounts, credit cards, lines of credit, mortgages, investments, and online services. The focus should be on those services that are relevant to students, such as special youth bank accounts, debit cards, and online banking.

The following terminology may be new to students, depending upon their life experience.
- **Self-service banking** is banking done over the Internet, by telephone, or at a banking machine that does not require the services of a teller.
- **Full-service banking** is banking that is done with the help of a teller.
- **Transactions** include any task involving money, such as a cash withdrawal, deposit, money transfer, pre-authorized payment, or a bill payment.
- **PIN** or **Personal Identification Number** is a secret numeric password that is used by a computer system to verify the identity of the user.

Two basic types of deposit accounts are **chequing** and **savings**. The two are different in that chequing accounts allow users to write cheques, whereas savings accounts usually offer a higher interest rate. Other characteristics depend on the type of savings or chequing account and on the financial institution. Most banks offer students accounts that have lower monthly fees or no service fees and may not require a minimum balance.

As students begin thinking about the type of account most suitable for them, they should consider whether they would prefer to pay bills online and frequently use bank machines, or if they would expect to make frequent deposits and prefer to do their transactions with a bank teller.

Financial institutions offer online banking as a viable option for their customers. There has been a transition from banking in person to banking over the telephone to banking using a computer. Online banking has both advantages and disadvantages. The convenience it offers simplifies life for some people. For others it may be a little more complex and intimidating.

Most financial institutions issue a **debit card** to account holders. They can be used at **automatic teller machines**, as well as to make purchases in stores that accept bank cards as payment. Many students may already have bank cards and be familiar with their uses. Nevertheless, consumer awareness about the potential dangers of bank cards is important, particularly for young people.

ATM service charges are the fees many banks and interbank networks charge for the use of their automated teller machines. In some cases, these fees are assessed solely for non-members of the bank; in other cases, they apply to all users.

Students should be aware that three types of consumer charges exist:
- **Regular account fee:** A service charge imposed by a consumer's financial institution to withdraw money at an ATM.

- **Surcharge:** A fee that may be imposed by the ATM owner and will be charged to the consumer using the machine.
- **Foreign fee:** A transaction fee or network access fee charged by the card issuer to the consumer for conducting a transaction outside of their network of machines.

Since its national launch in 1994, Interac Direct Payment has become so widespread that as of 2001, more transactions in Canada were completed using debit cards than cash. This popularity may be partially attributed to two main factors: the convenience of not having to carry cash, and the availability of automated bank machines and Direct Payment merchants on the network. Debit cards also have disadvantages for the consumer and the retailer. There is a spending limit, for example, on purchases and withdrawals. User fees can also apply, and the security may not be as high as with some other forms of payment.

Students should be aware of the dangers of someone else accessing their personal and financial information through online banking. This could lead to **identity theft**.

Personal information includes
- age, name, ID numbers, income, ethnic origin, or blood type
- opinions, evaluations, comments, social status, or disciplinary actions
- employee files, credit records, loan records, medical records, existence of a dispute between a consumer and a merchant

Financial information includes
- account numbers and balances
- tax returns
- loan and mortgage information
- credit card and debit card numbers
- credit rating
- passwords
- PIN numbers

Accounts that are accessed on a shared computer or accessed with a simple password are in particular danger of this type of theft. The table below outlines three security features.

Security Feature	Security Measure
Passwords	▪ disable automatic password-save features in the browsers and software used to access the Internet ▪ use unique passwords for each website visited
PIN	▪ PINs should never be shared ▪ PINs should never be given in response to email or telephone requests ▪ shield your PIN when using it ▪ choose a PIN that is not obvious ▪ do not write your PIN on the card ▪ vary your PIN on different cards ▪ contact your bank immediately if your card is lost or stolen
Encryptions	▪ use the highest level of encryption available in web browsers and in wireless communications

Students should also investigate ways that financial institutions ensure security for online and electronic banking. Most use security measures such as firewalls and cookies, several levels of login (e.g., passwords, PINs, authentication questions), instructions on how to report suspicious emails, chip technology in bank cards and credit cards, voiceprint verification for telephone banking, and cheque imaging.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- Begin the discussion about banking services by asking students if they have a bank account and, if so, which type. Discuss why they chose this type of account. Ask what benefits the account offers. Are there a number of free transactions per month? If they have a savings account, what is the interest rate?

Whole-Class/Group/Individual Assessment Tasks

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- Create a mind map that shows the types of transactions possible for each service: bank teller, ATM, online banking, telephone banking.

- For a monthly fee of $4.15 on a chequing account, you get
  - Eight self-service transactions. Additional self-service transactions cost $0.50 each.
  - Five full-service transactions. Additional full-service transactions cost $1 each.
  - Each use of the ATM not associated with this bank costs $1.25 plus the transaction fee, if applicable.

- Kelly did the following banking in one month:
  - wrote three cheques
  - paid six bills online
  - used her debit card six times
  - withdrew money three times from an ATM not associated with her bank

  What was the total monthly charge?

- Describe three advantages and three disadvantages of online banking.

- Using an ATM printout (example below) containing different customer service charges, identify the fee category and amount(s). Answer the questions below.

DIRECash ATM	
TERMINAL #	02016199
SEQUENCE #	19212
AUTH #	03241 00
DATE	18:45 08/20/2013
CARD NUMBER	XXXXXXXXXXXX012
DISPENSED AMOUNT	$200
REQUESTED AMOUNT	$200
FROM ACCOUNT	Chequing
TERMINAL FEE	$1.25
TOTAL AMOUNT	$201.25
BALANCE	$3057.85

  - What important information is provided by an ATM receipt?
  - Why do you think the card number was printed this way?
  - Why is it a good idea to keep your ATM receipt?

- On rare occasions, such as when the network system becomes overloaded, it is impossible to pay for purchases using debit cards or to access money at an ATM. Describe how you could be prepared for such an occasion.

- Create a way to present "Tips to Protect Your Personal and Financial Information." Some suggestions might be to create a pamphlet, a web page, a slideshow, or a short video.

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?
- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

SUGGESTED LEARNING TASKS

Consider the following sample instructional strategies when planning lessons.

- Have students work in small groups to brainstorm the following:
  - What are some transactions that students are likely to make?
  - Have you ever written a cheque? When might you need to write a cheque?
  - Why might you want a savings account?
  - What information should you consider about your financial needs so that you can make an informed choice when selecting a bank account?

- Local financial banking institutions will have brochures and websites outlining fees for various account options. Have students compile this information and consider their needs, and then determine which institution and which account would be best for them.

- Students can use brochures or the Internet to investigate one type of chequing account offered by one bank. They should answer the following questions:
  (a) What is the name of the bank and the name of the account?
  (b) How much is the monthly fee?
  (c) How many full-service transactions are included in the monthly fee?
  (d) How much does each additional full-service transaction cost?
  (e) How many self-service transactions are included in the monthly fee?
  (f) How much does each additional self-service transaction cost?

- Invite representatives from various financial institutions in your community to come to the school and explain account options and banking services to the students.

- Have students research account and fee options at various local banks and credit unions and present their findings to one another. Include information on how credit unions differ from banks.

- Engage students in a discussion about ATM service charge fees, prompting them to think about the cost to operate the machines compared to the cost of human resources in a bank.

- Provide students with a graphic organizer and have them research various banks and compare services.

- Students could first brainstorm a list of advantages and disadvantages of online banking and then use the Internet to complete their list. Some suggestions follow. These are not intended to be all-inclusive.

Advantages	**Disadvantages**
fast, efficient and effective	trust/security
convenient	steep learning curve
real-time monitoring of accounts	

- Have students brainstorm a list of advantages and disadvantages of debit card purchases and then use the Internet to add to their list.

- Have students do a search on the Internet to determine the features of a strong password. Using these features, have them create several strong passwords. Discuss.

- Discuss with students why it is important to keep personal records, and why they should also keep track of their bank charges.

- Discuss with students ways that they currently ensure the security of their personal and financial information and ways that they can increase its security.

- Ask students to describe two ways in which someone else could access their personal and financial information.

- Students can work in small groups to research ways that a financial institution protects the personal information of an account holder.

- Have students calculate the amount spent on ATM fees if they withdraw funds at other banking machines twice a week over a one-year period. Students could then discuss strategies to avoid paying these fees.

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- Automatic Teller Machine (ATM)
- chequing account
- debit card
- encryptions
- foreign fees
- full-service banking
- identity theft
- Interac
- Personal Identification Number (PIN)
- regular account fee
- savings account
- self-service banking
- surcharge
- transactions

Resources/Notes

Print

Math at Work 11 (Etienne et al. 2012)

- Chapter 5: Banking and Budgeting
  - Section 5.1
  - Skill Check

Notes

SCO N05 Students will be expected to demonstrate an understanding of credit options, including credit cards and loans. [CN, ME, PS, R]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N05.01 Compare advantages and disadvantages of different types of credit options, including bank and store credit cards, personal loans, lines of credit, and overdraft.
N05.02 Make informed decisions and plans related to the use of credit, such as service charges, interest, payday loans, and sales promotions, and explain the reasoning.
N05.03 Describe strategies to use credit effectively, such as negotiating interest rates, planning payment timelines, reducing accumulated debt, and timing purchases.
N05.04 Compare credit card options from various companies and financial institutions.
N05.05 Solve a contextual problem that involves credit cards or loans.
N05.06 Solve a contextual problem that involves credit linked to sales promotions.

Scope and Sequence

Mathematics at Work 10	Mathematics at Work 11	Mathematics at Work 12
–	**N05** Students will be expected to demonstrate an understanding of credit options, including credit cards and loans.	**N02** Students will be expected to solve problems that involve the acquisition of a vehicle by buying, leasing, or leasing to buy.

Background

Credit plays a big part in modern society. Knowledge of credit is important for financial well-being. Students need to have a good understanding of the advantages and disadvantages of different credit options, with an emphasis on how to use credit effectively, how to make sound financial decisions, and how to avoid it becoming a burden. Daily living habits, as well as long-term spending mistakes, are two factors that contribute to debt. It is important to understand that reducing debt is not as easy as creating it.

This outcome introduces students to various credit options. Students will investigate various credit options and use their financial knowledge in problem-solving situations involving credit cards and loans.

Different lending institutions offer availability to money at different costs to the consumer. There are a variety of credit options including credit cards, loans, lines of credit, and overdrafts. Each of these options has a number of advantages and disadvantages and students should be aware of these in order to make wise financial decisions. Credit card companies provide another credit option.

Many stores also offer credit opportunities, some with their own credit card systems, others by having customers pay off the purchase price plus interest over a period of time. Usually store credit has high interest rates.

Some businesses offer sales promotions in the form of **deferred payment plans** (store credit plans). Deferred payment plans allow customers to make a purchase, with payment due sometime in the future or multiple smaller payments can be made until the full amount has been paid. The use of a deferred payment model is considered one of the more common sales and marketing tools used by many companies. Essentially, the underlying concept is to buy now and pay later. When a consumer is unable to pay for the purchase today, but has a reasonable expectation of being able to provide payment in full by an agreed upon date in the future, the process of assuming deferred debt can make sense.

All forms of credit come with a cost. While certain credit offers seem extremely attractive, they often come with higher service charges, higher interest rates, and/or higher penalties. If students apply for a **payday loan**, they need to pay special attention to the agreement. Interest rates range from 10%–30%, and failing to repay the loan on time will result in a doubling of the interest rate for the next repayment schedule. Sales promotions, such as using a store credit plan, often have hidden administration charges and high-interest charges.

Students will explore and calculate the effect of **down payments**, length of borrowing period, varying interest rates on monthly payments, and total cost or **finance charge** over time. They should also understand the difference in charges between a **cash advance** and a charge for a purchase.

Bank loans involve **principal amounts**, **amortization periods**, and interest rates (simple and compound). A bank **line of credit** is an approved loan amount that gives quick access to money. **Overdraft protection** also offers a form of short-term loan, as it allows you to withdraw more money from your account than you have in it, up to a certain amount.

As a general rule, the longer the repayment period for a loan, the greater the amount of interest that is paid. The interest rate on a credit card affects finance charges when a balance is carried. The higher the interest rate, the higher the finance charges will be. Timing purchases to coincide with manufacturer discounts, clearance sales, and off-season discounts will also reduce the amount of credit required.

Emphasis should be placed on the importance of understanding the conditions of a credit card before signing an agreement. Researching credit options will help students use credit effectively.
When shopping around for a credit card, students should ask themselves,

- How much will I spend on the card each month?
- Will I be able to pay my balance in full each month?
- Would I benefit from reward programs? Will the benefits outweigh any associated fees?
- Am I prepared to pay an annual fee? What other fees apply?
- If I consistently carry a balance, what interest rate am I prepared to pay on my purchases and/or cash advances?
- Could I benefit from a low-rate credit card?
- Do I understand how credit card interest is charged?
- If I am considering a low, “introductory” interest rate, do I understand how that interest rate will change once the introductory period is over?
- Do I understand how payments will be applied to the charges on the card?
- What are my rights and responsibilities with a credit card?

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- Determine the tax on a purchase of $120. (Tax rate is 15%.)
- A 12-oz. bottle of barbecue sauce costs $1.54. A 16-oz. bottle of barbecue sauce costs $1.99. Which is the better buy?
- Chelsea bought stock in a company for $25. Two weeks later she sold it for $60. What was the percent increase in value?
- First estimate your answer, then solve for *x*. Compare the estimated and calculated answers to check if your answer is correct.

  (a) $\dfrac{268}{5 \text{ rolls}} = \dfrac{x}{1 \text{ roll}}$

  (b) $\dfrac{x}{300} = \dfrac{1}{1.56}$

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- Decide which credit option you would use to purchase the following items. Be able to support your choice.

Item	Cost ($)
iPod Touch	250
laptop computer	600
snowmobile	12,000
truck	40,000
house	250,000

- Research a bank-issued credit card to determine the annual fees, income requirements, credit limit, annual rate of interest charged, and other distinguishing features. Compare findings with those of classmates who researched different cards.

- What is the difference between paying with a debit card and paying with a credit card?

- Sarah's credit card has a balance at the beginning of March of $37.05. She charged $51.60 and $427.75 on the card that month. At the end of the month, interest was calculated on the unpaid balance from the beginning of March (12%), and she made the minimum payment (3% of the balance). In April, Sarah used her credit card for purchases of $31.50, $10.60, and $17.25. For the end of April, calculate the interest charged, the minimum payment, and the balance left on the card.

- Jarred can buy a used motorcycle for $2300 cash or make a down payment of 10% and make 30 monthly payments of $78.25. Calculate the finance charge and percent rate.

- For each of the following, calculate the **total installment price** and the finance charge for paying for the article over time. Sales tax is 15%.
  (a) Electric guitar: $279.98 or 15 monthly payments of $22.75
  (b) iPad: $388.88 or $19 down payment plus 24 payments of $16.99

- Rashim buys a home theatre system for $3495. This price includes the Nova Scotia taxes. He does not have the cash to buy it outright, so he takes advantage of the credit offered by the store. He must pay an administration fee of $49.25 up front, but no further payments for a year. After the year is up, he will start to make 48 monthly payments at $85.25.
  (a) Find the total amount he will have paid for this home theatre system through store credit.
  (b) What is the store's finance charge?
  (c) What percent is the finance charge of the total payment?
  (d) Rashim could get a personal loan from the bank at an interest rate of 8% per year, over 4 years.

  Should he choose this option instead?

- Xena and David bought their first dining room suite from McDonald's Used Furniture. Consider the following options and decide which method of payment would be the most economical. Show all of your calculations to justify your decision.
  (a) $100 down and 12 monthly payments of $125.
  (b) A bank loan for $1200 at 14% annual interest compounded monthly.
  (c) A credit card with a 19.9% annual interest rate and a minimum monthly payment of 4% of the unpaid balance.

- Answer the questions below that are related to the following deferred-payment plan.
  Doyle's Discount Store: No payments until 2015!!!!
  - I want to buy a $2000 big screen TV!
  - Suppose I don't have enough money yet, but I can use a store's credit plan, which states, "You don't have to make any payments until 2015." However, the fine print says
    > you have to pay taxes on the TV and an administrative fee of $50 up front; this fee is to process the credit application
    > once approved, you don't have to make a payment until 2015

> at the end of the term in 2015, you can either pay it off or make 48 monthly payments of $100 each month
  (a) What amount do I have to pay up front?
  (b) If I decide to make the 48 monthly payments how much will I have paid, in payments, after 48 months?
  (c) What is the total amount that I would have paid?
  (d) What would it have cost if I paid cash?
  (e) How much extra would I have paid if I used the credit plan?
  (f) Why would someone use a credit plan rather than saving and paying cash?

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?
- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Suggested Learning Tasks

Consider the following sample instructional strategies when planning lessons.

- It is important to provide authentic examples that are relevant to students. Application forms for credit cards can be found online, as well as in-store promotions.

- Introduce the various credit options to students through scenarios that are relevant to the student, such as a class trip or buying a car.

- To lead into a discussion about credit, students could brainstorm different types of credit. Have them construct a list of the advantages and disadvantages of using a credit card. Ask them to share ideas on how credit cards can be used effectively. Ask students if they know how old they must be to get a credit card and why this might be. Some suggested advantages and disadvantages of each credit option are included here.

Credit Option	Advantages	Disadvantages
Bank credit cards	▪ convenient ▪ no need to carry large amounts of cash ▪ can be used to pay bills ▪ more secure than some other options ▪ helps establish a credit history ▪ may provide extra product warranty or other rewards	▪ can lead to large debts ▪ account balance must be checked every month to guard against fraudulent charges ▪ can lead to identity theft
Store credit cards	▪ similar to bank credit cards ▪ introductory offers	▪ usually higher interest rates than bank credit cards
Loans	▪ lower interest rates ▪ higher amounts of money	▪ may need collateral ▪ more difficult to obtain
Lines of credit	▪ could be very low interest rates ▪ money always available	▪ temptation to spend money you do not have
Overdrafts	▪ prevents bounced cheques or insufficient funds charges	▪ interest charges can be very high ▪ service charge when used each month

- Provide three authentic credit card applications and have students complete them. (from LearningToGive.org at http://learningtogive.org/lessons/unit351/lesson4_attachments/2.html)

- Module 6 and 7 of *The City* provides a good resource for this outcome. (www.fcac-acfc.gc.ca/Eng/resources/educationalPrograms/Pages/Thecitya-Lazoneun.aspx#LifeinTheCity)

- Ask students to research a local store credit plan with special emphasis on the hidden details of the plan, such as administration charges and repayment details. They should present their findings to the class.

- Have students do a role-play where one student represents a credit card company and one represents a customer. The customer should negotiate a lower interest rate for their credit card.

- Students should brainstorm ideas that could convince creditors to give a customer a lower interest rate. Possible ideas include using the number of years the person has been a customer, the most recent number of consecutive on-time payments, credit score, and actual lower interest rates of other credit cards and offers.

- Students should brainstorm ways to reduce debt load. They can use the Internet to complete their list. Some suggestions follow:
  - Start a diary of daily spending habits. Cut back on expenses that are not necessities and find ways to spend less money on the necessities.

- Purchase items on credit that can be repaid in 30 days. By paying your credit card balances in full each month, you can avoid interest charges.
- Eliminate multiple merchant credit cards.
- Transfer debt on high-interest credit cards to low-interest credit cards to save money and reduce debt.
- Write daily reminders and notes in a spending diary, which will serve as a reminder to pay bills on time and avoid late fees.
- Ask credit card issuers if there are programs available to reduce interest, monthly payments, or even settle the balance amount completely.

▪ Students could apply their knowledge of credit cards and loans in a situation similar to the one below. Students would not be expected to do all of the calculations in this type of problem. They should understand, however, that loans have different repayment options, each resulting in different interest charges and penalties; but in general, shorter repayment periods result in lower interest charges.

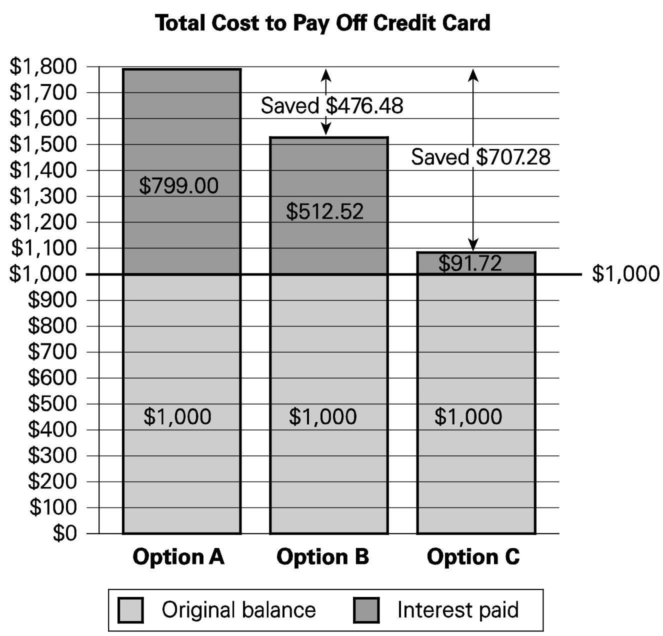
Credit Card Information			
Credit Card Balance:	$1000.00	Annual Interest Rate:	18.00%
Option A: What if you only make the minimum payment each month?			
Minimum Monthly Amount:	$10.00		
or			
Minimum Monthly Percentage:	3.0	(Whichever is greater)	
Based on the information that you've provided your first minimum payment amount is **$30.00**. Your minimum monthly payment will decrease as you pay off your balance.			
Option B: What if you make the minimum payment plus an additional amount each month?			
Additional Monthly Payment:		$5.00	
First minimum monthly payment amount:		+ $30.00	
Total		= $35.00	
Your balance will decrease more quickly than with Option A.			
Option C: What if you pay a fixed amount each month?			
Fixed Monthly Payment:	$100.00		
The higher the payment you make, the less interest you will pay, and the quicker your balance will be paid off.			

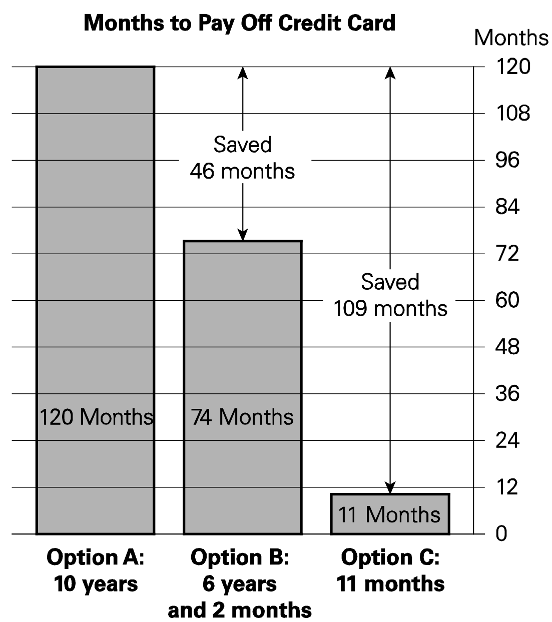
Calculation Results

	Option A: What if you only make the minimum payment each month?	Option B: What if you make the minimum payment plus an additional amount each month?	Option C: What is you pay a fixed amount each month?
Time to pay off	10 years	6 years and 2 months	11 months
Original balance	$1,000.00	$1,000.00	$1,000.00
Interest paid	$798.89	$512.52	$91.62
Total paid	$1,798.89	$1,512.52	$1,091.62
Amount saved	–	$286.37	$707.26
Time saved	–	3 years and10 months	9 years and 1 month

Results Summary

- **Option B:** If you pay **$5.00** more than your minimum payment each month, you will pay off your credit card balance **3 years and 10 months** sooner and you will save **$286.37** in interest.
- **Option C:** If you pay **$100.00** each month instead of the minimum payment, you will pay off your credit card balance **9 years and 1 month** sooner and you will save **$707.26** in interest.





- Students could investigate amounts of interest paid on a $10,000 loan by varying the repayment period (e.g., 24 months versus 60 months).

- Ask students to describe situations when they might use a payday loan. They should also identify situations when they would not.

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- amortization period
- cash advance
- credit
- credit card
- deferred payment plan
- down payment
- finance charge
- line of credit
- loan
- overdraft protection
- payday loan
- principal
- total installment price

Resources/Notes

Internet

- Lesson 4: Credit Cards (Handout 2) (LearningToGive.org 2014)
  http://learningtogive.org/lessons/unit351/lesson4_attachments/2.html)

- The City: A Financial Life Skills Resource (Financial Consumer Agency of Canada 2013)
  www.fcac-acfc.gc.ca/Eng/resources/educationalPrograms/Pages/Thecitya-Lazoneun.aspx#LifeinTheCity
  Modules 6 and 7 of "The City" provide good resources for this outcome.

Print

Math at Work 11 (Etienne et al. 2012)

- Chapter 5: Banking and Budgeting
  - Section 5.4
  - Skill Check
  - Test Yourself

Software

- Spreadsheet Software

Notes

Algebra
15–20 hours

GCO: Students will be expected to develop algebraic reasoning.

SCO A01 Students will be expected to solve problems that require the manipulation and application of formulas related to ▪ volume and capacity ▪ surface area ▪ slope and rate of change ▪ simple interest ▪ finance charges [C, CN, PS, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

A01.01 Solve a contextual problem involving the application of a formula that does not require manipulation.
A01.02 Solve a contextual problem involving the application of a formula that requires manipulation.
A01.03 Explain and verify why different forms of the same formula are equivalent.
A01.04 Describe, using examples, how a given formula is used in a trade or an occupation.
A01.05 Create and solve a contextual problem that involves a formula.
A01.06 Identify and correct errors in a solution to a problem that involves a formula.

Scope and Sequence

Mathematics 10	Mathematics at Work 11	Mathematics at Work 12
A01 Students will be expected to solve problems that require the manipulation and application of formulas related to perimeter, area, the Pythagorean theorem, primary trigonometric ratios, and income.	**A01** Students will be expected to solve problems that require the manipulation and application of formulas related to ▪ volume and capacity ▪ surface area ▪ slope and rate of change ▪ simple interest ▪ finance charges	–

Background

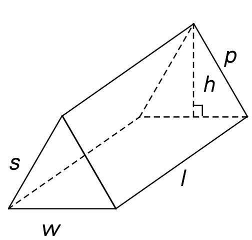
Note: This is not an outcome that can be taught in isolation but must be integrated as a foundational concept within each of the units in the course.

Students have been modelling and solving various forms of linear equations since Mathematics 7 and have practised manipulating these equations to solve for an unknown variable. In Mathematics at Work 10, students manipulated formulas related to perimeter, area, the Pythagorean theorem, primary trigonometric ratios, and income.

Preservation of equality will be further developed and the outcome addressed throughout this course as students apply formulas in a variety of contexts such as volume, capacity, surface area, slope and rate of change, simple interest, and finance charges.

Students will be expected to manipulate formulas, such as the following:

Concept	**Formula**
Surface area of a rectangular prism	$SA = 2lw + 2lh + 2wh$ or $SA = 2(lw + lh + wh)$
Surface area of a triangular prism	$SA = lw + wh + 2ls$ (see diagram below)
Surface area of a square-based pyramid	$SA = l^2 + 2ls$, where s = slant height
Surface area of a cylinder	$SA = 2\pi r^2 + 2\pi rh$ or $SA = 2\pi r(r+h)$
Surface area of a cone	$SA = \pi r^2 + \pi rs$ or $SA = \pi r(r+s)$
Surface area of a sphere	$SA = 4\pi r^2$
Volume of a rectangular prism	$V = lwh$ or V = area of base × height
Volume of a triangular prism	$V = \frac{1}{2}lwh$ or V = area of base × height
Volume of a square-based pyramid	$V = \frac{1}{3}b^2h$
Volume of a cylinder	$V = \pi r^2 h$
Volume of a cone	$V = \frac{1}{3}\pi r^2 h$
Volume of a sphere	$V = \frac{4}{3}\pi r^3$
Slope	$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$
Simple interest	$I = Prt$
Perimeter of a rectangle	$P = 2l + 2w$ or $P = 2(l + w)$
Circumference of a circle	$C = 2\pi r$ or $C = \pi d$
Area of a rectangle	$A = lw$
Area of a triangle	$A = \frac{bh}{2}$
Area of a circle	$A = \pi r^2$
Pythagorean theorem	$c^2 = a^2 + b^2$
Sine of angle A	$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$
Cosine of angle A	$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$
Tangent of angle A	$\tan A = \frac{\text{opposite}}{\text{adjacent}}$



Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

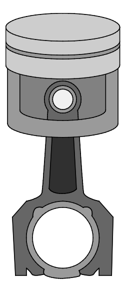
Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- Solve a given linear equation algebraically. For example, solve $\frac{1}{2}x + 14 = 3(x - 2)$.
- Identify and correct an error in a given incorrect solution of a linear equation.
- Given the area of the top of a circular engine piston is 25.43 cm^2, determine the diameter of the piston.



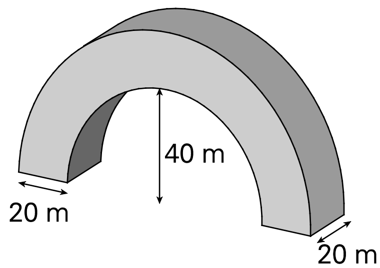
- The circumference of a CD is 28.26 cm. What is its diameter?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- Emma is training to be an Amazon Jungle tour guide. As part of her training, she must climb a slanted rope net into a tree that is 45 feet high. How far away from the tree is the net anchored if the net is 100 feet long? (State your answer in feet and inches.)
- The surface area of an official 5-pin bowling ball is approximately 459.96 cm^2. Determine the diameter of the bowling ball.

- Connie's Concrete Company is making an archway for the entrance to a building. What volume of concrete is needed to make the archway below?



- Find the error in each in each of the following and correct it.
  (a) A \$1000 investment earns simple interest at an annual rate of 1.3%. The interest earned in two years was calculated.
  $I = P \times r \times t$
  $I = 1000 \times 1.3 \times 2$
  $I = 2600$
  (b) A \$1000 investment earns simple interest at an annual rate of 1.2%. The interest earned in 18 months was calculated.
  $I = P \times r \times t$
  $I = 1000 \times 0.012 \times 18$
  $I = 216$
  (c) A \$1000 investment earns simple interest at an annual rate of 1.1%. The amount of time needed to earn \$400 in interest was calculated.
  $I = P \times r \times t$
  $400 = 1000 \times 0.011 \times t$
  $400 = 11 \times t$
  $400 - 11 = t$
  $389 = t$

- Identify and correct the errors in the following problem:
  – What is the slope of the line segment with endpoints A(4, 1) and B(1, 5)?

  Solution: slope = $\dfrac{5-1}{4-1}$

  slope = $\dfrac{4}{3}$

- The values A(4, 12) and B(14, 28) were substituted into the slope formula by Thor, Liam, and Sarah as follows:

  Thor: $m = \dfrac{28-12}{14-4}$ Liam: $m = \dfrac{12-28}{4-14}$ Sarah: $m = \dfrac{12-28}{14-4}$

  (a) Which of these three versions of substitution are/is correct? Explain.
  (b) What is an important rule to remember about the order of the coordinates when substituting into the slope formula?

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?
- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

SUGGESTED LEARNING TASKS

Consider the following sample instructional strategies when planning lessons.

- Ask students to make a cylinder by rolling a 8.5" × 11" sheet of paper lengthwise. Make another cylinder by rolling another 8.5" × 11" sheet of paper widthwise. Will the two cylinders have the same capacity? Explain.

- Have pairs of students select a volume formula. Each student is to rearrange the formula incorrectly and have their partner identify and correct the error.

- Have students show and explain why the surface areas of rectangular prisms, cones, and cylinders can be found using more than one formula.

SUGGESTED MODELS AND MANIPULATIVES

- calculator

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- formula
- equation
- expression
- substitution
- verify

Resources/Notes

Print

Math at Work 11 (Etienne et al. 2012)

- Chapter 1: Surface Area
  - Sections 1.1, 1.3, and 1.4
  - Skill Check
  - Test Yourself
  - Chapter Project
  - Games and Puzzles
- Chapter 2: Drawing and Design
  - Chapter Project
- Chapter 3: Volume and Capacity
  - Section 3.1. 3.2, 3.3, and 3.4
  - Skill Check
  - Test Yourself
- Chapter 5: Banking and Budgeting
  - Section 5.3 and 5.4
  - Skill Check
  - Test Yourself
- Chapter 6: Slope
  - Section 6.1, 6.2, and 6.3
  - Skill Check
  - Test Yourself
  - Chapter Project
- Chapter 7: Right Triangles and Trigonometry
  - Sections 7.1, 7.2, and 7.3
  - Skill Check
  - Test Yourself
  - Chapter Project

Notes

SCO A02 Students will be expected to demonstrate an understanding of slope ▪ as rise over run ▪ as rate of change ▪ by solving problems [C, CN, PS, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

A02.01 Describe contexts that involve slope (e.g., ramps, roofs, road grade, flow rates within a tube skateboard parks, ski hills).
A02.02 Explain, using diagrams, the difference between two given slopes (e.g., a 3:1 and a 1:3 roof pitch), and describe the implications.
A02.03 Describe the conditions under which a slope will be either 0 or undefined.
A02.04 Explain, using examples and illustrations, slope as rise over run.
A02.05 Verify that the slope of an object, such as a ramp or a roof, is constant.
A02.06 Explain, using illustrations, the relationship between slope and angle of elevation (e.g., for a ramp with a slope of 7:100, the angle of elevation is approximately 4°).
A02.07 Explain the implications, such as safety and functionality, of different slopes in a given context.
A02.08 Explain, using examples and illustrations, slope as rate of change.
A02.09 Solve a contextual problem that involves slope or rate of change.

Scope and Sequence

Mathematics at Work 10	Mathematics at Work 11	Mathematics at Work 12
G04 Students will be expected to demonstrate an understanding of primary trigonometric ratios (sine, cosine, tangent) by applying similarity to right triangles, generalizing patterns from similar right triangles, applying the primary trigonometric ratios, and solving problems.	**A02** Students will be expected to demonstrate an understanding of slope ▪ as rise over run ▪ as rate of change ▪ by solving problems	**A01** Students will be expected to demonstrate an understanding of linear relations by ▪ recognizing patterns and trends ▪ graphing ▪ creating tables of values ▪ writing equations ▪ interpolating and extrapolating ▪ solving problems

Background

In previous grades, students explored linear relations. This included work with tables of values and graphs (7PR01, 7PR02, 8PR01, 9PR02). While they have sketched graphs of linear relations using a table of values, this is the first formal opportunity students have to study slope in a mathematics course. Students discussed the meaning of variables and described how a change in one variable affected a change in the other. The *concept* of slope has been discussed, but the term **slope** has not yet been used. However, slope or steepness is a common day-to-day concept, and students may also have been introduced to slope in science class.

In this unit, students will connect the concept of slope to the idea of measuring the **rate of change**. To develop the concept of rate of change, students should be able to make its connection to slope using different forms of the same expression:

On a graph, slope can be determined by finding the $\frac{\text{rise}}{\text{run}}$ or the $\frac{\text{vertical change}}{\text{horizontal change}}$.

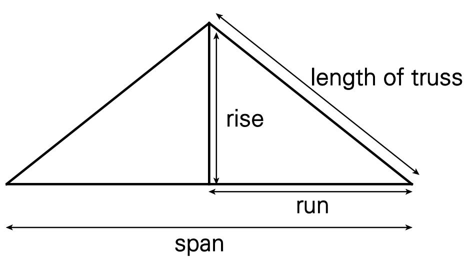
As a rate of change, slope can be expressed as $\frac{\text{change in } y\text{-values}}{\text{change in } x\text{-values}}$ or $\frac{\text{change in dependent variable}}{\text{change in independent variable}}$.

As an algorithm, slope can be determined using $\frac{y_2 - y_1}{x_2 - x_1}$ or as $\frac{y_1 - y_2}{x_1 - x_2}$.

This outcome will focus on describing slope and solving contextual problems using the rate of change formula. Students will gain a visual perspective of slope through diagrams and illustrations, calculate the rise over run ratio in diagrams, and apply the rate of change formula to real-world contexts. Throughout the unit, students will solve problems involving slope.

Students should be introduced to slope through discussion of real-world examples, such as building ramps, stairs, and roof trusses, in order to develop a concrete image of what slope represents. For example,

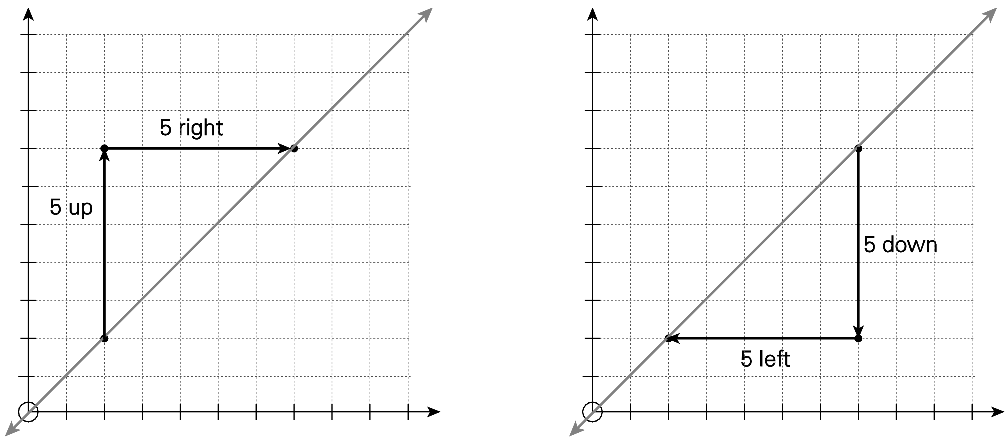
The slope = $\frac{\text{rise}}{\text{run}}$, and for a roof this is called the **pitch**. The span of a roof is twice as long as the run, so pitch = $\frac{\text{rise}}{\frac{1}{2}\text{span}}$ or rise = pitch × $\frac{1}{2}$ span.



Discuss concepts such as steepness and positive and negative slope using a variety of visuals. Discussion of the differences of various slopes should lead students to the concept of zero and undefined slope. The connection should be made that horizontal lines have a slope of 0 and vertical lines have an undefined slope.

Slope should first be developed in terms of rise over run. A task such as building staircases will help students develop an understanding of slope. As a warm-up, lay one metre stick flat on the ground and another one vertically against a wall. Ask students to explain, to a partner, the positions of these metre sticks. Explain to students that although it is difficult to describe slope using everyday language, it is easier to describe with mathematics.

The connection to rise over run can be made by providing students with grids depicting line segments of varying slopes and asking them to identify the rise and the run. Students should be cautioned to pay close attention to the direction they are moving and the sign of the rise and the run. The diagrams below show how students can calculate the slope by interpreting the rise and run differently but the slope will still be the same in both situations.



Students should work with graphs of lines with negative slopes as well as those with positive slopes.

Students worked with the primary trigonometric ratios in Mathematics at Work 10 (G04, A01). The idea that the slope of a line is the tangent of the angle of elevation should now be developed. When the tangent ratio (slope) is known, the inverse tangent function can be used to find the angle of elevation.

There are many real-world contexts where special consideration must be given to slope. Wheelchair ramps, ski slopes, and stairs, for example, need to be constructed using an appropriate slope with safety and functionality in mind. Students should discuss situations where steepness is important or where a small slope is more practical.

Slope is defined as the ratio of rise to run. It is calculated using two points on a line and the slope formula. While working with slope as rise over run, students should make the connection that slope is equal to the change in the dependent variable divided by the change in the independent variable. To apply the slope formula, students will have to link the dependent variable to the *y*-axis and the independent variable to the *x*-axis of a coordinate grid. Review of these terms and the coordinate grid, including how to plot and read points, may be necessary.

Students should learn that the slope of a line is equal to the change in the *y*-values divided by the change in the *x*-values, which can be determined using coordinates. This is referred to as **rate of change**. This should be developed using real-world contexts. For example, if a ski hill has a slope of one-quarter, students should understand that they drop one metre for every four metres they travel horizontally.

Some common student errors that occur when determining rate of change using this formula include inverting the formula, substituting coordinates incorrectly, and incorrect calculations.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

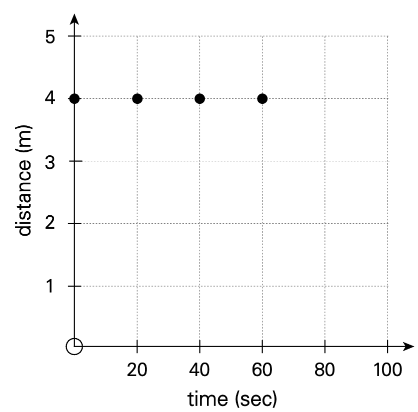
Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- A taxi cab charges the rates shown in the following table.

Length of trip (km)	5	10	15
Total cost ($)	9.25	15.50	21.75

(a) Plot these points on a coordinate grid.
(b) Discuss if these points should be joined.
(c) Explain why the graph does not start at the origin.
(d) From the graph, find the length of a trip that costs $25.
(e) From the graph, find the cost of a 12-km trip.

Given the graph to the right, complete the activities below.
(a) Create a table of values.
(b) Describe the pattern found in the graph.
(c) Describe a situation that the graph might represent.
(d) Write an equation that represents this situation.



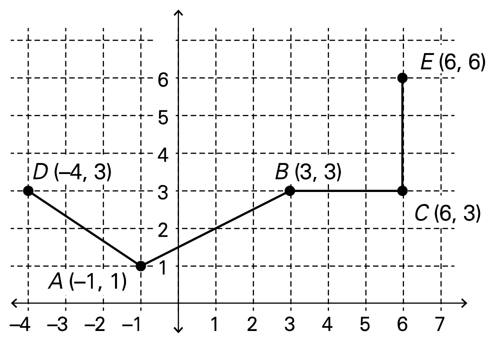
- Draw any right triangle. Label an acute angle with θ. Identify and measure its opposite and adjacent sides. Which trigonometric formula did you use to find θ? Find the measure of the angle.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- Create a collage of everyday objects that involve the concept of slope (possible examples include mountains, wheelchair ramps, and highway ramps).

- For the diagram shown below, identify which segment has a positive slope, a negative slope, a slope of 0, or a slope that is undefined.



  Explain why the slope of a horizontal line is 0 and why the slope of a vertical line is undefined.

- Determine the slope, m, of each line segment with the given end points.
  (a) *S* (3, 6) and *T* (5, 8)
  (b) *H* (4, 3) and *K* (4, 8)
  (c) *M* (9, 7) and *N* (11, 7)
  (d) *W* (2, 2) and *S* (4, 5)

- The recommended slope of a particular ramp is 1:3. If a ramp is to have a vertical height of 15 inches, what must be its horizontal length in order to meet this 1:3 ratio?

- Draw a line segment that has an endpoint at (2, 1) and a slope that is $\frac{2}{3}$.

- Describe real-world situations where positive, negative, zero, and undefined slopes are present.

- If you went skiing, would you prefer a hill with a slope of $\frac{1}{5}$ or $\frac{4}{5}$? Explain why.

- Would the graph of the data in this table show a constant slope?

x	***y***
1	2
3	3
5	5
7	8
9	12

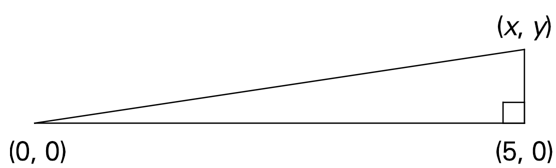
- Explain why the slope value is important for safety in large vehicle transportation.

- Identify and correct the errors in the following problem:
  - What is the slope of the line segments with endpoints *A* (4, 1) and *B* (1, 5)?

    Solution: slope = $\frac{5-1}{4-1}$

    slope = $\frac{4}{3}$

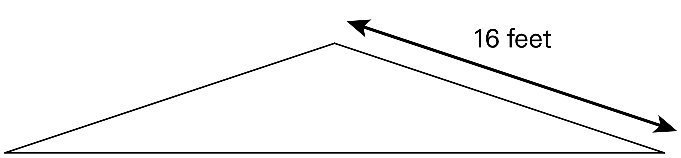
- Sally is running on a treadmill with no incline. She decides to increase the incline by two levels. Each incline level increases the track height by 1 unit.



  (a) What is the slope of the track before the incline?
  (b) What aspect of slope was changed to create the incline?
  (c) What is the slope of the track after the incline?
  (d) What is the angle of elevation of the track after the incline?
  (e) How would increasing the incline by three levels affect the angle of elevation?

- Recommended angles of stairs are between 30° and 35°. All stairs in a specific staircase are the same height and depth, the rise is 8'11", and the run is 12'. Is the angle of the stairway within the recommended range? If not, adjust either the rise or run so that the angle is within the recommended range.

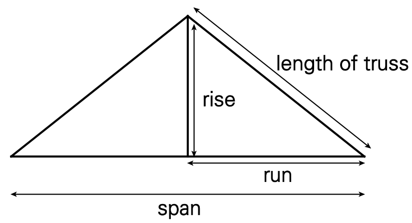
- Mark wants the pitch of the roof he is building to be $\frac{1}{3}$. Calculate the height of the truss given that the length is 16 ft.



- The span of a roof is 30 feet (most construction is done in imperial measure) and the pitch is $\frac{1}{3}$.

  What is the length of the roof?

$$\text{pitch} = \frac{\text{rise}}{\frac{1}{2}\text{span}}$$



- The span of a roof is 22 feet and the length is 12.3 feet. What is the pitch of the roof?

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?
- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Suggested Learning Tasks

Consider the following sample instructional strategies when planning lessons.

- When introducing slope, incorporate activities such as the following:
  - Students could measure various staircases and compare rise to run. Discuss the steepness of the staircases.
  - Ask students to examine the grade of a road. If a highway has a 6% grade for the next 8 km, for example, how far does it drop vertically over the 8 km travelled horizontally?

- Provide students with several examples of objects with varying slopes and have them place the items in order based on steepness.

- Ask students to research the importance of slope within an occupation and describe specific examples where the formula is used.

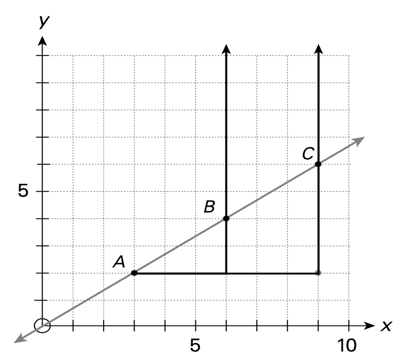
- The following can be done as a whole class task.
  - Using blocks, build one staircase with a slope of 1 (by placing together one block, then two blocks, then three, etc.) and another staircase with a slope of 2. Balance a metre stick on each staircase, and ask students how they might calculate the slope of the metre stick.
  - Create a diagram on grid paper illustrating the staircases within a coordinate grid.
  - Explain how to calculate the slopes of each of the metre sticks.
  - Compare the numerical values with the everyday expressions the class came up with during the warm-up task.

  As a follow-up, assign small groups of students a slope and have them build a model of a staircase with that slope. On grid paper, students can illustrate the staircase and show how they calculated the slope.

- Ask students to bring in visuals of objects such as ramps, roofs, ski hills, etc., and have them discuss the following in groups:
  (a) Which object has the steepest slope?
  (b) Which visual demonstrates a negative slope?
  (c) Arrange the visuals in order from least to greatest with respect to slope.
  (d) Do any of the visuals depict an undefined or 0 slope?

- Observation that the slope of the line is equal to the slope of the line segment joining any two points on the line is important to emphasize. Using a line segment, similar to the one below, enables students to verify that the slope between points *A* and *B* is the same as the slope between points *A* and *C* and between points *B* and *C*.

$m\ AB = m\ BC = m\ AC = \frac{2}{3}$



- A plumb bob is a weight at the end of a string, which is used to determine if an object is vertical (has undefined slope). Ask students to find occupations that use a plumb bob. They should present their findings to the class.

- Have students brainstorm various applications of slope and categorize the applications in regard to occupations and/or situations where slope would be applied.

- Take pictures of various roofs and compare pitch (slope) of the roofs. This can also be related to the angle of elevation. Geometers Sketchpad (Key Curriculum 2013) is an effective program to use with this task.

- Explore slope by using motion detectors. Have students move to create graphs with different slopes.

- Network with various tradespeople in regard to how they implement the concept of slope within their occupation.

- Ask students to demonstrate the relationship between slope and angle of elevation using a detailed drawing from a real-world context.

- To help students visualize the relationship between slope and the tangent ratio, draw two congruent triangles. On the first triangle, label the rise and run. On the second triangle, label the angle of elevation and the opposite and adjacent sides of the triangle. By comparing the two triangles, discuss with the class that the slope ratio is the same as the tangent ratio.

- An in-class task such as the following provides a quick check of a student's understanding of slope.
  - Ask students to draw the coordinate system.
  - Write down a slope of a line.
  - Ask students to position a piece of uncooked spaghetti on their paper to reflect the slope of the line.

  A quick assessment of student understanding can be performed by walking around the classroom and observing the steepness and direction of the spaghetti.

- Discuss the following with students.
  - If the rise stays the same, what effect would changing the run have on slope?
  - If the run stays the same, what effect would changing the rise have on slope?

- Collaborate with other teachers to find out the applications of slope that they would use in their workshop/course.

- Have students research the provincial guidelines for slopes of wheelchair ramps for home or for public buildings.

- Create a template of various pitches ($\frac{3}{12}$, $\frac{4}{12}$, $\frac{5}{12}$, and $\frac{6}{12}$). Put the 12 on the horizontal and then create various similar triangles for the different pitches. Have students check the pitch of different roofs in their neighbourhood using the template.

Suggested Models and Manipulatives

- calculators
- graph paper
- linking cubes

Mathematical Vocabulary

Students need to be comfortable using the following vocabulary.

- angle of elevation
- constant
- rate of change
- rise
- run
- slope
- undefined

Resources/Notes

Internet

- *bobvila.com: Help You Need for the Home You Want*, "The Plum Bob" (Bob Vila 2014)
  http://www.bobvila.com/articles/495-the-plumb-bob/#.Uxezo4WwX1W
  This online article describes the uses of a plumb bob.

- *bobvila.com: Help You Need for the Home You Want*, "How To: Use a Plum Bob" (Bob Vila 2014)
  http://www.bobvila.com/articles/how-to-use-a-plumb-bob/#Uxe2I4WwX1U
  This video shows a plumb bob in use.

- *WorkSafeBC*, "Part 26 Forestry Operations and Similar Activities, Section 26.16 Slope Limitations" (WorkSafe BC 2014)
  www2.worksafebc.com/Publications/OHSRegulation/Part26.asp#SectionNumber:G26.16
  This site contains regulations outlining the maximum slope on which forestry equipment can be operated.
- *MobilityBasics.ca*, "Wheelchair Ramp Safety & Standards" (Stewart 2014)
  wheelchair.ca/ramp.php
  This site contains wheelchair ramp safety and standards.

- *Occupational Health and Safety Act*, "Roof Work" (Province of Nova Scotia 2009)
  http://www.gov.ns.ca/just/regulations/regs/ohs296f.htm#roof
  Information on roof slope and fall protection.

- *All About Snow*, "Snow Avalanches" (National Snow & Ice Data Center 2013)
  nsidc.org/cryosphere/snow/science/avalanches.html
  Avalanche awareness information.

- *Professional Learning Newfoundland and Labrador*, "Slope" (Government of Newfoundland and Labrador 2013)
  www.k12pl.nl.ca/curr/10-12/math/math2202/classroomclips/slope.html
  The classroom clip on slope highlights a cross-curricular link to skilled trades.

- *Illuminations: Resources for Teaching Math*, "Rise-Run Triangles" (National Council of Teachers of Mathematics 2014)
  illuminations.nctm.org/Lesson.aspx?id=2570
  Algebra Lesson: Rise-Run Triangle

- *Roof Calculator Software* (Roofgenius.com 2014)
  http://roofgenius.com/roofpitch.htm
  For additional Information on pitch or slope of a roof.

Print

Math at Work 11 (Etienne et al. 2012)

- Chapter 6: Slope
  - Sections 6.1, 6.2, and 6.3
  - Skill Check
  - Test Yourself
  - Chapter Project

Notes

SCO A03 Students will be expected to solve problems by applying proportional reasoning and unit analysis. [C, CN, PS, R]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

A03.01 Explain the process of unit analysis used to solve a problem (e.g., given kmh and time in hours, determine how many kilometres; given revolutions per minute, determine the number of seconds per revolution).

A03.02 Solve a problem, using unit analysis.

A03.03 Explain, using an example, how unit analysis and proportional reasoning are related (e.g., to change kmh to km/min., multiply by 1 h/60 min. because hours and minutes are proportional [constant relationship]).

A03.04 Solve a problem within and between systems using proportions or tables (e.g., km to m or kmh to ft./sec.).

Scope and Sequence

Mathematics at Work 10	Mathematics at Work 11	Mathematics at Work 12
N01 Students will be expected to solve problems that involve unit pricing and currency exchange, using proportional reasoning. **M01** Students will be expected to demonstrate an understanding of the International System of Units (SI) by ▪ describing the relationships of the units for length, area, volume, capacity, mass, and temperature ▪ applying strategies to convert SI units to imperial units **M02** Students will be expected to demonstrate an understanding of the imperial system by ▪ describing the relationships of the units for length, area, volume, capacity, mass, and temperature ▪ comparing the American and British imperial units for capacity	**A03** Students will be expected to solve problems by applying proportional reasoning and unit analysis.	–

Mathematics at Work 10	Mathematics at Work 11	Mathematics at Work 12
▪ applying strategies to convert imperial units to SI units **M03** Students will be expected to solve and verify problems that involve SI and imperial linear measurements, including decimal and fractional measurements.		

Background

Students applied proportional reasoning to problems involving ratios in Mathematics 8 (8N05) and similar polygons in Mathematics 9 (9SS03). They also used proportional reasoning as they converted measurements within and between the imperial and SI systems in Mathematics at Work 10 (10M01, 10M02). It is very important to review those conversions.

This outcome requires that students use proportional reasoning as they solve problems that involve SI and imperial unit measurements. **Unit analysis** will also be used to verify that units in a conversion are correct.
To convert from one measurement to another, students need to understand the relationship between the units of measurement. To change kmh to km/min., for example, students can multiply by $\frac{1\text{ h.}}{60\text{ min.}}$ because hours and minutes are proportional. There is a constant relationship between them.

In the context of a problem, it may be necessary to convert given measurements into common units. When converting within or between SI and imperial units, unit analysis should be used to verify that units in a conversion are correct.

Unit analysis should be used, for example, if students have to convert 200 metres to kilometres.

$$200\text{ m} \times \frac{1\text{ km}}{1000\text{ m}} = 0.200\text{ km}$$

It is important for students to notice that the units have changed but the actual distance has not. The distance of 200 m is the same as 0.200 km. Prompt students to discuss the value of the ratio $\frac{1\text{ km}}{1000\text{ m}}$.
Since 1 km = 1000 m, the ratio is equal to 1. Students should recognize that when a numerical value is multiplied by 1, the value remains the same. This is the basis of unit conversion.

Students should clearly and methodically show how they have used unit analysis to make their conversions. For example, to determine which is faster, 45 miles/hour or 40 feet/second, students should show equivalencies and units.

Step 1: Convert mi./h. to mi./min.: $\frac{45\text{ mi.}}{1\text{ h.}} \times \frac{1\text{ h.}}{60\text{ min.}} = \frac{45\text{ mi.}}{60\text{ min.}} = 0.75\text{ mi./min.}$

Step 2: Convert mi./min. to mi./sec.: $\frac{0.75\text{ mi.}}{1\text{ min.}} \times \frac{1\text{ min.}}{60\text{ sec.}} = \frac{0.75\text{ mi.}}{60\text{ sec.}} = 0.0125\text{ mi./sec.}$

Step 3: Convert mi./sec. to ft./sec.: $\frac{0.0125 \text{ mi.}}{1 \text{ sec.}} \times \frac{5280 \text{ ft.}}{1 \text{ mi.}} = 66$ ft./sec.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

Assessing Prior Knowledge

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- Solve each proportion:
  (a) $\frac{3}{7} = \frac{a}{28}$
  (b) $\frac{b}{9} = \frac{3}{4}$
  (c) $\frac{5}{c} = \frac{15}{33}$

- Convert the following:
  (a) 42 cm = ____ inches
  (b) 26.2 km = ____ miles
  (c) 130 cm = ____ m
  (d) ____ g = 150 mg
  (e) 60 L = ____ mL
  (f) 3.25 km = ____ cm
  (g) ____ g = 0.68 kg
  (h) 4 ft. = ____ in.
  (i) 3 mi. = ____ yd.

- Serena used proportional reasoning to do the conversion: 0.78 kg = _____ mg.
  She wrote: $\frac{10\,000 \text{ mg}}{1 \text{ kg}} = \frac{? \text{ mg}}{0.78 \text{ kg}}$
  Has Serena made an error? If so, explain and complete correctly.

- A 12-oz. bottle of barbecue sauce costs $1.54. A 16-oz. bottle of barbecue sauce costs $1.99. Which is the better buy?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- A certain alloy appears bright red at a temperature of 560°C. What temperature is this in °F?
- A Canadian driving in the United States notices her speedometer is reading 80 kmh. What is her speed in mph?
- The cutting speed for a soft steel part in a lathe is 100 ft./min. Express in cm/s.
- A wood lot is 25 000 square feet. How many square metres is this? How many acres? How many hectares?
- How many minutes are there in 2.0 years?
- Often, vehicle fuel consumption is measured in mpg (miles per gallon). What unit is most commonly used in SI to measure fuel consumption?
- A car manufacturer states that the fuel efficiency of a car is 42 mi./gal. Determine this efficiency in SI standard units of L/km.

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?
- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Suggested Learning Tasks

Consider the following sample instructional strategies when planning lessons.

- Remind students to convert to the same units when determining the scale factor.
- Start with basic conversions before moving into compound units. An example of unit analysis used with simple units can be seen at *How to Convert Units—Unit Conversion Made Easy* (Schwanbeck 2009) www.youtube.com/watch?v=XKCZn5MLKvk.
- Encourage students to check the reasonableness of their answers when performing conversions. They should ask themselves if the answer makes sense. For example, students should understand that when they convert 300 metres to kilometres, the answer must be less than 300.
- Examples and problems given to students should be of conversions that would typically occur in our daily lives. Some examples are mpg and ft./sec.
- Network with various tradespeople in regards to how they implement the unit analysis within their occupation.
- Have students research various Olympic sports for winning speeds in sports of interest to them. They could chart these over several years.

Mathematical Vocabulary

Students need to be comfortable using the following vocabulary.

- proportional
- unit analysis

Resources/Notes

Internet

- *How to Convert Units—Unit Conversion Made Easy* (Schwanbeck 2009)
  www.youtube.com/watch?v=XKCZn5MLKvk
  An example of unit analysis used with simple units.

Print

Math at Work 11 (Etienne et al. 2012)

- Chapter 1: Surface Area
  - Sections 1.1 and 1.3
  - Skill Check
  - Test Yourself
- Chapter 2: Drawing and Design
  - Section 2.1
  - Skill Check
  - Chapter Project

- Chapter 3: Volume and Capacity
  - Sections 3.1 and 3.3
  - Skill Check
  - Test Yourself
- Chapter 5: Banking and Budgeting
  - Section 5.4
  - Skill Check
  - Test Yourself
- Chapter 6: Slope
  - Section 6.1

Notes

Statistics
10 hours

GCO: Students will be expected to develop statistical reasoning.

SCO S01 Students will be expected to solve problems that involve creating and interpreting graphs, including bar graphs, histograms, line graphs, and circle graphs. [C, CN, PS, R, T]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

S01.01 Determine the possible graphs that can be used to represent a given data set and explain the advantages and disadvantages of each.
S01.02 Create, with and without technology, a graph to represent a given data set.
S01.03 Describe the trends in the graph of a given data set.
S01.04 Interpolate and extrapolate values from a given graph.
S01.05 Explain, using examples, how the same graph can be used to justify more than one conclusion.
S01.06 Explain, using examples, how different graphic representations of the same data set can be used to emphasize a point of view.
S01.07 Solve a contextual problem that involves the interpretation of a graph.

Scope and Sequence

Mathematics at Work 10	**Mathematics at Work 11**	**Mathematics at Work 12**
–	**S01** Students will be expected to solve problems that involve creating and interpreting graphs, including bar graphs, histograms, line graphs, and circle graphs.	**S01** Students will be expected to solve problems that involve measures of central tendency, including mean, median, mode, weighted mean, and trimmed mean.

Background

Students encounter vast amounts of information that require organization, interpretation, and analysis so that they can process the data and make appropriate conclusions. They are frequently exposed to graphs in daily life. It is important that they are able to interpret the data presented to them to identify the potential of misleading information.

In Mathematics 3, students worked with bar graphs (3SP02). In Mathematics 6, they were introduced to line graphs and created, interpreted, and solved problems in relation to given data sets (6SP01, 6SP03). Circle graphs were constructed and analyzed in Mathematics 7 (7SP03). Work with bar graphs and line graphs was extended in Mathematics 8 (8SP01).

Initially, the focus should be on discussing the similarities and differences of bar graphs, line graphs, circle graphs, and histograms as they relate to a given data set. Actual construction of the graphs should be explored later. This will be the first time that students are exposed to a histogram, so a guided discussion will be required. As an introduction to these graphs, provide students with a relevant data set and the corresponding line graph, bar graph, circle graph, and histogram. Data sets may include class test results, weather patterns, or favourite television shows. The data set can be developed in class through discussion or surveys or it could be provided.

It is important that students be asked to evaluate various situations to determine and debate why a particular display is best suited to a specific type of data, or to a given context. For example, given a bar graph and a line graph, students should determine which is most appropriate to display the amount of water flowing into a container and justify their choice. Certain points of view may also be emphasized more strongly by using one type of graph over another. Students should also be able to discuss this in terms of **continuous** (data values on a graph that are connected) versus **discrete** (data values that are distinct) data sets.

Type of Graph	Advantages	Disadvantages
Circle Graph	▪ compares “part to whole” ▪ data is displayed as a percent ▪ size of sector can be easily compared to other sectors	▪ does not show actual amount or number for each category ▪ more difficult to draw ▪ data must have “part to whole” relationship ▪ too many categories makes it look crowded
Line Graph	▪ shows change over a period of time ▪ can easily see trends ▪ can be used to interpolate and extrapolate ▪ easy to draw	▪ limited to continuous data ▪ may be difficult to read accurately ▪ depending on scale ▪ comparisons between categories are not identified as quickly
Bar Graph	▪ shows number of items in specific categories ▪ easy to compare data ▪ easy to draw	▪ may be difficult to read accurately ▪ depending on scale ▪ trends are identifiable but not for purposes of interpolating and extrapolating
Histogram	▪ displays the shape of distribution ▪ trends are easily recognizable ▪ frequency count of items in each bin	▪ cannot read exact values ▪ more difficult to compare two data sets ▪ used only with continuous data

A review of the important characteristics and a description of how to create circle graphs, line graphs, and bar graphs will be needed. More detailed instruction will be required to create histograms. Students need to be aware of the characteristics of a good graph:

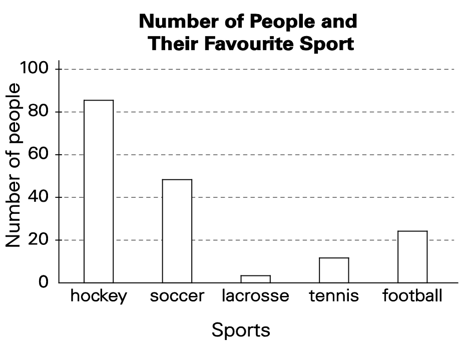
- accurately shows the fact
- complements or demonstrates arguments presented in the text
- has a title and labels
- shows data without altering the message of the data
- clearly shows any trends or differences in the data

Some of the attributes of a circle graph include the title, labels, and legend. Calculating percentages is an important aspect of creating circle graphs. Students may need practice in this area. They should be reminded of the formula to convert percentages to degrees: % × 360° = # of degrees. In addition, they should note that the total measure of the central angles if a circle is 360° and the percentages total 100%. It will be important for students to consider rounding to the nearest degree, and, because of this, recognizing that at times the angles will not total exactly 360°. The total should not be off by more than one degree, which will not make much difference in the overall graph.

Similarly, line graphs and bar graphs must have a title and labels. When constructing line graphs, special attention must be given to determining an appropriate scale with equal increments on each of the axes. A review of **independent** and **dependent** variables may be needed. When creating bar graphs, students will sort the information into appropriate categories and construct the bars.

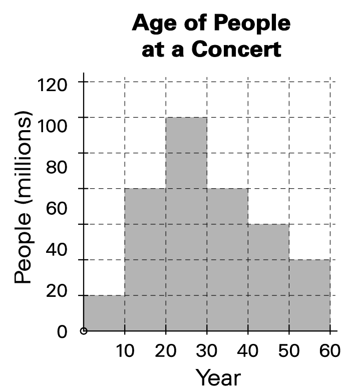
Graphing data that has been organized into **bins** (also called **intervals** or **classes**) results in a **histogram**. Students should be made aware that in histograms, the data elements are grouped and form a continuous range from left to right. The data is arranged in a **frequency table**. The **range** of the interval is used to determine the width of the bars, which should be neither too narrow nor too broad. Intervals are the same size, and the number of bins is usually kept to between 4 and 10. Often the bins and bin sizes will be determined by the context of the problem and the range of the data.

One approach to introducing histograms is to compare them to bar graphs and discuss the similarities and differences. Histograms are similar to bar graphs, except that histograms express continuous data and bar graphs express discrete data.



Frequency Table

Ages	Frequency
0–10	20
10–20	80
20–30	120
30–40	80
40–50	60
50–60	40



Students should be cautioned when sorting the data into bins. The number 20, for example, is normally included in the 20–30 bin. It is also acceptable to include it in the 10–20 bin. Although both conventions are acceptable, it is important for students to consistently follow one or the other convention. It is also important to note that graphing calculators and software such as GeoGebra (International GeoGebra Institute 2013) use the convention up to but not including the end value of the bin, and for some software, such as Excel, it is possible to predetermine the bin inclusion or exclusion of the final value.

For histograms, the values on the *x*-axis indicate the boundaries of the intervals. When drawing the histogram, the height of each bar is determined by the number of pieces of data that are included in that bin. Since the data in histograms is continuous, there are no spaces between the bars. All data must be included. If there is an interval that contains no pieces of data, a space is left where that bar would have been drawn. Like other data displays, all histograms must include a meaningful title, labels for the axes, and values to indicate the intervals of the data. The axes labels should include units where applicable (e.g., years, seconds, centimetres, percentages). The values for the intervals should be displayed at the limits of the bars.

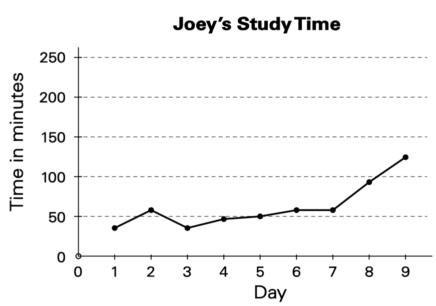
Emphasis should be placed on the interpretation of the data represented by the graph and not on the creation of the graph itself. Ensure that data used in the investigations of this outcome is relevant to students.

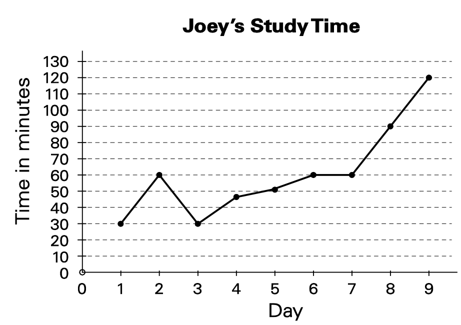
Once students have created various graphs, they will focus on analyzing the information displayed through **trends**, **interpolation**, and **extrapolation**. **Trends** in a graph are based on the context of the data set. Descriptors such as **increase**, **decrease**, **constant rate**, and **percent of data** can be used to describe the trend displayed in the graph. It is important that students relate it to the context of the problem. They are expected to identify values that fall within the given range of data points (**interpolation**) as well as ones that lie outside (**extrapolation**), while keeping in mind the trend of the data.

Interpolating is likely to be more accurate since the predicted value is bracketed by two known values. Extrapolating, on the other hand, is less reliable because a new trend could occur. Students may find extrapolating more challenging. Students should use their knowledge of trends, interpolation, and extrapolation to interpret a graph to solve a contextual problem. They will need to recognize patterns and trends represented in the graph and connect them to the context of the problem.

Students should be aware that different conclusions can be interpreted from the same graph. If a circle graph shows a large percentage of test scores below 50%, for example, one conclusion could be that students did not study enough for the test while another could be that the test was not fair.

Students should explore how changing certain aspects of a graph may change the perception of the information displayed. A common cause of misleading information on graphs stems from the choice of intervals on the vertical axis. Another cause is to begin the vertical axis numbering with something other than zero. Both situations may either over- or under-exaggerate increases or decreases. For example, the graphs below depict a situation where the choice of scale on the vertical axis impacts the effect of the graph. The scale used, for example, will affect how data is perceived. The two line graphs below depict Joey's study time. Both graphs are based on the same data set, but Joey appears to have studied more in the second graph because of the change in scale.





Discussion should take place regarding how the choice of certain graphs can lead to inaccurate judgments. Students' understanding of statistics is enhanced by evaluating the arguments of others. This is particularly important since advertising, forecasting, and public policy are frequently based on data analysis. The media is full of representations of data to support statistical claims. These can be used to stimulate discussion.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

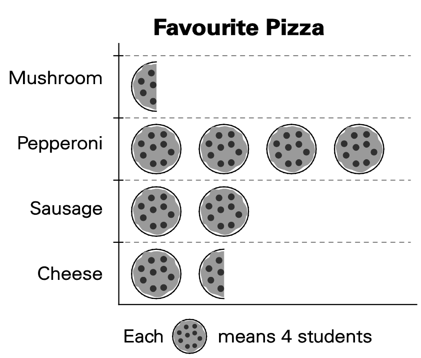
ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

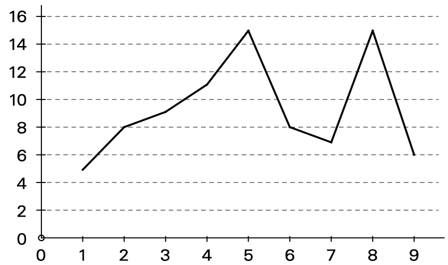
- Compare the information for the same given data shown on different types of graphs, including circle graphs, line graphs, bar graphs, double bar graphs and pictographs. Discuss the strengths and limitations of each graph.

- Change the format of a given graph, such as the size of the intervals, the width of bars, and the visual representation. Discuss how this might lead to misinterpretation of the data.
- Choose the type of graph that you would recommend to represent each situation below. Justify your choice.
  (a) Ray wants to compare the percent of students who go home for lunch to the percent of students who have lunch at school.
  (b) Ben's mother has put marks on a wall to track his growth since he was three years old.

- Use the data displayed on the pictograph below to create a different type of graph that you believe might be a more suitable representation. Defend your choice.



- Describe a situation that could be modelled by the following graph.



WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- Is the type of graph used to display the data in each scenario below appropriate? Why or why not?
  (a) Kait tracks her weight each week for a year. She represents the data using a line graph.
  (b) Patrice records the time it takes 20 people to run one kilometre. He represents the data using a bar graph.

- Over a two-month period, Michelle collected the following data.
  (a) How many times each week her dad fell asleep while watching television.
  (b) The weekly height of a tomato plant in the garden.
  (c) How many hours a day her brother spent eating, sleeping, playing video games, doing homework, and chatting online.

  Michelle wants to display her data for a school project. Which type of graph would you suggest for each data set? Justify your choice.

- The annual budgets for two families are given in the chart below. Compare the budgets of these two families.
  (a) What type of graph(s) would best display this data and compare the budgets? Why?
  (b) Using technology, create a graph or graphs to display the data.
  (c) What conclusions do you draw from the graphs about each family's budget? How do the graphs support your conclusions?

	Brown Family	**Smith Family**
Food	$3000	$2400
Housing	$4000	$3600
Operating Expenses	$2800	$2400
Clothing	$1200	$1400
Charities	$1000	$600
Medical Expenses	$1800	$800
Miscellaneous	$600	$1200
Savings	$600	$2600

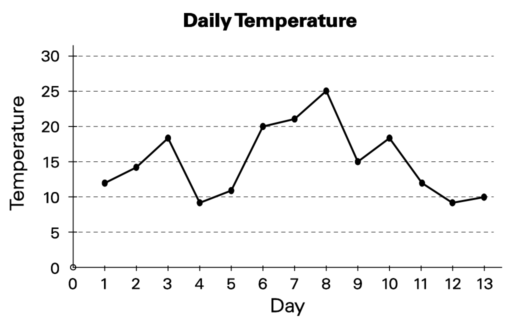
- The teacher of a mathematics class recorded the number of students who had a final mark in each of the following ranges.

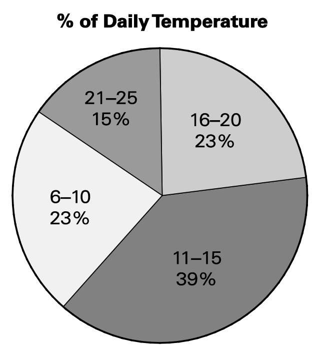
Mark	Number of Students
50–60	3
60–70	5
70–80	8
80–90	6
90–100	4

(a) Represent the data using a histogram.
(b) Represent the data using a circle graph.

- Analyze the following data and the accompanied graphs to answer the questions that follow.

Day	1	2	3	4	5	6	7	8	9	10	11	12	13
Temp	12	14	18	9	11	20	21	25	158	18	12	9	10





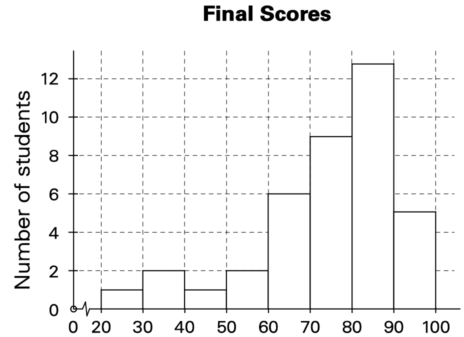
(a) Which graph can be used to determine the temperature for a particular day?
(b) Which graph can be used to determine the most frequent temperature range?
(c) Describe one advantage of using the line graph and one advantage of using the circle graph in this situation.

- In December the number of hours of bright sunshine recorded at 36 selected stations was as follows:

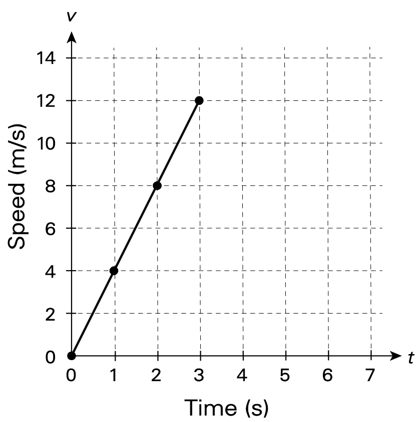
15	25	41	20	35	20	16	8	38	23	25	38
41	34	24	39	47	45	17	42	44	47	45	51
35	37	51	39	14	14	40	44	50	40	31	22

(a) Choose an interval and create a frequency table for the data.
(b) Use the grouped data to create a histogram (ensure that it includes a title and all labels).
(c) Choose a different interval and repeat (a) and (b).
(d) Compare the two histograms and explain which they feel is more useful.

- Final marks for Mathematics at Work 11 are displayed in the following histogram.
  (a) How many students are in the class?
  (b) How many students achieved a passing mark (60% or more)?
  (c) What is the highest mark?
  (d) Could this data be displayed effectively on a different type of graph? Explain.
  (e) What conclusions can be made from this graph?
  (f) If there are 86 students in total enrolled in this course, how many would you predict will pass the course?



- Using the following graph, extrapolate to predict the speed at 6 seconds.



- The following table, from Statistics Canada, lists the population (to the nearest thousand) of Nova Scotia as recorded by Census data 1971 to 2011.

Population of Nova Scotia (in thousands)

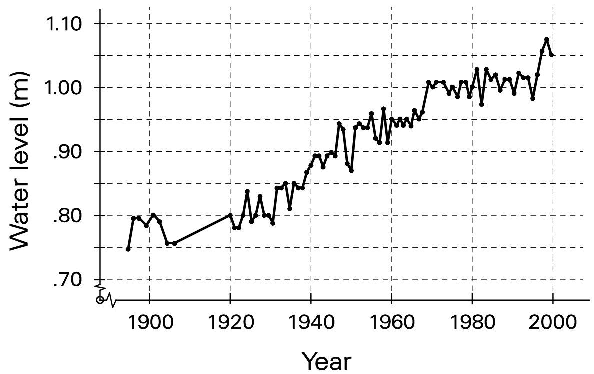
1971	789.0
1976	828.6
1981	847.4
1986	873.2
1991	899.9
1996	???
2001	908.0
2006	913.5
2011	921.7
2016	???
2021	???

(Source: Statistics Canada, 2011 Census of Population. Last modified: 2011-02-04.)

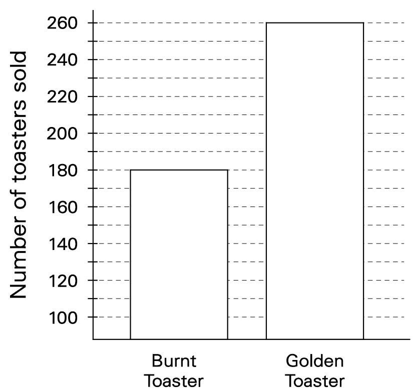
(a) The population for 1996 was not provided. Given the data, estimate the population for 1996.
(b) Based on the data provided, predict the population for 2016 and 2021.
(c) How confident are you about your prediction?
(d) What factors might affect the reasonableness of your predictions?

- Mary is getting in shape. The first day she does 9 sit-ups, the second day she does 13, the third day 17, and so on. Sketch a graph to represent this situation. If she continues in this way, how many sit-ups will she do on the 15th day? the 25th day? It is reasonable to continue this pattern forever?

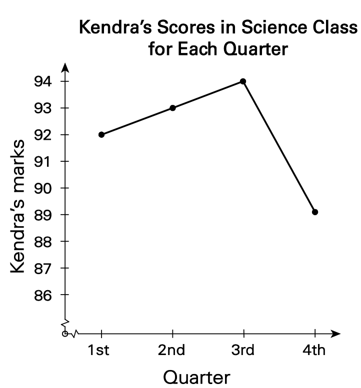
- The following graph shows the mean water level in Charlottetown over the past 110 years. Describe the trend in the mean water level over that time.



- Explain why the following statement is incorrect: "Sales of Golden Toaster were about double the sales of Burnt Toaster." What could be changed on the graph or added to make it less misleading?



- This graph shows that Kendra received a much lower grade in science class during the fourth quarter of the year than she did in the previous three quarters. Do you think Kendra should be worried by what appears to be such a large drop in her grades? Explain your reasoning.



- At an annual meeting for a certain company, the following information on profits was presented:

Year	Profit
1993	$340,000
1994	$350,000
1995	$370,000
1996	$410,000
1997	$450,000
1998	$465,000

  Make a graph to help support each of the following:
  (a) The company for the past six years has experienced a very small profit increase.
  (b) The company for the past six years has shown a large profit increase.

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should be used to inform instruction.

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?
- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

SUGGESTED LEARNING TASKS

Consider the following sample instructional strategies when planning lessons.

- While comparing types of graph, discussion questions could include the following:
  (a) Which of the graphs display the data in a manner that is easier to interpret? Why?
  (b) Which of the graphs display the data in a manner that is difficult to interpret? Why?
  (c) Which of the graphs would describe the data in terms of percent? Which of the graphs would describe the data in terms of number of students surveyed?

(d) How are the graphs similar? How are the graphs different? What are the limitations of each type of graph?
(e) Which graph best represents the data collected?

Students should learn that the suitability of the graph depends on the data set given. The choice of the graph is determined by what you want to analyze and interpret from the data.

- As a class, have students list advantages and disadvantages of the different types of graphs. Sample advantages and disadvantages of each type of graph are shown in the table below. This list is not intended to be exhaustive.

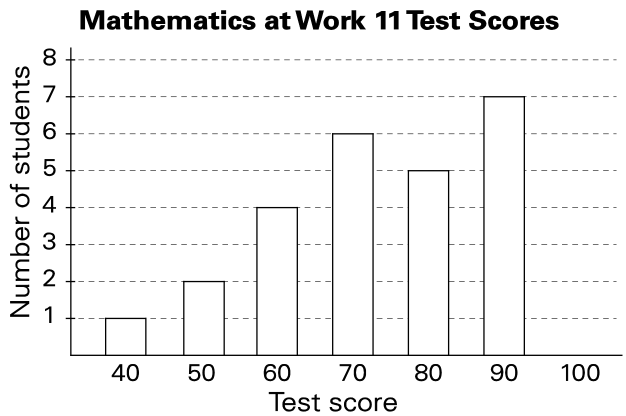
Type of Graph	**Advantages**	**Disadvantages**
Circle Graph	▪ compares “part to whole” ▪ data is displayed as a percent ▪ size of sector can be easily compared to other sectors	▪ does not show actual amount or number for each category ▪ more difficult to draw ▪ data must have “part to whole” relationship ▪ too many categories makes it look crowded
Line Graph	▪ shows change over a period of time ▪ can easily see trends ▪ can be used to interpolate and extrapolate ▪ easy to draw	▪ limited to continuous data ▪ may be difficult to read accurately ▪ depending on scale ▪ comparisons between categories are not identified as quickly
Bar Graph	▪ shows number of items in specific categories ▪ easy to compare data ▪ easy to draw	▪ may be difficult to read accurately ▪ depending on scale ▪ trends are identifiable but not for purposes of interpolating and extrapolating
Histogram	▪ displays the shape of distribution ▪ trends are easily recognizable ▪ frequency count of items in each bin	▪ cannot read exact values ▪ more difficult to compare two data sets ▪ used only with continuous data

- Have students use their knowledge of bar graphs to create histograms as outlined in the following example. The following data set represents the length, in millimetres, of various fish present in an aquarium: 9, 12, 14, 18, 22, 27, 29, 29, 44, 46, 47, 48, 50, 51, 52, 53, 64. If students examine this data, they should understand that a bar graph would not be an appropriate representation of the data.

- Have students summarize the kinds of data each type of graph would best display. They could generate a context for using each type of graph.

- Explore opportunities in other subject areas such as science or social studies to apply students' knowledge of histograms.

- Discuss with students the possible different conclusions that can be made about the information displayed in the graph below. Some sample conclusions might be, All marks are values in multiples of 10 or the histogram is drawn incorrectly.



- Collect various examples of graphs from newspapers, magazines, school data reports, etc.

- There are various websites that will create graphs such as Kids' Zone Learning with NCES, "Create a Graph" (Institution of Education Sciences 2013) at http://nces.ed.gov/nceskids/createagraph/.

- Use websites such as Statistics Canada (www.statcan.gc.ca) for background information, data, and lesson plans or NCTM's Illumination activities (2014)at http://illuminations.nctm.org.

- Use computer applications and graphing calculators to allow students to explore various data displays and be able to quickly modify the display without redrawing it manually.

- Students can compare various methods of displaying data and evaluating their effectiveness. Comparisons of scale adjustments to indicate such things as degree of growth or loss should be explored.

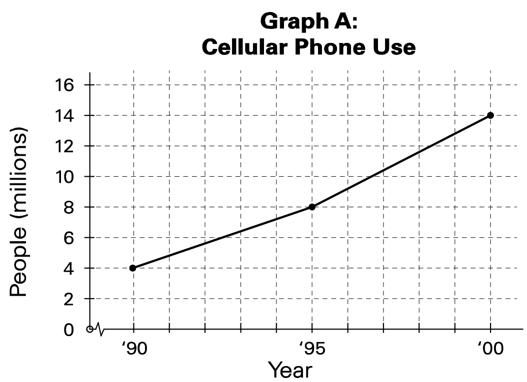
- Students could be asked to represent the same data in different formats and encouraged to consider whether one type of graph is a better means of representing the data than another.

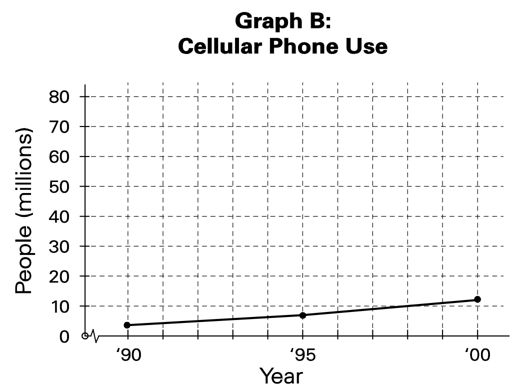
- Ask students to describe a situation in their daily lives where they would need to extrapolate information and explain why it would be useful.

- Encourage students who need help to interpolate and extrapolate values to use a ruler and draw a vertical line from the *x*-axis to the graphed line, and then draw a horizontal line to the *y*-axis.

- Check that students understand when it is reasonable to use interpolation or extrapolation and when it is not reasonable. Remind students that both are used to estimate values.

- The graphs below depict a situation where the choice of scale on the vertical axis impacts the effect of the graph. Discuss with students how the scale used affects how data might be perceived.





- Encourage students to think of ways and times that the media have published misleading graphs. Discuss with them when they have seen graphs that have been misleading.

Suggested Models and Manipulatives

- coloured pencils
- graph paper
- graphing technology
- hundred circle (copyable master)
- rulers
- sample graphs

Mathematical Vocabulary

Students need to be comfortable using the following vocabulary.

- bar graph
- bins
- circle graph
- classes
- continuous data
- discrete data
- extrapolate
- frequency table
- histogram
- interpolate
- line graph
- range
- trend

Resources/Notes

Internet

- *Kids' Zone Learning with NCES*, "Create a Graph" (Institution of Education Sciences 2013)
  http://nces.ed.gov/nceskids/createagraph/
  A website that will create graphs.

- *Statistics Canada* (Government of Canada 2014)
  www.statcan.gc.ca
  Statistics Canada has lots of data and suggested activities for teachers and students to use.

- *Illuminations: Resources for Teachers* (National Council of Teachers of Mathematics 2014)
  http://illuminations.nctm.org/ActivityDetail.aspx?ID=78
  Lesson plans or NCTM's Illumination activities.

- TinkerPlots Dynamic Data Exploration (Key Curriculum 2013)
  http://www.keycurriculum.com/products/tinkerplots

Print

Math at Work 11 (Etienne et al. 2012)

- Chapter 4: Interpreting Graphs
  - Sections 4.1, 4.2, and 4.3
  - Skill Check
  - Test Yourself
  - Chapter Project
- Chapter 5: Banking and Budgeting
  - Section 5.4

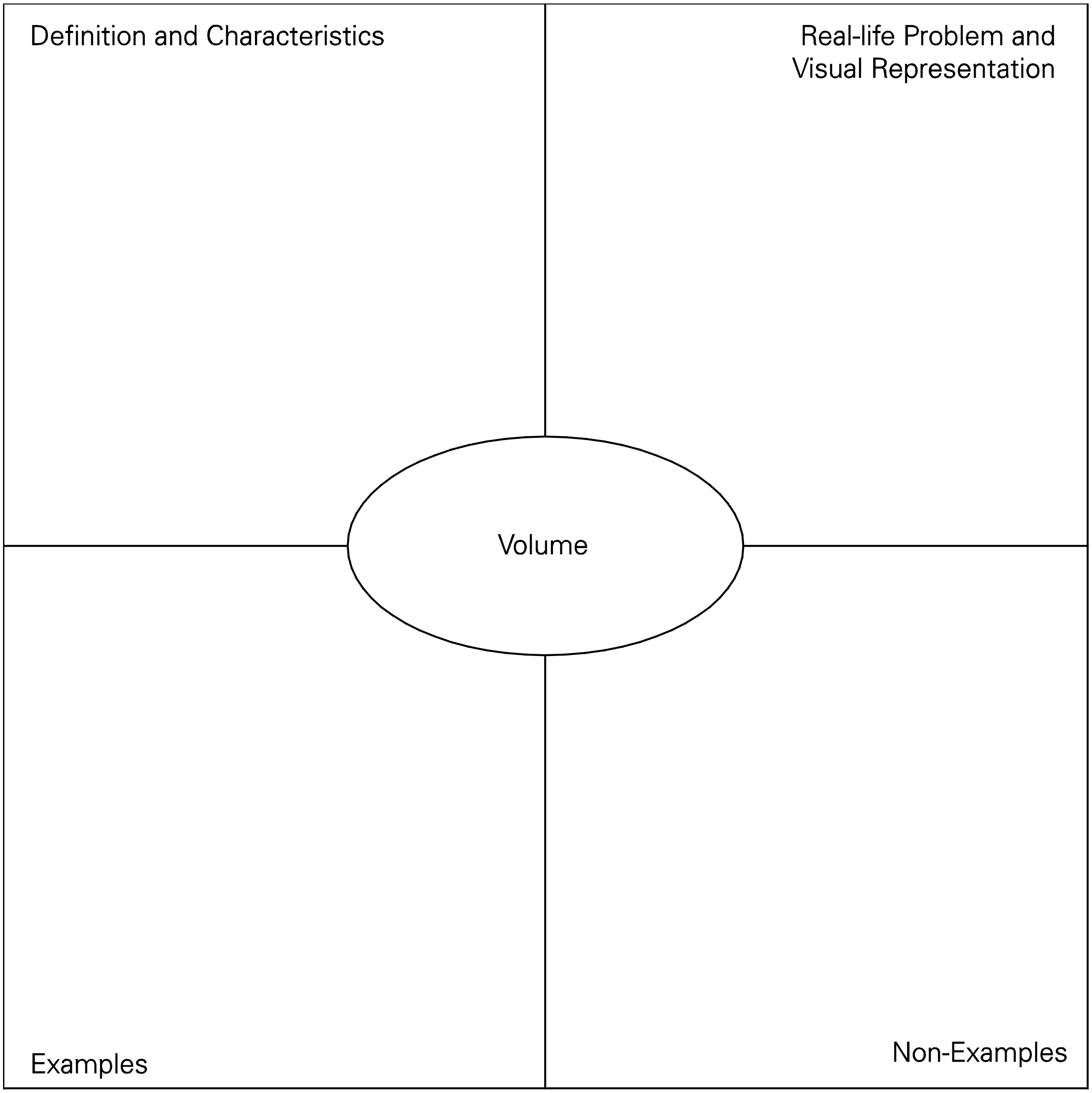
Software

- GeoGebra (International GeoGebra Institute 2013)
- Fathom (Fathom Applications Pty Ltd. 2013)

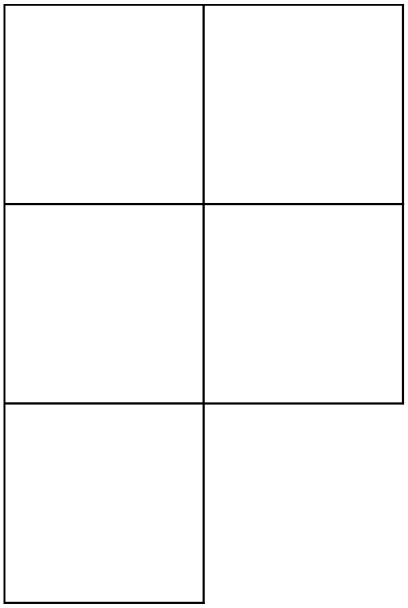
Notes

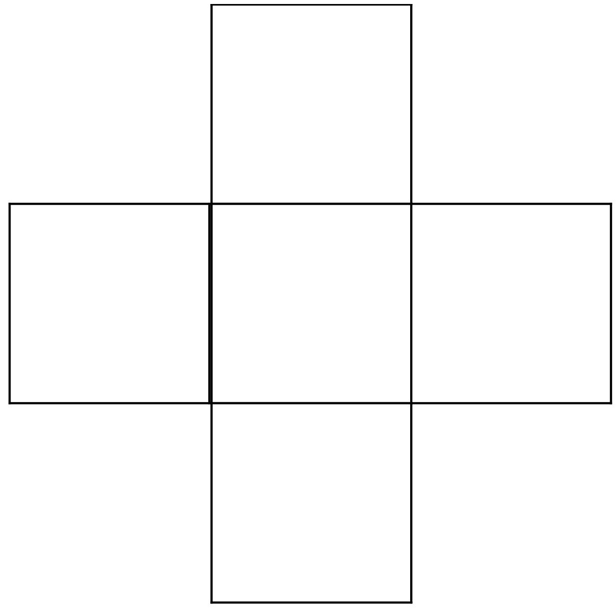
Appendix

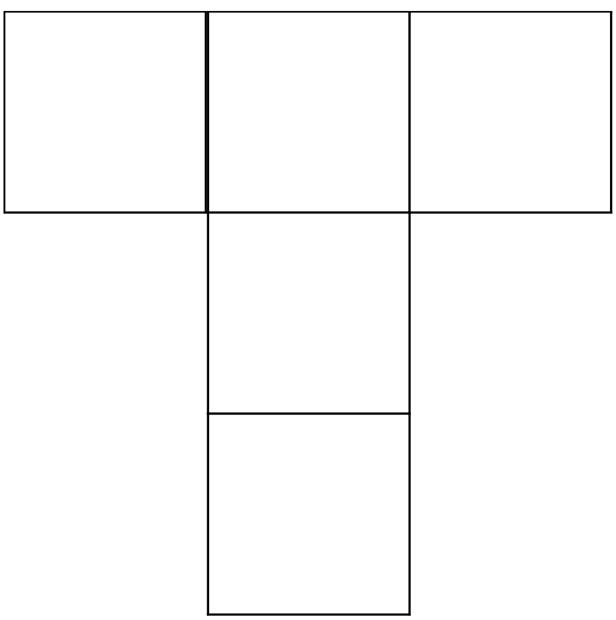
Frayer Model

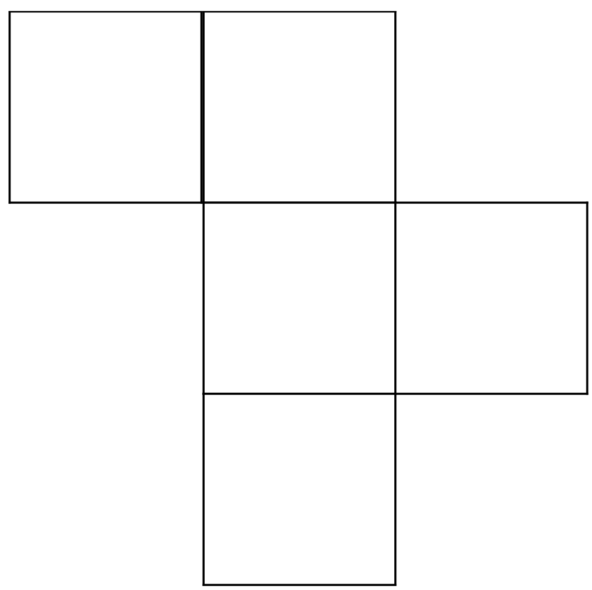


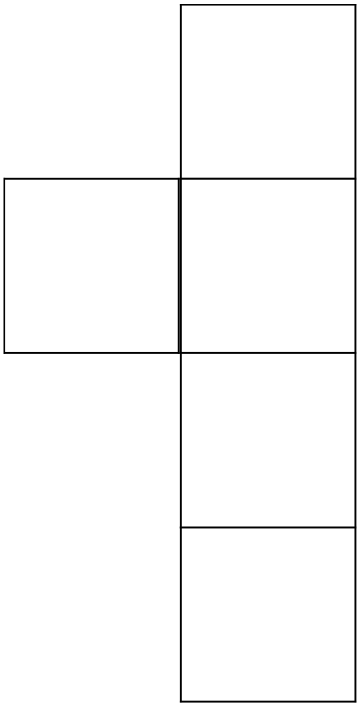
Pentominoes

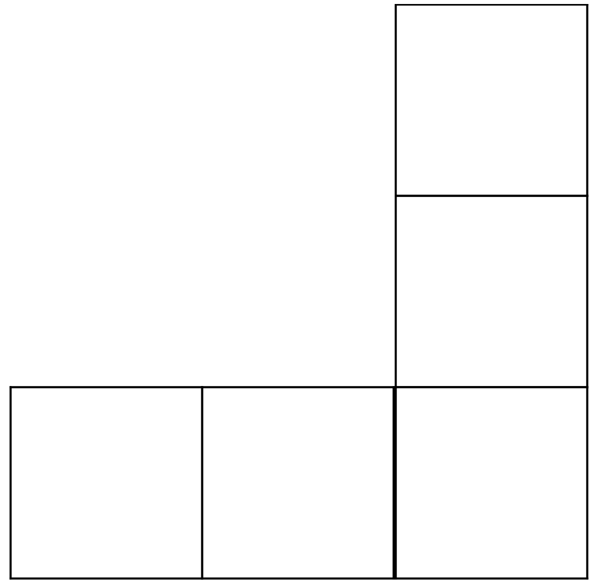


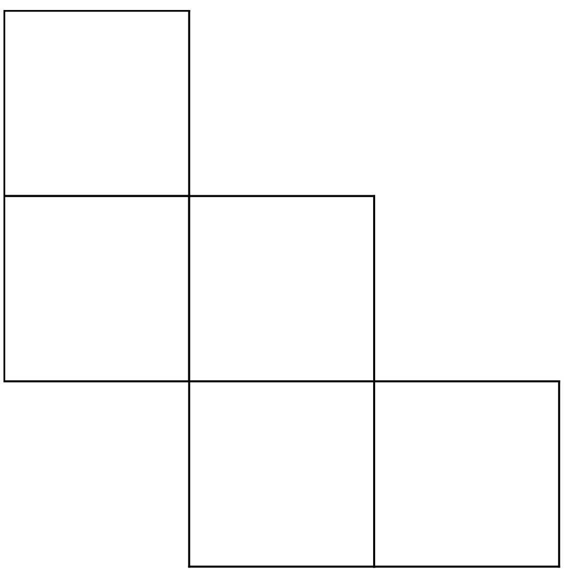


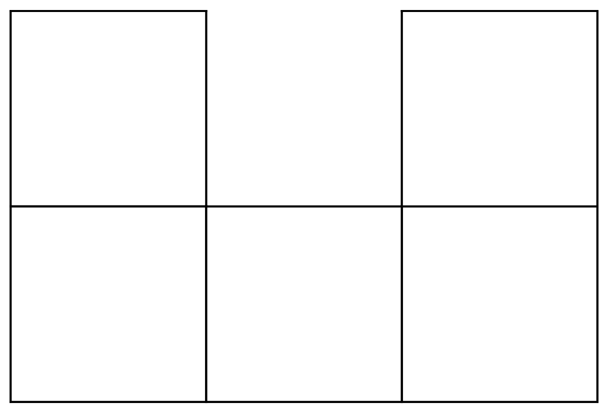


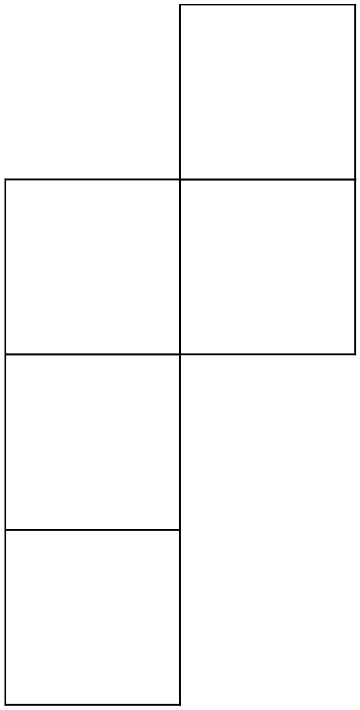


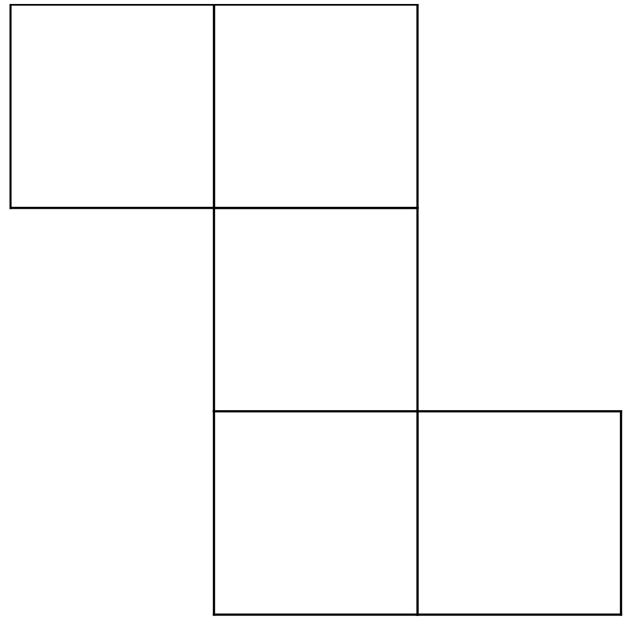


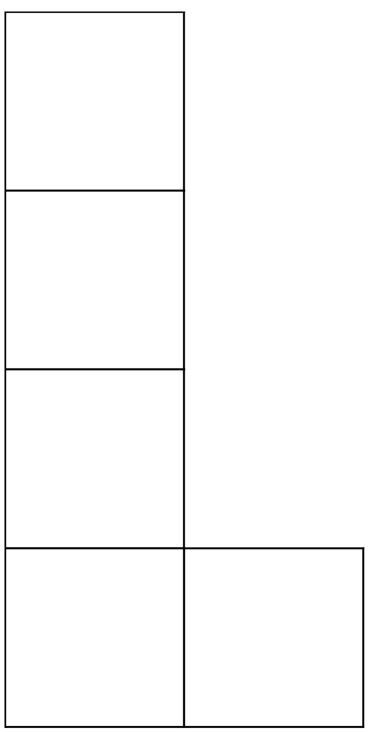


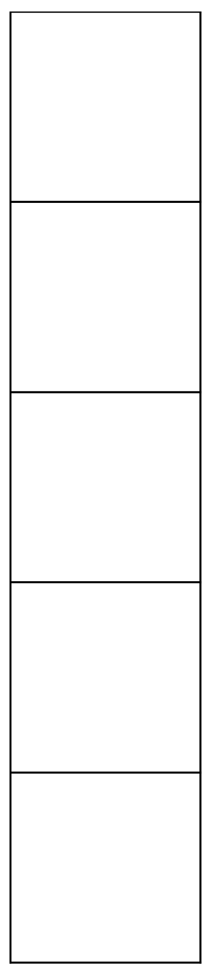








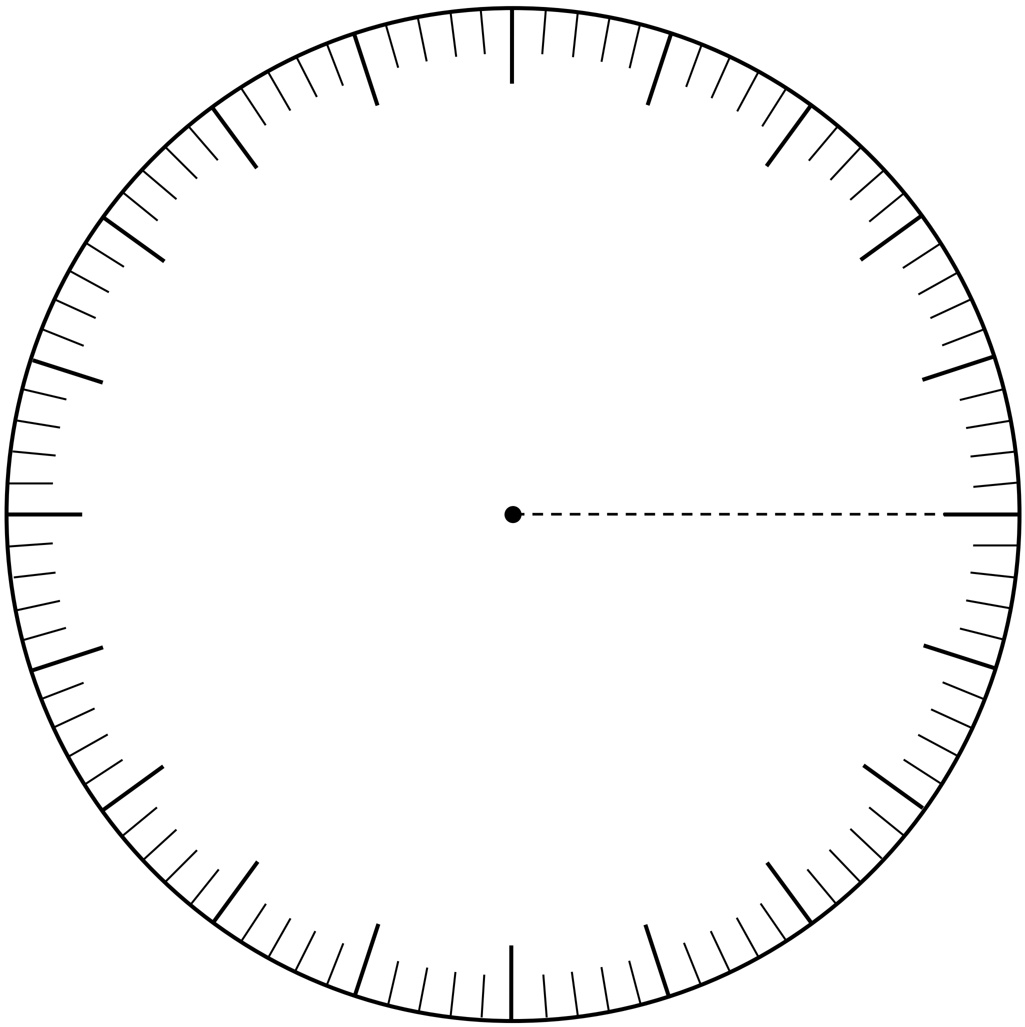




Hundred Chart

0	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

Hundred Circle



References

American Association for the Advancement of Science [AAAS-Benchmarks]. 1993. Benchmark for Science Literacy. New York, NY: Oxford University Press.

Armstrong, T. 1993. *Seven Kinds of Smart: Identifying and Developing Your Many Intelligences*. New York, NY: Plume.

Black, Paul, and Dylan Wiliam. 1998. “Inside the Black Box: Raising Standards Through Classroom Assessment.” *Phi Delta Kappan 80*, No. 2 (October 1998), 139–144, 146–148.

British Columbia Ministry of Education. 2000. *The Primary Program: A Framework for Teaching*. Victoria, BC: Province of British Columbia.

Caine, Renate Numella, and Geoffrey Caine. 1991. *Making Connections: Teaching and the Human Brain*. Reston, VA: Association for Supervision and Curriculum Development.

Coolmath.com. 2014. “Coolmath’s B-Cubed.” *Coolmath-Games.com*. www.coolmath-games.com/0-b-cubed/index.html.

Davies, Anne. 2000. *Making Classroom Assessment Work*. Courtenay, BC: Classroom Connections International, Inc.

Etienne, Steve, and Emily Kalwarowsky. 2013. *Financial Mathematics*. Whitby, ON: McGraw-Hill Ryerson.

Fibonicci. 2011. “Number Sequence Test—Medium.” *Fibonicci*. www.fibonicci.com/numeracy/number-sequences-test/medium.

Financial Consumer Agency of Canada. 2013. “The City: A Financial Life Skills Resource.” *Government of Canada*. www.fcac-acfc.gc.ca/Eng/resources/educationalPrograms/Pages/Thecitya-Lazoneun.aspx#LifeinTheCity.

Frankenstein, Marilyn. 1995. “Equity in Mathematics Education: Class in the World outside the Class.” *New Directions for Equity in Mathematics Education*. Cambridge, MA: Cambridge University Press.

Gardner, Howard E. 2007. *Frames of Mind: The Theory of Multiple Intelligences*. New York, NY: Basic Books.

GeoGebra. 2013. GeoGebra. International GeoGebra Institute. Computer Software. (Available at www.geogebra.org/cms/en.)

Google. 2014. “Google Maps.” *Google.ca*. https://maps.google.ca.

Gutstein, Eric. 2003. “Teaching and Learning Mathematics for Social Justice in an Urban, Latino School.” *Journal for Research in Mathematics Education 34*, No. 1. Reston, VA: National Council of Teachers of Mathematics.

Hall, Tracey E., Anne Meyer, and David H. Rose (Ed). 2012. *Universal Design for Learning in the Classroom: Practical Applications*. New York, NY: The Guilford Press.

Herzig, Abbe. 2005. “Connecting Research to Teaching: Goals for Achieving Diversity in Mathematics Classrooms.” *Mathematics Teacher*, Volume 99, No. 4. Reston, VA: National Council of Teachers of Mathematics.

Hope, Jack A., Larry Leutzinger, Barbara Reys, and Robert Reys. 1988. *Mental Math in the Primary Grades*. Palo Alto, CA: Dale Seymour Publications.

Hume, Karen. 2011. *Tuned Out: Engaging the 21st Century Learner*. Toronto, ON: Pearson Canada Inc.

Institution of Education Sciences, National Center for Education Statistics. 2013. *Kids' Zone: Learning with NCES*, "Create a Graph." Washington, DC: US Department of Education. http://nces.ed.gov/nceskids/createagraph.

Investor Education Fund. 2013. Get Smarter About Money.ca. *Investor Education Fund*. www.getsmarteraboutmoney.ca.

Jefferson Lab. 2014. "The Nim Number Game." *Jefferson Lab*. http://education.jlab.org/nim/index.html.

Katholieke Universiteit Leuven. 2014. *Humorous Implications*, "Numerical Puzzles." Declarative Languages and Artificial Intelligence. http://dtai.cs.kuleuven.be/projects/ALP/newsletter/archive_93_96/humour/index-num.html.

Key Curriculum. 2013. Geometer's Sketchpad. Whitby, ON: McGraw-Hill Ryerson. Computer Software.

Ladson-Billings, Gloria. 1997. "It Doesn't Add Up: African American Students' Mathematics Achievement." *Journal for Research in Mathematics Education 28*, No. 6. Reston, VA: National Council of Teachers of Mathematics.

LearningToGive.org. 2014. *Lesson 4: Credit Cards* (Handout 2). LearningToGive.org. http://learningtogive.org/lessons/unit351/lesson4_attachments/2.html.

MathIsFun.com. 2013. Math Is Fun, Enjoy Learning!, "Dots and Boxes Game." MathIsFun.com. www.mathsisfun.com/games/dots-and-boxes.html.

———. 2013. Math Is Fun, Enjoy Learning!, "Four In A Line." MathIsFun.com. www.mathsisfun.com/games/connect4.html.

———. 2013. Math Is Fun, Enjoy Learning!, "Tic-Tac-Toe." MathIsFun.com. www.mathsisfun.com/games/tic-tac-toe.html.

MindJolt Games. 2013. *MindJolt Games*. www.mindjolt.com.

National Council of Teachers of Mathematics. 2000. *Principles and Standards for School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

———. 2005. "Computation, Calculators, and Common Sense: A Position of the National Council of Teachers of Mathematics" (position paper, May 2005). Reston, VA: National Council of Teachers of Mathematics.

———. 2013. *Calculation Nation*. National Council of Teachers of Mathematics. http://calculationnation.nctm.org/Login.aspx?ReturnUrl=%2fGames%2fdefault.aspx.

———. 2013. *Illuminations: Resources for Teachers. National Council of Teachers of Mathematics*. http://illuminations.nctm.org.

———. 2013. *Illuminations: Resources for Teachers*, "Algebra Tiles." National Council of Teachers of Mathematics. http://illuminations.nctm.org.

———. 2013. *Illuminations: Resources for Teachers*, "Balancing Act." National Council of Teachers of Mathematics. http://illuminations.nctm.org/Activity.aspx?id=1576.

———. 2013. *Illuminations: Resources for Teachers*, "Rise-Run Triangles." National Council of Teachers of Mathematics. http://illuminations.nctm.org/Lesson.aspx?id=2570.

———. 2013. *Illuminations: Resources for Teachers*, "Isometric Drawing Tool." National Council of Teachers of Mathematics. http://illuminations.nctm.org/Activity.aspx?id=4182.

———. 2013. *Illuminations: Resources for Teachers*, "Building Height." National Council of Teachers of Mathematics. http://illuminations.nctm.org/LessonDetail.aspx?id=L764.

———. 2013. *Illuminations: Resources for Teachers*, "Construction of Clinometer." National Council of Teachers of Mathematics. http://illuminations.nctm.org/Lessons/BuildingHeight/BuildingHeight-OH.pdf.

National Snow & Ice Data Center. 2013. *All about Snow*, "Snow Avalanches." National Snow & Ice Data Center. http://nsidc.org/cryosphere/snow/science/avalanches.html.

New Brunswick Department of Education and Early Childhood Development. 2012. *Geometry, Measurement and Finance 10 Curriculum*. Fredericton, NB: Government of New Brunswick.

———. 2012. *Number, Relations and Functions 10 Curriculum*. Fredericton, NB: Government of New Brunswick.

———. 2014. *Financial and Workplace Mathematics 110 Curriculum*. Fredericton, NB: Government of New Brunswick.

Newfoundland and Labrador Department of Education. 2011. *Mathematics: Academic Mathematics 1201*, Interim Edition. St. John's, NL: Government of Newfoundland and Labrador.

———. 2011. *Mathematics: Applied Mathematics 1202*, Interim Edition. St. John's, NL: Government of Newfoundland and Labrador.

———. 2012. *Mathematics: Applied Mathematics 2202*, Interim Edition. St. John's, NL: Government of Newfoundland and Labrador.

Nova Scotia Department of Education. 2002. *Racial Equity Policy*. Halifax, NS: Province of Nova Scotia.

———. 2010. *Gifted Education and Talent Development*. Halifax, NS: Province of Nova Scotia.

———. 2011. *Racial Equity / Cultural Proficiency Framework*. Halifax, NS: Province of Nova Scotia.

OECD Centre for Educational Research and Innovation. 2006. *Formative Assessment: Improving Learning in Secondary Classrooms*. Paris, France: Organization for Economic Co-operation and Development (OECD) Publishing.

OriginalGames.com. 2014. *CombinationLock.com*. OriginalGames.com. www.combinationlock.com.

Orwell, George. n.d. 24 Game: (Double Digits)—Add, Subtract, Multiply, and Divide. Newmarket, ON: Spectrum Educational Supplies. (NSSBB #: 17232)

———. n.d. 24 Game: Factors and Multiples Game. Newmarket, ON: Spectrum Educational Supplies. (NSSBB #: 17310)

———. n.d. 24 Game: Fractions and Decimals. Newmarket, ON: Spectrum Educational Supplies. (NSSBB #: 23955)

———. n.d. 24 Game: Integers. Newmarket, ON: Spectrum Educational Supplies. (NSSBB #: 23956)

Prince Edward Island Department of Education and Early Childhood Development. 2011. *Prince Edward Island Mathematics Curriculum: Mathematics, MAT421A*. Summerside, PE: Government of Prince Edward Island.

———. 2011. *Prince Edward Island Mathematics Curriculum: Mathematics: MAT431A*. Summerside, PE: Government of Prince Edward Island.

———. 2012. *Prince Edward Island Mathematics Curriculum: Mathematics: MAT531A*. Summerside, PE: Government of Prince Edward Island.

Professional Learning K–12, Newfoundland and Labrador. 2013. "Trigonometry." Government of Newfoundland and Labrador. www.k12pl.nl.ca/curr/10-12/math/math2202/classroomclips/trig.html.

———. 2013. “Slope.” *Government of Newfoundland and Labrador*. www.k12pl.nl.ca/curr/10-12/math/math2202/classroomclips/slope.html.

———. 2013. *Professional Learning Newfoundland and Labrador*. Government of Newfoundland and Labrador. www.k12pl.nl.ca.

Province of Nova Scotia. 2009. *Occupational Health and Safety Act*, “Roof Work.” Province of Nova Scotia. www.gov.ns.ca/just/regulations/regs/ohs296f.htm#roof.

Public Broadcasting Service. 2014. *PBS Kids*, “123 Games.” PBS Kids. http://pbskids.org/games/numbers/html.

Quarry, P. 2009. *Mr. D ‘N’ T: Secondary Education and Design*. P. Quarry. www.mr-d-n-t.co.uk/index.htm.

Quebec Literacy Working Group. 2013. *Skills for Life*, “Unit 6 Managing My Money.” LEARN. www.learnquebec.ca/en/content/adult_ed/skills_for_life/unit6.html.

Rabbior, Gary. 2012. *Money and Youth: A Guide to Financial Literacy*. Toronto, ON: Canadian Foundation for Economic Education. (Available at http://moneyandyouth.cfee.org/en/thebook.php.)

RBC Royal Bank. 2013. “Create a Budget Calculator – Manage Your Debt Effectively.” *Royal Bank of Canada*. www.rbcroyalbank.com/products/personalloans/budget/budget-calculator.html.

Roofgenius.com. 2014. *Roof Calculator Software*. Roofgenius.com. http://roofgenius.comlroofpitch.htm.

Rubenstein, Rheta N. 2001. “Mental Mathematics beyond the Middle School: Why? What? How?” Mathematics Teacher, September 2001, Vol. 94, No. 6. Reston, VA: National Council of Teachers of Mathematics.

Samgine. 2011. “Number Puzzles and Math Puzzles.” *Samgine*. http://samgine.com/free/number-puzzles.

Schwanbeck, Greg. 2009. “How to Convert Units—Unit Conversion Made Easy.” YouTube. www.youtube.com/watch?v=XKCZn5MLKvk.

Science Education Resource Centre. 2013. *Starting Point: Teaching Entry Level Geoscience*. “How to Use Excel.” Carleton College. http://serc.carleton.edu/introgeo/mathstatmodels/xlhowto.html.

Shaw, J. M., and Cliatt, M. F. P. 1989. “Developing Measurement Sense.” In P.R. Trafton (Ed.), *New Directions for Elementary School Mathematics*. Reston, VA: National Council of Teachers of Mathematics.

SketchUp. 2013. *SketchUp 2013*. Google.com. (Available at www.sketchup.com.)

Statistics Canada. 2014. “Statistics Canada.” *Government of Canada*. www.statcan.gc.ca.

Steen, L.A. (ed.). 1990. *On the Shoulders of Giants: New Approaches to Numeracy*. Washington, DC: National Research Council.

Stewart, David. 2014. *MobilityBasics.ca*, “Wheelchair Ramp Safety & Standards.” David Stewart. http://mobilitybasics.ca/wheelchair-ramps/ramp.

System Improvement Group Alberta Education. 2006. *Western and Northern Canadian Protocol (WNCP) Consultation with Post-Secondary Institutions, Business and Industry Regarding Their Requirements for High School Mathematics: Final Report on Findings*. Edmonton, AB: Western and Northern Canadian Protocol (WNCP) for Collaboration in Education.

Tate, William F. 1995. “Returning to the Root: A Culturally Relevant Approach to Mathematics Pedagogy.” *Theory into Practice 34*, Issue 3. Florence, KY: Taylor & Francis.

The Futures Channel. 2014. “Designing Toy Cars.” *The Futures Channel*. http://thefutureschannel.com/videogallery/designing-toy-cars.

Tomlinson, Carol Ann. 2001. *How to Differentiate Instruction in Mixed-Ability Classrooms*. Alexandria, VA: Association for Supervision and Curriculum Development.

University of Cambridge. 2014. “Nrich: Enriching Mathematics.” *University of Cambridge*. http://nrich.maths.org.

Vila, Bob. 2014. “The Plumb Bob.” *Bobvila.com: Help You Need for the Home You Want*. www.bobvila.com/articles/495-the-plumb-bob/#.Uxezo4WwX1W.

———. 2014. “How To: Use a Plumb Bob.” *Bobvila.com: Help You Need for the Home You Want*. www.bobvila.com/articles/how-to-use-a-plumb-bob/#.Uxe2I4WwX1U.

Western and Northern Canadian Protocol (WNCP) for Collaboration in Education. 2006. *The Common Curriculum Framework for K–9 Mathematics*. Edmonton, AB: Western and Northern Canadian Protocol (WNCP) for Collaboration in Education.

———. 2008. *The Common Curriculum Framework for Grades 10–12 Mathematics*. Edmonton, AB: Western and Northern Canadian Protocol (WNCP) for Collaboration in Education.