Pre-Calculus 11

Outcomes

NOVA SCOTIA
NOUVELLE-ÉCOSSE

2022

Website References

Website references contained within this document are provided solely as a convenience and do not constitute an endorsement by the Department of Education of the content, policies, or products of the referenced website.

The department does not control the referenced websites and subsequent links, and is not responsible for the accuracy, legality, or content of those websites. Referenced website content may change without notice.

Regional Education Centres and educators are required under the Department's Public School Programs Network Access and Use Policy to preview and evaluate sites before recommending them for student use. If an outdated or inappropriate site is found, please report it to <curriculum@novascotia.ca>.

Pre-Calculus 11

Prepared by the Department of Education and Early Childhood Development

This is the most recent version of the current curriculum materials as used by teachers in Nova Scotia.

Pre-Calculus 11 Outcomes

AN01 Demonstrate an understanding of the absolute value of real numbers.

Performance Indicators:

AN01.01 Determine the distance of two real numbers of the form ±*a*, *a*∈*R*, from 0 on a number line, and relate this to the absolute value of a ($|a|$).
AN01.02 Determine the absolute value of a positive or negative real number.
AN01.03 Explain, using examples, how distance between two points on a number line can be expressed in terms of absolute value.
AN01.04 Determine the absolute value of a numerical expression.
AN01.05 Compare and order the absolute values of real numbers in a given set.

AN02 Solve problems that involve operations on radicals and radical expressions with numerical and variable radicands.

Performance Indicators:

AN02.01 Compare and order radical expressions with numerical radicands in a given set.
AN02.02 Express an entire radical with a numerical radicand as a mixed radical.
AN02.03 Express a mixed radical with a numerical radicand as an entire radical.
AN02.04 Perform one or more operations to simplify radical expressions with numerical or variable radicands.
AN02.05 Rationalize the denominator of a radical expression with monomial or binomial denominators.
AN02.06 Describe the relationship between rationalizing a binomial denominator of a rational expression and the product of the factors of a difference of squares expression.
AN02.07 Explain, using examples, that $(-x)^2 = x^2$, $\sqrt{x^2} = |x|, and\ \sqrt{x^2} \neq \pm x$.
AN02.08 Identify the values of the variable for which a given radical expression is defined.
AN02.09 Solve a problem that involves radical expressions.

AN03 Solve problems that involve radical equations (limited to square roots).

Performance Indicators:

AN03.01 Determine any restrictions on values for the variable in a radical equation.
AN03.02 Determine the roots of a radical equation algebraically, and explain the process used to solve the equation.
AN03.03 Verify, by substitution, that the values determined in solving a radical equation algebraically are roots of the equation.
AN03.04 Explain why some roots determined in solving a radical equation algebraically are extraneous.
AN03.05 Solve problems by modeling a situation using a radical equation.

AN04 Determine equivalent forms of rational expressions (limited to numerators and denominators that are monomials, binomials or trinomials).

Performance Indicators:

AN04.01 Compare the strategies for writing equivalent forms of rational expressions to the strategies for writing equivalent forms of rational numbers.
AN04.02 Explain why a given value is non-permissible for a given rational expression.
AN04.03 Determine the non-permissible values for a rational expression.
AN04.04 Determine a rational expression that is equivalent to a given rational expression by multiplying the numerator and denominator by the same factor (limited to a monomial or a binomial), and state the non-permissible values of the equivalent rational expression.
AN04.05 Simplify a rational expression.
AN04.06 Explain why the non-permissible values of a given rational expression and its simplified form are the same.
AN04.07 Identify and correct errors in a simplification of a rational expression, and explain the reasoning

AN05 Perform operations on rational expressions (limited to numerators and denominators that are monomials, binomials or trinomials).

Performance Indicators:

AN05.01 Compare the strategies for performing a given operation on rational expressions to the strategies for performing the same operation on rational numbers.
AN05.02 Determine the non-permissible values when performing operations on rational expressions.
AN05.03 Determine, in simplified form, the sum or difference of rational expressions with the same denominator.
AN05.04 Determine, in simplified form, the sum or difference of rational expressions in which the denominators are not the same and which may or may not contain common factors.
AN05.05 Determine, in simplified form, the product or quotient of rational expressions.
AN05.06 Simplify an expression that involves two or more operations on rational expressions.

AN06 Solve problems that involve rational equations (limited to numerators and denominators that are monomials, binomials or trinomials).

Performance Indicators:

AN06.01 Determine the non-permissible values for the variable in a rational equation.
AN06.02 Determine the solution to a rational equation algebraically, and explain the process used to solve the equation.
AN06.03 Explain why a value obtained in solving a rational equation may not be a solution of the equation.
AN06.04 Solve problems by modelling a situation using a rational equation.

T01 Demonstrate an understanding of angles in standard position [0° to 360°].

Performance Indicators:

T01.01 Sketch an angle in standard position, given the measure of the angle.

T01.02 Determine the reference angle for an angle in standard position.

T01.03 Explain, using examples, how to determine the angles from 0° to 360° that have the same reference angle as a given angle.

T01.04 Illustrate, using examples, that any angle from 90° to 360° is the reflection in the x-axis and/or the y-axis of its reference angle.

T01.05 Determine the quadrant in which a given angle in standard position terminates.

T01.06 Draw an angle in standard position given any point P (x, y) on the terminal arm of the angle.

T01.07 Illustrate, using examples, that the points P (x, y), P ($-x$, y), P ($-x$,$-y$), and P (x,$-y$) are points on the terminal sides of angles in standard position that have the same reference angle.

T02 Solve problems, using the three primary trigonometric ratios for angles from 0° to 360° in standard position.

Performance Indicators:

T02.01 Determine, using the Pythagorean theorem or the distance formula, the distance from the origin to a point P (x, y) on the terminal arm of an angle.

T02.02 Determine the value of $\sin\theta$, $\cos\theta$, or $\tan\theta$, given any point $P(x, y)$ on the terminal arm of angle θ.

T02.03 Determine, without the use of technology, the value of $\sin\theta$, $\cos\theta$, or $\tan\theta$, given any point $P(x, y)$ on the terminal arm of angle θ, where $\theta = 0°, 90°, 180°, 270°$, or $360°$.

T02.04 Determine the sign of a given trigonometric ratio for a given angle, without the use of technology, and explain.

T02.05 Solve, for all values of θ, an equation of the form $\sin\theta = a$ or $\cos\theta = a$, where $-1 \le a \le 1$, and an equation of the form $\tan\theta = a$, where a is a real number.

T02.06 Determine the exact value of the sine, cosine, or tangent of a given angle with a reference angle of 30°, 45°, or 60°.

T02.07 Describe patterns in and among the values of the sine, cosine, and tangent ratios for angles from 0° to 360°.

T02.08 Sketch a diagram to represent a problem.

T02.09 Solve a contextual problem, using trigonometric ratios.

T03 Demonstrate an understanding of angles in standard position, expressed in degrees and radians.

Performance Indicators:

T03.01 Sketch, in standard position, an angle (positive or negative) when the measure is given in degrees.

T03.02 Describe the relationship among different systems of angle measurement, with emphasis on radians and degrees.

T03.03 Sketch, in standard position, an angle with a measure of one radian.

T03.04 Sketch, in standard position, an angle with a measure expressed in the form $k\pi$ radians, where $k \epsilon Q$

T03.05 Express the measure of an angle in radians (exact value or decimal approximation), given its measure in degrees.

T03.06 Express the measure of an angle in degrees, given its measure in radians (exact value or decimal approximation).

T03.07 Determine the measures, in degrees or radians, of all angles in a given domain that are coterminal with a given angle in standard position.

T03.08 Determine the general form of the measures, in degrees or radians, of all angles that are coterminal with a given angle in standard position.

T03.09 Explain the relationship between the radian measure of an angle in standard position and the length of the arc cut on a circle of radius *r*, and solve problems based upon that relationship.

T04 Develop and apply the equation of the unit circle.

Performance Indicators:

T.04.01 Derive the equation of the unit circle from the Pythagorean theorem.

T.04.02 Describe the six trigonometric ratios, using a point P (x, y) that is the intersection of the terminal arm of an angle and the unit circle.

T.04.03 Generalize the equation of a circle with centre (0, 0) and radius r.

RF01 Factor polynomial expressions of the form:

$ax^2 + bx + c, a \neq 0$
$a^2b^2 - b^2y^2, a \neq 0, b \neq 0$
$a[f(x)]^2 + b[f(x)] + c, a \neq 0$
$a^2[f(x)]^2 - b^2[g(y)]^2, a \neq 0, b \neq 0$

where *a*, *b* and *c* are rational numbers.

Performance Indicators:

RF01.01 Factor a given polynomial expression that requires the identification of common factors.

RF01.02 Determine whether a given binomial is a factor for a given polynomial expression, and explain why or why not.

RF01.03 Factor a given polynomial expression of the form

- $ax^2 + bx + c, a \neq 0$
- $a^2x^2 - b^2y^2, a \neq 0$

RF01.04 Factor a given polynomial expression that has a quadratic pattern, including

- $a[f(x)]^2 + b[f(x)] + c, a \neq 0$
- $a^2[f(x)]^2 - b^2[g(y)]^2, a \neq 0, b \neq 0$.

RF02 Graph and analyze absolute value functions (limited to linear and quadratic functions) to solve problems.

Performance Indicators:

RF02.01 Create a table of values for $y = |f(x)|$, given a table of values for $y = f(x)$.

RF02.02 Generalize a rule for writing absolute value functions in piecewise notation.

RF02.03 Sketch the graph of $y = |f(x)|$; state the intercepts, domain, and range; and explain the strategy used.
RF02.04 Solve an absolute value equation graphically, with or without technology.
RF02.05 Solve, algebraically, an equation with a single absolute value, and verify the solution.
RF02.06 Explain why the absolute value equation $|f(x)| < 0$ has no solution.
RF02.07 Determine and correct errors in a solution to an absolute value equation.
RF02.08 Solve a problem that involves an absolute value function.

RF03 Analyze quadratic functions of the form and determine the:

- vertex
- domain and range
- direction of opening
- axis of symmetry
- *x*- and *y*-intercepts.

Performance Indicators:

RF03.01 Explain why a function given in the form $y = a(x - p)^2 + q$ is a quadratic function.
RF03.02 Compare the graphs of a set of functions of the form $y = ax^2$ to the graph of $y = x^2$, and generalize, using inductive reasoning, a rule about the effect of *a*.
RF03.03 Compare the graphs of a set of functions of the form $y = x^2 + q$ to the graph of $y = x^2$, and generalize, using inductive reasoning, a rule about the effect of *q*.
RF03.04 Compare the graphs of a set of functions of the form $y = (x - p)^2$ to the graph of $y = x^2$, and generalize, using inductive reasoning, a rule about the effect of *p*.
RF03.05 Determine the coordinates of the vertex for a quadratic function of the form $y = a(x - p)^2 + q$, and verify with or without technology.
RF03.06 Generalize, using inductive reasoning, a rule for determining the coordinates of the vertex for quadratic functions of the form $y = a(x - p)^2 + q$.
RF03.07 Sketch the graph of $y = a(x - p)^2 + q$, using transformations, and identify the vertex, domain and range, direction of opening, axis of symmetry, and *x*-intercepts and *y*-intercepts.
RF03.08 Explain, using examples, how the values of *a* and *q* may be used to determine whether a quadratic function has zero, one, or two *x*-intercepts.
RF03.09 Write a quadratic function in the form $y = a(x - p)^2 + q$ for a given graph or a set of characteristics of a graph.

RF04 Analyze quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including vertex, domain and range, direction of opening, axis of symmetry, x-intercept and y-intercept, and to solve problems.

Performance Indicators:

RF04.01 Explain the reasoning for the process of completing the square as shown in a given example.

RF04.02 Write a quadratic function given in the form $y = ax^2 + bx + c$ as a quadratic function in the form $y = a(x - p)^2 + q$ by completing the square.

RF04.03 Identify, explain, and correct errors in an example of completing the square.

RF04.04 Determine the characteristics of a quadratic function given in the form $y = ax^2 + bx + c$, and explain the strategy used.

RF04.05 Sketch the graph of a quadratic function given in the form $y = ax^2 + bx + c$.

RF04.06 Verify, with or without technology, that a quadratic function in the form $y = ax^2 + bx + c$ represents the same function as a given quadratic function in the form $y = a(x - p)^2 + q$.

RF04.07 Write a quadratic function that models a given situation, and explain any assumptions made.

RF04.08 Solve a problem, with or without technology, by analyzing a quadratic function.

RF05 Solve problems that involve quadratic equations

Performance Indicators:

RF05.01 Explain, using examples, the relationship among the roots of a quadratic equation, the zeros of the corresponding quadratic function, and the *x*-intercepts of the graph of the quadratic function.

RF05.02 Derive the quadratic formula, using deductive reasoning.

RF05.03 Solve a quadratic equation of the form $ax^2 + bx + c = 0$ by using strategies such as

- determining square roots
- factoring
- completing the square
- applying the quadratic formula
- graphing its corresponding function

RF05.04 Select a method for solving a quadratic equation, justify the choice, and verify the solution.

RF05.05 Explain, using examples, how the discriminant may be used to determine whether a quadratic equation has two, one, or no real roots, and relate the number of zeros to the graph of the corresponding quadratic function.

RF05.06 Identify and correct errors in a solution to a quadratic equation.

RF05.07 Solve a problem by

- analyzing a quadratic equation
- determining and analyzing a quadratic equation

RF06 Solve, algebraically and graphically, problems that involve systems of linear-quadratic and quadratic-quadratic equations in two variables

Performance Indicators:

RF06.01 Model a situation, using a system of linear-quadratic or quadratic-quadratic equations.
RF06.02 Relate a system of linear-quadratic or quadratic-quadratic equations to the context of a given problem.
RF06.03 Determine and verify the solution of a system of linear-quadratic or quadratic-quadratic equations graphically, with technology.
RF06.04 Determine and verify the solution of a system of linear-quadratic or quadratic-quadratic equations algebraically.
RF06.05 Explain the meaning of the points of intersection of a system of linear-quadratic or quadratic-quadratic equations.
RF06.06 Explain, using examples, why a system of linear-quadratic or quadratic-quadratic equations may have zero, one, two, or an infinite number of solutions.
RF06.07 Solve a problem that involves a system of linear-quadratic or quadratic-quadratic equations, and explain the strategy used.

RF07 Solve problems that involve linear and quadratic inequalities in two variables.

Performance Indicators:

RF07.01 Explain, using examples, how test points can be used to determine the solution region that satisfies an inequality.
RF07.02 Explain, using examples, when a solid or broken line should be used in the solution for an inequality.
RF07.03 Sketch, with or without technology, the graph of a linear or quadratic inequality.
RF07.04 Solve a problem that involves a linear or quadratic inequality.

RF09 Analyze arithmetic sequences and series to solve problems.

Performance Indicators:

RF09.01 Identify the assumption(s) made when defining an arithmetic sequence or series.
RF09.02 Provide and justify an example of an arithmetic sequence.
RF09.03 Derive a rule for determining the general term of an arithmetic sequence.
RF09.04 Describe the relationship between arithmetic sequences and linear functions.
RF09.05 Determine t_1, d, n, or t_n in a problem that involves an arithmetic sequence.
RF09.06 Derive a rule for determining the sum of n terms of an arithmetic series.
RF09.07 Determine t_1, d, n, or S_n in a problem that involves an arithmetic series.
RF09.08 Solve a problem that involves an arithmetic sequence or series.

RF10 Analyze geometric sequences and series to solve problems.

Performance Indicators:

RF10.01 Identify assumptions made when identifying a geometric sequence or series.
RF10.02 Provide and justify an example of a geometric sequence.
RF10.03 Derive a rule for determining the general term of a geometric sequence.
RF10.04 Determine t_1, r, n, or t_n in a problem that involves a geometric sequence.
RF10.05 Derive a rule for determining the sum of n terms of a geometric series.
RF10.06 Determine t_1, r, n, or S_n in a problem that involves a geometric series.
RF10.07 Generalize, using inductive reasoning, a rule for determining the sum of an infinite geometric series.
RF10.08 Explain why a geometric series is convergent or divergent.
RF10.09 Solve a problem that involves a geometric sequence or series.

RF11 Graph and analyze reciprocal functions (limited to the reciprocal of linear and quadratic functions).

Performance Indicators:

RF11.01 Compare the graph of $y = \frac{1}{f(x)}$ to the graph of $y = f(x)$.
RF11.02 Identify, given a function $f(x)$, values of x for which $y = \frac{1}{f(x)}$ will have vertical asymptotes; and describe their relationship to the non-permissible values of the related rational expression.
RF11.03 Graph, with or without technology, $y = \frac{1}{f(x)}$, given $y = f(x)$ as a function or a graph, and explain the strategies used.
RF11.04 Graph, with or without technology, $y = f(x)$, given $y = \frac{1}{f(x)}$ as a function or a graph, and explain the strategies used.