

# Pre-Calculus 12

*Foundational Outcomes*

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## Outcomes Framework Pre-Calculus 12 (2020-2021)

In September 2020, teachers will be working hard to create a space that is safe and welcoming for all learners no matter the location of their “classroom”. The first weeks will still be a time to establish a sense of community, engage learners in rich interactive experiences to promote critical thinking and create opportunities for collaboration and discussion. This is an opportune time to develop a culture and a climate for mathematics learning, conducive to collaboration, risk taking and inquiry.

The **Foundational Outcomes** identified in this document represent outcomes determined to be relevant for future learning in mathematics. Decisions about foundational outcomes were made in consultation with teachers, provincial mathematics team, Board and Regional Centre staff. The foundational outcomes are meant to guide teachers in making decisions about creating learning experiences that will prepare and engage their learners in a responsive way. However, a teacher’s professional judgment remains the most important guide to effectively responding to the needs of their learners.

Colour coding has been used to identify outcomes and indicators as foundational (**green**), optional (**orange**) or non-foundational (**red**) for the 2020-2021 school year.

**T03** Solve problems, using the six trigonometric ratios for angles expressed in radians and degrees.

**Performance Indicators:** all indicators

**T04** Graph and analyze the trigonometric functions sine, cosine and tangent to solve problems.

**Performance Indicators:**

- T04.01 Sketch, with or without technology, the graph of  $y = \sin x$ ,  $y = \cos x$ , or  $y = \tan x$ .
- T04.02 Determine the characteristics (amplitude, asymptotes, domain, period, range, and zeros) of the graph of  $y = \sin x$ ,  $y = \cos x$ , or  $y = \tan x$ .
- T04.03 Determine how varying the value of  $a$  affects the graphs of  $y = a \sin x$  and  $y = a \cos x$ .
- T04.04 Determine how varying the value of  $d$  affects the graphs of  $y = \sin x + d$  and  $y = \cos x + d$ .
- T04.05 Determine how varying the value of  $c$  affects the graphs of  $y = \sin(x + c)$  and  $y = \cos(x + c)$ .
- T04.06 Determine how varying the value of  $b$  affects the graphs of  $y = \sin bx$  and  $y = \cos bx$ .
- T04.07 Sketch, without technology, graphs of the form  $y = a \sin b(x - c) + d$  or  $y = a \cos b(x - c) + d$ , using transformations, and explain the strategies.

- T04.08 Determine the characteristics (amplitude, asymptotes, domain, period, phase shift, range and zeros) of the graph of a trigonometric function of the form  $y = a \sin b(x - c) + d$  or  $y = a \cos b(x - c) + d$ .
- T04.09 Determine the values of  $a$ ,  $b$ ,  $c$ , and  $d$  for functions of the form  $y = a \sin b(x - c) + d$  or  $y = a \cos b(x - c) + d$  that correspond to a given graph, and write the equation of the function.
- T04.10 Determine a trigonometric function that models a situation to solve a problem.
- T04.11 Explain how the characteristics of the graph of a trigonometric function relate to the conditions in a problem situation.
- T04.12 Solve a problem by analyzing the graph of a trigonometric function.

**T05** Solve, algebraically and graphically, first and second degree trigonometric equations with the domain expressed in degrees and radians.

**Performance Indicators:** all indicators

**T06** Prove trigonometric identities, using:

- reciprocal identities
  - quotient identities
  - Pythagorean identities
  - sum or difference identities
  - double-angle identities
- T06.01 Explain the difference between a trigonometric identity and a trigonometric equation.
  - T06.02 Verify a trigonometric identity numerically for a given value in either degrees or radians.
  - T06.03 Explain why verifying that the two sides of a trigonometric identity are equal for given values is insufficient to conclude that the identity is valid.
  - T06.04 Determine, graphically, the potential validity of a trigonometric identity, using technology.
  - T06.05 Determine the non-permissible values of a trigonometric identity.
  - T06.06 Prove, algebraically, that a trigonometric identity is valid.
  - **T06.07** Determine, using the sum, difference, and double-angle identities, the exact value of a trigonometric ratio.

**RF01** Demonstrate an understanding of operations on, and compositions of, functions.

**Performance Indicators:**

- **RF01.01** Sketch the graph of a function that is the sum, difference, product, or quotient of two functions, given their graphs.
- **RF01.02** Write the equation of a function that is the sum, difference, product, or quotient of two or more functions, given their equations.

- **RF01.03** Determine the domain and range of a function that is the sum, difference, product, or quotient of two functions.
- **RF01.04** Write a function  $h(x)$  as the sum, difference, product, or quotient of two or more functions.
- RF01.05 Determine the value of the composition of functions when evaluated at a point, including  $f[f(a)]$ ,  $f[g(a)]$ , and  $g[f(a)]$ .
- RF01.06 Determine, given the equations of two functions  $f(x)$  and  $g(x)$ , the equation of the composite function  $f[f(x)]$ ,  $f[g(x)]$ , and  $g[f(x)]$ , and explain any restrictions.
- RF01.07 Sketch, given the equations of two functions  $f(x)$  and  $g(x)$ , the graph of the composite function  $f[f(x)]$ ,  $f[g(x)]$ , and  $g[f(x)]$ .
- RF01.08 Write a function  $h(x)$  as the composition of two or more functions.
- **RF01.09** Write a function  $h(x)$  by combining two or more functions through operations on, and compositions of, functions

**RF02** Demonstrate an understanding of the effects of horizontal and vertical translations on the graphs of functions and their related equations.

**Performance Indicators:** all indicators

**RF03** Demonstrate an understanding of the effects of horizontal and vertical stretches on the graphs of functions and their related equations.

**Performance Indicators:** all indicators

**RF04** Apply translations and stretches to the graphs and equations of functions.

**Performance Indicators:** all indicators

**RF05** Demonstrate an understanding of the effects of reflections on the graphs of functions and their related equations, including reflections in the: x-axis, y-axis, and the line  $y=x$ .

**Performance Indicators:** all indicators

**RF06** Demonstrate an understanding of inverses of relations.

**Performance Indicators:** all indicators

**RF07** Demonstrate an understanding of logarithms.

**Performance Indicators:**

- RF07.01 Explain the relationship between logarithms and exponents.
- RF07.02 Express a logarithmic expression as an exponential expression and vice versa.
- **RF07.03** Determine, without technology, the exact value of a logarithm, such as  $\log_2 8$  and  $\ln e$ .

- **RF07.04** Estimate the value of a logarithm, using benchmarks, and explain the reasoning (e.g., since  $\log_2 8 = 3$  and  $\log_2 16 = 4$ ,  $\log_2 9$  is approximately equal to 3.1).

**RF08** Demonstrate an understanding of the product, quotient and power laws of logarithms.

**Performance Indicators:**

- RF08.01 Develop and generalize the laws for logarithms, using numeric examples and exponent laws.
- RF08.02 Derive each law of logarithms.
- RF08.03 Determine, using the laws of logarithms, an equivalent expression for a logarithmic expression.
- **RF08.04** Determine, with technology, the approximate value of a logarithmic expression, such as  $\log_2 9$  and  $\ln 10$ .

**RF09** Graph and analyze exponential and logarithmic functions.

**Performance Indicators:**

- RF09.01 Sketch, with or without technology, a graph of an exponential function of the form  $y = a^x$ ,  $a > 0$ .
- RF09.02 Identify the characteristics of the graph of an exponential function of the form  $y = a^x$ ,  $a > 0$ , including the domain, range, horizontal asymptote and intercepts, and explain the significance of the horizontal asymptote.
- **RF09.03** Sketch the graph of an exponential function by applying a set of transformations to the graph of  $y = a^x$ ,  $a > 0$ , and state the characteristics of the graph.
- RF09.04 Sketch, with or without technology, the graph of a logarithmic function of the form  $y = \log_b x$ ,  $b > 1$ .
- RF09.05 Identify the characteristics of the graph of a logarithmic function of the form  $y = \log_b x$ ,  $b > 1$ , including the domain, range, vertical asymptote and intercepts, and explain the significance of the vertical asymptote.
- **RF09.06** Sketch the graph of a logarithmic function by applying a set of transformations to the graph of  $y = \log_b x$ ,  $b > 1$ , and state the characteristics of the graph.
- **RF09.07** Demonstrate, graphically, that a logarithmic function and an exponential function with the same base are inverses of each other.

**RF10** Solve problems that involve exponential and logarithmic equations.

**Performance Indicators:**

- **RF10.01** Determine the solution of an exponential equation in which the bases are powers of one another.

- RF10.02 Determine the solution of an exponential equation in which the bases are not powers of one another, using a variety of strategies.
- RF10.03 Determine the solution of a logarithmic equation, and verify the solution.
- RF10.04 Explain why a value obtained in solving a logarithmic equation may be extraneous.
- **RF10.05** Solve a problem that involves exponential growth or decay.
- **RF10.06** Solve a problem that involves the application of exponential equations to loans, mortgages, and investments.
- **RF10.07** Solve a problem that involves logarithmic scales, such as the Richter scale and the pH scale.
- RF10.08 Solve a problem by modelling a situation with an exponential or a logarithmic equation.

**RF11** Demonstrate an understanding of factoring polynomials of degree greater than 2 (limited to polynomials of degree  $\leq 5$  with integral coefficients).

**Performance Indicators:** all indicators

**RF12** Graph and analyze polynomial functions (limited to polynomial functions of degree  $\leq 5$ ).

**Performance Indicators:** all indicators

**RF13** Graph and analyze radical functions (limited to functions involving one radical)

**Performance Indicators:** all indicators

**RF14** Graph and analyze rational functions (limited to numerators and denominators that are monomials, binomials or trinomials).

**Performance Indicators:** all indicators

**PC01** Apply the fundamental counting principle to solve problems.

**Performance Indicators:** all indicators

**PC02** Determine the number of permutations of elements taken at a time to solve problems.

**Performance Indicators:** all indicators

**PC03** Determine the number of combinations of different elements taken at a time to solve problems.

**Performance Indicators:** all indicators

**PC04** Expand powers of a binomial in a variety of ways, including using the binomial theorem (restricted to exponents that are natural numbers).

Performance Indicators: **all indicators**