Pre-Calculus 12

Outcomes

NOVA SCOTIA
NOUVELLE-ÉCOSSE

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Pre-Calculus 12

Prepared by the Department of Education and Early Childhood Development

This is the most recent version of the current curriculum materials as used by teachers in Nova Scotia.

Pre-Calculus 12 Outcomes

T03 Solve problems, using the six trigonometric ratios for angles expressed in radians and degrees.

Performance Indicators:

- T03.01 Determine, with technology, the approximate value of a trigonometric ratio for any angle with a measure expressed in either degrees or radians.
- T03.02 Determine, using a unit circle or reference triangle, the exact value of a trigonometric ratio for angles expressed in degrees that are multiples of 0°, 30°, 45°, 60°, or 90°, or for angles expressed in radians that are multiples of $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$ or $\frac{\pi}{2}$ 0, and explain the strategy.
- T03.03 Determine, with or without technology, the measures, in degrees or radians, of the angles in a specified domain, given the value of a trigonometric ratio.
- T03.04 Explain how to determine the exact values of the six trigonometric ratios, given the coordinates of a point on the terminal arm of an angle in standard position.
- T03.05 Determine the measures of the angles in a specified domain in degrees or radians, given a point on the terminal arm of an angle in standard position.
- T03.06 Determine the exact values of the other trigonometric ratios, given the value of one trigonometric ratio in a specified domain.
- T03.07 Sketch a diagram to represent a problem that involves trigonometric ratios.
- T03.08 Solve a problem, using trigonometric ratios

T04 Graph and analyze the trigonometric functions sine, cosine and tangent to solve problems.

Performance Indicators:

- T04.01 Sketch, with or without technology, the graph of $y = \sin x$, $y = \cos x$, or $y = \tan x$.
- T04.02 Determine the characteristics (amplitude, asymptotes, domain, period, range, and zeros) of the graph of $y = \sin x$, $y = \cos x$, or $y = \tan x$.
- T04.03 Determine how varying the value of a affects the graphs of $y = a \sin x$ and $y = a \cos x$.
- T04.04 Determine how varying the value of d affects the graphs of $y = \sin x + d$ and $y = \cos x + d$.
- T04.05 Determine how varying the value of c affects the graphs of $y = \sin (x + c)$ and $y = \cos (x + c)$.
- T04.06 Determine how varying the value of b affects the graphs of $y = \sin bx$ and $y = \cos bx$.
- T04.07 Sketch, without technology, graphs of the form $y = a \sin b(x - c) + d$ or $y = a \cos b(x - c) + d$, using transformations, and explain the strategies.
- T04.08 Determine the characteristics (amplitude, asymptotes, domain, period, phase shift, range and zeros) of the graph of a trigonometric function of the form $y = a \sin b(x - c) + d$ or $y = a \cos b(x - c) + d$.
- T04.09 Determine the values of $a, b, c,$ and d for functions of the form $y = a \sin b(x - c) + d$ or $y = a \cos b(x - c) + d$ that correspond to a given graph, and write the equation of the function.
- T04.10 Determine a trigonometric function that models a situation to solve a problem.
- T04.11 Explain how the characteristics of the graph of a trigonometric function relate to the conditions in a problem situation.
- T04.12 Solve a problem by analyzing the graph of a trigonometric function.

T05 Solve, algebraically and graphically, first and second degree trigonometric equations with the domain expressed in degrees and radians.

Performance Indicators:

T05.01 Verify, with or without technology, that a given value is a solution to a trigonometric equation.

T05.02 Determine, algebraically, the solution of a trigonometric equation, stating the solution in exact form, when possible.

T05.03 Determine, using technology, the approximate solution of a trigonometric equation in a restricted domain.

T05.04 Relate the general solution of a trigonometric equation to the zeros of the corresponding trigonometric function (restricted to sine and cosine functions).

T05.05 Determine, using technology, the general solution of a given trigonometric equation.

T05.06 Identify and correct errors in a solution for a trigonometric equation.

T06 Prove trigonometric identities, using:

- reciprocal identities
- quotient identities
- Pythagorean identities
- sum or difference identities
- double-angle identities

T06.01 Explain the difference between a trigonometric identity and a trigonometric equation.

T06.02 Verify a trigonometric identity numerically for a given value in either degrees or radians.

T06.03 Explain why verifying that the two sides of a trigonometric identity are equal for given values is insufficient to conclude that the identity is valid.

T06.04 Determine, graphically, the potential validity of a trigonometric identity, using technology.

T06.05 Determine the non-permissible values of a trigonometric identity.

T06.06 Prove, algebraically, that a trigonometric identity is valid.

T06.07 Determine, using the sum, difference, and double-angle identities, the exact value of a trigonometric ratio.

RF01 Demonstrate an understanding of operations on, and compositions of, functions.

Performance Indicators:

RF01.01 Sketch the graph of a function that is the sum, difference, product, or quotient of two functions, given their graphs.

RF01.02 Write the equation of a function that is the sum, difference, product, or quotient of two or more functions, given their equations.

RF01.03 Determine the domain and range of a function that is the sum, difference, product, or quotient of two functions.

RF01.04 Write a function $h(x)$as the sum, difference, product, or quotient of two or more functions.

RF01.05 Determine the value of the composition of functions when evaluated at a point, including $f[f(a)], f[g(a)]$,and $g[f(a)]$.

RF01.06 Determine, given the equations of two functions $f(x)$ and $g(x)$, the equation of the composite function $f[f(x)], f[g(x)]$,and $g[f(x)]$,and explain any restrictions.

RF01.07 Sketch, given the equations of two functions $f(x)$ and $g(x)$, the graph of the composite function $f[f(x)], f[g(x)]$,and $g[f(x)]$.

RF01.08 Write a function $h(x)$ as the composition of two or more functions.

RF02 Demonstrate an understanding of the effects of horizontal and vertical translations on the graphs of functions and their related equations.

Performance Indicators:

RF02.01 Compare the graphs of a set of functions of the form $y - k = f(x)$ to the graph of $y = f(x)$ and generalize, using inductive reasoning, a rule about the effect of *k*.

RF02.02 Compare the graphs of a set of functions of the form $y = f(x - h)$ to the graph of $y = f(x)$ and generalize, using inductive reasoning, a rule about the effect of *h*.

RF02.03 Compare the graphs of a set of functions of the form $y - k = f(x - h)$ to the graph of $y = f(x)$ and generalize, using inductive reasoning, a rule abo-ut the effects of *h* and *k*.

RF02.04 Sketch the graph of $y - k = f(x)$, $y = f(x - h)$, or $y - k = f(x - h)$ for given values of *h* and *k*, given a sketch of the function $y = f(x)$ where the equation of $y = f(x)$ is not given.

RF02.05 Write the equation of a function whose graph is a vertical and/or horizontal translation of the graph of the function $y = f(x)$.

RF03 Demonstrate an understanding of the effects of horizontal and vertical stretches on the graphs of functions and their related equations.

Performance Indicators:

RF03.01 Compare the graphs of a set of functions of the form $y = af(x)$ to the graph of $y = f(x)$ and generalize, using inductive reasoning, a rule about the effect of *a*.

RF03.02 Compare the graphs of a set of functions of the form $y = f(bx)$ to the graph of $y = f(x)$ and generalize, using inductive reasoning, a rule about the effect of *b*.

RF03.03 Compare the graphs of a set of functions of the form $y = af(bx)$ to the graph of $y = f(x)$ and generalize, using inductive reasoning, a rule about the effects of *a* and *b*.

RF03.04 Sketch the graph of $y = af(x)$, $y = f(bx)$ or $y = af(bx)$ for given values of *a* and *b*, given a sketch of the function $y = f(x)$ where the equation of $y = f(x)$ is not given.

RF03.05 Write the equation of a function, given its graph which is a vertical and/or horizontal stretch of the graph of the function $y = f(x)$

RF04 Apply translations and stretches to the graphs and equations of functions.

Performance Indicators:

RF04.01 Sketch the graph of the function $y - k = af[b(x - h)]$ for given values of *a*, *b*, *h*, and *k*, given the graph of the function $y = f(x)$, where the equation of $y = f(x)$is not given.

RF04.02 Write the equation of a function, given its graph that is a translation and/or stretch of the graph of the function $y = f(x)$

RF05 Demonstrate an understanding of the effects of reflections on the graphs of functions and their related equations, including reflections in the: *x*-axis, *y*-axis, and the line $y = x$.

Performance Indicators:

RF05.01 Generalize the relationship between the coordinates of an ordered pair and the coordinates of the corresponding ordered pair that results from a reflection in the *x*-axis, the *y*-axis, or the line *y* = *x*.

RF05.02 Sketch the reflection of the graph of a function $y = f(x)$ in the *x*-axis, the *y*-axis, or the line *y* = *x*, given the graph of the function $y = f(x)$ where the equation of $y = f(x)$is not given.

RF05.03 Generalize, using inductive reasoning, and explain rules for the reflection of the graph of the function $y = f(x)$ in the *x*-axis, the *y*-axis, or the line *y* = *x*.

RF05.04 Sketch the graphs of the functions $y = -f(x)$, $y = f(x)$ and $x = f(y)$, given the graph of the function $y = f(x)$ where the equation of $y = f(x)$ is not given.

RF05.05 Write the equation of a function, given its graph that is a reflection of the graph of the function $y = f(x)$ in the *x*-axis, the *y*-axis, or the line *y* = *x*.

RF06 Demonstrate an understanding of inverses of relations.

Performance Indicators:

RF06.01 Explain how the graph of the line $y = x$ can be used to sketch the inverse of a relation.

RF06.02 Explain how the transformation $(x, y) \rightarrow (y, x)$ can be used to sketch the inverse of a relation.

RF06.03 Sketch the graph of the inverse relation, given the graph of a relation.

RF06.04 Determine if a relation and its inverse are functions.

RF06.05 Determine restrictions on the domain of a function in order for its inverse to be a function.

RF06.06 Determine the equation and sketch the graph of the inverse relation, given the equation of a linear or quadratic relation.

RF06.07 Explain the relationship between the domains and ranges of a relation and its inverse.

RF06.08 Determine, algebraically or graphically, if two functions are inverses of each other.

RF07 Demonstrate an understanding of logarithms.

Performance Indicators:

RF07.01 Explain the relationship between logarithms and exponents.

RF07.02 Express a logarithmic expression as an exponential expression and vice versa.

RF07.03 Determine, without technology, the exact value of a logarithm, such as $\log_2 8$ and $\ln e$.

RF07.04 Estimate the value of a logarithm, using benchmarks, and explain the reasoning.

RF08 Demonstrate an understanding of the product, quotient and power laws of logarithms.

Performance Indicators:

RF08.01 Develop and generalize the laws for logarithms, using numeric examples and exponent laws.

RF08.02 Derive each law of logarithms.

RF08.03 Determine, using the laws of logarithms, an equivalent expression for a logarithmic expression.

RF08.04 Determine, with technology, the approximate value of a logarithmic expression, such as $\log_2 9$ and $\ln 10$.

RF09 Graph and analyze exponential and logarithmic functions.

Performance Indicators:

RF09.01 Sketch, with or without technology, a graph of an exponential function of the form $y = a^x$, $a > 0$.

RF09.02 Identify the characteristics of the graph of an exponential function of the form $y = a^x$, $a > 0$, including the domain, range, horizontal asymptote and intercepts, and explain the significance of the horizontal asymptote.

RF09.03 Sketch the graph of an exponential function by applying a set of transformations to the graph of $y = a^x$, $a > 0$, and state the characteristics of the graph.

RF09.04 Sketch, with or without technology, the graph of a logarithmic function of the form $y = \log_b x$, $b > 1$.

RF09.05 Identify the characteristics of the graph of a logarithmic function of the form $y = \log_b x$, $b > 1$, including the domain, range, vertical asymptote and intercepts, and explain the significance of the vertical asymptote.

RF09.05 Sketch the graph of a logarithmic function by applying a set of transformations to the graph of $y = \log_b x$, $b > 1$, and state the characteristics of the graph.

RF09.06 Demonstrate, graphically, that a logarithmic function and an exponential function with the same base are inverses of each other.

RF10 Solve problems that involve exponential and logarithmic equations.

Performance Indicators:

RF10.01 Determine the solution of an exponential equation in which the bases are powers of one another.

RF10.02 Determine the solution of an exponential equation in which the bases are not powers of one another, using a variety of strategies.

RF10.03 Determine the solution of a logarithmic equation, and verify the solution.

RF10.04 Explain why a value obtained in solving a logarithmic equation may be extraneous.

RF10.05 Solve a problem that involves exponential growth or decay.

RF10.06 Solve a problem that involves the application of exponential equations to loans, mortgages, and investments.

RF10.07 Solve a problem that involves logarithmic scales, such as the Richter scale and the pH scale.

RF10.08 Solve a problem by modeling a situation with an exponential or a logarithmic equation.

RF11 Demonstrate an understanding of factoring polynomials of degree greater than 2 (limited to polynomials of degree ≤ 5 with integral coefficients).

Performance Indicators:

RF11.01 Explain how long division of a polynomial expression by a binomial expression of the form $x - a, x \epsilon Z$ is related to synthetic division.

RF11.02 Divide a polynomial expression by a binomial expression of the form $x - a, x \epsilon Z$ using long division or synthetic division.

RF11.03 Explain the relationship between the linear factors of a polynomial expression and the zeros of the corresponding polynomial function.

RF11.04 Explain the relationship between the remainder when a polynomial expression is divided by $x - a, x \epsilon Z$ and the value of the polynomial expression at $x = a$ (remainder theorem).

RF11.05 Explain and apply the factor theorem to express a polynomial expression as a product of factors.

RF12 Graph and analyze polynomial functions (limited to polynomial functions of degree ≤ 5).

Performance Indicators:

RF12.01 Identify the polynomial functions in a set of functions, and explain the reasoning.

RF12.02 Explain the role of the constant term and leading coefficient in the equation of a polynomial function with respect to the graph of the function.

RF12.03 Generalize rules for graphing polynomial functions of odd or even degree.

RF12.04 Explain the relationship among the zeros of a polynomial function, the roots of the corresponding polynomial equation, and the *x*-intercepts of the graph of the polynomial function.

RF12.05 Explain how the multiplicity of a zero of a polynomial function affects the graph.

RF12.06 Sketch, with or without technology, the graph of a polynomial function.

RF12.07 Solve a problem by modeling a given situation with a polynomial function and analyzing the graph of the function.

RF13 Graph and analyze radical functions (limited to functions involving one radical)

Performance Indicators:

RF13.01 Sketch the graph of the function $y = \sqrt{x}$ using a table of values, and state the domain and range.

RF13.02 Sketch the graph of the function $y - k = a\sqrt{b(x - h)}$by applying transformations to the graph of the function $y = \sqrt{x}$ and state the domain and range.

RF13.03 Sketch the graph of the function $y = \sqrt{f(x)}$given the graph of the function $y = f(x)$and explain the strategies used.

RF13.04 Compare the domain and range of the function $y = \sqrt{x}$ to the domain and range of the function $y = f(x)$and explain why the domains and ranges may differ.

RF13.05 Describe the relationship between the roots of a radical equation and the *x*-intercepts of the graph of the corresponding radical function.

RF13.06 Determine, graphically, an approximate solution of a radical equation.

RF14 Graph and analyze rational functions (limited to numerators and denominators that are monomials, binomials or trinomials).

Performance Indicators:

RF14.01 Graph, with or without technology, a rational function.
RF14.02 Analyze the graphs of a set of rational functions to identify common characteristics.
RF14.03 Explain the behaviour of the graph of a rational function for values of the variable near a non-permissible value.
RF14.04 Determine if the graph of a rational function will have an asymptote or a hole for a non-permissible value.
RF14.05 Match a set of rational functions to their graphs, and explain the reasoning.
RF14.06 Describe the relationship between the roots of a rational equation and the *x*-intercepts of the graph of the corresponding rational function.
RF14.07 Determine, graphically, an approximate solution of a rational equation.