Mathematics 10 — Unit 3: Algebra and Number

Students will be expected to develop algebraic reasoning and number sense.

AN01 Students will be expected to demonstrate an understanding of factors of whole numbers by determining the prime factors, greatest common factor, least common multiple, square root, and cube root.

AN04 Students will be expected to demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials, and trinomials), concretely, pictorially, and symbolically.

AN05 Students will be expected to demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially, and symbolically.

SCO AN01 Students will be expected to demonstrate an understanding of factors of whole numbers by determining the prime factors, greatest common factor, least common multiple, square root, and cube root. [C, ME, R]			
[**C**] Communication	[**PS**] Problem Solving	[**CN**] Connections	[**ME**] Mental Mathematics and Estimation
[**T**] Technology	[**V**] Visualization	[**R**] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

AN01.01 Determine the prime factors of a whole number.
AN01.02 Explain why the numbers 0 and 1 have no prime factors.
AN01.03 Determine, using a variety of strategies, the greatest common factor or least common multiple of a set of whole numbers, and explain the process.
AN01.04 Determine, concretely, whether a given whole number is a perfect square, a perfect cube, or neither.
AN01.05 Determine, using a variety of strategies, the square root of a perfect square, and explain the process.
AN01.06 Determine, using a variety of strategies, the cube root of a perfect cube, and explain the process.
AN01.07 Solve problems that involve prime factors, greatest common factors, least common multiples, square roots, or cube roots.

Scope and Sequence

Mathematics 8	Mathematics 10	Grade 11 Mathematics Courses
N1 Students will be expected to demonstrate an understanding of perfect squares and square roots, concretely, pictorially, and symbolically (limited to whole numbers). **Mathematics 9** **N05** Students will be expected to determine the square root of positive rational numbers that are perfect squares.	**AN01** Students will be expected to demonstrate an understanding of factors of whole numbers by determining the prime factors, greatest common factor, least common multiple, square root, and cube root.	**RF02** Students will be expected to Students will be expected to demonstrate an understanding of the characteristics of quadratic functions, including vertex, intercepts, domain and range, and axis of symmetry. (M11)* **RF01** Students will be expected to factor polynomial expressions of the following form where *a*, *b*, and *c* are rational numbers. ▪ $ax^2 + bx + c, a \neq 0$ ▪ $a^2x^2 - b^2y^2, a \neq 0, b \neq 0$ ▪ $a(f(x))^2 + b(f(x)) + c, a \neq 0$ ▪ $a^2(f(x))^2 - b^2(g(y))^2, a \neq 0, b \neq 0$ (PC11)** **RF05** Students will be expected to solve problems that involve quadratic equations. (PC11)**

* M11—Mathematics 11 ** PC11—Pre-calculus 11

Background

Students in Mathematics 8 investigated square roots of whole numbers up to $\sqrt{144}$, including perfect squares and estimates of non-perfect squares. They would have explored these relationships concretely, pictorially, and symbolically for whole numbers.

In Mathematics 9, the study of square roots was extended to finding the square root of positive rational numbers that are perfect squares—including whole numbers, fractions, and decimals—using perfect squares as benchmarks to help students estimate.

The Mathematics 10 curriculum focuses on the factoring of whole numbers using a variety of strategies. This SCO serves as an introduction to factoring, greatest common factors, and least common multiples. Students will use a variety of strategies, including factoring, to determine the roots of perfect squares and perfect cubes.

Factors are numbers that are multiplied to get a product. For example, the factors of 12 are 1, 2, 3, 4, 6, 12. To factor a number means to write (express) the number as a product of its factors. To factor 12, a student may write 1 × 12, 2 × 6 (6 × 2), 3 × 4 (4 × 3), 2 × 2 × 3. To fully factor a number or to give the prime factorization of a number is to write the number as a product of prime factors. The prime factorization of 12 is 2 x 2 x 3. A review of prime numbers, composite numbers, and prime factorization may be necessary before students are introduced to prime factors.

The numbers 0 and 1 have no prime factors. When 1 is divided by a prime number, the answer is never a whole number; therefore, 1 has no prime factors. Zero is divisible by all prime numbers ($0 \div 2 = 0$, $0 \div 3 = 0$, etc.). This would appear to indicate that zero has an infinite number of prime factors. However, if 2 is a factor of 0, then so is the number zero. Since division by 0 is undefined, 0 has no prime factors.

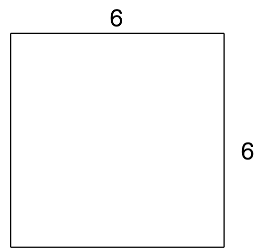
As an example of prime factorization, 24 can be expressed as a product of its prime factors: $24 = 2 \times 2 \times 2 \times 3$, or $24 = 2^3 \times 3$. To avoid confusion with the variable *x*, students should also be comfortable using a dot to indicate multiplication, as in $24 = 2 \bullet 2 \bullet 2 \bullet 3$.

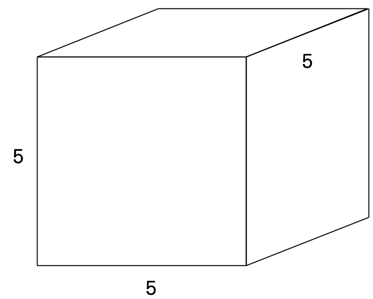
The product of two numbers of equal value is the square of those numbers. If the factors are whole numbers, the product is a perfect square. Conversely, two factors of equal value are the square roots of the square. For example, 25 is the square of 5 expressed symbolically as $5^2 = 25$ and the principle square root of 25 is 5 expressed symbolically as $\sqrt{25} = 5$.

Note: Since ($-5^2 = 25$) then there are actually two solutions for $x^2 = 25$. If $x^2 = 25$, then $x = \pm\sqrt{25} = \pm 5$.

Likewise, the product of three numbers of equal value is the cube of those numbers. If the factors are whole numbers, the product is a perfect cube. Conversely, three factors of equal value are the cube roots of the cube. For example, 27 is the cube of 3 expressed symbolically as $3^3 = 27$, and the cube root of 27 is 3 expressed symbolically as $\sqrt[3]{27} = 3$.

A perfect square can be represented by the area of a square. The length (measure) of each side of the square is the square root of the area. A perfect cube can be represented by the volume of a cube. The length (measure) of each side of the cube is the cube root of the volume.





Area = 36 square units
$\sqrt{36} = 6$
36 is a perfect square
6 is its square root

Volume = 125 cubic units
$\sqrt[3]{125} = 5$
125 is a perfect cube
5 is its cube root

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- Is 8 the square root of 16? Why or why not?
- Without using the square root function on a calculator determine the value of $\sqrt{196}$.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- Determine the first 10 prime numbers and explain your strategy for finding these numbers and for testing that they are prime.

- Draw a factor tree for 10, 60, and 120 to determine the prime factors.
  (For enrichment, have students work with larger numbers and encourage them to express the prime factors in simplified form. For example, the prime factors of 3300 are $2 \bullet 2 \bullet 3 \bullet 5 \bullet 5 \bullet 11$. These could be written as $2^2 \bullet 3 \bullet 5^2 \bullet 11$.)

- Explain the difference between listing the factors of a number and listing the multiples of a number.

- Play the following game.
  – Organize yourselves in pairs. Each of you will draw two playing cards (take out the face cards) to create a two-digit number. Player A determines the prime factors of the number and then adds up the prime factors to determine their score. Player B will continue the same procedure. The first player to reach 50 points wins.

- Play the following game.
  – Divide the class into two groups. Each team will appoint a spokesperson. Using a 10 × 10 grid as shown below, Team A will pick a number. (See Appendix A.10 for a copyable version of this hundred chart.) That number is then circled on the grid and cannot be chosen again by either team. Team A will be awarded the number of points equivalent to the number they picked. Team B will be awarded a point-value equivalent to the sum of all that number's factors, as identified by Team B and excluding the number itself, that have not already been counted. The numbers they are awarded are then circled on the grid and cannot be chosen again by either team. For example, if Team A picked 50, the number 50 would be circled on the grid and Team A would get 50 points and Team B would, ideally, identify the numbers 1, 2, 25, 5, and 10. If they identified all these numbers, none of which had already been selected, Team B would be awarded 1 + 2 + 25 + 5 + 10 or 43 points. In this case, Team A gained 7 points in that move. As the game progresses and more of the numbers have been claimed, strategies become more complex. The team with the most points when all numbers have been selected wins.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- Express each of the following numbers as a product of its prime factors.
  (a) 12
  (b) 28
  (c) 63

- Determine the greatest common factor (GCF) of each pair of numbers.
  (a) 15, 20
  (b) 16, 24
  (c) 28, 42

- In Tuesday's class, Erin has been practising determining the GCF for two integers. Her friend Anja asks her if she can determine the GCF of each pair of monomials. Erin decides to give it a try. Explain how she might find the GCF for each of the following pairs.
  (a) $4a$, $6a$
  (b) $2x^2$, $3x$
  (c) $12abc$, $3abc$
  (d) $9mn^2$, $8mn$
  (e) $6x^2y^2$, $9xy$

- Write 729 as a product of prime factors. Determine whether it is a perfect square and/or a perfect cube through grouping of the prime factors.
  (**Note:** $729 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$. The prime factors can be grouped as $(3 \cdot 3 \cdot 3)(3 \cdot 3 \cdot 3)$ to give the square of 27, or they can be grouped as $(3 \cdot 3)(3 \cdot 3)(3 \cdot 3)$ to give the cube of 9. The number 729 is both a perfect square and a perfect cube.)

- Pencils come in packages of 10. Erasers come in packages of 12. Jason wants to purchase the smallest number of pencils and erasers so that he will have exactly the same number of pencils as he has erasers. How many packages of pencils and how many packages of erasers should Jason buy?

- One trip around a track is 440 metres. One runner can complete one lap in 8 minutes, and the other runner can complete it in 6 minutes. How long will it take for both runners to arrive at their starting point together if they start at the same time and maintain their respective speeds?

- What is the length of the side of a square plot of land if its area is 1764 m^2?

- If the volume of a cube is 125 cm^3, what is the length of each side?

- An aquarium that is shaped like a cube has a volume of 216 m^3. It has glass on the bottom and four sides, but it has no top. The edges are reinforced with angle iron. What is the area of glass required? What is the length of angle iron required?

- A right rectangular prism measures 9 in. × 8 in. × 24 in. What are the dimensions of a cube with the same volume?

- Determine the cube root of 3375 in a variety of ways. This could include the use of prime factorization, the use of benchmarks, and/or the use of a calculator.

- A cube has a volume of 2744 cm^3. What is the diagonal distance through the cube from one corner to the opposite corner?

- The lowest whole number that is both a perfect square and a perfect cube is the number 1. Determine the next lowest whole number that is both a perfect square and a perfect cube, and explain your strategy.

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should be used to inform instruction.

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question
- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

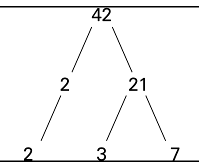
Consider the following sample instructional strategies when planning lessons.

- *Perfect Squares*
  - To review their current knowledge and ensure that students understand the relationship between a squared number and the shape of a square, use geo-boards and elastics or 1 cm graph paper and ask students if they can form a square that contains a given area. For example, ask students if they can draw a square on graph paper or construct a square on the geo-board with an area of 269 cm^2. Make sure to include area examples that are not perfect squares.

- *Perfect Cubes*
  - To review their current knowledge and ensure that students understand the relationship between a cubed number and the shape of a cube, use interlocking cubes and ask them if they can build a cube that contains a given volume. For example, ask students if they can construct a

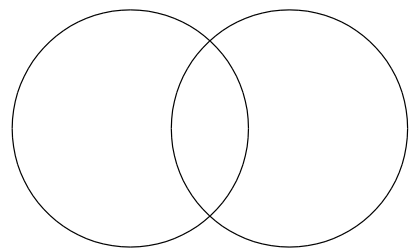
cube with the volume of 12 cm^3. Make sure to include volume examples that are not perfect cubes.

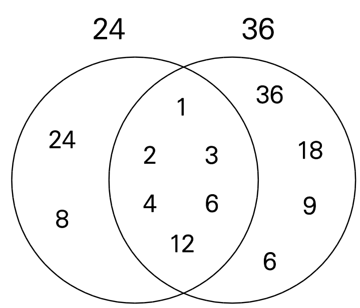
- *Factoring*
  - Students should become familiar with a variety of ways to find the prime factors of a whole number, including factor trees and repeated division by prime factors.
  - Students should be encouraged to use diagrams, manipulatives (such as counters), factor trees, and calculators to solve problems.

Factor Tree	**Manipulatives**
	Group 42 counters or interlocking cubes into 2 groups of 21 and then further group each group of 21 into 3 groups of 7. Since the groups can't be further broken down, the prime factorization would be $2 \bullet 3 \bullet 7$.

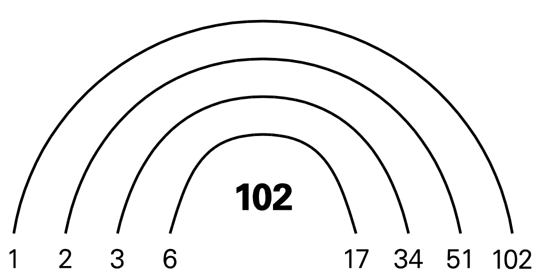
- *Greatest Common Factor (GCF)*
  - Students should explore a variety of ways to find the greatest common factor (GCF) of two or more numbers. For two numbers, one way to determine the GCF is to identify the prime factors common to both numbers, and then to take the product of these factors. For example, for 60 and 24, $60 = 2 \bullet 2 \bullet 3 \bullet 5$, and $24 = 2 \bullet 2 \bullet 2 \bullet 3$. Multiplying the factors in common gives $2 \bullet 2 \bullet 3 = 12$, so 12 is the GCF for 60 and 24.

  - A Venn diagram can be used to have students determine the greatest common factor. For students to use a Venn Diagram, they would
    - > write one number they wished to factor above the left-hand circle (e.g., students could write "24" above the circle)
    - > write the second number they wished to factor above the right-hand circle (e.g., they could write "36" above the right-hand circle)
    - > write all of the factors of 24 inside the circle labelled "24" and write all of the factors for 36 inside the circle labelled "36," placing the common factors of the two numbers inside the intersection of the two circles (In this example, 1, 2, 3, 4, 6, and 12 are common factors.)
    - > Conclude that the GCF would be the product of these common numbers (In this case it would be 12.)





  - If students have difficulty factoring a number, you could try using division facts to determine all the factors of each number and record the factors as a rainbow, or as a list of factors.



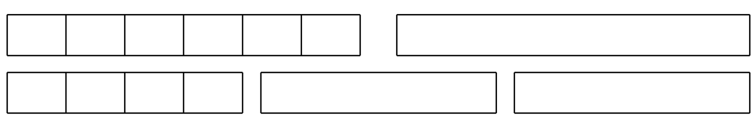
Factor 102
1×102
2×51
3×34
6×17

The factors of 102 are 1, 102, 2, 51, 3, 34, 6, and 17.

- *Least Common Multiple (LCM)*
  - Students should also be encouraged to explore a variety of ways to find the smallest multiple shared by two or more numbers (least common multiple or LCM). One method to determine the LCM is to compare the multiples of each number until a common multiple is found. For example, the multiples of 6 are 6, 12, 18, 24, 30, and the multiples of 10 are 10, 20, 30. The first multiple they have in common is 30, so this is the LCM.

  - Once students understand the concept of LCM using models and smaller numbers, have them extend this to finding the LCM of larger numbers using other methods, such as listing numbers and their multiples until the same multiple appears in all lists. For example, for the numbers 18, 20, and 30, the LCM is 180, which is the first multiple found in all three lists:
    Multiples of 18: 18, 36, 54, 72, 90, 108, 126, 144, 162, **180**, ...
    Multiples of 20: 20, 40, 60, 80, 100, 120, 140, 160, **180**, ...
    Multiples of 30: 30, 60, 90, 120, 150, **180**, ...

  - Have students use interlocking unit cubes to create multiples of two or more numbers for which they are determining the LCM. When the chains are of equal length, they will have found the LCM. For example, considering the numbers 6 and 4, two lengths of 6 is the same length as three lengths of 4. Therefore the LCM is 12. This will give students a visual image of the conceptual mechanics at work in determining the LCM.



- *Square and Cube Roots*
  - For large numbers, use prime factorization to determine square roots and cube roots. For example, after students determine the prime factors of the number, have them see if they can make equal groups of square roots or of cube roots as shown below.

    54: $2 \times 27 = 2 \times 3 \times 3 \times 3$

    $\sqrt{54} = \sqrt{2 \times 3 \times 3^2} = \sqrt{3^2}\sqrt{2 \times 3} = 3\sqrt{6}$

    54: $2 \times 27 = 2 \times 3 \times 3 \times 3$

    $\sqrt[3]{54} = \sqrt[3]{2 \times 3^3} = \sqrt[3]{3^3}\sqrt[3]{2} = 3\sqrt[3]{2}$

  - As an extension, have them do this with numbers that have 4th and 5th roots.

Suggested Models and Manipulatives

- colour tiles
- counters
- geo-boards and elastics
- interlocking cubes
- Venn diagrams

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- cube root
- greater common factor (GCF)
- least common factor (LCF)
- prime factors
- square root

Resources/Notes

Internet

- Funbrain, "Tic Tac Toe Squares" (Pearson 2013)
  www.funbrain.com/cgi-bin/ttt.cgi?A1=s&A2=17&A3=0
  Square Roots Tic Tac Toe
- Hotmath, "Number Cop." (Hotmath, Inc. 2013)
  http://hotmath.com/hotmath_help/games/numbercop/numbercop_hotmath.swf
  Prime Number Game
- "The Factor Game." (Burgis 2000)
  http://illuminations.nctm.org/tools/factor/index.html
  Factor Game
- Fun4theBrain, "Snowball Fight! - Least Common Multiple (LCM)" (Fun 4 The Brain 2013)
  http://fun4thebrain.com/beyondfacts/lcmsnowball.html
  LCM Game

Print

- *Foundations and Pre-calculus Mathematics 10* (Burglind et al., Pearson 2010)
  - Student Book
    - Chapter 3, Sections 1 and 2, pp. 132–149
    - Chapter 4, Section 1, pp. 204–206
  - Teacher Technology DVD
    - Teacher Resource
    - Blackline Masters
    - Smart Lessons
    - Animations
    - Dynamic Activities

Notes

SCO AN04 Students will be expected to demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials, and trinomials), concretely, pictorially, and symbolically. [CN, R, V]

[**C**] Communication	[**PS**] Problem Solving	[**CN**] Connections	[**ME**] Mental Mathematics and Estimation
[**T**] Technology	[**V**] Visualization	[**R**] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

(It is intended that the emphasis of this outcome be on binomial by binomial multiplication, with extension to polynomial by polynomial to establish a general pattern for multiplication.)

AN04.01 Model the multiplication of two given binomials, concretely or pictorially, and record the process symbolically.
AN04.02 Relate the multiplication of two binomial expressions to an area model.
AN04.03 Explain, using examples, the relationship between the multiplication of binomials and the multiplication of two-digit numbers.
AN04.04 Verify a polynomial product by substituting numbers for the variables.
AN04.05 Multiply two polynomials symbolically, and combine like terms in the product.
AN04.06 Generalize and explain a strategy for multiplication of polynomials.
AN04.07 Identify and explain errors in a solution for a polynomial multiplication.

Scope and Sequence

Mathematics 9	Mathematics 10	Pre-calculus 11
PR05 Students will be expected to demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2). **PR06** Students will be expected to model, record, and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially, and symbolically (limited to polynomials of degree less than or equal to 2).	**AN04** Students will be expected to demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials, and trinomials), concretely, pictorially, and symbolically.	**AN03** Students will be expected to solve problems that involve radical equations (limited to square roots). **AN04** Students will be expected to determine equivalent forms of rational expressions (limited to numerators and denominators that are monomials, binomials, or trinomials). **AN05** Students will be expected to perform operations on rational expressions (limited to numerators and denominators that are monomials, binomials, or trinomials).

Mathematics 9 (continued)	Mathematics 10 (continued)	Pre-calculus 11 (continued)
PR07 Students will be expected to model, record, and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree less than or equal to 2) by monomials, concretely, pictorially, and symbolically.		**AN06** Students will be expected to solve problems that involve rational equations (limited to numerators and denominators that are monomials, binomials, or trinomials).

Background

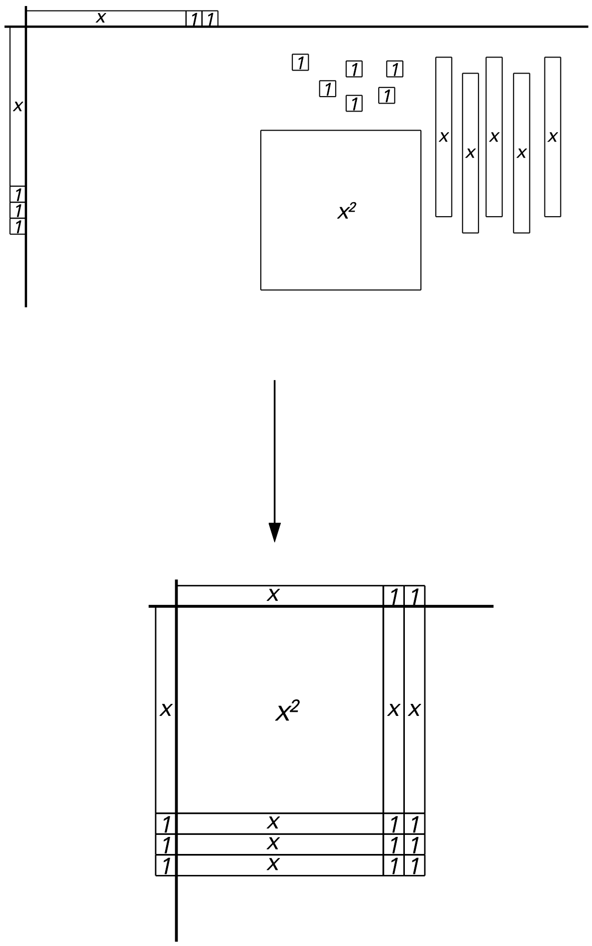
Terminology associated with polynomials was introduced in Mathematics 9. Teachers should continue to use these terms in context to allow students to integrate them into their vocabulary. This vocabulary includes **term**, **variable**, **constant**, **co-efficient**, **polynomial**, **degree of a term**, **degree of a polynomial**, **monomial**, **binomial**, and **trinomial**.

In Mathematics 9, polynomials, limited to degree 1 or 2, were added and subtracted, and polynomials were multiplied and divided—concretely, pictorially, and symbolically. Students multiplied powers with integer bases. They expressed polynomials with algebra tiles, with diagrams and pictures, and with symbols. Multiplication and division of polynomials were limited to polynomials with monomials.

In Mathematics 10, multiplication of polynomials is extended to polynomials by other polynomials. The goal of this outcome is for all students to become proficient in concrete, pictorial, and symbolic representations of the multiplication of polynomial expressions.

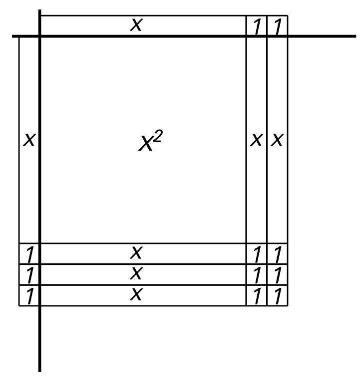
Algebra tiles and area models build an understanding of the concepts behind the symbols and **are not** considered optional for students who are able to master the more traditional symbolic models without these tools. Although there will be a greater reliance on symbolic representations as students progress into higher grades, proficiency with concrete and pictorial representations will help to build a deeper understanding of the concepts and their applications. Emphasis should be placed on the ability to switch between alternate representations, leading to a proficiency in symbolic representation.

The following illustrates the multiplication of the polynomial: $(x + 2)(x + 3)$, expressed concretely, pictorially, and symbolically. Be sure to review the values of each tile and how those tiles fit together to create the area.



	x	2
x	$x \bullet x = x^2$	$x \bullet 2 = 2x$
3	$3 \bullet x = 3x$	$(2)\,(3) = 6$

$(x + 2)\,(x + 3)$
$= x^2 + 2x + 3x + 6$
$= x^2 + 5x + 6$



$(x + 2)\,(x + 3)$
$= x^2 + 3x + 2 + 6$
$= x^2 + 5x + 6$

Concrete (with algebra tiles) ⟷ Pictorial (on paper) ⟷ Symbolic

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

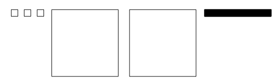
Students tasks such as the following could be completed to assist in determining students' prior knowledge.

- Is $4x + 3x^2 + 2$ equivalent to $3x^2 + 4x + 2$? Explain using a concrete model.

- Describe a real-life situation that would model the binomial expression $2x + 3$.

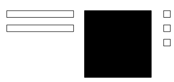
- From the following expressions, identify those that are equivalent to $-2y^2 + y - 3$.

  (a) $y - 3 - 2y^2$

  (b) 

  (c) $y^2 - 1 + 4y - 3y^2 - 3y - 2$

  (d) $-y^2 - 3$

  (e) 

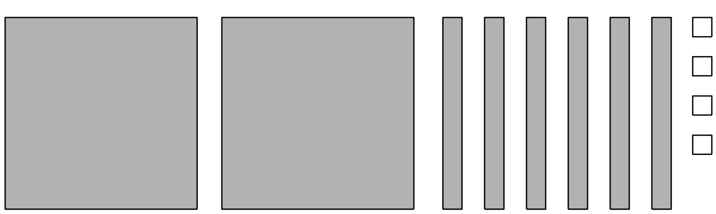
- Identify and correct the errors in the following:
  Step 1: $(2x^2 - 3x + 2) - (x^2 + x - 1)$
  Step 2: $2x^2 - 3x + 2 - x^2 + x - 1$
  Step 3: $x^2 - 2x - 1$

- Model how to determine the product or quotient for each of the following using algebra tiles or diagrams. Record the process symbolically
  (a) $3(2x - 1)$
  (b) $\dfrac{3x^2 - 6x}{-3x}$

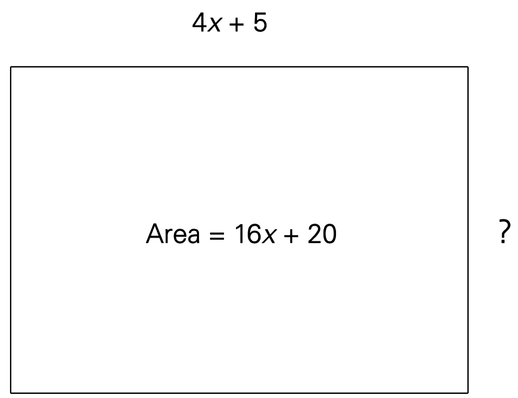
WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- Record the trinomial represented by these algebra tiles.

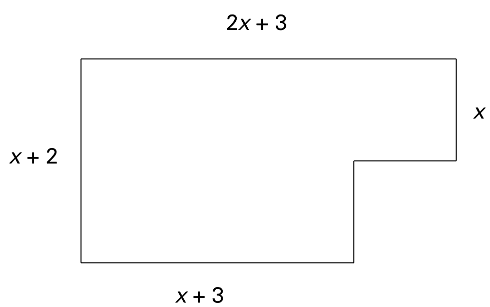


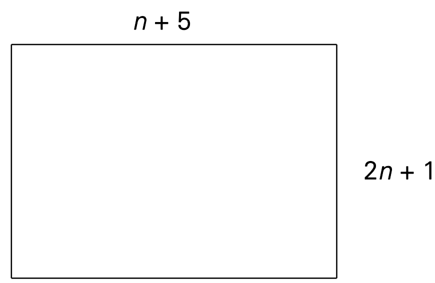
- Determine the value of the missing sides of the rectangle.



- Use the distributive property to multiply the following polynomials concretely with algebra tiles, pictorially and symbolically. Verify the product by substituting the variable with a number value.
  (a) $2(y + 3)$
  (b) $3b(4 + 2b)$
  (c) $2(x^2 + 5x + 4)$
  (d) $(x + 4)(x + 3)$
  (e) $(3x + 2)(x)$
  (f) $(5 + x)(2x + 1)$
  (g) $(6 + 2y)(1 + y)$
  (h) $(2x + 3)(x + 9)$

- Find an expression for the area of each of the following (diagrams are not drawn to scale):





- A classmate has missed the lesson on multiplying binomials. How would you explain to him the process used to determine the product of two binomials?

- Use a rectangle model to illustrate the multiplication of the following two digit numbers:
  (a) 17×14 (b) 16×11 (c) 21×12

- Use the rectangle model to multiply the following:
  (a) $(4x + 6)(3x - 5)$
  (b) $(2m + 2)(5m - 4)$
  (c) $(-3x + 2)(2x - 7)$

- Use algebra tiles to model the binomial products and record your answers.
  (a) $(x + 3)(x + 5)$
  (b) $(x + 1)(x - 4)$
  (c) $(x - 2)(x - 5)$

- Use the distributive property to find the product of the following binomials:
  (a) $(x + 5)(x + 4)$
  (b) $(d - 1)(d + 14)$
  (c) $(2 + f)(f - 7)$
  (d) $(9 - w)(5 - w)$
  (e) $(k + 10)(k - 40)$

- Complete the following:
  (a) How many terms are created when $(x + 1)(x + 2)$ is multiplied?
  (b) When the product of $(x + 1)(x + 2)$ is written as $x^2 + 2x + 1x + 2$, two terms can be combined. Will this always be the case when two binomials are multiplied together?
  (c) How many terms are created when determining the product of $(x + 1)(x^2 + 4x + 2)$? How many pairs of like terms can be combined? Will this always be the case when a binomial is multiplied by a trinomial?
  (d) How many terms are created when determining the product of $(x + 1)(x^3 + x^2 + 4x + 2)$? How many sets of like terms can be combined? Will this always be the case when a binomial is multiplied by a fourth-degree polynomial?
  (e) What pattern can you find in the above answers?

- Rima solved the following multiplication problem:
  $(3x + 4)(x + 3)$
  $= 3x + 9x + 4x + 12$
  $= 16x + 12$
  (a) Is Rima's answer correct?
  (b) Rima checked her work by substituting $x = 1$. Was this a good choice to verify the multiplication?
  (c) When verifying work, what numbers should be avoided? Why?

- Complete the following:
  (a) Why is $(x + 4)(x^2 - 3x + 2) = (x^2 - 3x + 2)(x + 4)$?
  (b) How can you check that $(b - 1)(b - 2)(b - 3) = b^3 - 6b^2 + 11b - 6$?
  (c) Find a shortcut for multiplying $(x + 5)^2$. Why does this work? Will the same type of shortcut work for multiplying $(x + 5)^3$?

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?

- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

Note: Caution should be exercised when teaching students to multiply two binomials. Instructors may have previously used the FOIL (**F**irsts, **O**utsides, **I**nsides, **L**asts) acronym. This should be avoided. Students need to understand that, while having an organized plan to manage binomial multiplication is important, the order of multiplication is *not* important. They need to note that each term from the first expression must be multiplied by each and every term from the second expression. Some students may generalize the FOIL acronym to a binomial by a trinomial and miss multiplying some of the terms.

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

- Present students with a selection of binomial multiplied by binomial expressions with the product included. Ask students to find the pattern or the relationship between the questions and their product. Have students discover strategies on their own by showing them repeated examples until they identify the pattern. Once each student has come up with a strategy, have him or her test it to make sure it applies to a multitude of examples.

- After students have completed the binomial multiplication, have them verify their solutions through substitution. For example, for $(x + 2)(x + 3) = x^2 + 5x + 6$

  Substitute $x = 3$ to verify

left side	**right side**
$(x + 2)(x + 3)$	$x^2 + 5x + 6$
$= (3 + 2)(3 + 3)$	$= 3^2 + 5(3) + 6$
$= (5)(6)$	$= 9 + 15 + 6$
$= 30$	$= 30$

- Have students explore the multiplication of various types of binomials, and ask them to develop a strategy for multiplying binomials of this type.
  (a) $(x - a)(x + a)$
  (b) $(x + a)^2$
  (c) $(x - a)^2$
  (d) $(ax - b)(ax + b)$
  (e) $(ax + b)^2$
  (f) $(ax - b)^2$

SUGGESTED MODELS AND MANIPULATIVES

- algebra tiles

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- binomial
- co-efficient
- constant
- degree of a polynomial
- degree of a term
- monomials
- multiply polynomials
- polynomial
- substitution
- term
- trinomials
- variable

Resources/Notes

Internet

- Illuminations: Resources for Teaching Math, "Algebra Tiles." (National Council of Teachers of Mathematics 2013)
  http://illuminations.nctm.org/ActivityDetail.aspx?ID=216

Print

- *Foundations and Pre-calculus Mathematics 10* (Burglind et al., Pearson 2010)
  - Student Book
    - Chapter 3, Sections 4, 5, 6, and 7, pp. 157–187
  - Teacher Technology DVD
    - Teacher Resource
    - Blackline Masters
    - Smart Lessons
    - Animations
    - Dynamic Activities

Notes

SCO AN05 Students will be expected to demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially, and symbolically. [C, CN, R, V]			
[**C**] Communication	[**PS**] Problem Solving	[**CN**] Connections	[**ME**] Mental Mathematics and Estimation
[**T**] Technology	[**V**] Visualization	[**R**] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

AN05.01 Determine the common factors in the terms of a polynomial, and express the polynomial in factored form.
AN05.02 Model the factoring of a trinomial, concretely or pictorially, and record the process symbolically.
AN05.03 Factor a polynomial that is a difference of squares, and explain why it is a special case of trinomial factoring where $b = 0$.
AN05.04 Identify and explain errors in a polynomial factorization.
AN05.05 Factor a polynomial, and verify by multiplying the factors.
AN05.06 Explain, using examples, the relationship between multiplication and factoring of polynomials.
AN05.07 Generalize and explain strategies used to factor a trinomial.
AN05.08 Express a polynomial as a product of its factors.

Scope and Sequence

Mathematics 9	Mathematics 10	Grade 11 Mathematics Courses
PR05 Students will be expected to demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2). **PR06** Students will be expected to model, record, and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially, and symbolically (limited to polynomials of degree less than or equal to 2). **PR07** Students will be expected to model, record, and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree less than or equal to 2) by monomials, concretely, pictorially, and symbolically.	**AN05** Students will be expected to demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially, and symbolically.	**RF02** Students will be expected to demonstrate an understanding of the characteristics of quadratic functions, including vertex, intercepts, domain and range, and axis of symmetry. (M11)* **AN04** Students will be expected to determine equivalent forms of rational expressions (limited to numerators and denominators that are monomials, binomials, or trinomials). (PC11)** **AN05** Students will be expected to perform operations on rational expressions (limited to numerators and denominators that are monomials, binomials, or trinomials. (PC11)**

Mathematics 9 (continued)	Mathematics 10 (continued)	Grade 11 Mathematics Courses (continued)
		AN06 Students will be expected to solve problems that involve rational equations (limited to numerators and denominators that are monomials, binomials, or trinomials. (PC11)** **RF01** Students will be expected to factor polynomial expressions of the following form where *a*, *b* and *c* are rational numbers. ▪ $ax^2 + bx + c$, $a \neq 0$ ▪ $a^2x^2 - b^2y^2$, $a \neq 0$, $b \neq 0$ ▪ $a(f(x))^2 + b(f(x)) + c$, $a \neq 0$ ▪ $a^2(f(x))^2 - b^2(g(y))^2$, $a \neq 0$, $b \neq 0$ (PC11)** **RF05** Students will be expected to solve problems that involve quadratic equations. (PC11)**

* M11—Mathematics 11
** PC11—Pre-calculus 11

Background

The concept of factoring whole numbers was introduced earlier in this unit in AN01, and the multiplication of binomials and trinomials was addressed in AN04.

For this outcome, students will develop an understanding of common factors and trinomial factoring in the form $ax^2 + bx + c$ where b and c are integers. For purposes of differentiation, $a = 1$, or $a > 1$. Trinomial factoring will include perfect squares and the difference of squares.

Exploration of common factors and trinomial factoring should begin with factoring as the inverse of multiplication; the introduction should employ algebra tiles and the area model. There should be a progression from concrete to pictorial to symbolic representations to build an understanding of the concepts behind the symbols. Mastering concrete and pictorial representations *should not be considered optional* for students who are able to master symbolic models without these tools.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

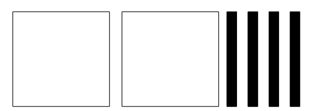
Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

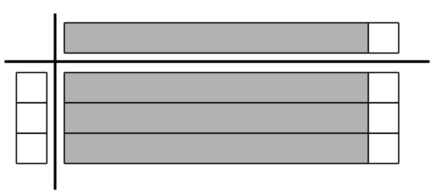
ASSESSING PRIOR KNOWLEDGE

Students tasks such as the following could be completed to assist in determining students' prior knowledge.

- Given a polynomial, identify the terms **degree**, **variables**, **coefficients**, and **constants** in each. Use a model to represent the polynomial. Use a chart (provided by the teacher) similar to the one below and insert any polynomial expression or model into that chart, thus completing the remaining cells.

Expression	# of terms	Degree	Variables	Coefficient(s)	Constant	Model
3	1	0	none	none	3	
$x^2 + 2x - 3$	3	2	x	1, 2	−3	
$4x - 2x^2 + 3$	3	2	x	4, −2	3	

- Explain how the model shown below can be used to determine the answer to $\dfrac{3x+3}{3}$.



WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) for either assessment for learning or assessment of learning.

- Fill in the blanks to make a true statement:
  (a) $12x + 18y = (\square)(2x + 3y)$
  (b) $3x^2 - 5x = (\square)(3x - 5)$
  (c) $4ab + 3ac = (\square)(4b + 3c)$
  (d) $3y^2 + 18y = 3y(y + \square)$
  (e) $14a - 12b = 2(\square - 6b)$
  (f) $x^2 + 6x + \square = (x + \square)(x + \square)$
  (g) $x^2 + \square x + 12 = (x + \square)(x + \square)$
  (h) $x^2 - \square x + 5 = (x + \square)(x + \square)$
  (i) $x^2 - \square x - 12 = (x - \square)(x + \square)$
  (j) $2x^2 - x - 6 = (2x + \square)(x - \square)$
  (k) $\square x^2 + \square x + \square = (3x + 1)(2x + 5)$

Note: Questions (a) to (k) represent increasing levels of difficulty.

- Determine two different trinomials that both have (x + 3) as a factor. Verify your answer by substituting a specific value, such as $x = 2$, for the variable.

  Note: Repeat this activity with a variety of factors.

- Factor the following expressions. Verify your answer by substituting a specific value for the variable.
  (a) $x^2 - 9$
  (b) $12x^2 - 12x + 3$
  (c) $9x^2 - 4y^2$
  (d) $y^2 - 16$
  (e) $1 - 64t^2$
  (f) $x^2 + 6x + 9$
  (g) $15 - x - 2x^2$
  (h) $4m^2 - 25$

- Ask students to use algebra tiles to factor the following expressions and explain why some expressions cannot be factored.
  (a) $5m^2 - 35m$
  (b) $3x^4 + x^2$
  (c) $15e^2g^5 - 20e^7g^2$

- Explain why $x^2 + 3x + 4$ cannot be factored.

- How many integer values are there for k for which $x^2 + kx + 24$ is factorable?

- Given $s^2 - 3s - 10$, find and correct the mistake in the factoring below.

  Sum = –3, product = –10
  numbers are –2 and 5
  $(s - 2)(s + 5)$

- Given $2x^2 + 6x - 40$, find and correct the mistakes in the factoring below.

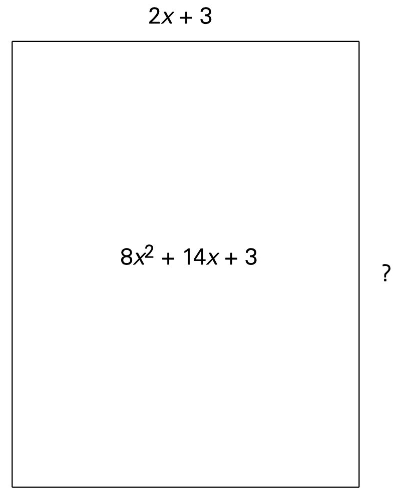
  $2x^2 + 6x - 40$
  $= 2(x^2 + 3x - 40)$
  $= 2(x - 8)(x + 5)$

- A rectangle has a product given by the expression $15x^2 - 7x - 30$. If the product represents the area of the rectangle in square centimetres, complete the following:
  (a) Find an expression for the length and width of the rectangle.
  (b) Find the length of the rectangle given that its width is 4 cm.

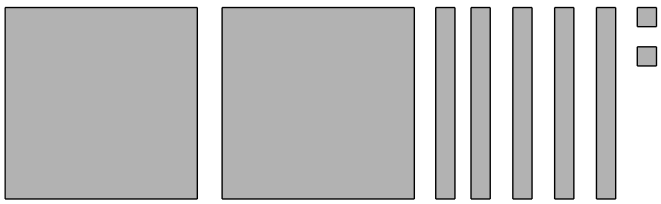
- If $6x^2 + kx - 7$ is factorable, how many different integer values are there for k? Explain.

- Give an example of a polynomial that is factored completely and an example of one that is not. Explain the differences between them.

- The area of a rectangle is represented by the product $8x^2 + 14x + 3$ square units. If the length of one side is $(2x + 3)$ units, ask students to determine the width of the other side. How did knowing one of the factors help the students determine the other factor?



- Factor the trinomial represented by these algebra tiles and record the result (shaded represents positive).

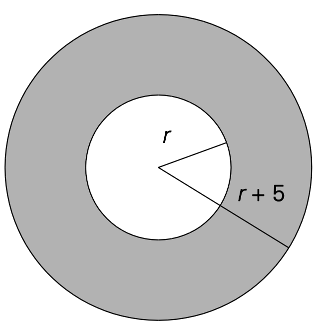


- Using the binomial factor that you have been given, find one or more trinomials, displayed around the classroom, that has your binomial as a factor. Then, find the person who has the other factor for one of "your" posted trinomials. Both of you should stand next to this trinomial.

  Note: To do this activity, create some cards with factors for the students and some larger cards with trinomials that can be posted around the room.

- In each trinomial, how is the coefficient of the linear term related to the product of the coefficient of the squared term and the constant term?
  (a) $4x^2 - 12x + 9$
  (b) $25x^2 - 20x + 4$
  (c) $16x^2 + 24x + 9$

- The diagram shows two concentric circles with radii r and $r + 5$. Write an expression for the area of the shaded region and factor the expression completely. If $r = 4$ cm, calculate the area of the shaded region to the nearest tenth of a square centimetre.



- Determine two values of n that allow the polynomial $25b^2 + nb + 49$ to be a perfect square trinomial. Use them both to factor the trinomial.

- Explain why $x^2 - 81$ can be factored but $x^2 + 81$ cannot be factored.

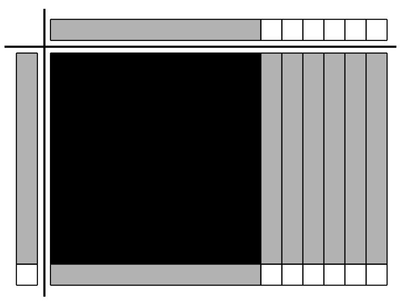
- Find a shortcut for multiplying $(x + 5)(x - 5)$. Explain why this works.

- Explain why $(x + 3)^2 \neq x^2 + 9$.

- Now that you have factored the different types of polynomials, which type of factoring do you find easier and why? Which type of factoring do you find the most difficult and why?

- *Factoring Bingo:* You have a blank bingo card and 24 polynomial expressions in factored form. Randomly write one expression in each square on your card. Allow for a free space. Show a polynomial (in expanded form) and match it to the corresponding factored form on your bingo card and cross it off. Traditional bingo alignments win—horizontal, vertical, and diagonal. Options such as four corners may also be used. (**Note:** This game can be varied such that students are provided with only the polynomial in expanded form or with a mixture of polynomials in factored and expanded form.)

- Explain how the model shown below can be used to determine the answer for $\dfrac{x^2 + 7x + 6}{x + 1}$.



FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

- To encourage deep understanding, it is recommended that there is a progression from concrete to pictorial to symbolic representations. Complexity should be gradually increased. For example, begin with binomials with positive values, extend to binomials with negative values, then mixed values, and finally, use trinomials with coefficients greater than 1 as the highest level of understanding.

- Avoid teaching factoring as an algorithm. Take the time to develop an understanding and have students discover patterns and rules rather than presenting a process.

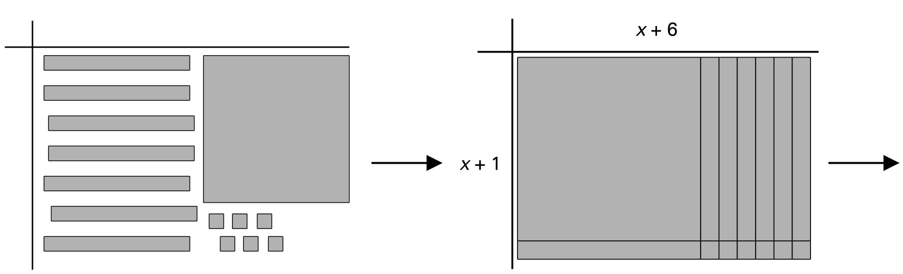
- As opposed to explicitly teaching students that factoring is the opposite of the distributive property, encourage students to come up with their own strategies through the use of repeated examples. Be sure to have students test their strategies to ensure they work with various examples. When introducing a difference of two squares, again, have students discover the rule as opposed to explicitly teaching it.

- Algebra placemats can be laminated and provided as individual workspaces for students when working with algebra tiles. Alternatively (or additionally), laminated cardstock can be used with dry-erase markers to visually represent work.

- Factoring can be introduced as the inverse of multiplication using algebra tiles and the area model. Starting with the product to be factored, algebra tiles can be arranged as a rectangle, the dimensions of which will be the factors.

- To factor a polynomial using an algebra placemat, students will assemble the tiles into a rectangular shape. The factors are the dimensions of the rectangle. When there is more than one solution, this rectangle could be arranged differently. For example, to factor $4x + 2$, arrange tiles on the placemat as a rectangle of dimensions 2 and $2x + 1$. The sides of the rectangle are its factors, so $4x + 2 = 2(2x + 1)$.

- For example, to factor $x^2 + 7x + 6$, arrange tiles on the placemat as a rectangle of dimensions $(x + 1)$ and $(x + 6)$. The sides of the rectangle are its factors; therefore



$x^2 + 7x + 6 = (x + 1)(x + 6)$

SUGGESTED MODELS AND MANIPULATIVES

- algebra tiles

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- difference of squares
- factored form
- factoring
- perfect square trinomial

Resources/Notes

Internet

- Mangahigh.com. “The Wrecks Factor” (Blue Duck Education 2013)
  www.mangahigh.com/en_us/games/wrecksfactor
  Factoring Game
- Polynomial Bingo (activity), Institute for Math and Science Education UA Fort Smith (Roberta Parks, n.d.)
  http://makingmathfun.wikispaces.com/file/view/Polynomial+Factoring+Bingo.pdf
  Factoring Bingo

Print

- *Foundations and Pre-calculus Mathematics 10* (Burglind et al., Pearson 2010)
  - Student Book
    - Chapter 3, Sections 3, 4, 5, 6, and 8, pp. 157–181, 188–197
  - Teacher Technology DVD
    - Teacher Resource
    - Blackline Masters
    - Smart Lessons
    - Animations
    - Dynamic Activities

Notes