Mathematics 10 — Unit 5: Relation and Functions

NOVA SCOTIA
NOUVELLE-ÉCOSSE

Students will be expected to develop algebraic and graphical reasoning through the study of relations.

RF01 Students will be expected to interpret and explain the relationships among data, graphs, and situations.

RF02 Students will be expected to demonstrate an understanding of relations and functions.

RF04 Students will be expected to describe and represent linear relations, using words, ordered pairs, tables of values, graphs, and equations.

RF05 Students will be expected to determine the characteristics of the graphs of linear relations, including the intercepts, slope, domain, and range.

RF07 Students will be expected to determine the equation of a linear relation to solve problems, given a graph, a point and the slope, two points, and a point and the equation of a parallel or perpendicular line.

RF08 Students will be expected to solve problems that involve the distance between two points and the midpoint of a line segment.

RF09 Students will be expected to represent a linear function, using function notation.

Specific Curriculum Outcomes

Process Standards Key

[**C**] Communication	[**PS**] Problem Solving	[**CN**] Connections	[**ME**] Mental Mathematics and Estimation
[**T**] Technology	[**V**] Visualization	[**R**] Reasoning	

RF01 Students will be expected to interpret and explain the relationships among data, graphs, and situations. [C, CN, R, T, V]

RF02 Students will be expected to demonstrate an understanding of relations and functions. [C, R, V]

RF03 Students will be expected to demonstrate an understanding of slope with respect to rise and run, line segments and lines, rate of change, parallel lines, and perpendicular lines. [PS, R, V]

RF04 Students will be expected to describe and represent linear relations, using words, ordered pairs, tables of values, graphs, and equations. [C, CN, R, V]

RF05 Students will be expected to determine the characteristics of the graphs of linear relations, including the intercepts, slope, domain, and range. [CN, PS, R, V]

RF06 Students will be expected to relate linear relations to their graphs, expressed in

- slope-intercept form ($y = mx + b$)
- general form ($Ax + By + C = 0$)
- slope-point form ($y - y_1) = m(x - x_1$)

[CN, R, T, V]

RF07 Students will be expected to determine the equation of a linear relation to solve problems, given a graph, a point and the slope, two points, and a point and the equation of a parallel or perpendicular line. [CN, PS, R, V]

RF08 Students will be expected to solve problems that involve the distance between two points and the midpoint of a line segment. [C, CN, PS, T, V]

RF09 Students will be expected to represent a linear function, using function notation. [CN, ME, V]

RF10 Students will be expected to solve problems that involve systems of linear equations in two variables, graphically and algebraically. [CN, PS, R, T, V]

SCO RF01 Students will be expected to interpret and explain the relationships among data, graphs, and situations. [C, CN, R, T, V]			
[**C**] Communication	[**PS**] Problem Solving	[**CN**] Connections	[**ME**] Mental Mathematics and Estimation
[**T**] Technology	[**V**] Visualization	[**R**] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

RF01.01 Graph, with or without technology, a set of data, and determine the restrictions on the domain and range.
RF01.02 Explain why data points should or should not be connected on the graph for a situation.
RF01.03 Describe a possible situation for a given graph.
RF01.04 Sketch a possible graph for a given situation.
RF01.05 Determine, and express in a variety of ways, the domain and range of a graph, a set of ordered pairs, or a table of values.

Scope and Sequence

Mathematics 9	**Mathematics 10**	**Grade 11 Mathematics Courses**
PR02 Students will be expected to graph linear relations, analyze the graph, and interpolate or extrapolate to solve problems.	**RF01** Students will be expected to interpret and explain the relationships among data, graphs and situations.	**RF01** Students will be expected to model and solve problems that involve systems of linear inequalities in two variables. (M11)* **RF02** Students will be expected to demonstrate an understanding of the characteristics of quadratic functions, including vertex, intercepts, domain and range, and axis of symmetry. (M11) **RF02** Students will be expected to graph and analyze absolute value functions (limited to linear and quadratic functions) to solve problems. (PC11)** **RF04** Students will be expected to analyze quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including vertex, domain and range, direction of opening, axis of symmetry, *x*- and *y*-intercepts and to solve problems. (PC11)

* M11—Mathematics 11 ** PC11—Pre-calculus 11

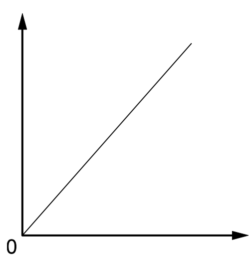
Background

Throughout the middle grades, students investigated plotting points on graphs and on a Cartesian plane, as well as interpolating and extrapolating from a given graph. In Mathematics 9, students used a table of values to create linear graphs of real-life situations and used those graphs to extrapolate and interpolate data. In Mathematics 10, they will describe a possible situation for a given graph and sketch a graph for a given situation. They will analyze a variety of graphs, including non-linear types. For example, students will be exposed to distance-time graphs and speed-time graphs.

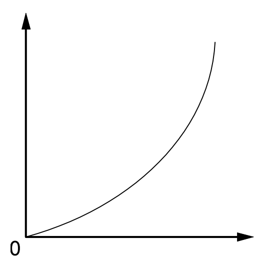
Students need to develop an awareness of the following concepts:

- A graph is an effective way to show the relationship between two quantities.
- A constant rate of change is represented graphically by a straight line, and the steepness of the line indicates the rate at which one quantity is changing in relation to the other.

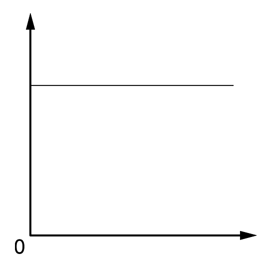
Not all relationships are represented by straight lines. It is, therefore, essential that students realize that a curve shows that the rate of change is not constant. A horizontal line means that there is no rate of change, since every value on the horizontal axis is related to the same value on the vertical axis.



Constant rate of change.



Rate of change is not constant.



No rate of change.

Students will interpret data given to them in various forms such as table of values, or real-life situations. They will be able to sketch a graph from a set of data or for a given situation, and conversely will be able to describe a situation given a graph.

Discrete Data: Data are discrete when values have a finite or limited number of possible values, such as the number of students in a class, number of tickets sold, hourly wage, or number of items that were purchased. The plotted points are not joined together.

Continuous Data: Data are continuous for an interval when there are an infinite number of possible data points within that specific interval, such as temperature or time. The graphical representation is represented by connected points.

In Mathematics 7, students studied the concept of central tendency and were introduced to range in the context of statistics (one variable). In Mathematics 9, students solved and graphed linear inequalities (9PR04). They are familiar with the inequality signs, $>$, $\geq$, $\leq$, $<$. In Mathematics 10, this outcome now introduces the concepts of the domain and range. **Note:** Range is now introduced in the context of functions (two variables). For a given relationship, students will determine restrictions on the domain (the set of all independent variables or *x*-values) and the range (the set of all dependent variables or *y*-values).

Students will now use the inequality signs to express the domain and range given the different representations of a function. Students will become comfortable expressing domain and range using words, set notation, and interval notation. This is the first time that students have been exposed to set notation and interval notation. Students will continue to use set notation and interval notation in Mathematics 11.

Assessment, Teaching, and Learning

Assessment Strategies

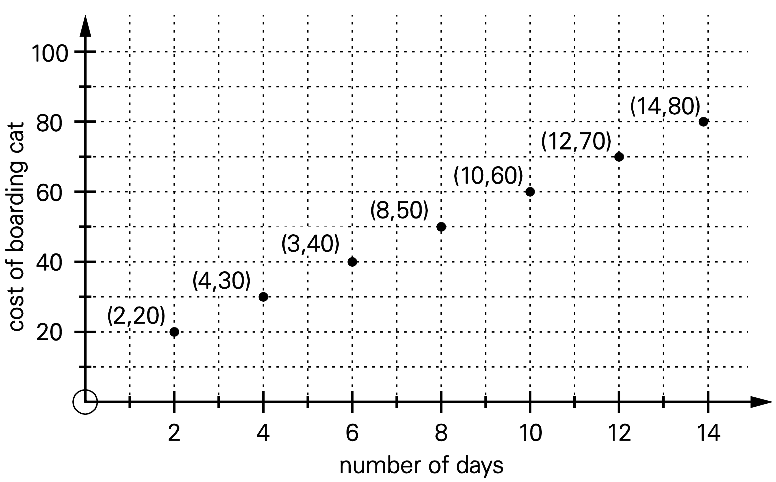
Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

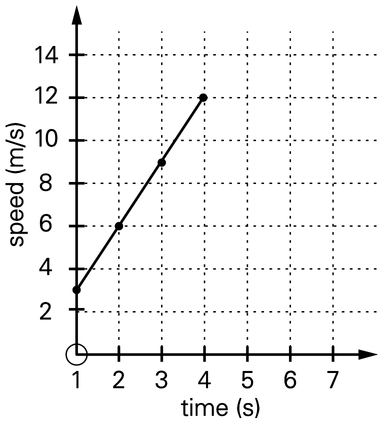
Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- Cassia decides to pay a neighbour to board her cat, Sir Fluff, while she is away visiting relatives. She represents the cost on the graph shown below.



  (a) What is the cost, per day, for Cassia to have her neighbour take care of Sir Fluff?
  (b) If she extends her visit to three weeks (21 days), how much is Cassia going to pay her neighbour for taking care of Sir Fluff?

- Given the following graph, describe the pattern. Describe a situation that could result in the graph.



- Determine if the values in the following table satisfy the corresponding inequality.

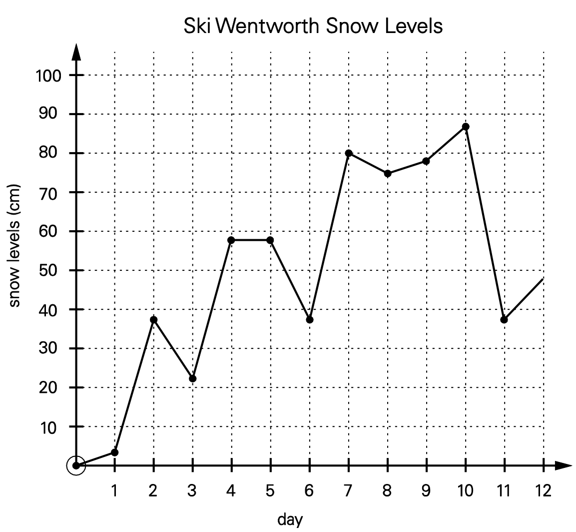
Inequality	**Values**
$x > 3$	5, 7, 9, 10
$-3x + 12 < 36$	−9, −10, −15.2
$\frac{x}{4} + 6 \geq -2$	$-10, 15, \frac{2}{3}, 7$

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

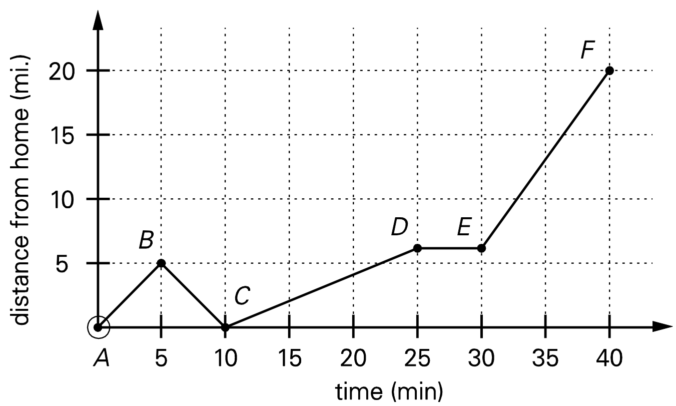
- A photographer charges a sitting fee of \$20 and \$1.50 for every photograph ordered.
  (a) Graph the situation using at least 5 points.
  (b) Explain why the points are not connected.
  (c) Is it possible that a person would be charged \$30? Justify your answer.

- Write a paragraph interpreting the graph shown below.



- For each of the following situations, do a rough sketch of the relation and include a reasonable domain and range.
  (a) When you turn on a hot water faucet, the temperature of the water depends on how many seconds the water has been running. Sketch a graph of temperature versus time.
  (b) Until the end of adolescence, your height depends on your age. Sketch a graph of height versus age.
  (c) The number of holiday cards sold depends on the time of year. Sketch a graph of the number of cards sold versus the month of the year using holidays your family or community members celebrate. (These may include days such as Easter, Ramadan, Diwali, Hannukah, Mabon, Buddha Day, and Chinese New Year. If you cannot think of any, or if you do not celebrate any holidays, use birth dates as a data set.)
  (d) You put some ice cubes in a glass and fill it with cold water on a summer day. Sketch a graph of the temperature of the water versus the time it is sitting on a table.
  (e) The time of sunset depends on the time of the year. Sketch a graph of the time of sunset versus time of the year.

- The following graph shows Duane leaving home at point *A* and going to a party at point *F*.



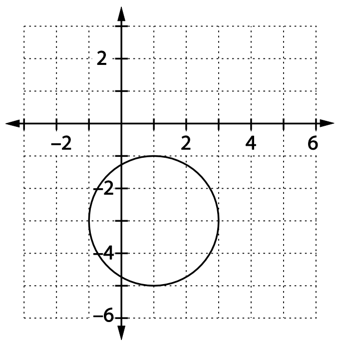
  (a) What was the slowest speed? What was the maximum speed?
  (b) What could segment *BC* represent? Explain your reasoning.
  (c) What could segment *DE* represent? Explain your reasoning.
  (d) Describe a scenario to represent the graph.

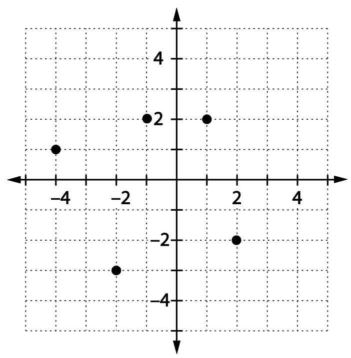
- The following data represents the sales of a song on iTunes in Nova Scotia during a seven-day period.

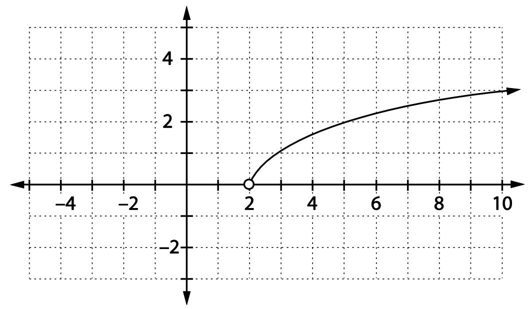
Day	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Number sold in hundreds	5	7	9	11	13	15	17

  (a) Explain why the relation is a function.
  (b) Is the data continuous or discrete? Explain why.
  (c) Draw a graph to represent the data.
  (d) Write the domain and range of the function.

- Write the domain and range for each of the following, using, where applicable,
  - words
  - a list
  - set notation
  - interval notation

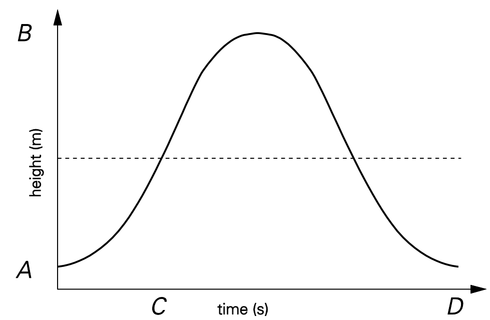






- Alexis is running in a marathon. She runs at a consistent rate. The distance, *d*, in km, that she is from the finish line in terms of time, *t*, in hours, since she began the race can be described by the equation $d = 20 - 2.5t$. If the domain of the relation is all real numbers between 0 and 8, determine the range. Explain the significance of the domain and range in terms of the context of this question.

- A Ferris wheel has a diameter of 30 m, with the centre 18 m above the ground. It makes one complete rotation every 60 seconds. The graph to the right shows the height of one of the chairs on the Ferris wheel starting at the lowest point.
  (a) What are the values of *A*, *B*, *C,* and *D*. What do they represent?
  (b) What are the domain and range of the graph?



- This table shows the population of four Nova Scotia communities in 2012.

Community	Population (2011)
Halifax	390 100
Truro	45 900
New Glasgow	35 800
Kentville	26 400

  (a) Describe the relation in words.
  (b) Represent this relation as a set of ordered pairs.
  (c) Represent this relation as an arrow diagram.

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

- Ensure students can complete the following tasks in order to demonstrate an understanding of how graphs and data are related to life:
  - Explain the situation that a graph represents.
  - Graph the data when given an explanation of a particular situation.
  - Create a graph with the axes labeled *x* and *y* and then exchange their graph with someone else and have them develop a scenario for the given graph.
  - Explain situations to their peers to show their understanding.

- Rather than using *x* and *y*, emphasis should be given to labelling the axes of the graph to represent the given situation. This can be emphasized using a distance/time graph. If distance represents the distance from home, for example, interpreting the graph would be different than if distance represents the distance from school. Encourage students to use labels that clearly describe what is being represented by the graph and to define what each variable represents.

- To help students develop their understanding of various relationships, they should use technology as well as paper and pencil. Graphing data using technology, students can use a graphing calculator or a variety of software, such as spreadsheets, Autograph (Eastmond Publishing Ltd. 2013), Smart Notebook Math Tools (SMART Technologies 2013), MimioStudio's Mimio Math Tools (2013), or Geometer's Sketchpad (Key Curriculum 2013). This would be a good opportunity to review whether or not data points should be connected in a given scenario. Arrows on graphs indicate that the graph continues; students should be encouraged to use them as needed.

- Some students have difficulty expressing a relation in words. Have each student choose one ordered pair in the relation and then write a sentence involving the two elements. For example, (dime, 0.10) could be interpreted as “A dime has a value of $0.10.” or “Ten cents is the value of one dime.”

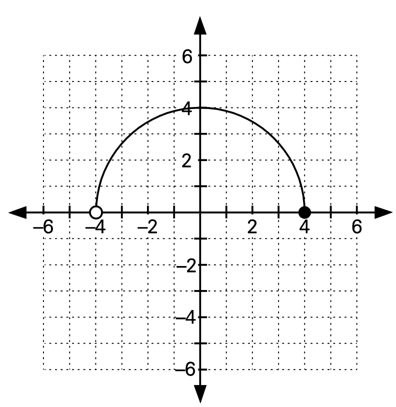
- On distance-time or speed-time graphs, some students will interpret a line segment that rises from left to right as a person travelling up a hill. Remind them that a line segment that rises from the left to the right indicates that both variables are increasing. Therefore, on a distance-time graph, a line segment like this indicates that the distance travelled is increasing as time increases. On a speed-time graph, it shows that the speed is increasing as time increases.

- It is important to provide students with a variety of problems that use both discrete and continuous data. Students should be given a variety of opportunities to develop their critical-thinking skills as they determine which numbers are reasonable in a given context, including examples of discrete and continuous data. Remind them that a list of all data points is only possible if the data is discrete.

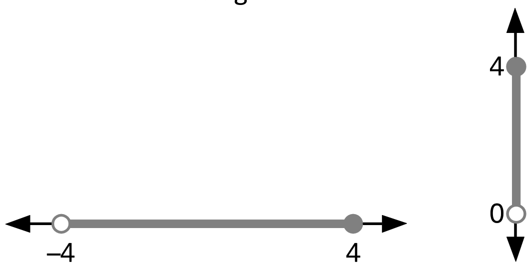
- Domain and range must be explored through data, graphs, and given situations. Students should understand which numbers are “reasonable” in any given context. For example negative values for a measurement of width would not make sense.

- As students study functions and their graphs, it is important that they understand that certain properties of a graph can provide information about a given situation. The general shape of the graph, the scale, and the starting and ending points are important features. Whether a line segment is horizontal, slopes upward to the right, or slopes downward to the right should also be addressed. Curves are used, for example, when the change in the independent and dependent variable is not constant. Students should be given an opportunity to reflect on and discuss the following:
  (a) Why do some graphs pass through the origin but other graphs do not?
  (b) What does a horizontal line represent on a speed-time graph?
  (c) What does a horizontal line represent on a distance-time graph?
  (d) What does a segment sloped upward to the right represent on a distance-time graph?
  (e) When should the data points be connected? How do we determine if the data is continuous or discrete?

- Students often have difficulty determining the domain and range of various graphs. Before students use set notation and interval notation, they may find it helpful to orally explain the domain and range and then proceed to translate their answer into words. Consider the following graph:



- Shading the relevant sections of the horizontal and vertical axis to visualize the restrictions on the independent and dependent variables can be a very powerful tool. This would be a good lead into interval notation. Interval notation uses different brackets to indicate an interval. Students often make fewer errors using this notation as it is not dependent on inequality signs.



- Students can then be exposed to set notation. A common student error occurs when the incorrect inequality sign is used. Set notation should indicate whether the data is continuous or discrete. The inequality $x < 2$, for example, may not indicate if the data is continuous or discrete unless the symbol for the number set is specified. A review of number systems may be necessary here.

- The following table illustrates the different ways of expressing the domain and range.

	Domain	**Range**
Words	Set of all real numbers between –4 and 4, not including –4 but including 4	Set of all real numbers between 0 and 4, inclusive
Set Notation	$\{x \mid -4 < x \leq 4, x \in \mathbf{R}\}$ **This is usually read as** *x* such that *x* is greater than –4 and less than or equal to 4, and is a member of the Real number system (**R**)	$\{y \mid 0 \leq y \leq 4, y \in R\}$ **This is usually read as** *y* such that *y* is greater than or equal to 0 and less than or equal to 4, and is a member of the Real number system (**R**)
List	No list (continuous data)	No list (continuous data)
Interval Notation	$(-4, 4]$	$[0, 4]$
Alternate Interval Notation (used in IB)	$]-4, 4]$	$[0, 4]$

Suggested Models and Manipulatives

- graph paper

Mathematical Vocabulary

Students need to be comfortable using the following vocabulary.

- continuous data
- discrete data
- domain
- interval notation
- range
- set notation

Resources/Notes

Print

- *Foundations and Pre-calculus Mathematics 10* (Burglind et al., Pearson 2010)
  - Student Book
    - > Chapter 5, Sections 1, 2, 3, 4, and 5, pp. 254–299
  - Teacher Technology DVD
    - > Teacher Resource
    - > Blackline Masters
    - > Smart Lessons
    - > Animations
    - > Dynamic Activities

Software

- Autograph (Eastmond Publishing Ltd. 2013)
- Geometer's Sketchpad (Key Curriculum 2013)
- MimioStudio (Mimio 2013)
- Smart Notebook (SMART Technologies 2013)
- Spreadsheet software

Notes

SCO RF02 Students will be expected to demonstrate an understanding of relations and functions. [C, R, V]			
[**C**] Communication	[**PS**] Problem Solving	[**CN**] Connections	[**ME**] Mental Mathematics and Estimation
[**T**] Technology	[**V**] Visualization	[**R**] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

RF02.01 Explain, using examples, why some relations are not functions but all functions are relations.
RF02.02 Determine if a set of ordered pairs represents a function.
RF02.03 Sort a set of graphs as functions or non-functions.
RF02.04 Generalize and explain rules for determining whether graphs and sets of ordered pairs represent functions.

Scope and Sequence

Mathematics 9	Mathematics 10	Grade 11 Mathematics Courses
PR01 Students will be expected to generalize a pattern arising from a problem-solving context using linear equations and verify by substitution. **PR02** Students will be expected to graph linear relations, analyze the graph, and interpolate or extrapolate to solve problems.	**RF02** Students will be expected to demonstrate an understanding of relations and functions.	**RF02** Students will be expected to demonstrate an understanding of the characteristics of quadratic functions, including vertex, intercepts, domain and range, and axis of symmetry. (M11)* **RF02** Students will be expected to graph and analyze absolute value functions (limited to linear and quadratic functions) to solve problems. (PC11)** **RF03** Students will be expected to analyze quadratic functions of the form $y = a(x - p)^2 + q$ and determine the vertex, domain and range, direction of opening, axis of symmetry, x- and y-intercepts. (PC11) **RF04** Students will be expected to analyze quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including vertex, domain and range, direction of opening, axis of symmetry, *x*- and *y*-intercepts and to solve problems. (PC11)

* M11—Mathematics 11
** PC11—Pre-calculus 11

Background

In Mathematics 8, students examined the various ways to describe a relation (8PR01). When given a linear relation, they represented the relation using ordered pairs, tables of values, and graphs. In Mathematics 9, the focus was on writing an expression or equation given the pictorial, oral, or written form of the relation. Students graphed linear relations and used interpolation and extrapolation to solve problems. They were exposed to discrete and continuous data (9PR02). In Mathematics 10, students learn that functions are a specific type of relation. They are also introduced to the terms **domain** and **range** in the context of a graph. This is the first time the concept of a function is introduced.

As students work with patterns, tables, and graphs, they should realize that a relation can be represented in a variety of ways and that each representational form is a viable way to explore a problem. A relation can be described using the following:

- arrow diagrams
- equations
- graphs
- ordered pairs
- table of values
- word

This will be an introduction to the concept of relations and functions. Given a graph or a table of values, students should be able to determine and explain the difference between a relation and a function.

Relation: There is a relationship that exists between x (the independent variable) and y (the dependent variable) where for every value of x in a relation there is *at least* one corresponding value of y.

Function: There is a relationship that exists between x (the independent variable) and y (the dependent variable) where for every value of x there is *only one* corresponding value of y.

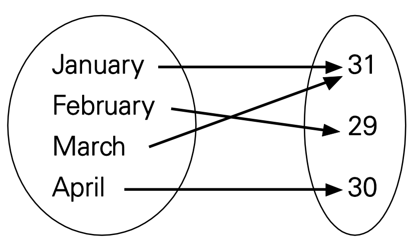
Relations can be represented in various forms as shown below.

Examples of functions:

Table
Points on a graph.

x	***y***
2	4
3	6
4	10
5	10
6	10

Mapping or Arrow Diagram
Number of days in the month.

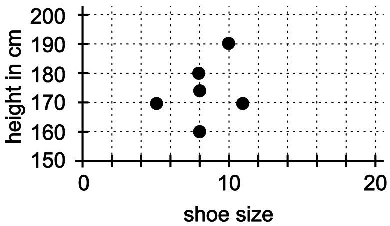


Ordered Pairs
Vehicle that has this number of wheels.

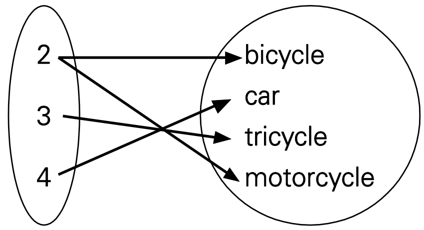
{(unicycle, 1), (bicycle, 2), (motorcycle, 2), (tricycle, 3), (car, 4)}

Examples that are not functions:

Graph
Shoe size versus height.



Mapping or Arrow Diagram
Number of wheels on a vehicle.



Ordered Pairs
Name and home of students at a workshop.

{(Marie, Ottawa), (Cheng, Toronto), (Matthew, Halifax), (Saadia, Bathurst), (Mathieu, Rivière-du-Loup)}

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

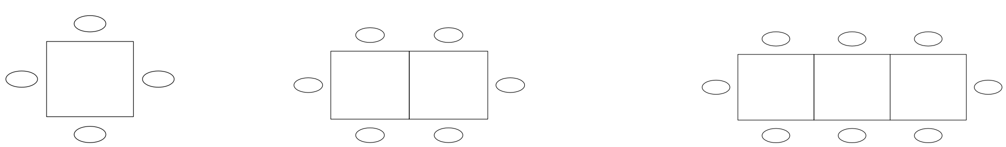
Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- When placing chairs around a table, Amad sees that he can place 4 chairs around one table; if he pushes two tables together, Amad can place 6 chairs around the two tables; if he pushes 3 tables together, he can place 8 chairs as shown in the diagram below.

  (a) Describe in words how to determine the number of chairs if you know the number of tables by completing the sentence, The number of chairs can be found by _______.
  (b) How many chairs will Amad need if 8 tables are pushed together to form a row at a banquet?
  (c) Write an equation that describes the number of chairs needed in terms of the number of tables that have been pushed together.

- Your class is planning a trip to the Shubenacadie Wildlife Park. The school will have to pay $200 for the bus plus $5 per student. Explain how you would find the cost for 42 students.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- Below are two groups of tables. The first group lists four data sets of relations that are functions and the second group four data sets of relations that are not functions. Express each of the relations as a graph, as an arrow diagram, and as a set of ordered pairs. Describe to a partner how you would determine whether each representation is a function or not.

Relations that are functions.

x	*y*
2	1
4	2
6	7
8	3
10	4

x	*y*
1	2
2	4
3	6
4	8
5	10

x	*y*
0	–2
1	4
2	–3
3	5
4	6

x	*y*
1	7
2	–1
3	2
4	6
5	4

Relations that are not functions.

x	*y*
1	2
2	4
3	6
4	8
1	10

x	*y*
2	1
4	2
2	7
8	3
10	4

x	*y*
3	–2
1	4
2	–3
3	5
4	6

x	*y*
1	7
5	–1
3	2
4	6
5	4

- Using real-life examples, create two relations to share with a partner, each with a different format (table, arrow diagram, graph, set of ordered pairs). One must be a function and one a non-function. Your partner must then explain which relation is the function and which is not.

- Determine if the following sets of ordered pairs represent functions.
  (a) (2, 4), (3, 6), (4, 6), (5, 6), (6, 12), (7, 14)
  (b) (−3, 7), (0, 10), (3, 13), (3, –5), (6, 16), (9, 19)

- Give an example of a graph or set of ordered pairs that represents a function. Use the definition of a function to support your answer.

- Give an example of a graph or set of ordered pairs that does not represent a function. Use the definition of a function to support your answer.

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?

- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

- Begin by asking students to think about situations where there may be more than one answer for a corresponding question. Examples would be, What number, when squared, gives you 25? and What is the length of songs that sell for 99 cents on iTunes?

- Then ask students to think about situations where there is only one possible answer for a question. Examples would be, What number, when doubled, gives you 22? or How much tax do you pay when you buy a $100 pair of sneakers? These type of relationships are called functions.

- Students should develop the idea that all functions are relations but not all relations are functions.

- Students will use a variety of personal strategies to determine if a relation is a function.
  - It is important that you don't tell the students directly how to determine if a relation is a function
  - Using the definition of a function, students can determine if a set of ordered pairs is a function by looking for repeated values of the independent variable.
  - As students graph relations, they should be encouraged to develop a visual representation of what is and isn't a function.
  - Once students have constructed their various strategies, they will be ready to discover that a graph is not a function when any vertical line intersects the graph at more than one point, indicating that one input value has more than one output value. The vertical line test can then be used to sort a set of graphs as functions or non-functions.
  - When using different strategies, encourage students to explain their reasoning as to why a relation does or does not represent a function.

- Tables and graphs of functions or non-functions can be photocopied and distributed among students to discuss and determine whether they are functions or non-functions.

SUGGESTED MODELS AND MANIPULATIVES

- graph paper
- measuring tapes

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- extrapolate
- function
- interpolate
- relation
- vertical line test

Resources/Notes

Internet

- Professional Learning K–12, Newfoundland and Labrador (Professional Learning NL 2013)
  www.k12pl.nl.ca
  The professional learning site provides a classroom clip of students finding the different representations of a linear relation and completing a puzzle.

Print

- *Foundations and Pre-calculus Mathematics 10* (Burglind et al., Pearson 2010)
  - Student Book
    - Chapter 5, Sections 1 and 2, pp. 256–275
    - Chapter 5, Section 5, pp. 287–297
  - Teacher Technology DVD
    - Teacher Resource
    - Blackline Masters
    - Smart Lessons
    - Animations
    - Dynamic Activities

Notes

SCO RF04 Students will be expected to describe and represent linear relations, using words, ordered pairs, tables of values, graphs, and equations. [C, CN, R, V]			
[**C**] Communication	[**PS**] Problem Solving	[**CN**] Connections	[**ME**] Mental Mathematics and Estimation
[**T**] Technology	[**V**] Visualization	[**R**] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

RF04.01 Identify independent and dependent variables in a given context.
RF04.02 Determine whether a situation represents a linear relation, and explain why or why not.
RF04.03 Determine whether a graph represents a linear relation, and explain why or why not.
RF04.04 Determine whether a table of values or a set of ordered pairs represents a linear relation, and explain why or why not.
RF04.05 Draw a graph from a set of ordered pairs within a given situation, and determine whether the relationship between the variables is linear.
RF04.06 Determine whether an equation represents a linear relation, and explain why or why not.
RF04.07 Match corresponding representations of linear relations.

Scope and Sequence

Mathematics 9	Mathematics 10	Grade 11 Mathematics Courses
PR01 Students will be expected to generalize a pattern arising from a problem-solving context using linear equations and verify by substitution. **PR02** Students will be expected to graph linear relations, analyze the graph, and interpolate or extrapolate to solve problems.	**RF04** Students will be expected to describe and represent linear relations, using words, ordered pairs, tables of values, graphs, and equations.	**RF02** Students will be expected to demonstrate an understanding of the characteristics of quadratic functions, including vertex, intercepts, domain and range, and axis of symmetry. (M11)* **RF02** Students will be expected to graph and analyze absolute value functions (limited to linear and quadratic functions) to solve problems. (PC11)**

* M11—Mathematics 11
** PC11—Pre-calculus 11

Background

Since Mathematics 7, students have solved contextual problems using linear equations. Students have also represented linear data in a variety of forms, such as tables, graphs, equations, or situations.

In Mathematics 9, students used patterns in the tables of values and graphs to recognize that a linear relation occurs when there is a constant change in the independent and dependent variable (9R01). Examples were limited to increments of 1 in the independent variable. This will now be extended to include increments other than 1.

Students previously represented relations in a variety of ways. In this outcome, students will focus on linear relations and will move interchangeably among the various representations.

In Mathematics 10, students will determine if a relationship is linear or not by using a variety of methods: common differences, graph shape, degree of the equation, and creating a table of values. They will learn to identify which variable is independent and which is dependent. Students will find the slope in each of the various forms studied and recognize that, in any linear relationship, the slope is constant, no matter which representation is being used.

Students will determine whether a relation is linear or non-linear. Using a table of values or ordered pairs, they will check to see if the changes in the independent and dependent variable are constant. Linear relations have graphs that are straight lines. Comparison of a table and its graph enables students to recognize that the constant change in the independent variable represents the horizontal change in the graph. Likewise, the constant change in the dependent variable represents the vertical change in the graph. This constant change will be introduced to students as rate of change.

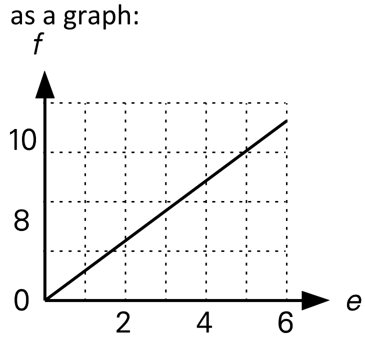
As students work with the various representations of relations, they should develop an understanding of the connections between them. Alternate representations can strengthen students' awareness of symbolic expressions and equations.

For example, exploring one relationship using a variety of representations:

- Suppose that data were gathered comparing the length of an ear compared to the length of a person's face. It was determined that the length of the face was approximately triple that of the ear. This could be expressed
  - *in words*: Three times the length of your ear, e, is equal to the length of your face, f, from chin to hairline.
  - *as an equation*: $f = 3e$
  - *as a set of ordered pairs*: (4, 12), (4.5, 13.5), (5, 15), (5.5, 16.5), (6, 18), (6.5, 19.5)
  - as a Table of Values:

Ear Length, *e* (cm)	**Face Length, *f* (cm)**
4.0	12.0
4.5	13.5
5.0	15.0
5.5	16.5
6.0	18.0
6.5	19.5

  - as a graph:



In Mathematics 9, students graphed linear relations represented as equations by creating a table of values. For a linear relation, they recognized that a constant change in the independent variable resulted in a constant change in the dependent variable. Encourage students to distinguish between linear and non-linear equations without graphing. Students can also explore whether equations are linear, for example, by observing the degree of the equation.

Students will draw the conclusion that there are a number of ways to determine whether a relation is a linear relation or a non-linear relation.

- Linear relations have graphs that are straight lines.
- In the table of values of a linear relation, values of y increase or decrease by a constant amount as values of x increase or decrease by a constant amount.
- When a linear relation is written as an equation, it will contain one or two variables, and there will be no term with a degree higher than one.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- Given the equation, $y = 2x + 5$, describe this relation in words. Make up a problem that could be solved using this equation.

- Write a linear equation to represent the pattern in the given table of values.

Time (s)	0	1	2	3	4
Height (m)	2	3.5	5	6.5	8

  (a) Describe a context for the equation.
  (b) How would you describe the rate of change in this situation?

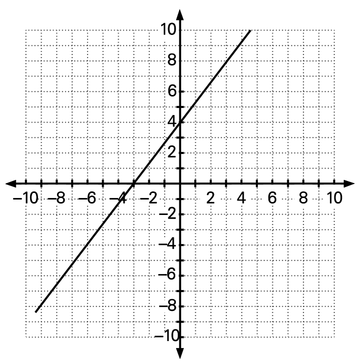
WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

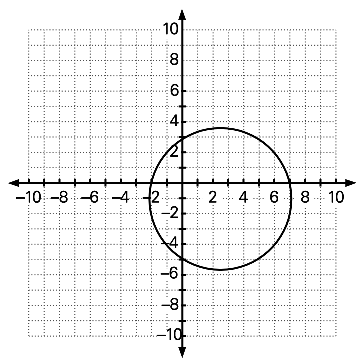
Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

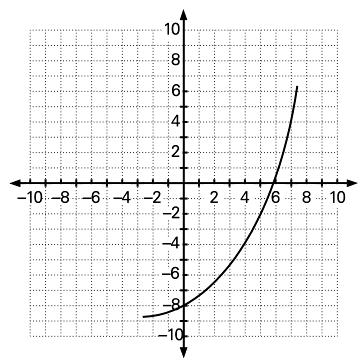
- Using the stocks page from the newspaper (either that the teacher has posted on the wall or by using the Internet to access the TSX), pick a stock company of your choice and then record and graph the value of the stock over a one-month period. Use your graph to determine
  - linear and/or non-linear sections
  - the slope of the line of best fit for the linear sections (determined by eye)

  Discuss with your classmates what factors might affect the value of the stock and relate this to current events. (Alternatively, students could look at the history of that stock over the previous month and extrapolate data to determine if they should invest in this stock by studying slopes).

  Which of the following relations are linear?







Time (months)	Height (inches)
0	22
6	23
12	25
18	27
24	32

Price ($)	Taxes ($)
10	1.50
60	9.00
110	16.50
160	24.00
210	31.50

Area (sq. ft.)	Cost ($)
100	600
150	900
200	1200
225	1350
250	1500

- Which of the following relations are linear?
  (a) {(2, 10), (4, 15), (6, 20), (8, 25), (10, 30), 12, 35)}
  (b) {(0, 1), (20, 2), (40, 4), (60, 8), (80, 1), (100, 32)}
  (c) $x^2 - 5x + 3 = y$
  (d) $x + 5 = 13$
  (e) $y = 23$
  (f) $5 + x^3 = 2x + 1$
  (g) $y = 3x + 12$

- Create a table of values for each of the following functions, graph the data, and determine if the function represents a linear relation.
  (a) $y = -4x + 7$
  (b) $d = t^2 + t - 2$
  (c) $g = 0.5t + 8$

- Determine if the following situations represent a linear relation and explain your reasoning.
  (a) A local taxi company charges a flat rate of $3.75 plus $0.75 per kilometre.
  (b) Paola is using blocks to build a tower. She starts with three blocks and adds two blocks to get each successive tower.
  (c) An investment increases in value by 10% each year.

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

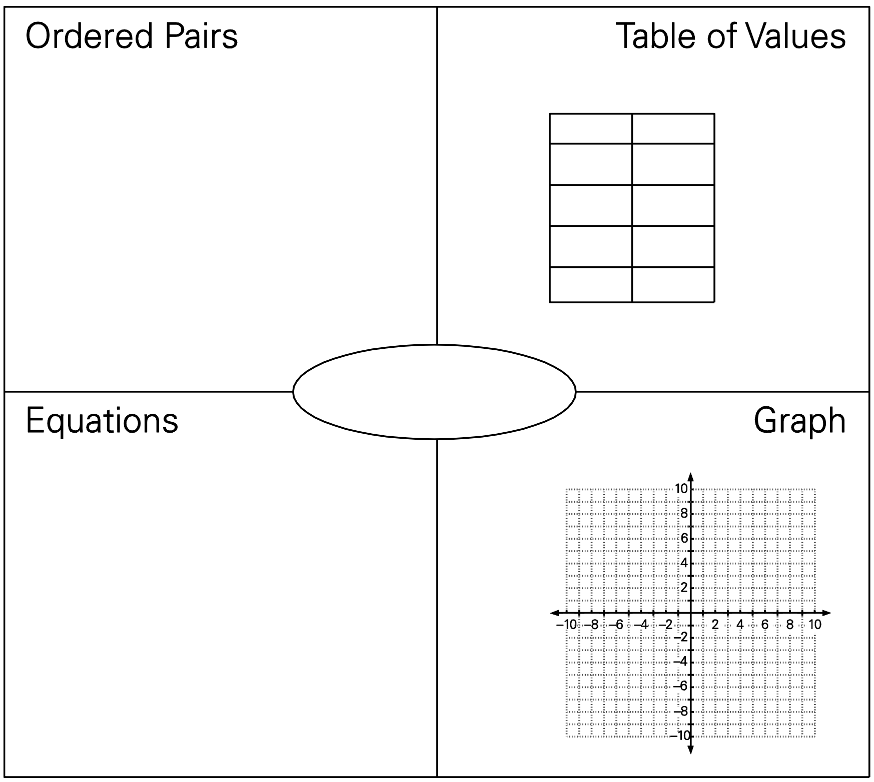
Guiding Question

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

- Provide students with data from various real-life situations and have them determine which relations are linear and which are non-linear. Have them explain what this means.

- Provide the students with one of the following: an equation, a set of ordered pairs or table of values, a graph, or a word description of a situation. Have the students generate the other three representations. For example,
  - given the equation $y = 3x - 5$, create a table of values or a set of ordered pairs, a graph, and a word description of a situation described by the equation
  - An organizer such as the one shown below could be used. (See Appendix B for a copyable version.)



- Give students a list of situations and relationships, providing time for them to work together to determine if the situation would be a linear or non-linear relation. For example, the relationship between step length and distance travelled is linear, but the relationship between length of sides and area of a square is non-linear.

- Have students compare linear equations and non-linear equations. Provide linear and non-linear equations and have students complete a table of values as well as a graph for each. For example, equations could include

  $y = x + 2$ $\quad$ $y = 6x - 7$ $\quad$ $y = x$ $\quad$ $y = x^2 + 2x + 1$ $\quad$ $y = x^2$

  Technology may also be used to illustrate the graphs.

SUGGESTED MODELS AND MANIPULATIVES

- graph paper

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- common difference
- degree
- dependent variable
- independent variable
- linear

Resources/Notes

Print

- *Foundations and Pre-calculus Mathematics 10* (Burglind et al., Pearson 2010)
  - Student Book
    - Chapter 5, Sections 2, 5, and 6, pp. 264–273, 287–310
  - Teacher Technology DVD
    - Teacher Resource
    - Blackline Masters
    - Smart Lessons
    - Animations
    - Dynamic Activities

Internet

- EducationWorld, Connecting to Math in Real Life (Petti 2013)
  www.educationworld.com/a_curr/mathchat/mathchat019.shtml
  Data sets.

Notes

SCO RF05 Students will be expected to determine the characteristics of the graphs of linear relations, including the intercepts, slope, domain, and range. [CN, PS, R, V]			
[**C**] Communication	[**PS**] Problem Solving	[**CN**] Connections	[**ME**] Mental Mathematics and Estimation
[**T**] Technology	[**V**] Visualization	[**R**] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

RF05.01 Determine the intercepts of the graph of a linear relation, and state the intercepts as values or ordered pairs.
RF05.02 Determine the slope of the graph of a linear relation.
RF05.03 Determine the domain and range of the graph of a linear relation.
RF05.04 Sketch a linear relation that has one intercept, two intercepts, or an infinite number of intercepts.
RF05.05 Identify the graph that corresponds to a given slope and y-intercept.
RF05.06 Identify the slope and *y*-intercept that correspond to a given graph.
RF05.07 Solve a contextual problem that involves intercepts, slope, domain, or range of a linear relation.

Scope and Sequence

Mathematics 9	Mathematics 10	Grade 11 Mathematics Courses
PR02 Students will be expected to graph linear relations, analyze the graph, and interpolate or extrapolate to solve problems.	**RF05** Students will be expected to determine the characteristics of the graphs of linear relations, including the intercepts, slope, domain, and range.	**RF02** Students will be expected to demonstrate an understanding of the characteristics of quadratic functions, including vertex, intercepts, domain and range, and axis of symmetry. (M11)* **RF03** Students will be expected to analyze quadratic functions of the form $y = a(x – p)^2 + q$ and determine the vertex, domain and range, direction of opening, axis of symmetry, *x*- and y-intercepts. (PC11)** **RF04** Students will be expected to analyze quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including vertex, domain and range, direction of opening, axis of symmetry, and *x*- and *y*-intercepts and to solve problems. (PC11)

Mathematics 9	Mathematics 10	Grade 11 Mathematics Courses
		RF05 Students will be expected to solve problems that involve quadratic equations. (PC11) **M01** Students will be expected to solve problems that involve the application of rates. (M11)

* M11—Mathematics 11
** PC11—Pre-calculus 11

Background

Students had experience interpolating or extrapolating information to solve problems using linear relations in Mathematics 9.

For this outcome, students will identify and express intercepts as values or ordered pairs (for example, either *y*–intercept of 2 or (0, 2) could be used to express the same intercept). Students will also explain what the intercepts represent when given the graph. Students will graph linear relations when given one intercept and slope, or when given both intercepts.

Students should work with situations wherein the rate of change is positive, negative, or zero.

Students can use graphs to determine the coordinates of the *x*- and *y*-intercepts. Ensure that students represent the horizontal and vertical intercepts as ordered pairs, (x, 0) and (0, y) respectively. Encourage students to explain what a horizontal and vertical intercept means in a contextual problem. For example, the vertical intercept may be the initial value in a context and the horizontal intercept may be the end value in a context.

Students will apply their mathematical learning about relations to determine the characteristics of a linear relation. They will revisit domain and range as it applies to linear graphs. Students will also use rate of change and vertical intercepts to identify graphs and vice versa. This work should help strengthen the connections between the various representations and the rate of change and vertical intercepts. Students should recognize, for example, that if the rate of change is positive, the line segment will slant upward to the right.

Students have now been exposed to rate of change and the vertical intercept of a linear relation. This would be a good opportunity to make a connection between a linear graph and its equation within the context of a problem. Students will not be exposed to the slope-intercept form until the next section.

Note: It is important not to introduce students to the equation of a line by talking about $y = mx + b$. This rote use of a formula tends to promote memorization rather than understanding.

A sketch of a linear relation may have one, two, or an infinite number of intercepts. Encourage students to explore the three possibilities by sketching graphs and discussing their findings. A line that lies on an axis, for example, has an infinite number of intercepts with that axis. A horizontal or vertical line that does not lie on an axis has only one intercept. An oblique line has two intercepts (one *x*-intercept and one y-intercept). When students are provided with a visual representation of the situations, students gain a better understanding of the concept.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- You have just purchased a new cell phone. The phone plan costs $10 per month and $0.10 per minute. Create a graph to represent the situation. Using the graph, estimate the cost of sending 100 text messages.

- When Nelson is driving his all-terrain vehicle (ATV) on a long distance off-road trek, he observes the amount of gas remaining in his tank over a period of time.

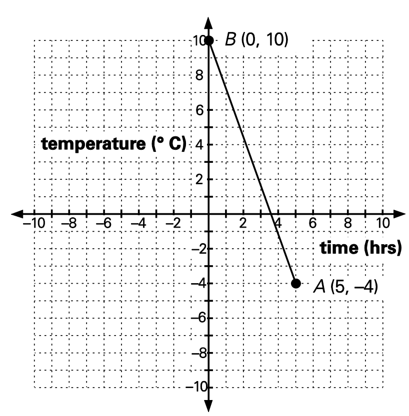
Distance travelled (km)	50	100	150	200	250
Amount of gas remaining in tank (L)	18	14	10	6	2

(a) Determine the amount of gas in the tank when Nelson began his trek.
(b) How far can Nelson travel before he runs out of gas?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- The graph to the right represents the temperature of sea water placed in a freezer. Explain the meaning of the slope, what Point A represents, what the x- and y-intercepts represent, and the domain and range.



Note: The slope represents the rate at which the temperature is decreasing. The y-intercept represents the temperature of the water when it is placed in the freezer. The x-intercept indicates the temperature after 4 hours. Point *A* represents the time and temperature at which the water begins to freeze. (Sea water freezes at –4°C.)

- Jian has a monthly cell phone plan represented by the equation $C(n) = 0.15n + 25$ where C is the total charge and n represents the number of text messages sent and received.
  (a) Explain why the equation represents a linear relation.
  (b) State the rate of change. What does it represent?

- Parvana wants to sell her car. The cost to place an advertisement in the newspaper is \$15.30. This includes three lines of text and a picture. Each additional line of text would cost \$2.50. Write an equation to represent the linear function that would represent this situation.

- Many students participate in the 5 km race for charity. Adrienne donates \$30 of her own money. She also collects \$20/km run in pledges.
  (a) Determine an appropriate domain and range for this situation, and use a table of values to graph the function.
  (b) Do either the *x*-intercept or *y*-intercept have any meaning in this situation? Explain.
  (c) Write the function as a linear equation in two variables.

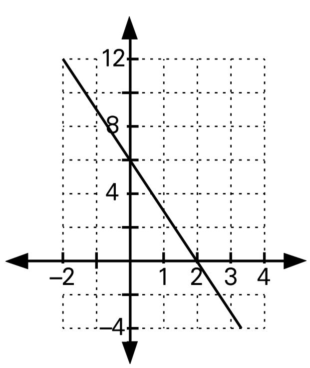
- Determine the *x*- and *y*-intercepts for each of the following:
  (a) $2x - 3y = 12$
  (b) $3x - 24 = 8y$
  (c) $5x - 2y + 12 = 0$

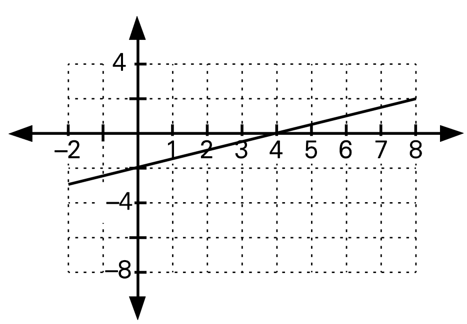
- When Marie is driving her truck on a long distance off-road trek, she observes the amount of gas remaining in her tank over a period of time.

Distance travelled (km)	50	100	150	200	250
Amount of gas remaining in tank (L)	35	30	25	20	15

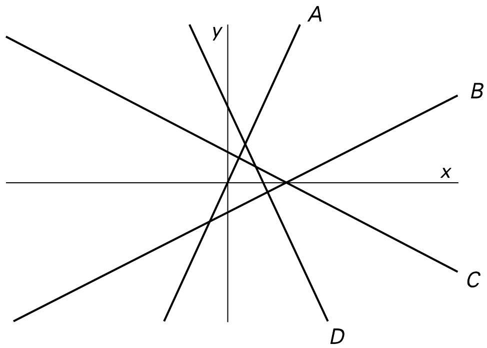
  (a) Determine the *x*-intercept for this data and explain its meaning.
  (b) Determine the *y*-intercept for this data and explain its meaning.

- State the slope and intercepts for each of the following:

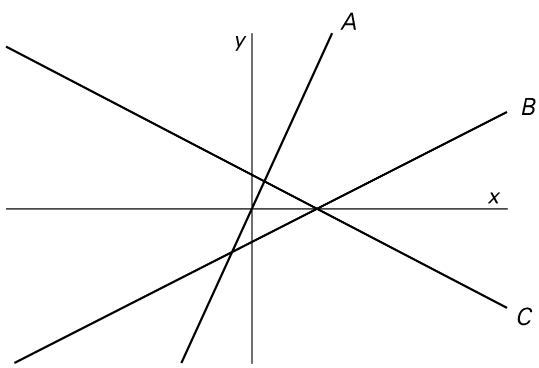




- The graphs shown below have slopes of 2, $\frac{1}{2}$, $-\frac{1}{2}$, and –2. Match the slope to each of the graphs.



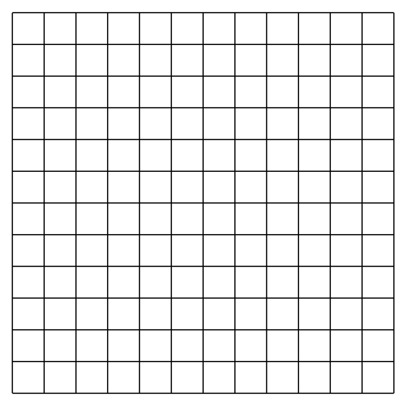
- The graphs shown below have *y*-intercepts of 2, 0, and –2. Match the *y*-intercept to each of the graphs.



- A hot tub contains 1600 L of water and is emptied at a constant rate of 20 L per minute.

(a) Complete the table of values and sketch a graph to represent the relation.
(b) Determine the *y*-intercept. What does this value represent in this context?
(c) Determine the *x*-intercept. What does this value represent in this context?
(d) Determine the slope. What does this value represent in this context?
(e) State the domain and range and explain their importance in terms of the context of situation.

Time (min)	**Volume (L)**
0	
20	
40	
60	
80	
100	



Follow-up on Assessment

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?

- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

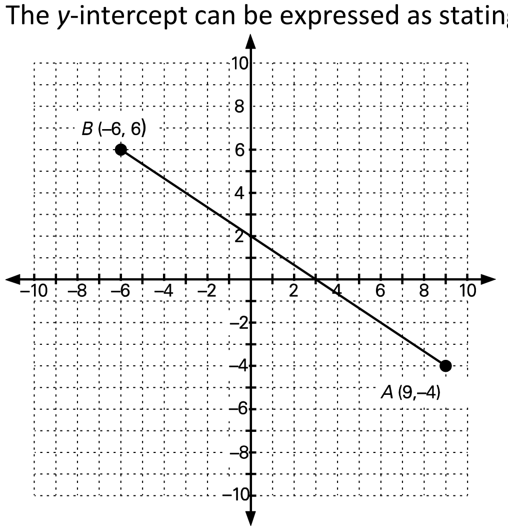
Consider the following sample instructional strategies when planning lessons.

- A quick review of the various ways to determine slope would be advisable. Students could be presented with graphs of linear equations and be required to state the slopes, intercepts, and the domain and range. When expressing domain and range that contain end points, students should express domain and range in either set and/or interval notation:

  Domain: $\{x \mid -6 \leq x \leq 9, x \in R\}$ and $[-6, 9]$

  Range: $\{y \mid -4 \leq y \leq 6, y \in R\}$ and $[-4, 6]$

  The x-intercept can be expressed as stating, x-intercept at 3 or (3, 0).
  The y-intercept can be expressed as stating, y-intercept at 2 or (0, 2).



- Students sometimes mistakenly identify the vertical intercept as that on the *x*-axis. Reinforce with students that vertical axis is the *y*-axis and horizontal axis is the *x*-axis.

- Students should be provided with examples in which characteristics of linear relations are applied to real-life situations. When students see a connection between their mathematical learning and real life, their understanding is enhanced. Furthermore, they should see examples in which both intercepts are meaningful and other examples where only one of the intercepts makes any sense in context.

SUGGESTED MODELS AND MANIPULATIVES

- graph paper

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- intercept
- *x*-intercept
- *y*-intercept

Resources/Notes

Print

- *Foundations and Pre-calculus Mathematics 10* (Burglind et al., Pearson 2010)
  - Student Book
    - Chapter 5, Section 7, pp. 311–323
    - Chapter 6, Sections 1, 4, 5, and 6, pp. 330–343, 357–387
  - Teacher Technology DVD
    - Teacher Resource
    - Blackline Masters
    - Smart Lessons
    - Animations
    - Dynamic Activities

Notes

SCO RF07 Students will be expected to determine the equation of a linear relation to solve problems, given a graph, a point and the slope, two points, and a point and the equation of a parallel or perpendicular line. [CN, PS, R, V]			
[**C**] Communication	[**PS**] Problem Solving	[**CN**] Connections	[**ME**] Mental Mathematics and Estimation
[**T**] Technology	[**V**] Visualization	[**R**] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

RF07.01 Determine the slope and y-intercept of a given linear relation from its graph, and write the equation in the form $y = mx + b$.
RF07.02 Write the equation of a linear relation, given its slope and the coordinates of a point on the line, and explain the reasoning.
RF07.03 Write the equation of a linear relation, given the coordinates of two points on the line, and explain the reasoning.
RF07.04 Write the equation of a linear relation, given the coordinates of a point on the line and the equation of a parallel or perpendicular line, and explain the reasoning.
RF07.05 Graph linear data generated from a context, and write the equation of the resulting line.
RF07.06 Determine the equation of the line of best fit from a scatter plot using technology and determine the correlation.
RF07.07 Solve a problem, using the equation of a linear relation.

Scope and Sequence

Mathematics 9	Mathematics 10	Pre-calculus 11
PR01 Students will be expected to generalize a pattern arising from a problem-solving context using linear equations and verify by substitution.	**RF07** Students will be expected to determine the equation of a linear relation to solve problems, given a graph, a point and the slope, two points, and a point and the equation of a parallel or perpendicular line.	**RF03** Students will be expected analyze quadratic functions of the form $y = a(x - p)^2 + q$ and determine the vertex, domain and range, direction of opening, axis of symmetry, and *x*- and *y*-intercepts. **RF04** Students will be expected to analyze quadratic functions of the form $y = ax^2 + bx + c$ to identify characteristics of the corresponding graph, including vertex, domain and range, direction of opening, axis of symmetry, and *x*- and *y*-intercepts, and to solve problems.

Background

Students have been exposed to patterns through interpretation of graphs of linear relations in earlier grades. From a pictorial pattern, students should be able to identify and write the pattern rule and create a table of values in order to write an expression that represents the situation. When an oral or written pattern is given, students should be able to write an expression directly from that pattern.

Students have learned to identify and match linear equations with their respective graphs and have also learned to find slope and points from graphs. This outcome introduces students to writing equations using the slope-point form of linear equations when given a graph from which they will identify a point on the line and the slope, points on the graph, and/or the slope directly.

For a scatter plot, the line of best fit will be the linear relation that most closely describes the data. The equation of this line and its correlation should be determined using technology (such as the graphing calculator or other technology). Within this context, students should discuss and explore the concept of correlation and how it varies.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

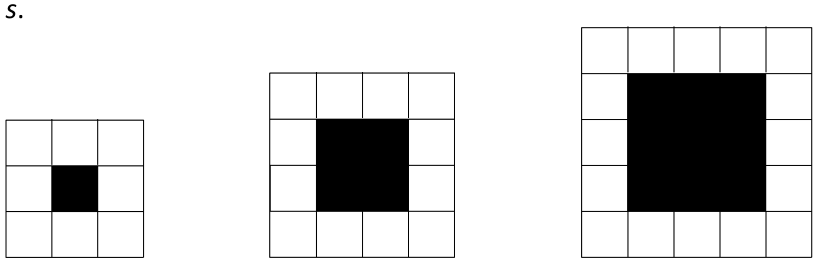
Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

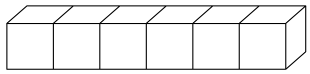
ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- Write a sentence that explains the pattern that connects the number of bricks, *b*, to the side length, *s*.
- Write an expression representing the number of bricks, *b*, around a square fire-pit with side length, *s*.



Determine an expression describing the surface area, A, obtained when a number of cubes, n, are linked together to form a train such as the one shown below.



WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

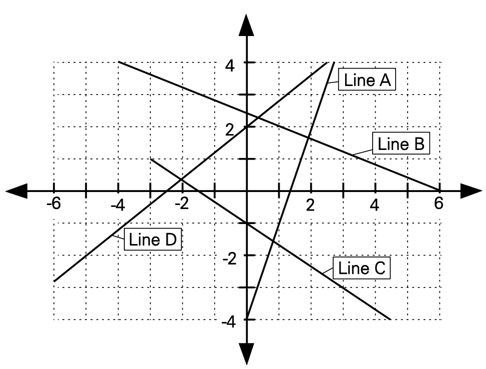
- Kuniko wanted to go on a student exchange to Mexico, but she only had \$56 dollars in her bank account. She got a job walking dogs in her neighbourhood for \$50 a week. If she saves every penny she makes, write the equation to best represent the total amount of money she will have in her bank account after a certain number of weeks. Using this equation, determine how many weeks it will take her to save \$2000.

  Extension: Kuniko's friend Carmen started with \$250 and was paid the same amount. Calculate the number of weeks it would take Carmen to save \$2000.

  (**Note:** This is a good place to discuss discrete and continuous data and whether a linear relation could represent both.)

- Create a design using a combination of horizontal, vertical, and oblique lines. Transfer this design to a coordinate grid and determine the equations of the lines needed to generate the design.

- State equations for each of the following graphs:



- Determine the equation of a line passing through (2, 6) and parallel to the line $2x + 3y = 12$

- Determine the equation of a line passing through (–1, –4) and perpendicular to $y = 3x - 1$.

- Students from Ingonish are planning a trip to Kejimkujik National Park. If 50 students go, it will cost \$1200. If 80 students go, the cost will be \$1500. Ask students to graph this data and write the equation of the resulting line. If 76 students plan on taking the trip, ask students how much it will cost.

- The cost of a taxi ride is given by $C = 3.20 + 1.75\,k$, where C is the cost in \$ and k is the number of kilometres. Ask students to find the cost to travel 12 kilometres.

- While surfing the Internet, you find a site that claims to offer “the most popular source for the cheapest DVDs anywhere.” Unfortunately, the website is not clear about how much they charge for each DVD and how much is charged for shipping and handling, but it does give you the following information:

Number of DVDs Ordered	1	2	3
Total Cost (includes S&H)	\$15	\$24	\$33

(a) Plot the data shown in the table and write an equation for the line.
(b) Your friend says that he can get a dozen DVDs from this website for \$90. Is he correct? Explain.
(c) How much would it cost to order 50 DVDs from this website?

- Trevor is not sure whether to write a linear equation using slope-intercept form or slope-point form. Depending on the information given, is one more efficient than the other? Explain your reasoning.

- The following chart shows the average prices for a loaf of bread and a bag of flour at a local convenience store for a period of three months. It seems reasonable that a higher price of flour would result in a higher cost for a loaf of bread. Determine the line of best fit to describe the price of a loaf of bread in terms of the price of flour.

Month	**Bag of Flour**	**Loaf of Bread**
1st month	\$3.50	\$2.89
2nd month	\$3.55	\$2.92
3rd month	\$3.75	\$3.03

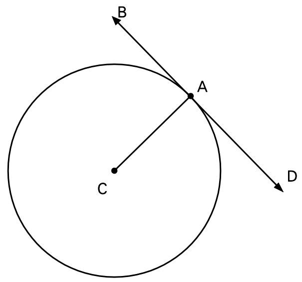
- Write the equation of the line given the following information:
  - the line has a slope of $\frac{1}{2}$ and passes through (–3, 6).
  - the line passes through (–2, 6) and (4, –8).
  - the line passes through (4, 10) and is parallel to the line $y = 2x - 4$.
  - the line passes through (2, 5) and is perpendicular to $y = -\frac{1}{4}x + 9$.

- Julie has played the game Flow Free and has logged her personal best results as shown in the chart below.

Time (minutes)	$\frac{1}{2}$	1	2	4
Number of puzzles solved	5	9	16	30

  - When determining a line of best fit, will you expect the correlation coefficient to be positive or negative? Explain.
  - Determine the line of best fit and use it to predict how many puzzles Julie could solve in 3 minutes.

- The line *BD* is tangent to the circle at point *A* (2, 4). If the centre of the circle is *C* (–1, 1), write the equation of the tangent line *BD*.

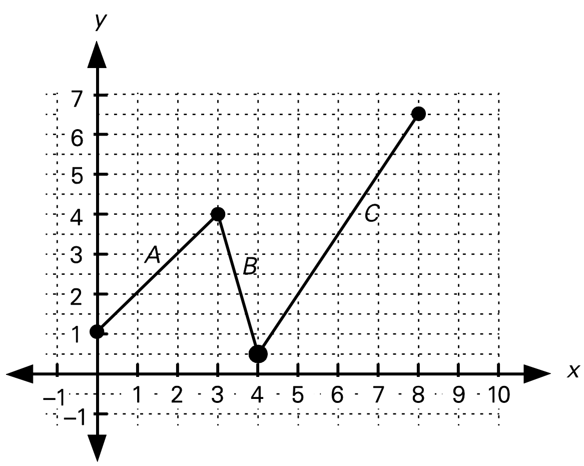


(RF7.2, RF7.3, RF7.6)

- Find the equation of a line with the same x-intercept as the line $2x + 3y + 6 = 0$ and is parallel to the line $2x - 5y - 7 = 0$.

- Find the equation of a line with the same x-intercept as the line $x - 2y = 5$ and the same y-intercept as the line $5x - 2y = 8$.

- The graph shown below is made up of linear segments *A*, *B*, and *C*. Write an equation in slope-point form for the line that contains each segment.



Follow-up on Assessment

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

- Give the students several graphs (one with a positive slope, one with a negative slope, one parallel to the *x*-axis, and one parallel to the *y*-axis). Working with a partner, have students determine the slope-intercept, the slope-point, and the general forms for each of the graphs.

- When given the slope and the coordinates of a point on the line, students will have to find the *y*-intercept in order to write the equation in slope-intercept form. Students have a choice of strategies they can use when they are asked to find the *y*-intercept. One approach involves graphing. Students can plot the given point and use the slope to find the *y*-intercept. This is an efficient method when the *y*-intercept is an integral value. Students should be exposed to the algebraic method of determining the *y*-intercept through the use of substitution. The graphing method provides students with a nice visual, but the algebraic method is more effective in finding any *y*-intercept.

- Students will progress from writing the equation of a linear relation in slope-intercept form, given the slope and a point on the line, to writing the equation given the coordinates of two points on a line.

- It is important for students to recognize that the equation written in slope-intercept form is dependent on the slope. Students will determine the slope using a graph or the slope formula. Once the slope is determined, students can then find the *y*-intercept using the graphical or algebraic methods developed earlier. Prompt students to have a discussion about how much information is necessary in order to determine the equation of a line in slope-intercept form and then demonstrate why it does not matter which point is substituted into the equation to find the *y*-intercept.

- Students should practise various ways of finding a point and the slope, depending on the information given, in order to determine the equation of a linear relation.

- To determine a point on the line you could do one of the following:
  > Select a point from the graph.
  > The student is given the *x*- and *y*-coordinates of a point on the line.
  > An *x*- or *y*-intercept is determined from a given equation by substituting either $x = 0$ (for the *y*-intercept) or $y = 0$ (for the *x*-intercept).
  > A table of values for the line is provided to the student from which any point can be selected.
- To determine the slope of the line you could do one of the following:
  > A value for slope is given directly.
  > Any two points on the line are selected and the rise and run between the points is determined to give the slope.
  > The *x*- and *y*-coordinates for two points are given, and the slope formula is used to calculate the slope.
  > A parallel line is given, and the slope is determined from this line.
  > A perpendicular line is given, and its slope determined. The negative reciprocal is the slope of the line.
  > The line is horizontal, and therefore, the slope is zero.
  > The line is vertical, and therefore, the slope is undefined.

- Depending on the information given, students can express the equation of the linear relation in slope-intercept, general, or slope-point form. Students should be encouraged to determine which form best suits a given situation.

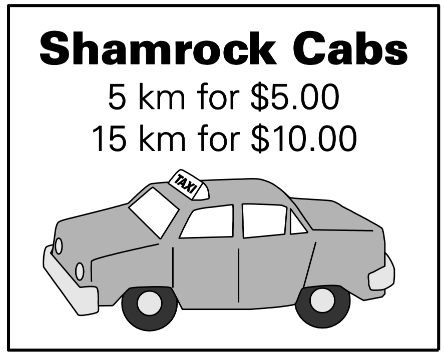
- Encourage students to verify their equation is correct by selecting a point that is on the line and then checking to see if it satisfies the equation. Students should examine various graphs, including horizontal and vertical lines.

- Students should recognize that, when given the slope and a point on the line (other than the *y*-intercept), writing equations in slope-point form requires less algebraic steps than writing equations in slope-intercept form. Encourage students to choose the more appropriate form of an equation when specific information is given. For example, the slope-intercept form indicates the slope of the line and its *y*-intercept; the slope-point form indicates the slope and its coordinates of a point on the line. Although both equations look different, they still represent the same function. Students should be exposed to examples wherein the equation of a line written in slope-point form is rearranged and expressed in slope-intercept form.

- Ask students to participate in the activity Relation Match. For this activity, students work in groups of two or more. Each group should be given a deck of cards containing 13 cards with an equation in slope-intercept form and 13 cards with a graph. The dealer shuffles all the cards and deals them out. Students match the graph and the equation. Remove the matches and place them face up on the table. Next, a player draws a card from their partner. They should locate the matching equation/graph and add it to their pairs. Students take turns drawing cards from their partner's hands. The game continues until all matches are made.

- To participate in the game Slope Relay, students should work in teams. Draw or display two coordinate grids on the white boards. Read two points and have students race to plot the points, draw the correct line, and determine its equation.

- Students should be exposed to situations in which two points are generated from a given context. Once students have generated the equation of a linear relation, they can use it to solve a variety of problems. Consider the advertisement, to the right, posted on a community bulletin board.



  Students will determine the equation that represents this linear relationship between the distance travelled and the cost of the cab ride. Encourage them to use the equation to then solve a particular problem. For example, if Paola travels 82 km, how much will she have to pay?

- Students can model a real-life situation using a linear relation in slope-point form. An understanding of what the slope represents, and making a connection between the data and the context of the problem, makes the mathematics more meaningful for students. When asked to use the function developed to solve a particular problem, students can substitute the value directly into the equation and solve for the unknown variable.

  For example, students could be given the following situation: The temperature in a sauna is rising at a constant rate of 2°F every 5 minutes. Ten minutes after Bevan enters the sauna, the temperature is 80°F. Determine when the temperature will reach 120°F. It is simplest to find the slope-point form of the equation since the slope and a point have been given.

  $$T - 80 = \frac{2}{5}(m - 10)$$
  $$120 - 80 = \frac{2}{5}(m - 10)$$
  $$40 = \frac{2}{5}(m - 10)$$
  $$100 = m - 10$$
  $$110 = m$$

  Therefore, without rearranging the equation to slope-intercept form, students are able to determine the answer to the question.

- Scatter plots are one of three topics covered in the Pearson Canada WNCP *Nova Scotia Curriculum Companion*, available as a free download in the Mathematics 10 folder on the Grade 10 Learning Commons moodle.

- When exploring scatter plots, the following site provides an interactive program that allows students to quickly plot points and explore the effect on the *r* value, as they plot points closer to and further from the line of best fit. Multiple Linear Regression activity: www.shodor.org/interactivate/activities/Regression

SUGGESTED MODELS AND MANIPULATIVES

- graph paper
- linking cubes

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- correlation (positive and negative)
- scatter plot

Resources/Notes

Internet

- Math Playground, "Save the Zogs" (MathPlayground.com 2013)
  www.mathplayground.com/SaveTheZogs/SaveTheZogs.html
  Linear equations game
- The Computational Science Reference Desk (CSERD 2013)
  www.shodor.org/interactivate/activities/Regression
  Multiple Linear Regression activity

Print

- *Foundations and Pre-calculus Mathematics 10* (Burglind et al., Pearson 2010)
  - Student Book
    - > Chapter 6, Sections 4, 5, and 6, pp. 357–387
  - Teacher Technology DVD
    - > Teacher Resource
    - > Blackline Masters
    - > Smart Lessons
    - > Animations
    - > Dynamic Activities

- *Nova Scotia Curriculum Companion* document found on Moodle (Available only to Nova Scotia teachers)

Notes

SCO RF08 Students will be expected to solve problems that involve the distance between two points and the midpoint of a line segment. [C, CN, PS, T, V]			
[**C**] Communication	[**PS**] Problem Solving	[**CN**] Connections	[**ME**] Mental Mathematics and Estimation
[**T**] Technology	[**V**] Visualization	[**R**] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

RF08.01 Determine the distance between two points on a Cartesian plane using a variety of strategies.
RF08.02 Determine the midpoint of a line segment, given the endpoints of the segment, using a variety of strategies.
RF08.03 Determine and endpoint of a line segment, given the other endpoint and the midpoint, using a variety of strategies.
RF08.04 Solve a contextual problem involving the distance between two points or midpoint of a line segment.

Scope and Sequence

Mathematics 9	Mathematics 10	Grade 11 Mathematics Courses
M01 Students will be expected to solve problems and justify the solution strategy using circle properties including the following: ▪ The perpendicular from the centre of a circle to a chord bisects the chord. ▪ The measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc. ▪ The inscribed angles subtended by the same arc are congruent. ▪ A tangent to a circle is perpendicular to the radius at the point of tangency.	**RF08** Students will be expected to solve problems that involve the distance between two points and the midpoint of a line segment.	—

Background

Students will develop both the distance formula and midpoint formula by building on their former knowledge and understanding of *xy*-coordinates on a Cartesian plane, to which they were introduced in Mathematics 6. They will also build on their understanding of the Pythagorean theorem, which they used to solve problems in Mathematics 8 and 9.

Teachers should take care to develop a clear understanding of the distance formula and the midpoint formula as distinct concepts. Experience has shown that students frequently confuse these two formulas.

Developing an understanding of the distance formula as a means of determining the distance between two points will build on students' understanding of the Pythagorean theorem.

Explorations can begin with determining the length of the special cases of horizontal or vertical lines on a Cartesian plane in which the distance is the difference between the two *x*-values (horizontal) or the two *y*-values (vertical). The distance formula is the general form of a rearrangement of the Pythagorean formula for finding the length of the hypotenuse. The formula for the distance between two points *A* and *B* is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

The development of an understanding of the midpoint formula is based on a different concept—that of the mean of two values. On a Cartesian plane, the midpoint of a line is found by determining the mean of the *x*- and the *y*-coordinates of the two end points of the line. If the endpoints are $P(x_1, y_1)$ and $Q(x_2, y_2)$, the coordinates for the midpoint will be the mean between the two *x*-values and the mean between the two *y*-values, or

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$

In Mathematics 9, students worked with circles and their various properties. They know that the perpendicular from the centre of a circle to a chord bisects the chord, and that the tangent to a circle is perpendicular to the radius at the point of tangency. Given this information, they are able to extend their use of the distance and midpoint formulas to work with circle diameters and chord lengths and to solve related problems.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions

- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- How much horizontal distance is necessary in order for straight ramp 8 m long to be built to a height of 1.5 m?

- In a computer catalogue, a computer monitor is listed as being 22 inches. This distance is the diagonal distance across the screen. If the screen measures 12 inches in height, what is the actual width of the screen to the nearest inch?

- Plot the points *A* (3, 5); *B* (–2, 4); *C* (–1, –3); and *D* (4, –2) on graph paper. What type of quadrilateral is *ABCD*?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

- The line segment *AB* is formed by joining the ordered pairs *A* (–4, 3) and *B* (2, –3).
  (a) Determine the length of the line segment *AB*.
  (b) Find the coordinates of the midpoint *AB*.

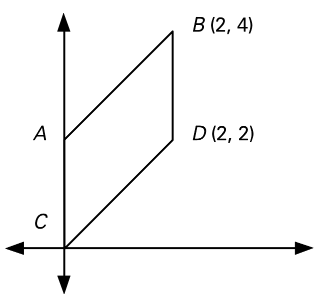
- A triangle has the vertices *A* (–3, 1), *B* (1, 7), and *C* (5, 1).
  (a) Find the perimeter.
  (b) Classify the triangle as scalene, isosceles, or equilateral.

- Show that points *P* (5, –1), *Q* (2, 8), and *R* (–2, 0) lie on a circle with a centre of *C* (2, 3).

- One endpoint of a line segment is (–4, 3). The midpoint is (–3, 6). Find the other endpoint.

- Given two points, *A* and *B*, find the point $\frac{1}{3}$ of the way from *A* to *B*.

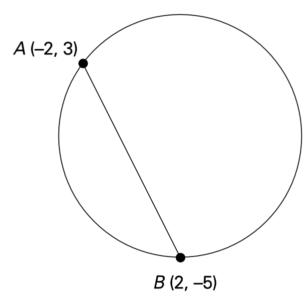
- What is the distance between *B* and the midpoint $\overline{CD}$?



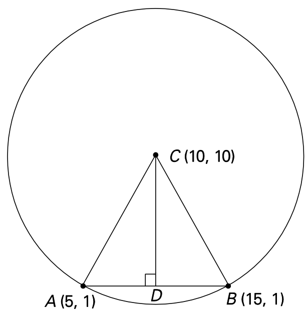
- Given that *AB* has the midpoint, *M* (2, –3) and an endpoint, *B* (–5, 1), what are the coordinates of *A*?

- Points *A* (3, 9) and *B* (12, 15) are joined to form *AB*. What is the midpoint of *AB*?

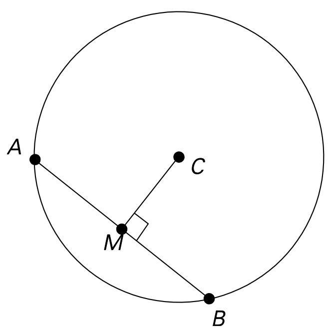
- Given the diagram shown to the right, find the equation of the perpendicular bisector of the chord.



- Given the following diagram, in which C is the centre of the circle and D is the midpoint of the chord AB, determine the length of CD.



- Given that C is the centre of the circle and AB is a chord, if AB is 24 cm and CM is 5 cm, what is the length of the diameter of the circle?



- If AB and CD are perpendicular bisectors of each other, and CD has endpoints C (5, –2) and D (–13, 12), what are the coordinates of the point of intersection of AB and CD?

FOLLOW-UP ON ASSESSMENT

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions

- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?

- How will the diverse learning needs of students be met?

SUGGESTED LEARNING TASKS

Effective instruction should consist of various strategies.

Guiding Question

- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

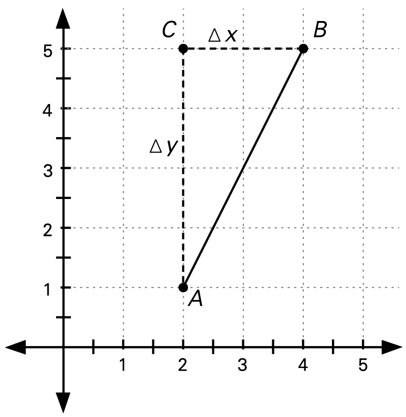
Consider the following sample instructional strategies when planning lessons.

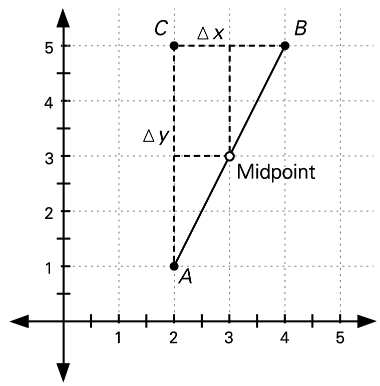
- The length and midpoint of a line are two of three topics covered in the Pearson Canada WNCP *Nova Scotia Curriculum Companion*, available in the Mathematics 10 folder on the Grade 10 Learning Commons moodle.
- Students should be given a chance to develop formulae rather than receiving them at the beginning of the process. The following activity can set the stage for this understanding.

- You are planning a trip across Canada, you have to travel through all 10 provinces and 3 territories.
  (a) Find the total distance between the start and finish that you will travel.
  (b) Split your trip up into 8 days, find the distance you will travel each day. Try to pick locations you would want to visit.
  (c) You must make a pit stop every day for gas and food. You have to stop exactly half-way every day. Find your stopping town (your midpoint).
  (See Appendix A.11 for a copyable version of this map.)

- With their knowledge of Pythagorean theorem, students should be able to determine the length of a line segment on graph paper (see below). Section 4.2, *Mathematical Modeling*, Book 3 (Barry et al. 2002), can be used as a supplemental resource for this unit.

- A similar diagram can be used to develop an understanding of the midpoint formula in a specific case and then for the general case, using the mean of *x*-coordinates and the mean of the *y*-coordinates.





$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

SUGGESTED MODELS AND MANIPULATIVES

- graph paper
- map of Canada (Appendix A.10)
- rulers

MATHEMATICAL VOCABULARY

Students need to be comfortable using the following vocabulary.

- chord
- midpoint
- perpendicular bisector
- tangent line to circle

Resources/Notes

Print

- *Nova Scotia Curriculum Companion* document found on Moodle
- *Mathematical Modeling, Book 3,* Section 4.2 (Barry et al. 2002)

Notes

SCO RF09 Students will be expected to represent a linear function, using function notation. [CN, ME, V]			
[**C**] Communication [**T**] Technology	[**PS**] Problem Solving [**V**] Visualization	[**CN**] Connections [**R**] Reasoning	[**ME**] Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have met the corresponding specific curriculum outcome.

RF09.01 Express the equation of a linear function in two variables, using function notation.
RF09.02 Express an equation given in function notation as a linear function in two variables.
RF09.03 Determine the related range value, given a domain value for a linear function.
RF09.04 Determine the related domain value, given a range value for a linear function.
RF09.05 Sketch the graph of a linear function expressed in function notation.

Scope and Sequence

Mathematics 9	Mathematics 10	Grade 11 Mathematics Courses
PR01 Students will be expected to generalize a pattern arising from a problem-solving context using linear equations and verify by substitution.	**RF09** Students will be expected to represent a linear function, using function notation.	**RF outcomes** Function notation required for all RF outcomes. (M11*, PC11)**

* M11—Mathematics 11
** PC11–Pre-calculus 11

Background

In Mathematics 8, students solved linear equations of the form $Ax + B = C$ (8PR02). They will use this skill when they are given the value of the dependent variable and are asked to solve for the independent variable.

This outcome provides students with an introduction to function notation for linear functions A function gives each input value (x) a unique corresponding output value (y). The $f(x)$ notation can be thought of as another way of representing the y-value.

Students should make a connection between the input and output values and ordered pairs as visualized on a graph. For example, the notation $f(2) = 5$ indicates that the point with coordinates (2, 5) lies on the graph of $f(x)$.

In function notation such as $f(x)$, the f is an arbitrary name, but g and h are commonly used, as in $g(x)$ and $h(x)$. The letter in the parentheses indicates the independent variable used when the function is represented by an equation. For example, writing $A(r) = \pi r^2$ indicates A is the name of the function and r is the independent variable.

Students will be expected to express an equation in two variables using function notation. For example, the equation $y = 4x – 1$ can be written as $f(x) = 4x – 1$. Conversely, they will express an equation in function notation as a linear function in two variables. For example, $h(t) = -3t + 1$ can be written as $h = -3t + 1$. Often the function name is related to a context as in this example, in which the function name is $h(t)$, for a problem that involves the height (h) of an object at a certain time (t).

Students will determine the range value given the domain value. For example, if students are given $f(x) = 5x - 7$, they should be able to determine $f(1)$. Conversely, students will determine the domain value given the range value. For example, for the function $f(x) = 4x - 1$, where $f(x) = 3$, students will solve for x.

While students will not study the composition of functions at this time, the input for a function can be another function. For example, students could be asked to determine the simplified expression for $f(x + 1)$ if $f(x) = 3x - 5$.

Throughout this unit, students have sketched graphs using a variety of methods. Students will now be exposed to graphing linear functions with an emphasis on function notation. They will graph linear functions using the following:
- The horizontal and vertical intercepts.
- Rate of change and vertical intercept.

Assessment, Teaching, and Learning

Assessment Strategies

Assessment for learning can and should happen most days as a part of instruction. Assessment of learning should also occur frequently. A variety of approaches and contexts could be used for assessing the learning of all students—as a class, in groups, and individually.

Guiding Questions
- What are the most appropriate methods and activities for assessing student learning?
- How will I align my assessment strategies with my teaching strategies?

ASSESSING PRIOR KNOWLEDGE

Student tasks such as the following could be completed to assist in determining students' prior knowledge.

- Given the equation $h = 2t + 5$, describe this relation in words. Make up a problem that could be solved using this equation.

- Given that the cost of having a yard raked can be described in terms of time (as shown in the chart below), determine an equation that will describe that cost in terms of time.

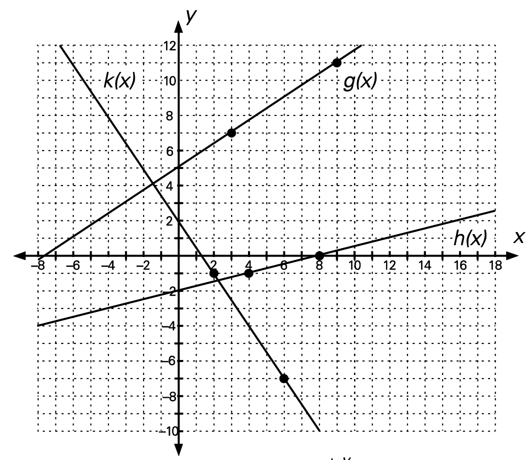
Time (hours)	2	3	3.5	4
Cost ($)	50	70	80	90

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample student tasks and activities (that can be adapted) as either assessment for learning or assessment of learning.

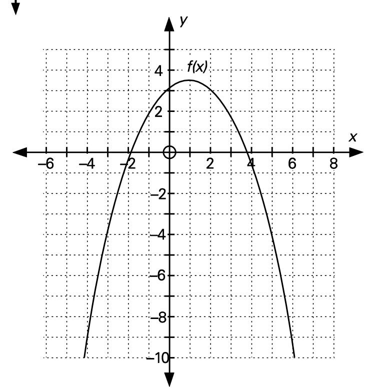
- Given $g(x) = 2x - 3$, $h(x) = 7 - 3x$, and $k(x) = x + 1$, calculate the following
  - (a) $g(15)$
  - (b) $h(28)$
  - (c) x when $k(x) = 17$
  - (d) $g(x + 2)$
  - (e) $h(5x)$
  - (f) $k(2a + 1)$
  - (g) x when $g(x) = k(x)$
  - (h) x when $h(2x) = k(3x - 2)$

- Use the graphs shown to the right in order to determine
  - (a) the values of $k(2)$, $h(4)$, and $g(0)$
  - (b) where $h(x) = 0$



- Evaluate the following:
  - (a) $d(t) = 3t + 4$, determine $d(3)$
  - (b) $f(x) = x^2 - 2x - 24$, determine $f(-2)$
  - (c) $h(t) = 4t^2 - 3t$, determine $h(1) + h(-2)$.
  - (d) $f(x) = 5x - 11$, find the value of x that makes $f(x) = 9$.
  - (e) $g(x) = -2x + 5$, find the value of x that makes $g(x) = -7$.

- Given the function $f(x)$, shown to the right, find
  - (a) $f(-2)$
  - (b) $f(2)$
  - (c) x such that $f(x) = -9$



- The perimeter of a rectangle is $P = 2l + 2w$. If it is known that the length must be 6 ft., then the perimeter is a function of the width. Write this function using function notation.

- The volume of a cylinder is given by the equation $V = \pi r^2 h$. If it is known that the radius is 5 cm, then the volume is a function of the height of the cylinder. Write this function using functional notation.

- If $C(x)$ is linear and $C(5) = 11$ and $C(12) = 25$, sketch the graph of $C(x)$.

- For a circle, $A(r) = \pi r^2$ and $C(r) = 2\pi r$, determine the values of $A(3)$ and $C(3)$ and state what these values represent.

- For a cube, $V(\text{x}) = x^3$ and $A(x) = 6x^2$, determine x when
  - (a) $V(x) = 512$
  - (b) $A(x) = 864$
  - (c) Explain what your answers represent.

Follow-up on Assessment

The evidence of learning that is gathered from student participation and work should inform instruction.

Guiding Questions
- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?
- What are some ways students can be given feedback in a timely fashion?

Planning for Instruction

Planning for a coherent instructional flow is a necessary part of an effective mathematics program.

Guiding Questions
- Does the lesson fit into my yearly/unit plan?
- How can the processes indicated for this outcome be incorporated into instruction?
- What learning opportunities and experiences should be provided to promote achievement of the outcomes and permit students to demonstrate their learning?
- What teaching strategies and resources should be used?
- How will the diverse learning needs of students be met?

Suggested Learning Tasks

Effective instruction should consist of various strategies.

Guiding Question
- How can the scope and sequence be used to determine what prior knowledge needs to be activated prior to beginning new instruction?

Consider the following sample instructional strategies when planning lessons.

- Many students seem puzzled by function notation well after it has been introduced. They often ask, Why can't we just write $y = 2x + 1$ instead of $f(x) = 2x + 1$? To motivate students to use function notation and improve understanding, try using multi-variable functions instead of single-variable functions in introducing this notation.

- You can pose the following type of question: Suppose you are texting your friend Alek and ask him for the dimensions of a bookshelf that he has offered to give you. He replies with a list of three numbers: 12 in., 20 in., 24 in. Do you know the specific height, width, and depth of the bookshelf?
  - What is missing in the above scenario is the "function definition." The function definition tells you what the function does, how many parameters (or "arguments") the function requires as inputs, what those parameters are, and in what order they occur. In this example, the following information is probably what you wanted to know: Dimension (depth, height, width).

- Later in mathematical studies, functions will often be described in terms of multiple variables. Functional notation thus provides the clearest, most concise method possible to communicate information. For example, the volume of a cylinder might be described as a function $V(r,h) = \pi r^2 h$ and you could thus concisely ask for $V(3, 4)$ when you wanted to find the volume of a cylinder with a radius of 3 and a height of 4.

- Another view of functional notation that may assist students in seeing its value as a concise way of communication is to describe three equations.
  - Line 1: $y = 2x + 3$
  - Line 2: $y = 5 – 4x$
  - Line 3: $y = 1.5x + 6$

  Next, ask a student to find the y-value when $x = 1$. They would also need to know what equation was to be used. To clarify, you would need to say, "Find the y-value in the equation $y = 5 – 4x$ when $x = 1$."

  Using functional notation, however, simplifies the problem.
  - Line 1: $f(x) = 2x + 3$
  - Line 2: $g(x) = 5 – 4x$
  - Line 3: $h(x) = 1.5x + 6$

  Asking students to find $g(1)$ requires no further clarification.

  Functional notation also has the added benefit of crossing language barriers, since $g(1) = ?$ does not depend on the understanding of any language other than mathematics.

- A virtual function machine can also be used to clarify the idea of functional notation. Alternately, you can create function machines simply. Just collect a few containers: a coffee can, a mason jar, and a box would serve the purpose. Label each of these containers with an equation, such as $C(n) = 2n + 1$, $M(n) = n + 5$ and $B(n) = n^2$. Ensure that the students are able to describe what each function machine does.
  - The coffee can would serve the function of doubling a number and adding 1.
  - The mason jar would add 5 to the number.
  - The box would square the number.

  At this point, you take a piece of paper and put a number such as 7 on three separate pieces of paper. Drop one of the pieces of paper in the coffee can and stir, shake, or agitate it. Ask the students what number will result when the 7 has had time to be transformed by the coffee can function. (Make sure you have placed the answer 15 in the coffee can prior to this lesson). Then describe how the function machine is processing the number 7 (doubling to 14 and then adding 1 to 15) and finally draw the result out of the tin. Repeat this process with the mason jar (yielding a 12) and the Box (yielding a 49).

  You may wish to plan for another input number as well. If you are able to manage the sleight of hand necessary, you can present this as a bit of magic and have some fun with functional notation. This process effectively emphasizes functional notation and provides a visual connection that students are likely to remember.

- A common student error occurs when the parentheses are mistakenly used as multiplication in function notation. For example, $f(4) = 8$ does not mean $4f = 8$. It is important for students to make the connection that this is a place holder which represents the domain value.

- Another error occurs when students are given a function such as $h(t) = 4t - 3$ where $h(t) = 18$ and are asked to determine t. Students often substitute the given value for the independent variable rather than the dependent variable. This would be a good opportunity to reiterate the purpose and meaning of function notation.

- A review of rearranging equations would be helpful since function notation requires the linear relation to be solved for the y-variable.

- It is important that to ensure that students make the connection between the input and output values and ordered pairs. For example, the notation $f(2) = 5$ indicates that the point with coordinates (2, 5) lies on the graph of $f(x)$. In this way, students visually recognize both the input and output of the function.

- At least two points are necessary to draw a line. Students will be exposed to finding the vertical intercept by evaluating $f(0)$ and finding the horizontal intercept by solving $f(x) = 0$. As an alternative, students can graph a line using the rate of change from the equation and the vertical intercept. Examples of linear functions with restricted domain and range should also be included.

Suggested Models and Manipulatives

- three empty containers

Mathematical Vocabulary

Students need to be comfortable using the following vocabulary.

- function notation
- input value
- output value

Resources/Notes

Internet

- The Computational Science Education Reference Desk, "Number Cruncher" (CSERD 2013)
  http://shodor.org/interactivate/activities/NumberCruncher
  Function Machine Virtual Manipulative

Print

- *Foundations and Pre-calculus Mathematics 10* (Burglind et al., Pearson 2010)
  - Student Book
    - Chapter 5, Section 2, pp. 264–273; Section 5, pp. 287–297; Section 7, pp. 311–323
  - Teacher Technology DVD
    - Teacher Resource
    - Blackline Masters
    - Smart Lessons
    - Animations
    - Dynamic Activities

Notes