Mathematics 7
Unit 2: Geometry: Transformations

G02, G03

SCO G02: Students will be expected to identify and plot points in the four quadrants of a Cartesian plane, using integral ordered pairs. [C, CN, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

G02.01 Label the axes of a four quadrant Cartesian plane and identify the origin.
G02.02 Identify the location of a given point in any quadrant of a Cartesian plane using an integral ordered pair.
G02.03 Plot the point corresponding to a given integral ordered pair on a Cartesian plane with units of 1, 2, 5, or 10 on its axes.
G02.04 Draw shapes and designs in a Cartesian plane using given integral ordered pairs.
G02.05 Create shapes and designs, and identify the points used to produce the shapes and designs, in any quadrant of a Cartesian plane.

Scope and Sequence

Mathematics 6	**Mathematics 7**	**Mathematics 8**
G05 Students will be expected to identify and plot points in the first quadrant of a Cartesian plane using whole number ordered pairs.	**G02** Students will be expected to identify and plot points in the four quadrants of a Cartesian plane, using integral ordered pairs.	**PR01** Students will be expected to graph and analyze two-variable linear relations. **G02** Students will be expected to demonstrate an understanding of the congruence of polygons under a transformation.

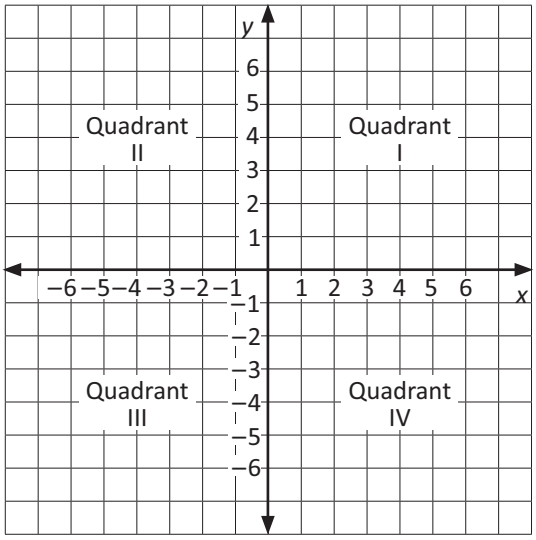
Background

In Mathematics 6 (G02), students were introduced to the Cartesian plane. They identified and plotted whole number ordered pairs, and drew shapes and designs limited to the first quadrant. Experiences with horizontal and vertical integral number lines in both Mathematics 6 and Mathematics 7 prepared students to extend plotting skills to work in all four quadrants of a Cartesian plane. Students should already be familiar with key terms such as coordinate plane, ordered pairs, origin, *x*-axis, *y*-axis, x-coordinates and y-coordinates. Continued use of appropriate terminology is important. Each of the achievement indicators associated with this outcome have previously been addressed in Mathematics 6, restricted to the first quadrant. Plotting ordered pairs accurately is an important skill for performing and describing transformations in learning outcome 7G03, and for graphing equations in Patterns and Relations.

Students should plot data points in all four quadrants. Each ordered pair of integers represents a position on a four quadrant Cartesian plane. The scale of the axes will need to be determined based on the magnitude of the coordinates. Students should be exposed to a variety of scales including those with intervals of 1, 2, 5, and 10.

The horizontal number line in the Cartesian number line is termed the *x*-axis, and the vertical number line the *y*-axis. The point at (0, 0) is called the "origin". The scale is chosen based on the numbers in the situation. The four main areas are named "quadrants". They are numbered 1 to 4 (often expressed as Roman numerals I to IV) in a counter-clockwise direction beginning with the positive coordinates.

A common error when identifying and plotting points is to reverse the order of the *x*-coordinate and the *y*-coordinate. To avoid making this mistake, students should label the *x*- and *y*-axes of a Cartesian plane.



Situations which might be modelled using 4-quadrant graphs include:

- daily high and low temperatures for different days plotted as coordinates;
- mathematical relationships (e.g., a number vs. its double) plotted as coordinates;
- locations, as blocks north, south, east, and west from the town centre plotted as coordinates.

Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to plot points on grids with different scales (e.g., intervals of ones, twos, fives, tens).
- Have students create "join-the-dots" pictures on a coordinate grid to reinforce locating coordinates. After they draw their pictures on a grid, they list the coordinates in order of connection. The list of coordinates can be given to other students who then use them to recreate the picture.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Plot the following points on a grid: A (–3, 2), B (1, 2), C (–3, –2). Determine what the coordinates of a fourth point (D) would be in order to create the square ABCD when the 4 points are connected.
- Determine an appropriate grid scale for plotting the following points: (–35, 30), (15, 30), (–20, –20), and (30, –20). Create the grid and plot the points. Explain why you chose that scale.
- Plot points A: (–2, 4) and B: (3, 4). Join the points to create line segment AB. What is the distance between A and B?
- Write the coordinates to draw a simple picture. Trade coordinates to see if your partner can follow your instructions to produce the picture you have in mind.

Planning for Instruction

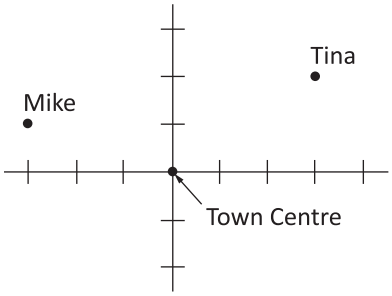
CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

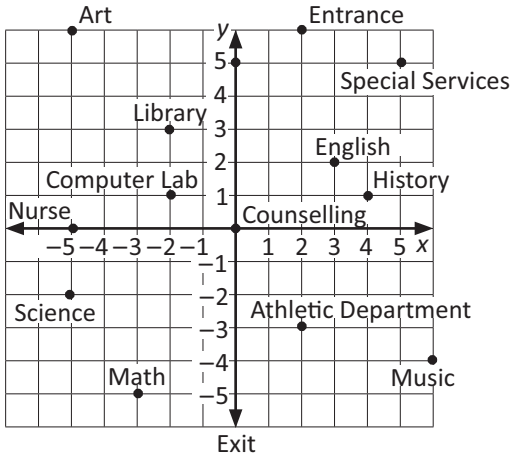
- Use four geo-boards linked together to represent the four quadrants.
- Provide students with a variety of points that may require them to change the scale on the coordinate grid. For example, (–35, 40) would require students to scale by 5 or 10 instead of a scale factor of 1.
- Create a grid on the floor where students can physically move to identified coordinates. You can integrate the Cartesian plane with the study of geography by using the coordinates on a map of the world. The equator could be represented as the *x*-axis, and 0° longitude could be represented as the *y*-axis. Using the map, ask students to determine the number of degrees (in relation to both the *x*-axis and the *y*-axis) between two cities. Coordinates can be determined with any type of map. You could, for example, use a highway map or a topological map of the area around your school or community.

SUGGESTED LEARNING TASKS

- Show a map (graph) like the one below. How many blocks north of town centre does Tina live? How many blocks east? Write Mike's location as an ordered pair.

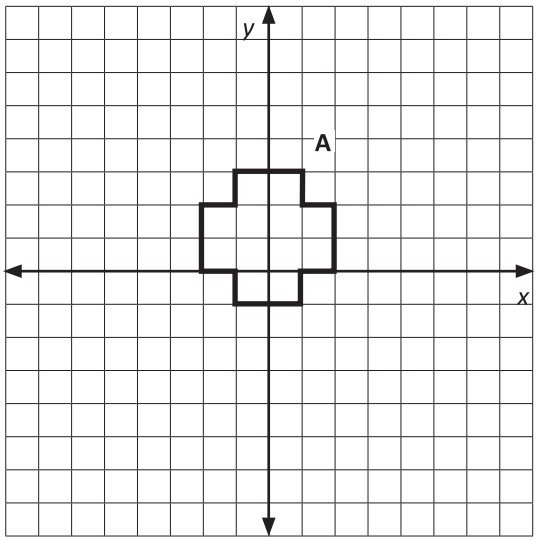


- Plot 10 points in quadrant 1 for which the difference between the first and second coordinate is 3. This will create a line. Find ordered pairs with one or more negative coordinates that are on that line.
- Plot 10 points for which the first coordinate is the opposite of the second (e.g., (5,–5)). Describe the pattern you see and explain why you might have expected that pattern.
- Plot 10 points on a Cartesian plane. In pairs, take turns trying to find each other's points similar to the game of Battleship.
- Create drawings using all four quadrants of the coordinate grid. Then provide other students with a list of the vertices, in order, for each drawing created. The other students would subsequently re-create the drawings.
- Research why coordinate planes are often called Cartesian planes. Write a brief paragraph explaining your findings.
- Use the coordinate plane below to answer the questions that follow. It shows a map of the rooms in a junior high school.



  - Jessica is in the room located at (5, 5). What room is she in? Describe in words how to get to the nurse from this point.

  - Jessica's next class is 8 units to the right and 2 units up on the map from the nurse. In what room is Jessica's next class? Find the ordered pair that represents the location of that room.
  - Lucas is in the music room, but his next class is in the library.
  - Give Lucas directions on how to get to the library.
- Answer questions similar to the following: Given sets of points as ordered pairs, such as A(1, 3), B(–1, 3), C(–1, 2), D(–2, 2), E(–2, –1), F(2, –1), G(2, 2) and H(1, 2), plot them on a coordinate plane and join those points to create a shape.
- Using shapes drawn on a coordinate plane, identify the locations of the vertices.



- Create a simple shape, such as a polygon, on quadrant I of a coordinate grid. Then uses the plot to generate a list of ordered pairs. Exchange lists with a partner, plot the points on the partner's list, and connect the points in the order given to create the polygon. Verify each other's work.
- Create your own designs on grid paper and list the ordered pairs. If needed help, use pattern blocks to build a design, and then tracing it onto grid paper. For a display of coordinate picture designs and a student gallery of completed pictures, refer to the *PlottingCoordinates.com* website for "CoordinArt News": (www.plottingcoordinates.com/coordinartnews.html).
- For each set of ordered pairs below, choose an appropriate scale and label the *x*-axis and the *y*-axis. Plot the ordered pairs, label the points, and draw a line to connect the points in order and connect the last point with the first. State the quadrant(s) in which the figure is located.
  - A(5, 5) B(3, 5) C(3, 3) D(5, 3) The figure is in quadrant(s) _____
  - A(–6, 8) B(–9, 3) C(1, 3) D(4, 8) The figure is in quadrant(s) _____
  - A(10, 12) B(–15, –8) C(22, –22) The figure is in quadrant(s) _____
  - A(–30, –25) B(25, 40) C(–30, 40) The figure is in quadrant(s) _____

SUGGESTED MODELS AND MANIPULATIVES

- geo-boards
- graph paper
- maps

MATHEMATICAL LANGUAGE

Teacher	Student
▪ axes	▪ axes
▪ Cartesian plane	▪ Cartesian plane
▪ coordinates	▪ coordinates
▪ ordered pair	▪ ordered pair
▪ origin	▪ origin
▪ *x*-axis	▪ *x*-axis
▪ *y*-axis	▪ *y*-axis

Resources

Print

Math Makes Sense 7 (Garneau et al. 2007)

- Unit 8: Geometry (NSSBB #: 2001640)
  - > Section 8.5 Graphing on a Coordinate Grid
  - > Section 8.6 Graphing Translations and Reflections
  - > Section 8.7 Graphing Rotations
  - > Unit Problem: Design the Cover
- *ProGuide* (CD; Word Files) (NSSBB #: 2001641)
  - > Assessment Masters
  - > Extra Practice Masters
  - > Unit Tests
- *ProGuide* (DVD) (NSSBB #: 2001641)
  - > Projectable Student Book Pages
  - > Modifiable Line Masters

Digital

- "Welcome to Graph Mole," *FunBased Learning* (Dun 2007): http://funbasedlearning.com/algebra/graphing/points2
- "CoordinArt News," *PlottingCoordinates.com* (PlottingCoordinates.com 2010): www.plottingcoordinates.com/coordinartnews.html

SCO G03: Students will be expected to perform and describe transformations (translations, rotations, or reflections) of a 2-D shape in all four quadrants of a Cartesian plane (limited to integral number vertices). [C, CN, PS, T, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

G03.01 Identify the coordinates of the vertices of a given 2-D shape on a Cartesian plane.

G03.02 Describe the horizontal and vertical movement required to move from a given point to another point on a Cartesian plane.

G03.03 Describe the positional change of the vertices of a given 2-D shape to the corresponding vertices of its image as a result of a transformation, or successive transformations, on a Cartesian plane.

G03.04 Determine the distance between points along horizontal and vertical lines in a Cartesian plane.

G03.05 Perform a transformation or consecutive transformations on a given 2-D shape, and identify coordinates of the vertices of the image.

G03.06 Describe the positional change of the vertices of a 2-D shape to the corresponding vertices of its image as a result of a transformation or a combination of successive transformations.

G03.07 Describe the image resulting from the transformation of a given 2-D shape on a Cartesian plane by identifying the coordinates of the vertices of the image.

Scope and Sequence

Mathematics 6	**Mathematics 7**	**Mathematics 8**
G03 Students will be expected to perform a combination of translation(s), rotation(s), and/or reflection(s) on a single 2-D shape, with and without technology, and draw and describe the image. **G04** Students will be expected to perform a combination of successive transformations of 2-D shapes to create a design and identify and describe the transformations. **G06** Students will be expected to perform and describe single transformations of a 2-D shape in the first quadrant of a Cartesian plane (limited to whole number vertices).	**G03** Students will be expected to perform and describe transformations (translations, rotations, or reflections) of a 2-D shape in all four quadrants of a Cartesian plane (limited to integral number vertices).	**G02** Students will be expected to demonstrate an understanding of the congruence of polygons under a transformation.

Background

Students have studied transformational geometry in previous grades. They used the informal terms slides, flips, and turns in 2G04 but were later introduced to the formal mathematical language in 5G04. When students first began studying these transformations, they worked with concrete shapes on a flat surface. In Mathematics 5, students worked with a single transformation (G03, G04). In Mathematics 6, this was extended to a combination of transformations (G03, G04). They also worked with a single transformation in the first quadrant of a Cartesian plane (G06).

In translations, reflections, and rotations, the shapes and their images are congruent, but their orientation on the plane and/or their location on the plane may change, depending on the shape and the type of transformation. These transformations, which preserve size and shape, are known as isometries (from the Greek iso "shape" and metry "measure"). Transformations are studied using 2-D shapes, which are called "pre-images," and the resultant shape after a transformation, called the "image." The pre-images are named by their vertices (ABC) and the images are labelled (A′B′C′), read as A prime, B prime, C prime, and so on. Successive images are labelled with additional prime marks (A′′B′′C′′), read as A double-prime, B double-prime, C double-prime, and so on. Labelling the vertices should move clockwise after identifying the initial vertex.

In Mathematics 7, students extend their skills to work with successive transformations in all four quadrants of the Cartesian plane. Students regularly encounter 2-D transformations represented in design patterns and computer graphics. They are evident on logos, fabric patterns, frieze patterns, wallpaper, architectural design, landscape design, and so on. Transformations can be used to create interesting symmetrical patterns. In addition, 2-D transformations on Cartesian planes can be used to represent physical movements in a single plane (e.g., sports plays, rides at a fair, traffic routes). Movie animation is created using motion geometry. Examples can be viewed online. The learning experiences suggested for outcome G03 will help students develop their understanding and appreciation of the transformations existing around them, enhance their problem-solving skills and spatial sense, and prepare them for further studies in algebra and geometry.

Transformational changes can be described by identifying the type of transformation, the changes in orientation of the shape or position of the vertices of the shape, the horizontal and vertical movement, or the new *x*- and *y*-coordinates of the vertices of the image, or by stating the change in (*x*- and *y*-coordinates) between the pre-image and its image. When describing transformations, students should be able to recognize a given transformation as a reflection, a translation, a rotation, or some combination of these. In addition, when given a pre-image and its image, students should be able to describe

- a translation, using words and notation describing the translation (e.g., ΔA′B′C′ is the translation image of ΔABC) (Given a pre-image students should be able to say, for example that ΔABC has been translated 2 units to the right and 3 units up to produce its image ΔA′B′C′. Continue to remind students that they must describe the horizontal change first and the vertical change second.)
- a reflection, by determining the location of the line of reflection (Reflections should be limited to use of the *x*-axis or *y*-axis as reflection lines. When describing the reflection, the proper language to use is "a reflection in the *x*-axis" and not "a reflection across the *x*-axis.")
- a rotation, using degree or fraction-of-turn measures, both clockwise and counterclockwise, and identify the location of the centre of a rotation (A centre of rotation may be located on the pre-image (such as at a vertex) or off the shape.)

Successive transformations are defined as the same transformation being applied one after the other. This means that the transformation is first applied to the pre-image resulting in $image_1$ and then the same transformation is now applied to $image_1$ resulting in $image_2$. The prime and double prime notation will be used to label each image.

Consecutive transformation follows the same idea as successive transformation but uses a combination of different transformations. If A(2, –2) is reflected in the *x*-axis, for example, the resulting image is (A′(2, 2)). If A′ is then translated 4 units left and 2 units up, this results in A′′(–2, 4).

The language that is used when describing consecutive or successive transformations is "followed by." For the example, A (2,-2), the question to ask students would be, "What is the resulting image when A (2,-2) is reflected in the *x*-axis followed by a translation 4 units to the left and 2 units up? The common mistake students make is to apply both transformations to the pre-image. Using the terminology "followed by" will help students use the correct image for the second transformation.

Students should be able to describe the single positional change that maps the pre-image directly to $image_2$. In this example the single transformation that will map A onto A′′ is translation 4 units left and 6 units up. Students should also be able to describe the single positional change by comparing the vertices of the pre-image with their corresponding vertices in the final image.

Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

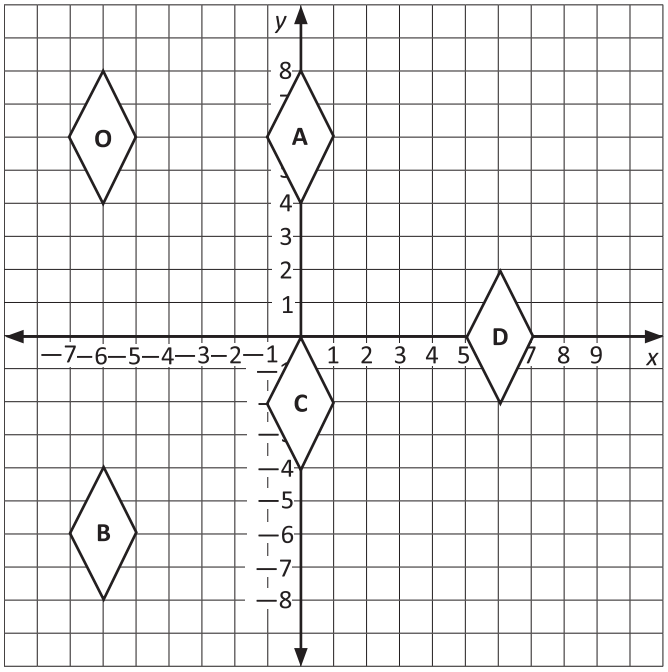
All of the following tasks are to be done in the first quadrant only:

- Ask students to prove that a 2-D shape and its transformation image are congruent.
- Ask students to draw a shape (pre-image), translate it, and then describe the positional change that results in the image.
- Present students with three pictures (pre-image, $image_1$, and $image_2$) on grid paper after two transformations were performed on them. Ask students to predict what two transformations were performed. Could this have been done in more than one way? Could this have been done by a single transformation?
- Provide students with a 2-D shape and have them follow the given successive transformations to determine the image.
- Ask students to explain the transformations shown in a pattern, such as fabric, wallpaper, or other designs.
- Invite students to investigate such questions as,
  - If a shape undergoes a translation followed by another translation, does it matter in which order they take place?
  - Does a reflection followed by a translation produce the same result as the translation followed by the reflection?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Sketch a quadrilateral (pre-image) on a Cartesian plane. Label and record the coordinates of its vertices.
  - Translate the quadrilateral 3 units to the right and 2 units up.
  - Label and record the coordinates of the corresponding vertices of the image.
  - Compare the coordinates of the pre-image with the coordinates of its image and record your observations.
  - Use your observations to predict the coordinates if the pre-image is translated 3 units left and 3 units down.
- Determine what happens to a plotted pre-image if all the first coordinates are switched with the corresponding second coordinates (e.g., A (3, –2) becomes A′(–2, 3)).
  - Describe where the image of each of these points would be located following a half-turn rotation about the origin: P (–3, –5), Q (3, 6), R (–2, 4).
- ΔABC has the following coordinates: A (1, 2), B (3, 5), and C (4, 0).
  - Reflect the triangle in the horizontal axis and label the coordinates for ΔA′B′C′.
  - Reflect ΔA′B′C′ in the vertical axis and label the coordinates for ΔA′′B′′C′′.
  - Discuss ΔA′′B′′C′′ in relation to ΔABC. Is Δ ABC congruent to Δ A′′B′′C′′? Explain. Has the orientation of the transformation of ΔABC changed? Explain.
- If O is the original shape, and A, B, C, and D are images of O, ask students to do the following.
  - Identify the coordinate pairs of the vertices for object O and its images.
  - Describe the movement required to move from O to each of its images.



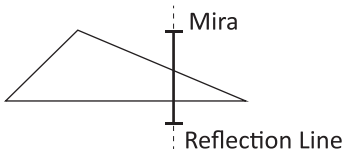
- Respond to the following: If a shape undergoes two transformations, one after the other, does it matter in what order they are applied? Will you get the same final image either way?

Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Have students use notation such as "the reflection image of A (3, 5) is A' (–3, 5)" and say "the reflection image of A (3, 5) is A *prime* (–3, 5)."
- Use grid or dot paper to represent the four-quadrant coordinate plane. Paper folding and the Mira (transparent mirror) are encouraged in working with reflections. When using paper folding, students can fold on the reflection line and trace the image figure. The Mira can be placed on the reflection line, as shown, and students can trace the image from the reflection that appears in the Mira.



- Have students explore rotations where the turn centre of is located off the given pre-image. Many students may still require the use of tracing paper to assist them in placing the transformation of the pre-image.
- Use computer programs, such as Geometer's Sketchpad or GeoGebra, when available, to assist work with transformations.
- Explore tessellations as a context for applying transformations. It may also be interesting to study art that involves repetitive patterning.

SUGGESTED LEARNING TASKS

- Use a geoboard to create a pre-image and a transformation of that image. Exchange geoboards with a partner, and have that partner explain the transformation, using specific transformation language, and to describe a transformation that would move the new image back to the pre-transformational position. This process can be repeated using different figures, transformations, and combinations of transformations.
- Reflect a triangle in a line and then reflect it in another line that is parallel to the first. Compare the final image with the pre-image. Describe *one* transformation that would move the transformation back to its original image position.
- Work in groups to complete this project: Select a piece of your favourite music, and choreograph a dance to accompany the music. The dance should include at least two examples of each transformation (translation, reflection, and rotation). Perform the dance in class while other students identify the various transformations.

- Using grid paper, answer the following: You are employed by a graphic design firm that creates graphic designs for companies who manufacture wallpaper, wrapping paper, tile, and fabric. Your supervisor has assigned you to develop a new design, using the following design elements:
  - it must use at least two types of transformations
  - it must use at least two colours
  - it must be extended to cover at least 75% of the grid

Create a design and write an explanation of how you created your design.

- Provide grid paper with some shapes already sketched on the paper and provide specifications for some transformations. Have students perform the transformations and label the images. In place of the sketches, provide the ordered pairs for the vertices of the pre-images and have students plot the pre-images and images.
- Have students sketch pre-images, specify transformations, prepare a key, and exchange papers with a partner, who will create the images. Then have students return the papers to each other, verify responses, and discuss any discrepancies.

SUGGESTED MODELS AND MANIPULATIVES

- coordinate graph paper
- geo-boards*
- geometry sets
- Geometer's Sketchpad
- grid paper
- Miras
- tracing paper/wax paper

*also available in *Interactive Math Tools* (Pearson n.d.)

MATHEMATICAL LANGUAGE

Teacher	Student
▪ 2-D	▪ 2-D
▪ Cartesian plane	▪ Cartesian plane
▪ centre of rotation	▪ centre of rotation
▪ clockwise	▪ clockwise
▪ combination	▪ combination
▪ congruent	▪ congruent
▪ coordinates	▪ coordinates
▪ counter-clockwise	▪ counter-clockwise
▪ image	▪ image
▪ isometries	
▪ line of reflection	▪ line of reflection
▪ quadrant	▪ quadrant
▪ reflection	▪ reflection
▪ rotation	▪ rotation
▪ shape	▪ shape
▪ successive	▪ successive
▪ transformation	▪ transformation
▪ translation	▪ translation
▪ vertex	▪ vertex
▪ vertices	▪ vertices

Resources

Print

Developing Thinking in Geometry (Johnston-Wilder and Mason 2005), 154–158

Teaching Student-Centered Mathematics, Grades 5–8, Volume 3 (Van de Walle and Lovin 2006b), 209–215

Math Makes Sense 7 (Garneau et al. 2007)

- Unit 8: Geometry (NSSBB #: 2001640)
  - Section 8.6 Graphing Translations and Reflections
  - Section 8.7 Graphing Rotations
  - Technology: Using a Computer to Transform Shapes
  - Unit Problem: Design the Cover
- *ProGuide* (CD; Word Files) (NSSBB #: 2001641)
  - Assessment Masters
  - Extra Practice Masters
  - Unit Tests
- *ProGuide* (DVD) (NSSBB #: 2001641)
  - Projectable Student Book Pages
  - Modifiable Line Masters

Digital

- "33.01 The Coordinate Plane." *Grade 7: The Learning Equation Math.* (Reed 2000): http://staff.argyll.epsb.ca/jreed/math7/strand3/3301.htm (Directions are displayed with the applet.)
- "Shape, Space and Measures," *KS3 Bitesize* (BBC 2015): www.bbc.co.uk/education/topics/zvhs34j. (In this computer game, students choose a mirror line or rotation points to reflect a pentagon house onto its shadow.)
- "Symbol Rotation Patterns," *Wolfram Demonstrations Project.* (Wolfram Demonstrations Project and Contributors 2015): http://demonstrations.wolfram.com/SymbolRotationPatterns/
- "Transmographer," *Interactive*. (Shodor 2015): www.shodor.org/interactivate/activities/Transmographer/

- GeoGebra (International GeoGebra Institute 2015): www.geogebra.org/cms/en
- The Geometer's Sketchpad (Key Curriculum Press 2013; NSSBB #: 50474, 50475, 51453)