# Mathematics 7
# Unit 4: Patterns and Relations

**N01, PR01, PR02, PR04, PR05**

| **SCO N01:** Students will be expected to determine and explain why a number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10, and why a number cannot be divided by 0.<br>[C, R] | | | |
|---|---|---|---|
| **[C]** Communication | **[PS]** Problem Solving | **[CN]** Connections | **[ME]** Mental Mathematics and Estimation |
| **[T]** Technology | **[V]** Visualization | **[R]** Reasoning | |

## Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

**N01.01** Determine if a given number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10, and explain why.
**N01.02** Sort a given set of numbers based upon their divisibility using organizers such as Venn and Carroll diagrams.
**N01.03** Determine the factors of a given number using the divisibility rules.
**N01.04** Explain, using an example, why numbers cannot be divided by 0.

## Scope and Sequence

| Mathematics 6 | Mathematics 7 | Mathematics 8 |
|---|---|---|
| **N03** Students will be expected to demonstrate an understanding of factors and multiples by<br>• determining multiples and factors of numbers less than 100<br>• identifying prime and composite numbers<br>• solving problems using multiples and factors | **N01** Students will be expected to determine and explain why a number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10, and why a number cannot be divided by 0. | **N01** Students will be expected to demonstrate an understanding of perfect squares and square roots, concretely, pictorially, and symbolically (limited to whole numbers).<br><br>**N02** Students will be expected to determine the approximate square root of numbers that are not perfect squares (limited to whole numbers). |

## Background

Exploration of the divisibility rules serves as an excellent opportunity to extend number sense. Knowledge of divisibility rules provides a valuable tool for mental arithmetic and general development of operation sense. A clear grasp of divisibility helps students identify factors and understand relationships between numbers, solve problems, sort numbers, work with fractions, understand percentages and ratios, and work with algebraic equations. When students can easily identify factors, they can readily identify prime and composite numbers, common factors and multiples, and find both the greatest common factor (GCF) and the least common multiple (LCM). Understanding divisibility enhances students' ability to rename fractions with common denominators and to rewrite fractions in simplest terms, thereby making it easier to compare fractions and to perform operations with fractions. If a dividend can be divided by a divisor to yield a quotient that is a whole number (with no remainder), then the dividend is considered to be divisible by that divisor. 36 is **divisible** by 4 because it gives a quotient of 9, with no remainder. The divisor and dividend are whole numbers.

If a dividend is divisible by a divisor, that divisor is a factor of the dividend. 36 is divisible by 4, therefore 4 is a factor of 36. If the divisor is a factor of the dividend, then the dividend is a multiple of the divisor. 36 is a multiple of 4.

In earlier grades, students built their understanding of sequences and number patterns. These patterns are used to develop division of larger numbers. It is assumed that students can

recognize number patterns in tables

extend a table of values using a pattern

describe the relationships among terms in a table.

If students understand place value and have facility in using mental mathematics strategies and facts, it will be easier for them to find patterns in multiples of factors, to add the digits of multiples, and to recognize numbers that are divisible by a particular factor. Proficiency with these skills will help students to discover the divisibility rules, understand and explain why divisibility rules work, and use divisibility rules effectively to determine divisibility.

Students have explored patterns in a multiplication chart in Mathematics 4 and learned about factors and multiples in Mathematics 6. Building on these, students can discover the divisibility rules for 2, 5, and 10. Also, once students understand divisibility for 2 and 3, they can use this knowledge to develop a means of testing for divisibility by 6. This should be seen as a problem solving opportunity for students. They can also explore whether this strategy will always work for other numbers such as 8 and 10.

In Mathematics 7, students are not expected to do prime factorization. It is a performance indicator in Mathematics 8 in the study of squares and square roots, and a learning outcome in the Mathematics 10 course.

To avoid an arbitrary rule for not being able to divide by 0, use patterns for multiplication and division.

For example, $\frac{12}{4}=3$ because $3\times 4=12$

$\frac{10}{5}=2$ because $5\times 2=10$

$\frac{0}{4}=0$ because $0\times 4=0$

but $\frac{4}{0}=?$ because ? × 0 = 4 (There is no answer)

The divisibility rules described below are for teacher information. It is not expected that students will complete a table like this. The suggested order for instruction of divisibility rules is 2, 5, 10, 3, 6, 9, 4, and 8.

| Divisibility Rules for Common Factors—Suggested Teaching Order | | | |
|---|---|---|---|
| **Divisible by** | **Rule** | **Explanation** | **Examples and Non-examples** |
| 2 | The number is even, that is, the ones digit is 2, 4, 6, 8, or 0. | Even numbers are composed of groups of 2. Therefore, it is necessary only to examine the units (or ones) place when determining divisibility by 2. | 238 is divisible by 2 because the digit in the ones place (8) is even<br><br>89 is not divisible by 2 because the digit in the ones place (9) is odd. |
| 5 | The ones digit is 0 or 5. | Use place value logic. All multiples of 5 end in 0 or 5.<br>(0, 5, 10, 15, 20, 25, 30, 35, …) | 130 is divisible by 5 because it ends in 0.<br><br>89 is not divisible by 5 because 89 does not end in 0 or 5. |
| 10 | The ones digit is 0. | All multiples of 10 have 0 in the ones place. | 130 is divisible by 10 because the digit in the ones place (0) is 0.<br><br>89 is not divisible by 10 because the digit in the ones place (9) is not 0. |
| 3 | The sum of the digits is divisible by 3.<br><br>Continually add the digits until you end up with a single digit that is divisible by 3 (will ultimately result in a total of 3, 6, or 9). | Use place value and the logic of remainders.<br><br>Each hundred can be divided into 33 groups of 3 and leaves 1 unit remaining.<br><br>Each ten divides into three groups of 3 and leaves 1 unit remaining.<br><br>The ones are already individual units. Add all the remaining units (or remainders). If this sum divides evenly by 3, the original number is divisible by 3.<br><br>For example 573 can be written as<br><br>$5 \times 100 + 7 \times 10 + 3$<br>$= 5\,(99 + 1) + 7\,(9 + 1) + 3$<br>$= 5 + 7 + 3$<br>$= 15$<br>$= 6$<br>and we know that 6 is divisible by 3. | 354 is divisible by 3 because<br>$3 + 5 + 4 = 12$<br>$1 + 2 = 3$<br><br>The sum of the digit is 3 which is divisible by 3. Therefore 354 is divisible by 3.<br><br>238 is not divisible by 3 because<br>$2 + 3 + 8 = 13$<br>$1 + 3 = 4$<br><br>4 is not divisible by 3. Therefore 238 is not divisible by 3. |

| Divisibility Rules for Common Factors—Suggested Teaching Order | | | |
|---|---|---|---|
| **Divisible by** | **Rule** | **Explanation** | **Examples and Non-examples** |
| 6 | The number is even and divisible by 3, that is, the number has both 2 and 3 as factors (is divisible by both 2 and 3). | Every second multiple of 3 is also a multiple of 2. Therefore, if the number is divisible by both prime factors 2 and 3, it must also be divisible by 6 because two groups of 3 make a group of 6. | 426 is divisible by 6 because it is divisible by both 2 (it is an even number) and 3 (the sum of its digits is divisible by 3)<br><br>153 is not divisible by 6 because it is not divisible by 2 (it is an odd number), but it is divisible by 3 (the sum of its digits is divisible by 3). |
| 9 | The sum of the digits is divisible by 9. The number is divisible by 3 twice. (Remember that 9 is 3 × 3.) | Use place value and the logic of remainders.<br><br>Each hundred can be divided into 11 groups of 9 and leaves 1 unit remaining.<br><br>Each ten divides into one group of 9 and leaves 1 unit remaining.<br><br>The ones are already individual units.<br><br>Add all the remaining units (or remainders). If this sum divides evenly by 9, or by 3 twice, the original number is divisible by 9. | 351 is divisible by 9 because dividing 3 hundreds by 9 leaves 3 units remaining, 5 tens leaves 5 units remaining, and 1 unit is the remainder in the units place.<br><br>Add up the remainders:<br>3 + 5 + 1 = 9. Since the remainders are divisible by 9, the entire number is also divisible by 9.<br><br>418 is not divisible by 9 because dividing 4 hundreds by 9 leaves 4 units remaining, 1 ten leaves 1 unit remaining, and 8 units are the remainder in the units place. Add up the remainders: 4 + 1 + 8 = 13. Since 13 is not divisible by 9, the entire number is not divisible by 9. |
| 4 | The number formed by the tens and ones digits is divisible by 4. That is, the number formed by the tens and ones digits is divisible by 2 twice. | Use place value logic.<br><br>100 is the smallest place value position divisible by 4 (100 ÷ 4 = 25). Any number greater than 100 can be expressed as *x* number of hundreds , therefore, only the number formed by the digits in the tens and units places must be examined. | 524 is divisible by 4 because 100 is divisible by 4. Therefore 5 x 100 is divisible by 4. 24 is divisible by 2 twice which means it is divisible by 4.<br><br>490 is not divisible by 4. Although 4 × 100 is divisible by 4, 90 is not divisible by 2 twice, therefore it is not divisible by 4. |
| 8 | The number formed by the hundreds, tens, and ones digits is divisible by 8. That is, the number formed by the hundreds, tens, and ones digits is divisible by 2 three times. | Use place value logic. 1000 is the smallest place value position divisible by 8 (1000 ÷ 8 = 125). Any number larger than 1000 can be expressed as *x* number of thousands Therefore, only the number formed by the digits in the hundreds, tens, and units places must be examined. | 7480 is divisible by 8 because 480 is divisible by 2 three times (480 ÷ 2 = 240; 240 ÷ 2 = 120; 120 ÷ 2 = 60).<br><br>220 is not divisible by 8 because 220 is not divisible by 2 three times. (220 ÷ 2 = 110, 110 ÷ 2= 55, 55 is not divisible by 2). Therefore 220 is not divisible. |

# Assessment, Teaching, and Learning

## Assessment Strategies

### ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

Students should be able to select from a repertoire of mental mathematics strategies for addition, subtraction, multiplication, and division.

Have students describe and apply mental mathematics strategies, such as

- skip-counting from a known fact
- using doubling or halving
- using repeated doubling

Ask students to identify a number with five factors.

Have students draw diagrams (such as rectangles or factor trees) to show why a given number is or is not prime (e.g., 10, 17, 27).

### WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

Determine if a given number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10.

Determine the factors of a given number using the divisibility rules.

Explain, using an example, why a number cannot be divided by 0.

Create a 3-digit number that is divisible by both 4 and 5. Is it also divisible by 2 and 8?

Complete the number by filling in each blank with a digit. Explain how you know your answer is correct:

- 26_ is divisible by 10
- 154_ is divisible by 2
- _6_ is divisible by 6
- 26_ is divisible by 3
- 1_2 is divisible by 9
- 15_ is divisible by 4

There will be 138 people at a party. Determine if the host can fill tables of 5. Tables of 6? Support your answer by using divisibility rules.

Choose three numbers that are divisible by both 6 and 9. Find the smallest number, other than 1, by which the chosen numbers are divisible. Share your answers with the class and discuss.

Each of Eli's four friends has a code number. Keile's number is divisible by 3, 5, and 8. Max's number is divisible by 2 and 3. Jennifer's number is divisible by 4 and 5, but not 3. Ben's number is divisible by

3 and 5, but not 8. Eli receives a message with the code number 5384 from one of his four friends. Determine who sent the message

Determine if each statement is true or false and identify an example that supports your answer:

All numbers divisible by 6 are divisible by 3.

Some, but not all, numbers divisible by 6 are divisible by 3.

No numbers divisible by 6 are divisible by 3.

All numbers divisible by 3 are divisible by 6.

Some, but not all, numbers divisible by 3 are divisible by 6.

No numbers divisible by 3 are divisible by 6.

Explain why it is not possible to calculate 12 ÷ 0.

The principal of Great School has to determine the number of classes of grade 7 students in her school. Discuss the divisibility rules that can be used to determine the possible number of classes there could be if there are 240 grade 7 students and all classes have an equal number of students.

Complete the following table and explain how the table shows division by 0 is not possible.

| Division Statement | Related Multiplication Statement |
|---|---|
| 6 ÷ 2 = 3 | 3 × 2 = 6 |
| 10 ÷ 5 = 2 | 2 × 5 = ___ |
| 14 ÷ 2 = ___ | 2 × 7 = 14 |
| 15 ÷ ___ = 5 | 3 × 5 = 15 |
| ___ ÷ 8 = 3 | 3 × 8 = ___ |
| 12 ÷ 0 = _____ | 0 × ___ = 12 |

## Planning for Instruction

### CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

Organize instruction so that the students develop the divisibility rules themselves through investigations.

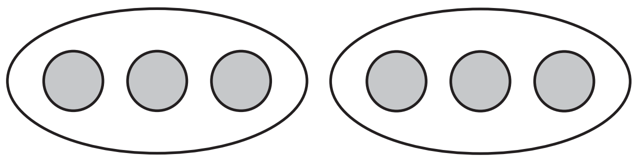
Explore why some divisibility rules work, such as the divisibility rule for 4.

Show students numbers at random. Ask students to determine by what numbers they are divisible.

Use a hundred chart to explore patterns of multiples.

Using the “repeated subtraction” meaning of division will help students understand why numbers cannot be divided by 0. Given 20 ÷ 5, for example, students should understand that 5 can be subtracted from 20 four times (20 – 5 – 5 – 5 – 5 = 0). Given 6 ÷ 0, students should determine that no matter how many times they subtract 0, they will still have 6. Since there is no answer, 6 ÷ 0 is undefined.

Dividing by 0 can also be visualized using counters. For example, given 6 ÷ 3, how many groups of 3 are in 6? Students separate 6 counters into 2 groups of 3.



Given 6 ÷ 0, students will see that it is not possible to separate the 6 counters into groups of 0. The following are ways to show a number cannot be divided by zero:

Using a calculator to divide a number by zero results in an error message.

Applying the action of division results in an impossible situation.

Example:

If you have a quantity x, how many groups of zero can you make? You would be trying to make groups of zero forever. If you had to share a quantity into zero groups, you would have no groups to share the quantity with. Both scenarios are impossible. Using the pattern and logic of related facts provides no solution to dividing by zero. Multiplication and division are inverse operations. Think of related statements such as the following:

4 x 2 = 8 and 8 ÷ 4 = 2

0 x ? = 8 and 8 ÷ 0 = ? (There is no answer.)

**SUGGESTED LEARNING TASKS**

Explore the use of a calculator as a tool to test for divisibility. Students should realize that the test for divisibility on a calculator involves dividing to see if the quotient is a whole number, not a decimal.

Explore divisibility rules for 3, 6, and 9. Write the first 10 multiples of 3. What do you notice about the number? If no student mentions the sum of the digits, ask them to find the sum of the digits and describe what they notice. Which numbers in the same list are divisible by 6? What do you notice about these numbers? Test the conclusions, using numbers such as 393, 504, and 5832.

Sort a given set of numbers based upon their divisibility using organizers, such as Venn and Carroll diagrams.

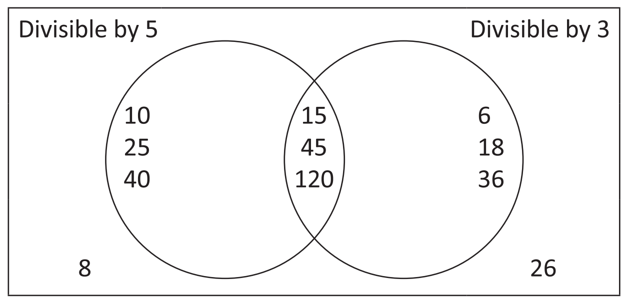
Based on what you have discovered, do you think there should be a divisibility rule for zero? Explain your response.

Have students complete the following table with different numbers:

| Number | Is it divisible by 2? How do you know? | Is it divisible by 3? How do you know? | Is it divisible by 4? How do you know? | Is it divisible by 5? How do you know? | Is it divisible by 6? How do you know? | Is it divisible by 8? How do you know? | Is it divisible by 9? How do you know? | Is it divisible by 10? How do you know? |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | | |
| | | | | | | | | |

Create a Carroll diagram or a Venn diagram to sort the following numbers based on the divisibility rules for 3 and 5: 6, 8, 10, 15, 18, 25, 26, 36, 40, 45, 120. Extension: Which numbers are also divisible by 15?

| **Carroll Diagram** | Divisible by 3 | Not Divisible by 3 |
|---|---|---|
| Divisible by 5 | 15 45 120 | 10 25 40 |
| Not Divisible by 5 | 6 36 18 | 8 26 |




**SUGGESTED MODELS AND MANIPULATIVES**

- calculator
- colour tiles
- hundred chart
- Carroll diagram
- Venn diagram

## MATHEMATICAL LANGUAGE

| Teacher | Student |
| --- | --- |
| Carroll diagram | Carroll diagram |
| dividend | dividend |
| divisibility | divisibility |
| divisible | divisible |
| divisor | divisor |
| factor | factor |
| multiple | multiple |
| product | product |
| quotient | quotient |
| Venn diagram | Venn diagram |

# Resources/Notes

## Print

*Math Makes Sense 7* (Garneau et al. 2007)
Unit 1: Patterns and Relations (NSSBB #: 2001640)

- Section 1.1 Patterns in Division
- Section 1.2 More Patterns in Division

*ProGuide* (CD; Word Files) (NSSBB #: 2001641)

- Assessment Masters
- Extra Practice Masters
- Unit Tests

*ProGuide* (DVD) (NSSBB #: 2001641)

- Projectable Student Book Pages
- Modifiable Line Masters

| **SCO PR01:** Students will be expected to demonstrate an understanding of oral and written patterns and their equivalent linear relations.<br>[C, CN, R] | | | |
|---|---|---|---|
| **[C]** Communication<br>**[T]** Technology | **[PS]** Problem Solving<br>**[V]** Visualization | **[CN]** Connections<br>**[R]** Reasoning | **[ME]** Mental Mathematics and Estimation |

## Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

**PR01.01** Formulate a linear relation to represent the relationship in a given oral or written pattern.
**PR01.02** Provide a context for a given linear relation that represents a pattern.
**PR01.03** Represent a pattern in the environment using a linear relation.

## Scope and Sequence

| Mathematics 6 | Mathematics 7 | Mathematics 8 |
|---|---|---|
| **PR01** Students will be expected to demonstrate an understanding of the relationships within tables of values to solve problems.<br><br>**PR02** Students will be expected to represent and describe patterns and relationships, using graphs and tables. | **PR01** Students will be expected to demonstrate an understanding of oral and written patterns and their equivalent linear relations. | **PR01** Students will be expected to graph and analyze two-variable linear relations. |

## Background

Mathematics is often referred to as the science of patterns, as patterns permeate every mathematical concept and are found in everyday contexts. Patterns are prevalent in plant and animal life, as well as in the physical world. They are evident in the arts, music, structures, movement, time, and space. Our number system is rooted in pattern, and an understanding of pattern is the basis of mathematical concepts in every strand of mathematics. Someone who possesses the ability to identify a pattern and its symbolic relation can solve a problem that previously seemed insurmountable.

Some characteristics of patterns include the following. Patterns include repeating patterns and increasing and decreasing patterns. Repeating patterns have a “core,” which is the part that repeats. Increasing and decreasing patterns are evident in a wide variety of contexts, including arithmetic and geometric situations. Arithmetic patterns, such as 11, 8, 5, 2, … are patterns that are formed by adding or subtracting the same number each time. Geometric patterns, such as 2, 6, 18, 54, … are patterns that are formed by multiplying or dividing by the same number each time.

Patterns represented concretely or pictorially can also be written as number patterns, where numbers represent the quantity in each step of the pattern. In Mathematics 5 students used patterns to generalize relationships. How a term value changes from one term to the next defines the **recursive relationship** in the pattern. It explains what is done to one number to determine the next number in the pattern. An expression that explains what you do to the term number to get the corresponding term value in the pattern describes the **functional relationship** in the pattern and is known as the pattern rule. Students continue to investigate these relationships in Mathematics 6 through tables and graphs. Patterns can be represented in different ways. The recursive and functional relationships can be seen in the different representations of a pattern. Determining the expression that represents the functional relationship can be challenging, and will require persistence to be able to determine the term value from the term number. Each type of representation provides a different view and a different way to think about the relationships. Encourage students to work toward identifying the functional relationship in each type of representation.

To increase students' ability to think symbolically, begin with more obvious relations and teach students to ask themselves increasingly complex questions about the relations (e.g., What remains the same? What changes? By how much does it change? Is this true for every term in the sequence? How can that idea be represented? What happens if … ?).

The following process for representing patterns flows from the concrete or pictorial to the symbolic. It is important to guide students through the process.

**Concrete or pictorial representation:** The context of the pattern exists in the physical pattern itself, and can be represented concretely or pictorially. Students can examine the physical terms to determine what remained the same in each term and what changed. Playing with colour or arrangement of patterns can often help students to see the constant and the changing aspects of a pattern.

Example:

| | | | | | | |
|---|---|---|---|---|---|---|
| Pictorial representation of a pattern | ★★<br>★ | ★★<br>★★<br>★ | ★★<br>★★<br>★★<br>★ | ★★<br>★★<br>★★<br>★★<br>★ | | |

In this example, two stars are added to the top of each new term. This pattern can be extended by adding 2 stars each time. Students should be asked to describe the patterns they see in words and share their thinking orally with the class. Include descriptions that represent both recursive thinking and functional thinking. Pictorial representations can be represented in a table of values.

**Tables of values:** Charts or tables of values display numeric representation of the pattern values, and may also be used to record the changes between the terms. These representations, as illustrated in the following example, facilitate numeric comparisons. They can be presented in horizontal or vertical format.

Example:

| | | | | | | |
|---|---|---|---|---|---|---|
| Term number (*n*)<br>(the position for the term in the sequence) | 1 | 2 | 3 | 4 | 5 | 6 |
| Term value (*v*)<br>(the number of items in the term) | 3 | 5 | 7 | 9 | | |

Reading across in the table above and seeing that each term value increases by 2 is *recursive thinking.* Using the term number and the corresponding term value to determine the pattern rule is *functional thinking.* Ask, "What can be done to the term number to get the term value?" "What changes?" "What stays the same?" In the pictorial representation above the single star is the same in each term and the number of groups of two stars changes. The pattern rule here is, multiply the term number by 2 and add 1. Ask students what the 2 represents in the pattern and what the 1 represents.

The relationship can be expressed as the expression $2n + 1$ or as the equation $2n + 1 = v$, where $n$ = the term number and $v$ = the term value.

This outcome should be done in conjunction with PR02.

# Assessment, Teaching, and Learning

## Assessment Strategies

### ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

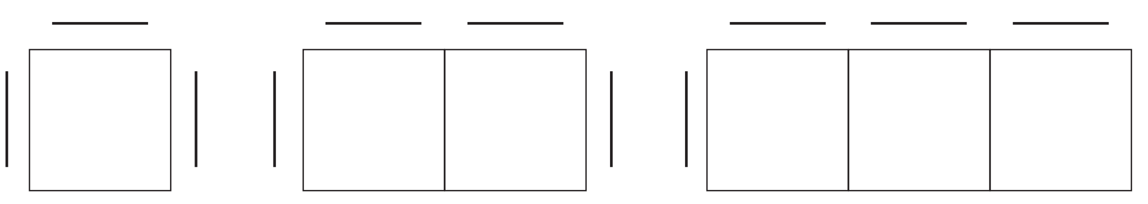
Ask students to fill in the missing values in a given table and explain their thinking.

| | | | | |
|---|---|---|---|---|
| Term number | 1 | 2 | 3 | ? |
| Term value | 4 | 8 | ? | 16 |

Mary walked 3 km on Monday. Each day that week, she walked 2 more km than the day before. Ask students to create a table of values for this data, describe the pattern, and make a graph.

Ask students to refer to the following table of values to answer these questions:

| Number of tables | Number of chairs |
|---|---|
| 1 | 4 |
| 2 | 6 |
| 3 | 8 |
| 4 | 10 |
| 5 | 12 |



- Describe the pattern rule for the number of chairs you would need for the tables. Explain your thinking.
- Use this rule to predict the number of chairs for 10 tables.

## WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

Describe a real-life situation that could be represented by 3*p* + 4.

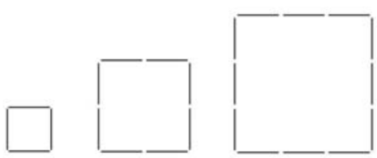
Create a pattern and represent it in five ways.

For the following diagrams,

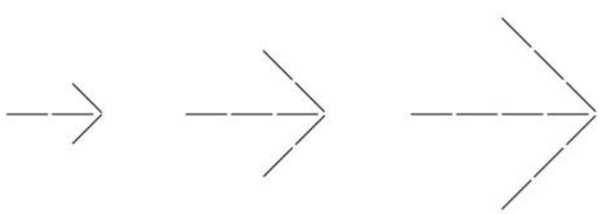
- construct and extend the pattern with concrete objects
- describe the pattern rule in their own words (for example, Start with 2 and add 1 each time)
- Describe how to determine any term value using the pattern rule
- use the pattern to determine the 30$^{th}$ term value
- develop a table
- generate a graph
- write an equation to describe the linear relation

a) (The pattern rule is used to describe the number of circles in each term.)

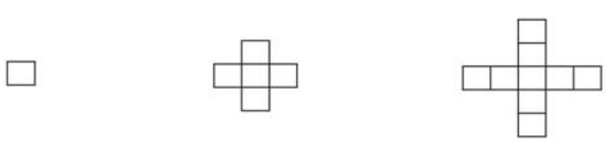
b) (The pattern rule is used to describe the number of toothpicks in each term.)

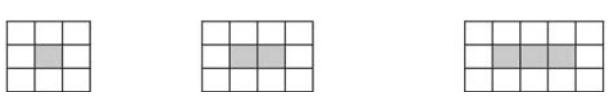


c) (The pattern rule is used to describe the number of toothpicks in each term.)



d) (The pattern rule is used to describe the number of white squares in each term.)



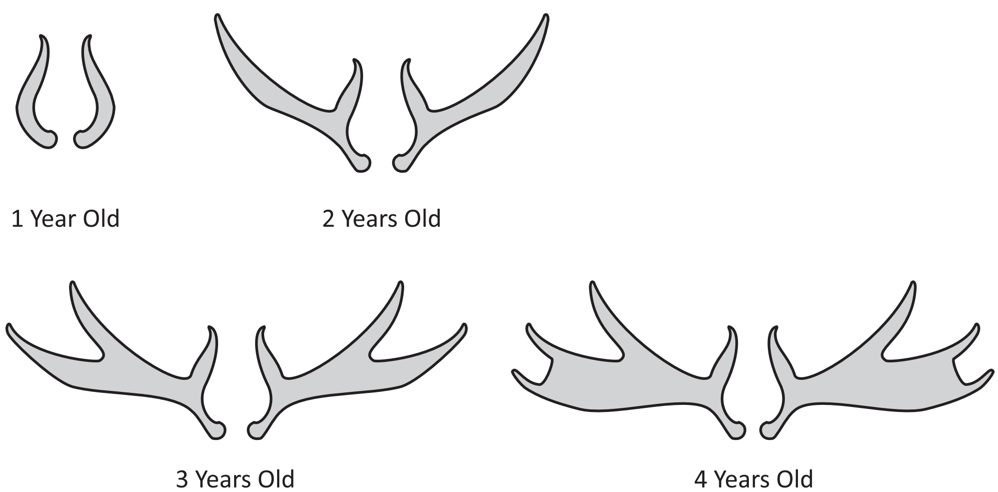


Continue the pattern and complete the table to show the pattern. Describe the relationship between the variables, write an equation for the linear relation, and graph the table of values.

Cost of Flowers in a Vase


| Number of flowers (f) | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Cost in dollars (c) | | | | | | | |

Number of Tips on Moose Antlers




| Age of moose in years (*a*) | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Number of tips on antlers | | | | | | | |

## Planning for Instruction

### Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

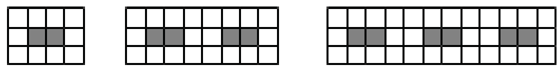
Ask students to describe patterns and rules orally and in writing before using algebraic symbols. Provide opportunity to connect the concrete and pictorial representations to symbolic representations as well as connecting the symbolic representations to pictorial and concrete representations.

Provide examples of growth patterns that are arithmetic and geometric.

Encourage students to draw diagrams and create tables of values to assist them in visualizing the relationship when formulating linear relations representing oral or written patterns.

Give examples, prior to asking students to provide contexts to represent linear relations. The expression 10h + 2, for example, could represent the amount of money a person makes if they are paid $10 per hour plus a $2 bonus.

Students could investigate a number of patterns that may be expressed using linear relations, such as the black-and-white tile pattern for kitchen flooring, as shown below. Students should be able to construct a table of values showing the number of black tiles and the number of white tiles in the first five designs, describe the pattern, and write an equation.



| Number of black tiles ($b$) | Number of white tiles ($w$) |
|---|---|
| 2 | 10 |
| 4 | 20 |
| 6 | 30 |
| 8 | 40 |
| 10 | 50 |

**Suggested Learning Tasks**

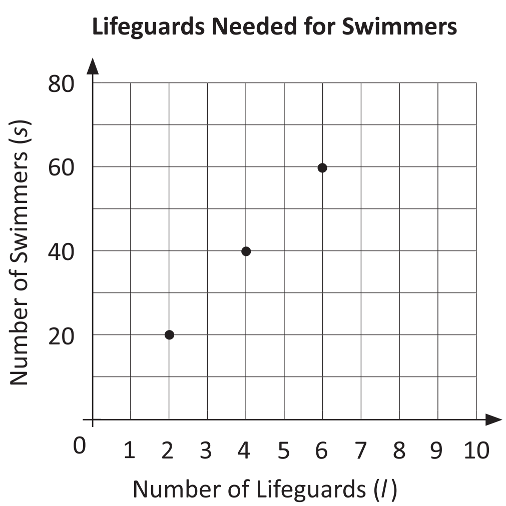
Study the tables below, describe the relationship between the variables, write an equation, and then graph the relation and describe the graph.

Cost of renting a scooter

| Number of hours ($n$) | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Cost ($c$) | 50 | 70 | 90 | 110 | 130 |

A taxi charges a base fare of $4, plus $1 for every kilometre travelled. This can be represented by the equation $c = k + 4$ ($c$ = cost, $n$ = number of km). Make a table of values showing the total cost for the first 5 km. You can graph the table of values and describe the pattern. How much would a 10 km taxi ride cost?

Using the graph below, have students perform the following tasks:




How many swimmers would be allowed for 10 lifeguards?

How many lifeguards would be needed for 50 swimmers?

Describe the pattern in words.

Write an equation for the number of ($l$) lifeguards needed for ($s$) swimmers.

Create a context that can be represented by the following number pattern.

89, 74, 62, 53, 47, …, …, …

### SUGGESTED MODELS AND MANIPULATIVES

colour tiles
linking cubes
counters
toothpicks

### MATHEMATICAL LANGUAGE

| Teacher | Student |
|---|---|
| algebraic expression | algebraic expression |
| arithmetic patterns | arithmetic patterns |
| decrease | decrease |
| discrete data | discrete data |
| explicit relationship | explicit relationship |
| increase | increase |
| geometric patterns | geometric patterns |
| increasing | increasing |
| decreasing | decreasing |
| linear graph | linear graph |
| linear relation | linear relation |
| | linear relationship |
| predict | predict |
| recursive relationship | recursive relationship |
| relationships | relationships |
| repeating | repeating |
| table of values | table of values |
| term | term |
| term number | term number |
| term value | term value |
| unknown term | unknown term |
| values | values |

# Resources

## Print

*Math Makes Sense 7* (Garneau et al. 2007)
Student Book Unit 1 Patterns and Relations (NSSBB #: 2001640)

- Section 1.3 Algebraic Expressions
- Section 1.4 Relationships in Patterns
- Unit Problem: *Fund Raising*

*ProGuide* (CD; Word Files) (NSSBB #: 2001641)

- Assessment Masters
- Extra Practice Masters
- Unit Tests

*ProGuide* (DVD) (NSSBB #: 2001641)

- Projectable Student Book Pages
- Modifiable Line Masters

| | | | |
|---|---|---|---|
| **SCO PR02:** Students will be expected to create a table of values from a linear relation, graph the table of values, and analyze the graph to draw conclusions and solve problems.<br>[C, CN, PS, R, V] | | | |
| **[C]** Communication<br>**[T]** Technology | **[PS]** Problem Solving<br>**[V]** Visualization | **[CN]** Connections<br>**[R]** Reasoning | **[ME]** Mental Mathematics and Estimation |

## Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

**PR02.01** Create a table of values for a given linear relation by substituting values for the variable.
**PR02.02** Create a table of values, using a linear relation, and graph the table of values (limited to discrete elements).
**PR02.03** Sketch the graph from a table of values created for a given linear relation, and describe the patterns found in the graph to draw conclusions (e.g., graph the relationship between $n$ and $2n + 3$).
**PR02.04** Describe, using everyday language, in spoken or written form, the relationship shown on a graph to solve problems.
**PR02.05** Match a given set of linear relations to a set of graphs.
**PR02.06** Match a given set of graphs to a given set of linear relations.

## Scope and Sequence

| Mathematics 6 | Mathematics 7 | Mathematics 8 |
|---|---|---|
| **PR01** Students will be expected to demonstrate an understanding of the relationships within tables of values to solve problems.<br><br>**PR02** Students will be expected to represent and describe patterns and relationships, using graphs and tables. | **PR02** Students will be expected to create a table of values from a linear relation, graph the table of values, and analyze the graph to draw conclusions and solve problems. | **PR01** Students will be expected to graph and analyze two-variable linear relations. |

## Background

Remind students that patterns are represented in many equivalent forms and one form can be translated into another. Each different representation provides a different view of the same pattern. The more views students see, the greater their understanding of the pattern will be. A word description of the pattern can be used to make a physical representation of the pattern, as well as a table of values, and/or a graph. The *x*- and *y*-values of a graph can also be represented in a word description of a pattern, and can be used to form an algebraic equation.

Student investigations of linear relations should start with models, followed by oral and written descriptions. Where appropriate, use manipulatives to foster a better understanding of the concept.

Work on patterns continues in PR02 as students create a table of values from a linear relation and sketch the graph. The word linear may cause confusion but should become clear as students graph the

table of values and see the linearity of the points. This outcome should be done in conjunction with PR01.
A linear pattern can be described using a table of values. For example, the number pattern 3, 5, 7, 9, 11 … expressed in a table of values is

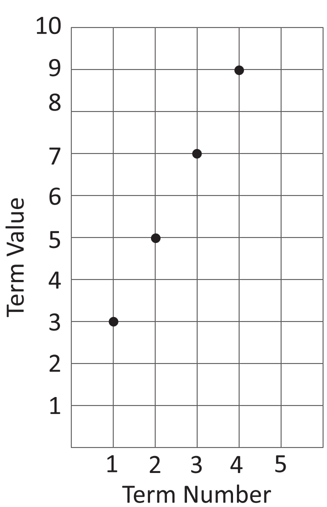
| Term number (n) | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Term value (v) | 3 | 5 | 7 | 9 | 11 |

A variable such as *n* is used to represent an unknown quantity. It is important to note that students have been using a symbol (open frame in the form of a circle, square, triangle, etc.) to represent an unknown value since Mathematics 4, and variables were formally introduced in Mathematics 5.

Students use tables to organize the information that a pattern provides. When using tables, it is important for students to realize that they are looking for the relationship between the term number, represented by a variable, and the term value, represented by an expression.

A graph provides a picture of a relationship in a pattern. It provides clear evidence of whether the values are increasing or decreasing, how quickly the change is happening, and the increment of change. Relationships can be described by articulating these changes. In this outcome, students work with graphs, sketching the graph from a table of values, or analyzing given graphs. The analysis of graphs should include creating stories that describe the relationship depicted and constructing graphs based on a context that involves changes in related quantities. When students are describing a relationship in a graph they should use appropriate mathematical language.

**Note:** In the previous equation, $v = 2n + 1$, and 2 is the coefficient and 1 is the constant.




In Mathematics 7, students work with discrete data. Since *n* and *v* in the above equation refer specifically to the term number and term value, both of which are natural numbers (1, 2, 3, 4, ...). As a result, the graph of the relationship is the points and no line should be drawn through these points. When assessing student performance, keep in mind that students are limited to discrete elements and linear relations, and do not include powers and exponents.

Students should be given opportunities where appropriate, to interpolate (finding a point between two known points), as well as to extrapolate (finding a point that lies beyond the known data). This terminology is not the focus here. Students should be able to describe the general pattern of the graph (e.g., it goes upward to the right with the points in a straight line).

As an extension, students could explore if an ordered pair satisfies a given equation by plotting the points to see if it follows the pattern, and substituting it into the equation.

# Assessment, Teaching, and Learning

## Assessment Strategies

### ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to create a graph to display the relationship between the number of tricycles and the numbers of wheels. They should represent this data in a table of values as well.

### WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

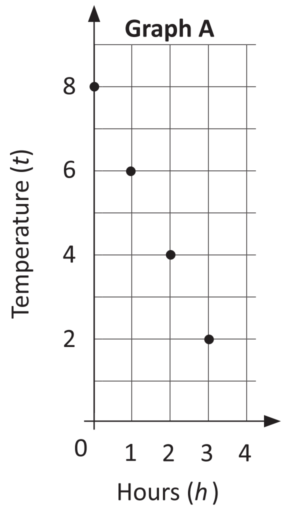
Study the tables below, describe the relationship between the variables and write a linear relation and then graph the relation and describe the graph.

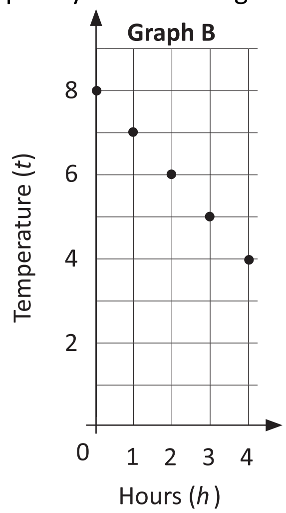
**Songs on an MP3 Player**

| | | | | | |
|---|---|---|---|---|---|
| Number of Pop songs (*p*) | 1 | 2 | 3 | 4 | 5 |
| Number of Rock songs (*r*) | 3 | 6 | 9 | 12 | 15 |

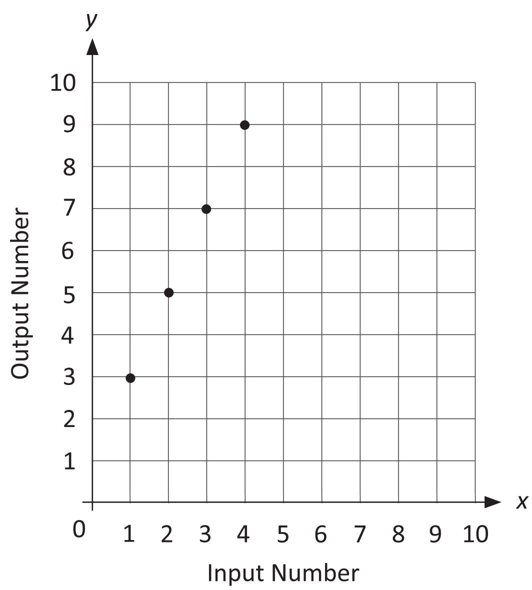
On Monday, the morning temperature was 8 degrees. The temperature drops 2 degrees each hour. Faith says that Graph A shows this relationship and it can be written as $y = 8 – 2x$. The next day the morning temperature is 8 degrees. The temperature drops 1 degree each hour. Faith says that Graph B shows this relationship and it can be written as $y = 8 – x$.

Determine if she is correct and explain your reasoning




Determine which relations can be matched with the graph. Explain your reasoning.




*x* represents the input number
*y* represents the output number

$y = 2x + 1$
$y = x + 2$
The y value is equal to double the x value increased by 1.
The y value is equal to double the x value decreased by 1.

Describe a real-life situation that could be represented by $3p + 4$.

## Planning for Instruction

### Choosing Instructional Strategies

Consider the following strategies when planning daily lessons.

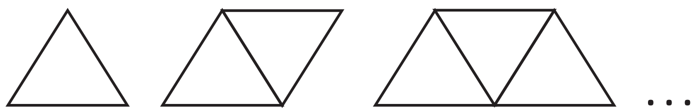
Encourage students to draw diagrams and create tables of values to help them visualize the linear relations that represent oral or written patterns.

Provide experiences representing the same pattern in multiple ways; i.e., model, diagram, table of values, graph, and expression.

Use real-world contexts as much as possible. Have students represent the pattern by formulating a linear relation.

### SUGGESTED LEARNING TASKS

- Continue the pattern for up to seven triangles with the diagram below.



| Term number: ($n$) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | … | 10 | … | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Total number of line segments ($t$) | | | | | | | | … | | … | |

Complete the chart to show pattern growth.

Describe in writing how the pattern grows.

Write an equation to show the relationship between the term number and the number of line segments.

Draw a graph to show the pattern.

Does it make sense to join the points? Explain the shape of the graph.

Provide students with the pattern: 97, 94, 91, 87… Continue the pattern for the next three numbers.

Describe, in words, how the pattern grows. Write the equation.

Provide students with the following table which shows the relationship between the number of passengers on a tour bus and the total cost of providing boxed lunches.

| Passengers ($p$) | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Cost of lunches ($c$) | 5 | 10 | 15 | 20 | 25 |

Explain how the lunch cost is related to the number of passengers.

Write an equation for finding the lunch cost ($c$) for the number of passengers ($p$).

Use the equation to find the cost of lunch if there were 25 people on the tour.

Draw a graph to show the relationship in the table of values.

How many people were on the bus if the tour-bus leader spent $200 on box lunches?

Provide students with a table of values and a graph and ask if they match. Have them justify their thinking.

For the following diagrams,

construct and extend the pattern with concrete objects

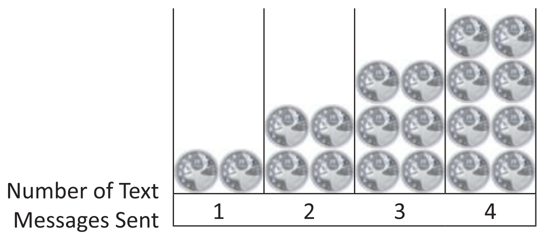
draw a representation of the pattern

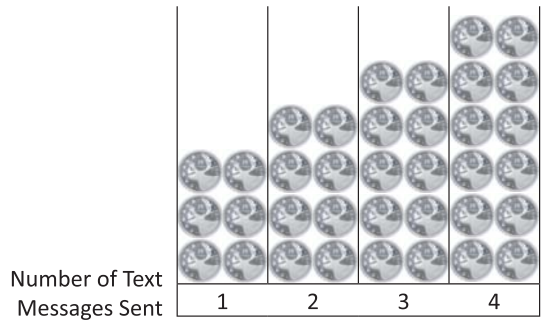
describe the pattern in their own words

develop a table

generate a graph

wr
ite an
equati
on




**SUGGESTED MODELS AND MANIPULATIVES**

linking cubes

colour tiles

toothpicks,

counters

### MATHEMATICAL LANGUAGE

| Teacher | Student |
|---|---|
| algebraic expression | algebraic expression |
| arithmetic patterns | arithmetic patterns |
| decrease | decrease |
| discrete data | discrete data |
| explicit relationship | |
| increase | increase |
| geometric patterns | geometric patterns |
| increasing | increasing |
| linear graph | linear graph |
| linear relationship | linear relationship |
| predict | predict |
| recursive relationship | |
| relationships | relationships |
| repeating | repeating |
| table of values | table of values |
| term | term |
| term number | term number |
| term value | term value |
| unknown term | unknown term |
| values | values |
| variable | variable |

# Resources/Notes

## Print

*Math Makes Sense 7* (Garneau et al. 2007)
Student Book Unit 1 Patterns and Relations (NSSBB #: 2001640)

- Section 1.5 Patterns and Relationships in Tables
- Section 1.6 Graphing Relations
- Unit Problem: Fund Raising

*ProGuide* (CD; Word Files) (NSSBB #: 2001641)

- Assessment Masters
- Extra Practice Masters
- Unit Tests

*ProGuide* (DVD) (NSSBB #: 2001641)

- Projectable Student Book Pages
- Modifiable Line Masters

| **SCO PR04:** Students will be expected to explain the difference between an expression and an equation.<br>[C, CN] | | | |
|---|---|---|---|
| **[C]** Communication | **[PS]** Problem Solving | **[CN]** Connections | **[ME]** Mental Mathematics and Estimation |
| **[T]** Technology | **[V]** Visualization | **[R]** Reasoning | |

## ▪ Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

**PR04.01** Identify and provide an example of a constant term, numerical coefficient, and variable in an expression and an equation.
**PR04.02** Explain what a variable is and how it is used in a given expression.
**PR04.03** Provide an example of an expression and an equation and explain how they are similar and different.

## ▪ Scope and Sequence

| Mathematics 6 | Mathematics 7 | Mathematics 8 |
|---|---|---|
| **PR03** Students will be expected to represent generalizations arising from number relationships using equations with letter variables.<br><br>**PR04** Students will be expected to demonstrate and explain the meaning of preservation of equality concretely, pictorially, and symbolically. | **PR04** Students will be expected to explain the difference between an expression and an equation. | – |

## ▪ Background

Students first learned the term equation in Mathematics 1 when they were introduced to number sentences. The term expression was introduced in Mathematics 2, and the terms coefficient and constant were introduced in Mathematics 5.

Student's previous experience with representing patterns, equality, and inequality should assist them in understanding the difference between the expressions and equations.

An expression is a mathematical phrase composed of number(s) and/or variable(s).

An equation is a statement that two quantities or expressions are equal in value or equivalent.

The major difference between an equation and an expression is that an equation expresses equality through the use of an equal sign.

$3 + 4$ and $3 + y$ are examples of expressions

$3 + 4 = 7$ and $3 + y = 7$ are examples of equations

Variables can have different uses. Students may think that a variable represents a specific unknown value such as in the equation $2n + 12 = 34$. However in an expression, such as $3n – 1$, and an equation, such as $y = 2x + 1$, the variable can have more than one value and therefore varies. It is important for students to understand this distinction. Also students are asked to use variables to make generalizations such as the nth term in a pattern or determining a formula. In this case the variable is used as a pattern generalizer and can have more than one value. It is also important for students to understand that equations like $x + 6 = 10$ can also be written as $10 = x + 6$ without changing the equality or balance. Students need experiences seeing the variable written on either side of the equation.

A numerical coefficient is a quantity (usually a numerical constant), by which a variable is multiplied in an expression or an equation; for example, in the algebraic equation $2p = 10$, the numerical coefficient is 2. Students should work with expressions with a numerical coefficient of 1, as well as with fractional coefficients. Students sometimes have difficulty identifying that the numerical coefficient is 1 in expressions such as $x + 5$. In the equation $\frac{1}{2}$ k = 6, k is the variable, 6 is the constant term and $\frac{1}{2}$ is the numerical coefficient. It may be beneficial for some students to rewrite an equation like $\frac{k}{2}$ + 6 = 10 as $\frac{1k}{2}$ + 6 = 10 so that they can clearly see that the numerical coefficient is $\frac{1}{2}$.

It is important that students understand mathematical conventions, and regularly use lower case letters for variables. Outcome PR04 should be done in conjunction with outcome PR05.

# Assessment, Teaching, and Learning

## ▪ Assessment Strategies

### ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students’ prior knowledge.

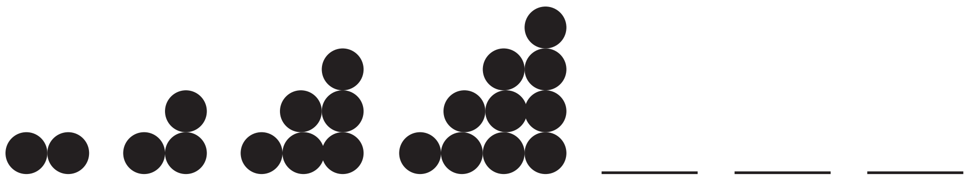
Provide students with a number of equations, such as 25 + 13 = n + 25, and ask them to determine the value of the variable that makes a true statement. Observe whether the students misinterpret the equal sign, or the commutative property by answering 38. Include the four operations in the equations provided.

Ask students to use a balance to model the solution to single-variable, one-step equations such as the following:

$13 + n = 20$ $\quad$ $49 = 7p$ $\quad$ $8t = 40$ $\quad$ $m \div 5 = 7$

Invite students to write and explain the formula for finding the perimeter of any regular polygon (equilateral triangle, square, regular pentagon, regular hexagon, etc.) using variables.

Provide students with pictures or models of the first three steps of an increasing pattern. Invite students to extend the pattern for several more steps, record the pattern in a table, and look for the relationship. Ask them to write the relationship as an expression and use the expression to predict entries at any step.

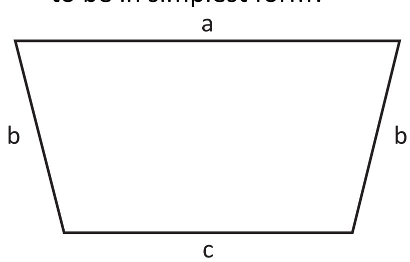


| **Term Number** | **Term Value** |
|---|---|
| 1 | 2 |
| 2 | 3 |
| 3 | 6 |
| 4 | 10 |

**WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS**

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

Write an expression to represent the perimeter of the following figure. Do you consider your expression to be in simplest form?




Complete the following tables.

| Algebraic Expression | Expression in Words | Variable | Numerical Coefficient | Constant |
|---|---|---|---|---|
| $3b - 1$ | One less than three times a number | $b$ | 3 | –1 |
| $x + 3$ | | | | |
| $\frac{n}{2} + 4$ | | | | |
| $5 - 7y$ | | | | |
| $6 + n$ | | | | |

| Algebraic Equation | Equation in Words | Variable | Numerical Coefficient | Constant |
|---|---|---|---|---|
| $3b + 1 = 7$ | One more than three times a number is seven | $b$ | 3 | 1 and 7 |
| $16 = x + 7$ | | | | |
| $\frac{n}{25} + 3 = 7$ | | | | |
| $9 = 15 - 2y$ | | | | |
| $11 = 4 + n$ | | | | |

Write each statement as an expression

a number increased by 14

the quotient of a number and 4

the difference between 35 and a number 75 take away a number

the product of a number and 7

a number decreased by 7

four times a number

Change each statement above so that it could be represented by an equation.

Write an algebraic expression that has a variable *h*, numerical coefficient 4, and constant term 11.

Which of the following are expressions? Which are equations? How are they similar? How are they different?

$2 - x$

$20 = 5v$

$\frac{h}{3} = 4$

$w + 7$

$10 = b + 5$

$7 + a = 9(2)$

Write the algebraic expression that would represent the situation.

Chris worked a certain number of hours yesterday and 8 more hours today.

Loretta earned $10.50 for each hour she worked.

Noel has $20 in his pocket. He earns $11 for each hour he works.

## Planning for Instruction

### CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

Encourage students to describe patterns and rules orally and in writing before using algebraic symbols.

Provide opportunity for students to connect the concrete and pictorial representations to symbolic representations as well as connecting the symbolic representations to pictorial and concrete representations.

Ask students to provide examples of algebraic equations and examples of algebraic expressions. Ask them what it is about the expressions that make them algebraic. Ask students to explain what it is that makes the examples they created equations or expressions. Ask whether an algebraic expression or an algebraic equation can be demonstrated using a balance. Ask students to explain or demonstrate.

## SUGGESTED LEARNING TASKS

Create a set of algebraic expressions and equations on index cards. Create a matching set of cards with the "word" forms of each expression or equation. Randomly distribute the cards among the class. Invite students to find the person whose card matches theirs (e.g., $6k + 3$ would match with "three more than six times a number"). The cards could also be used as a "Concentration Game" in which cards are turned over two at a time and the player determines if they "match." If the cards do not match, the cards are turned back over and the next player takes a turn.

Provide students with some expressions and/or equations in algebraic form, such as

$4p – 5 = b$ $4p – 5$
$p + 5$ $4p – 5 = 55$

Describe the ways in which they are similar and ways in which they differ. Which are equations and which are expressions? Explain why.

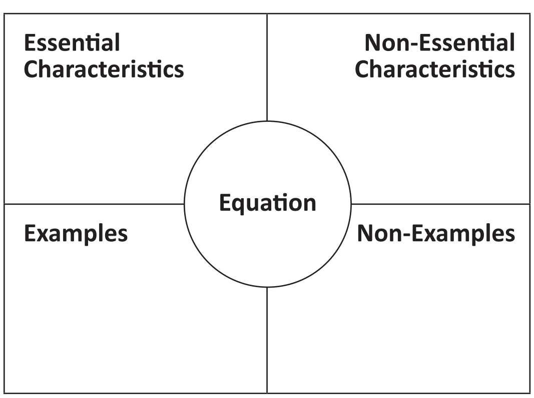
Create a contextual story problem for each equation or expression.

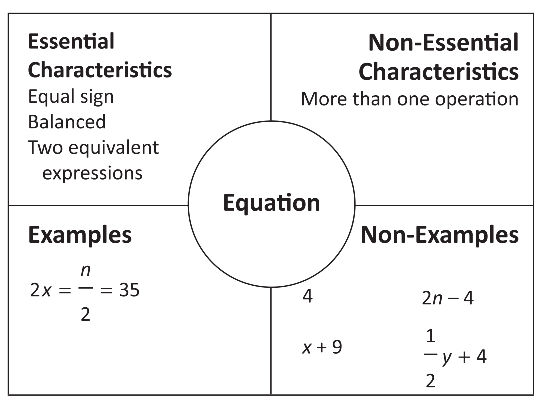
Work collaboratively to create a classroom chart as shown below.

| Algebraic Expression | Expression in Words | Variable | Numerical Coefficient | Constant |
|---|---|---|---|---|
| $3b – 1$ | one less than three times a number | $b$ | 3 | –1 |
| | | | | |

| Algebraic Equation | Equation in Words | Variable | Numerical Coefficient | Constant |
|---|---|---|---|---|
| $3b + 1 = 7$ | one more than three times a number is seven | $b$ | 3 | 1 and 7 |
| | | | | |

Ask students to complete Frayer Model concept maps for expressions and for equations. Once students complete the maps, have them share their ideas with others, modifying their maps as necessary to incorporate new information.



## SUGGESTED MODELS AND MANIPULATIVES

balance
virtual balance

## MATHEMATICAL LANGUAGE

| Teacher | Student |
|---|---|
| equation<br>expression<br>numerical coefficient<br>unknown<br>variable | equation<br>expression<br>numerical coefficient<br>unknown<br>variable |

# Resources

## ▪ Print

*Making Mathematics Meaningful to Canadian Students, K–8* (Small 2008), 587–588

*Making Mathematics Meaningful to Canadian Students, K–8*, Second Edition (Small 2013), 626–628

*Teaching Student-Centered Mathematics, Grades 5–8,* Volume Three (Van de Walle and Lovin 2006b), 279–281

*Math Makes Sense 7* (Garneau et al. 2007)
Unit 1: Patterns and Relations (NSSBB #: 2001640)

- Section 1. 3 Algebraic Expressions
- Section 1.7 Reading and Writing Expressions
- Section 1.8 Solving Equations Using Algebra Tiles
- Unit Problem: Fund Raising

Unit 6: Equations (NSSBB #: 2001640)

- Section 6.1 Solving Equations
- Unit Problem: Choosing a Digital Music Club

*ProGuide* (CD; Word Files) (NSSBB #: 2001641)

- Assessment Masters
- Extra Practice Masters
- Unit Tests

*ProGuide* (DVD) (NSSBB #: 2001641)

- Projectable Student Book Pages
- Modifiable Line Masters

## ▪ Internet

*National Library of Virtual Manipulatives* (Utah State University 2015): http://nlvm.usu.edu/en/nav/vlibrary.html

*Illuminations: Resources for Teaching* Math (National Council of Teachers of Mathematics 2015): http://illuminations.nctm.org

| **SCO PR05:** Students will be expected to evaluate an expression given the value of the variable(s). [CN, R] | | | |
|---|---|---|---|
| **[C]** Communication | **[PS]** Problem Solving | **[CN]** Connections | **[ME]** Mental Mathematics and Estimation |
| **[T]** Technology | **[V]** Visualization | **[R]** Reasoning | |

## ▪ Performance Indicator

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

**PR05.01** Substitute a value for an unknown in a given expression and evaluate the expression.

## ▪ Scope and Sequence

| Mathematics 6 | Mathematics 7 | Mathematics 8 |
|---|---|---|
| **PR03** Students will be expected to represent generalizations arising from number relationships using equations with letter variables.<br><br>**PR04** Students will be expected to demonstrate and explain the meaning of preservation of equality concretely, pictorially, and symbolically. | **PR05** Students will be expected to evaluate an expression given the value of the variable(s). | – |

## ▪ Background

We can use symbols to represent a pattern. A variable is a symbol that represents an unknown quantity. Students are familiar with variables in formulas, such as area = base *x* height, A = bh. Students might relate variables to things that change over time that are part of their own experiences, such as their height.

Some letters used as variables may be confusing to students as they have more than one meaning. For example, x may be mixed up with the multiplication symbol or m may be confused with metres. It is important that an expression such as 3*m*, with no space between the 3 and the m, is read as “a number multiplied by 3,” or “3 times *m*.” If it is written as 3 m, with a space between the 3 and the m, then the m represents a quantity such as metres and 3 m could be read as 3 metres. When evaluating algebraic expressions, ensure that students understand the meaning of such notations.

It is also possible for students to confuse the placement of variables when writing expressions or equations; for example, if there are 6 notebooks (n) for each student (s), they might write $s = 6n$, instead of $n = 6s$. Reading each of these equations as a sentence will help students determine which equation correctly represents the context.

To evaluate an algebraic expression, students substitute a number for the variable and carry out the computation. Using real-life situations relevant to students will help with this.

Draw students' attention to expressions such as $4h + 8$, where multiplication is used. If $h = 5$, for example, a common student error is writing 45 + 8 instead of 4(5) + 8 or $4 \cdot 5 + 8$.

Students should also be aware that division is often represented as a fraction, such as $8 - \frac{m}{2}$ or $8 - m \div 2$. Outcome PR04 should be done in conjunction with outcome PR05.

# Assessment, Teaching, and Learning

## ▪ Assessment Strategies

### ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

Ask students to determine the value of the variable to make a true statement.

$18 + n = 31$ $81 = 9 \times t$ $8 \times w = 56$ $x \div 6 = 7$

### WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (which can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

Determine which expression has the greatest value if $p = 8$.

$p + 7$
$2p$
$10 - p$
$8 \div p$
$3p - 12$
$2 + 2p$

Evaluate the following:

$2k + 5$ when $k = 21$
$5 + 4m$ when $m = 4.2$
$\frac{y}{4} + 22$ when $y = 60$
$-3 + 5q$ when $q = 2$

Explain the steps used to evaluate the following expressions for the given value of the variable:

$3p + 5$, for $p = 1$
$2m - 3$, for $m = 6$

## ▪ Planning for Instruction

### CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

Provide opportunity to connect the concrete and pictorial representations to symbolic representations, as well as connecting the symbolic representations to pictorial and concrete representations. For example, introduce the concept of algebraic expressions and equations using real-life examples. For example, if you pay a basic fee of $20 a month for a cell phone and are charged $0.90 for each text message, your monthly bill could be determined using the expression $20 + 0.90h$.

**SUGGESTED LEARNING TASKS**

Evaluate the following expressions if $n = 5$ and $v = 2$.

$6n$

$3 + v$

$4(n + 2)$

$10v$

$n + 23$

$7v + 8$

$\frac{25}{n}$

$35 - 2v$

$6n - \frac{12}{2}$

Alisha gets paid $9 an hour to babysit. She gets a bonus of $5 per hour if she works past 10 pm. The expression $9m + 5$ represents what Alisha got paid last night. What does the variable in this expression represent? What is the constant in this expression? What does it represent? If she babysat from 5:00 pm till midnight, how much did she earn last night?

**SUGGESTED MODELS AND MANIPULATIVES**

algebra tiles

linking cubes

**MATHEMATICAL LANGUAGE**

| Teacher | Student |
|---|---|
| evaluate<br>substitution<br>unknown<br>variable | evaluate<br>substitution<br>unknown<br>variable |

# Resources

- ## Print

*Math Makes Sense 7* (Garneau et al. 2007)
Unit 1: Patterns and Relations (NSSBB #: 2001640)

Section 1. 3 Algebraic Expressions

Section 1.4 Relationships in Patterns

Section 1.8 Solving Equations Using Algebra Tiles

Unit Problem: Fund Raising

*ProGuide* (CD; Word Files) (NSSBB #: 2001641)

Assessment Masters

Extra Practice Masters

Unit Tests

*ProGuide* (DVD) (NSSBB #: 2001641)

Projectable Student Book Pages

Modifiable Line Masters