Mathematics 8

Unit 5: Percent, Ratio, and Rate

N03, N04, N05

SCO N03 Students will be expected to demonstrate an understanding of and solve problems involving percents greater than or equal to 0%. [CN, ME, PS, R, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N03.01 Provide contexts where a percentage may be between 0% and 1%, between 1% and 100%, and more than 100%.
N03.02 Represent a given fractional percentage using concrete materials and pictorial representations.
N03.03 Represent a given percentage greater than 100% using concrete materials and pictorial representations.
N03.04 Determine the percentage represented by a given shaded region on a grid, and record it in decimal, fractional, and percent form.
N03.05 Express a given percent in decimal or fraction form.
N03.06 Express a given decimal in percent or fraction form.
N03.07 Express a given fraction in decimal or percent form.
N03.08 Solve a given problem involving percents mentally, with pencil and paper, or with technology, as appropriate.
N03.09 Solve a given problem that involves finding the percent of a percent.

Scope and Sequence

Mathematics 7	**Mathematics 8**	**Mathematics 9**
N03 Students will be expected to solve problems involving percents from 1% to 100% (limited to whole numbers).	**N03** Students will be expected to demonstrate an understanding of and solve problems involving percents greater than or equal to 0%.	—

Background

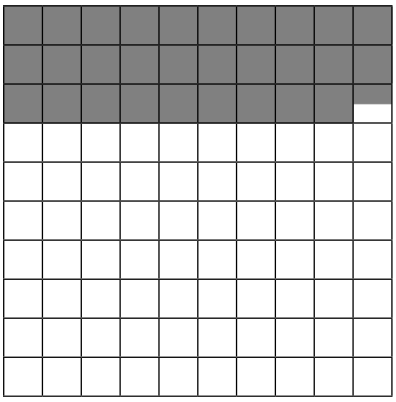
Percentages are ratios that compare a number to 100. Percentages can range from 0 to higher than 100. In Mathematics 7 (N03) students represent a quantity as a percentage, fraction, decimal, or ratio. Percentages have the same value as their fraction, decimal, and ratio equivalent, and this can be useful in solving problems with percentages.

In Mathematics 7 (N03), students worked with percentages from 1% to 100%. In Mathematics 8, students examine contexts where percentages can be greater than 100% or less than 1% (fractional percentages).

Students should be able to move flexibly between percentage, fraction, and decimal equivalents in problem solving situations. For example, when finding 25% of a number, it is often much easier to use

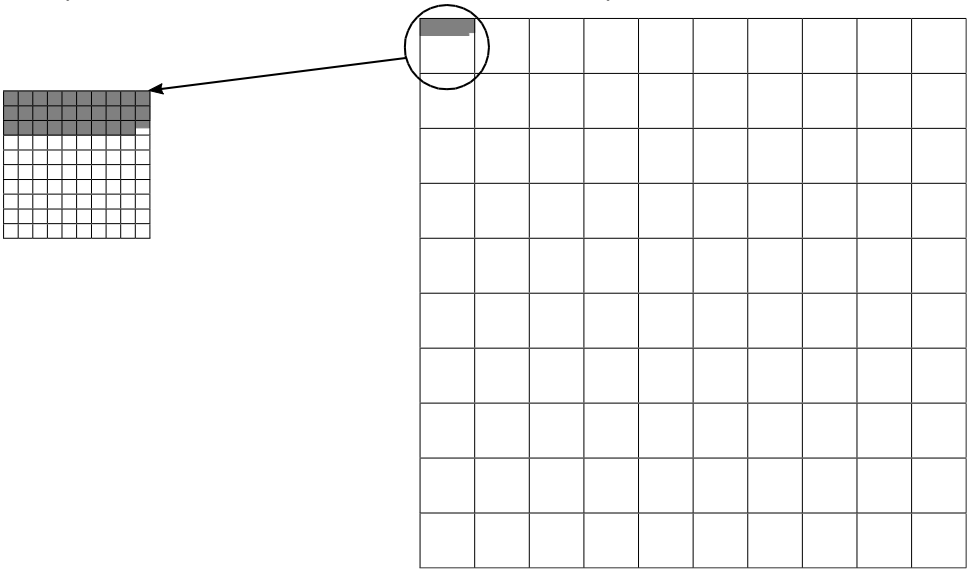
$\frac{1}{4}$ and then divide by 4 as a means of finding or estimating the percentage. If students can express fractions and decimals as hundredths, the term “percent” can be substituted for the term “hundredths.” The fraction $\frac{3}{2}$ can be expressed in hundredths, $\frac{150}{100}$ which has a decimal equivalent of 1.5, which is equivalent to 150%.

In previous grades, when working with whole number percentages from 1% to 100%, students represented them using 10 x 10 grid paper. In Mathematics 8, this is expanded to percentages between 0% and 1%, percentages greater than 100%, as well as other fractional percentages. Begin with a 10 x 10 grid to represent percentages. If the entire grid represents 100%, then each small square represents 1%. For fractional percentages that are easily recognizable, e.g., 0.5%, shade one-hald of one small square. To represent 29.5%, use grid paper and shade in 29 small squares and one-half of another small square.



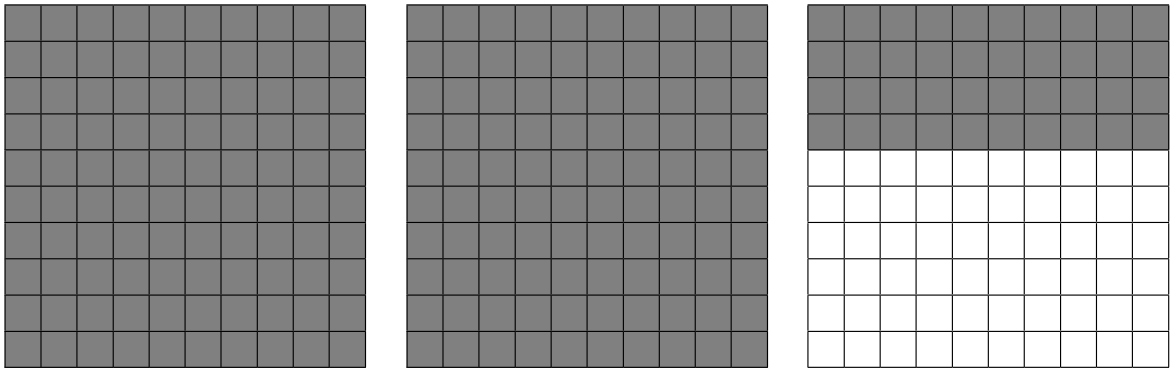
Fractional percentages less than 1% can be represented by zooming in on the 1% square, further subdividing it and shading in the appropriate area.

To represent 0.28%, the 1% is subdivided into 100 parts and 28 blocks out of 100 are shaded.



To represent $\frac{2}{3}$ %, the 1 % is subdivided into 3 parts and 2 are shaded.

Percentages greater than 100% are represented using more than one 10 x 10 grid chart. The diagram below represents 240%.



In this diagram, two full hundred grid charts and 40 blocks of another hundred grid chart are shaded.

The skills students learned in 7N03 taught students to convert between percentage, fraction, and decimal equivalents for whole number percentages between 1% and 100%. They will apply these skills to fractional percentages between 0% and 1%, percentages greater than 100%, as well as other fractional percentages.

Fractional percentages between 0% and 1% must be developed at a sensible pace. There is sometimes a tendency among students to see the percentage 0.1% as the decimal 0.1. It is important to distinguish the difference in these two forms. Similarly, students may confuse $\frac{3}{4}$ % with 75%. The hundreds and hundredths grid charts will help distinguish these differences. Given a shaded region on a grid, students will be expected to express the shaded region in fraction, decimal, or percentage form.

Another strategy that can be used when dealing with percentages greater than 100% and between 0% and 1% is patterning. For example:

Percent	Decimal	Fraction
0.3%	0.003	$\frac{3}{1000}$
3%	0.03	$\frac{3}{100}$
30%	0.3	$\frac{3}{10}$
300%	3	$\frac{3}{1}$

Percent	Decimal	Fraction
70%	0.7	$\frac{7}{10}$
7%	0.07	$\frac{7}{100}$
0.7%	0.007	$\frac{7}{1000}$
0.07%	0.0007	$\frac{7}{10000}$

Fractional and decimal percentages can be related to benchmark percentages. For example, 0.25% means one-fourth of 1%. If you know 1% of 400 is 4, then 0.25% of 400 would be a one-fourth of 4 or 1. It is also important to recognize that 1% can be a little or a lot depending on the size of the whole. For example, 1% of all of the population of a city is a lot of people compared to 1% of the students in a class.

Students will continue to create and solve problems that they explored in Mathematics 7, which involve finding a, b, or c in a relationship of a% of b = c using estimation and calculation. However, the problem-solving situations will be more varied. As an application, students will be required to apply percentage increase and decrease in problem situations for self, family, and communities, in which percentages greater than 100 or fractional percentages are meaningful. They will apply their knowledge of percentages to find a number when a percent of it is known, and to find the percent of a percent.

A common example of combined percents is addition of percents, such as taxes. Students encounter combined percentages every day when they buy items at stores and pay sales tax. Although this tax appears to be just one percentage, it is a “harmonized sales tax” (HST), which includes both federal and provincial sales tax rates.

Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students’ prior knowledge.

- Ask students to change each of the following to a percentage mentally and to explain their thinking:
  $\frac{2}{5}$ $\frac{4}{25}$ $\frac{6}{50}$ $\frac{8}{20}$
- Ask students to estimate the percent for each of the following and to explain their thinking:
  $\frac{7}{48}$ $\frac{5}{19}$ $\frac{7}{20}$
- Give students 10 × 10 grid and ask them to shade percentages from 1 to 100 percent.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Estimate the percent for each fraction. Explain your reasoning.
  - $\frac{125}{85}$
  - $\frac{99}{95}$
  - $\frac{2}{230}$

- Use mental mathematics to solve the following:
  - If 2% of a number is 4, what is the number?
  - What would 11.5% of that number be? (Hint: think 10% + 1% + 0.5%)
- Provide students with 10 × 10 grid paper and have them shade in amounts to represent given percentages greater than 100% (e.g., 124%, 101%, 150%).
- Ask students to explain the meaning of the following scenario statements and give reasons for their explanations.
  - When your coach tells you to "give 110%," what does she mean?
  - What is the chance that the principal will give you the day off school because of your smile?
  - A newspaper article includes 200% in its headline. Give a situation to which the article may be referring.
  - The Cape Breton Screaming Eagles handed out T-shirts to the first 100 fans at a hockey game. This represented $\frac{1}{2}$ % of the fans who attended that game.
  - The school reached 150% of its goal in collecting food items for the local food hamper.
- Show students a hundredths grid that is shaded and have them record the percent, decimal, and fraction that is represented by the shading.
- Ask students to respond to the following: Jill predicted that the chance of Maple Academy winning the championship game against Evergreen Collegiate is 0.50%. Which school do you think Jill attends? Explain your choice.
- Have students express a variety of percentages, decimals, and fractions in all three forms. This could be done in a chart. They could also represent these by shading in a grid or using materials.

Percent	Decimal	Fraction
146%		
	0.003	
		$\frac{140}{100}$

- Ask students to solve the following problems and explain their reasoning:
  - Superstar basketball sneakers, which regularly sell for $185, were marked down by 25%. To further improve sales, the discount price was reduced by another 15%. What was the final selling price? Explain why this is not the same as a 40% discount. What would the difference be?
  - About 0.6% of Nova Scotia's population lives in Wolfville. The population of Nova Scotia is about 750 000. What is the population of Wolfville? If the population in Wolfville increases by 1000 when students attend Acadia University, what percent increase would this be?

- The price of a $250 video game console was increased by 25%. After two weeks the price was reduced by 25%. Explain why the final price is not $250.

- Ask students to respond to the following:
  - Two stores offer different discount rates as follows:
  - Store A: 50% off, one day only.
  - Store B: 25% off one day, followed by 25% off the reduced price the second day.
  - Which store has the better sale?

  - A jacket cost $100. The discount on the jacket is 15%. However you must also pay 15% sales tax. Would the jacket cost you $100, less than $100, or more than $100? Explain your reasoning.
  - Charlie works part-time at a local fast-food restaurant. On his next pay cheque, he will receive a 5% increase in pay on top of a 10% performance bonus. Charlie tells his friends he is receiving a 15% raise in pay. Is he correct? Explain.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

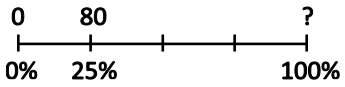
Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

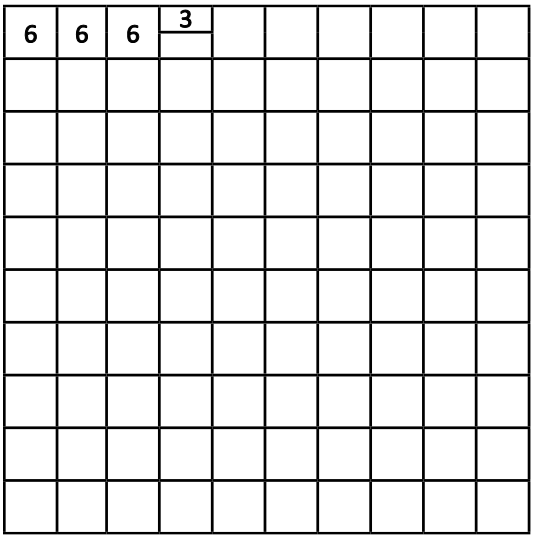
Consider the following strategies when planning daily lessons.

- Discuss with students the relevance of percentages in real-world applications (e.g., sales tax, discounts, sports statistics). Compile with students a list of situations where percentages are used. This list may include, but is not limited to:
  - test marks (78% on a science test)
  - sales tax (15% tax on all sales)
  - discount (25% off all purchases)
  - probability (10% chance of rain)
  - athletic statistics (scored 25% of shots on goal)
- Discuss with students situations that may result in percents greater than 100%. Ask students questions such as the following:
  - What percentage has the cost of soda pop increased when today's cost is compared to the cost 50 years ago?
  - What percentage has the cost of a rare collectable item increased compared to its original value?
- Discuss with students situations that may result in percents between 0% and 1%. Ask students questions such as the following:
  - What is the percent chance that it will snow in August?

- Use a visual model for benchmark percentages like 10%, 25%, or 50% in questions, such as the following:
  - 25% of a number is 80. What is the number?



- Employ a variety of strategies to calculate the percentage of a number:
  - Represent with 10 × 10 grid:
    - > $3\frac{1}{2}$ % of 600



  - Partition the percent:
    - > 110% of 80:

      100% of 80 = 80
      10% of 80 = 8
      8 + 80 = 88

  - Represent the percent as a decimal equivalent and multiply:
    - > finding 110% of 80:

      = 1.1 × 80 = 88
    - > finding 0.5% of 800, find 1% and halve it:

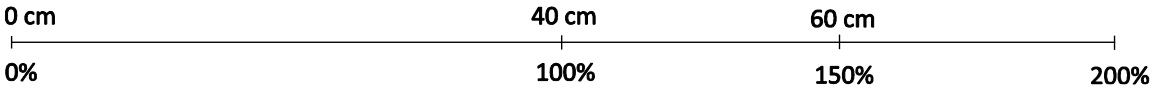
      1% of 800 = 8

      $\frac{1}{2}$ of 8 = 4
    - > changing to a fraction and dividing:

      25% of 60 = $\frac{1}{4}$ × 60 = 60 ÷ 4 = 15
    - > using a proportion:

$$0.7\% \text{ of } 300 = \frac{0.1}{100} = \frac{x}{80}$$

- Use a 10 x 10 grid to represent 100%. Show examples of percentages greater than 100% such as, 260% and have students shade the grids to represent the amount or ask students to identify the percent shaded.
- Use a metre stick to represent 100%. Use more than one metre stick to show examples of percents greater than 100%.
- Use a double number line (vertical or horizontal) to solve percentage problems. For example, a length of 40 cm is increased by 50%. What is the new length?



Suggested Learning Tasks

- As a decimal 140% = 1.40. Use patterning to write the following percents in decimal form.
  - 14%
  - 1.4%
  - 0.14%
- As a fraction 0.09% = $\frac{9}{10000}$, use patterning to write the following percents in fraction form.
  - 0.9%
  - 9%
  - 90%
  - 900%
- Substitute a variety of numbers in the following problems and solve.
- A school has a total enrolment of (527 or 200) students.
  - Yesterday (11.5% or 15%) were absent. How many came to school?
  - The current enrolment is 250% of the enrolment fifteen years ago. What was the enrolment then?
- Estimate the percent increase represented in the following problem. Derek's father said, "In my day, I could buy a chocolate bar and a soft drink for 25¢. Using your knowledge of the cost of these items today, what is the percent increase?
- Use a double number line to model: The student council president was elected with 215 votes. If she received 58% of the votes cast, about how many votes were cast?
- Construct a model to represent the following: Jimmy has an hour and 20 minutes to finish 5 tasks. What percentage of time can Jimmy spend on each task if each task takes an equal amount of time?
- Your friend was absent from school when your teacher explained fractional percentages. When he was studying for his test, he said that $\frac{1}{2}$ % was 0.5 as a decimal. How would you use a model or diagram to help him to understand the mistake he made?
- Trina received an 80% on a recent math test. If she answered 48 questions correctly, how many questions were on the test?

- Adam increased his song list by 40%. If he had 300 songs originally, how many songs does he now have?
- Shawn earned $85 and spent $15. What percent of his money did he spend?
- Last week the canteen sold 60 sandwiches. This week they sold 48 sandwiches. Calculate the percent decrease. How can you check that the percent decrease is correct?
- Copy and complete the following table.

Percent	Decimal	Fraction
148%		
$\frac{7}{20}$ %		
26.4%		
	2.65	
	0.003	
	0.254	
		$\frac{8}{5}$
		$\frac{1}{250}$
		$\frac{3}{8}$

- Catherine said that her amount of homework increased 400% when it went from one half hour of work to two hours of work. Do you agree? Explain.
- Cyril collects hockey cards. He had 150 cards in his collection. His birthday was in June and his friends gave him hockey cards as presents which increased his collection by 20%. At Christmas his hockey card collection increased by another 15%. How many cards are in his collection after this 15% increase?
- If  represents 50%, what diagram represents 250%?
- If  represents 125%, what diagram represents 75%

SUGGESTED MODELS AND MANIPULATIVES

- Calculator*
- coins
- fraction circles*
- fraction pieces
- fraction strips
- hundredths grid
- thousandths grid
- metre stick
- number line / double number line*
- open number lines
- various objects for counting (e.g., beans, counters)

*also available in *Interactive Math Tools* (Pearson n.d.

MATHEMATICAL LANGUAGE

Teacher	Student
▪ combined percent ▪ decimal ▪ fraction ▪ fractional percent ▪ percent ▪ percentage ▪ ratio ▪ GST ▪ PST ▪ HST	▪ combined percent ▪ decimal ▪ fraction ▪ fractional percent ▪ percent ▪ percent increase ▪ percent decrease ▪ ratio ▪ GST ▪ PST ▪ HST

Resources

Print

Math Makes Sense 8 (Baron et al. 2008; NSSBB #: 2001642)

- Unit 5: Percent, Ratio, and Rate
  - Section 5.1: Relating Fractions, Decimals, and Percents
  - Section 5.2: Calculating Percents
  - Section 5.3: Solving Percent Problems
- *ProGuide* (CD; Word Files) (NSSBB #: 2001643)
  - Assessment Masters
  - Extra Practice Masters
  - Unit Tests
- *ProGuide* (DVD) (NSSBB #: 2001643)
  - Projectable Student Book Pages
  - Modifiable Line Masters

Math Matters: Understanding the Math You Teach, Second Edition (Chapin and Johnson 2006), 149–164.

Teaching Student-Centered Mathematics, Grades 5–8, Volume Three (Van de Walle and Lovin 2006), 175.

Proportional Reasoning (Erickson 2000; NSSBB #: 23187)

Digital

- "Interactive Hundredths Grid [unnamed]," *McGraw-Hill Education* (McGraw-Hill Education 2015): www.glencoe.com/sites/common_assets/mathematics/ebook_assets/vmf/VMF-Interface.html.

SCO N04 Students will be expected to demonstrate an understanding of ratio and rate. [C, CN, V]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N04.01 Explain the multiplicative relationship found within a ratio.
N04.02 Represent a two-term ratio from a given context concretely and pictorially and record using the forms 3:5 or 3 to 5.
N04.03 Express a three-term ratio from a given context in the forms 4:7:3 or 4 to 7 to 3.
N04.04 Express a part-to-part ratio as a part-to-whole fraction.
N04.05 Identify and describe ratios and rates (including unit rates) from real-life examples and record them symbolically.
N04.06 Express a given rate using words or symbols.
N04.07 Express a given ratio as a percentage, and explain why a rate cannot be represented as a percentage.

Scope and Sequence

Mathematics 7	**Mathematics 8**	**Mathematics 9**
N03 Students will be expected to solve problems involving percents from 1% to 100% (limited to whole numbers). **SP04** Students will be expected to express probabilities as ratios, fractions and percents.	**N04** Students will be expected to demonstrate an understanding of ratio and rate.	**N03** Students will be expected to demonstrate an understanding of rational numbers by comparing and ordering rational numbers and solving problems that involve arithmetic operations on rational numbers.

Background

Students have had previous experiences with **ratios**. In Mathematics 6 (N05), they defined, represented, and interpreted ratios of two numbers presented to them concretely. In Mathematics 7 (N03), they solved proportions within problem-solving situations involving percentage. In Mathematics 8, students will build on and extend their knowledge of ratios. They will also be introduced to **rate**. They will describe and record rates using real-life examples. Experiences with problem-solving situations using unit rates and unit prices should lead them to make connections between mathematics and everyday life.

Understanding that a ratio is a multiplicative comparison between two quantities is critical to the development of proportional reasoning. Encourage students to look for the multiplicative relationships within a ratio. For example, if two cans of blue paint are going to be mixed with six cans of yellow paint to produce green paint, the ratio of blue paint to yellow paint is 2:6. It can be said that the number of cans of blue paint is $\frac{1}{3}$ the number of cans of yellow paint, or that the number of cans of yellow paint is three times the number of cans of blue paint. These are examples of the multiplicative relationships found in this ratio. Students should be able to express these relationships in words. Students that understand the multiplicative relationship will be able to state other amounts of blue cans of paint and yellow cans of paint that will produce the same shade of green.

If a student describes the relationship between the number of blue cans of paint and yellow cans of paint this way; the number of blue cans of paint is four less than the number of yellow cans of paint, they see an additive comparison and not a multiplicative comparison. An additive comparison is not a ratio and students need to be able to think multiplicatively before they can be successful solving problems that will use proportional reasoning.

Ratios are classified by the type of comparison, either **part-to-part** (comparison of one part of the whole to another part of the same whole), **part-to-whole** (comparison of one part to the whole) for comparing measures or number or measures of the same type. Consider the following examples: 13 people rode the rollercoaster; 2 of them were girls. The ratio of girls to people was 2:13. This gives an example of a part-to-whole ratio: In an orchard with 80 trees, 43 of the trees were apple trees and the rest were pear trees. The ratio of apple trees to pear trees was 43:37, which represents a part-to-part ratio.

Part-to-whole relationships are fractions and can also be written in fractional form. Therefore 2:13 could also be written as $\frac{2}{13}$. Students need to understand that all fractions are also ratios but not all ratios are fractions.

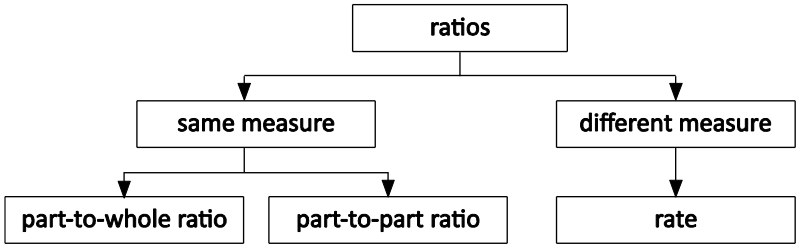
It should be emphasized that only part-to-whole ratios are expressed using fraction notation because a part-to-whole relationship is a fraction. For example, Daniel gets a hit 2 out of every 9 times he is at bat. The ratio of hits to bats can be written as 2 to 9, 2:9, or $\frac{2}{9}$.

Examples are also common using **three-term ratios**. If 13 people rode the rollercoaster and 2 of them were girls, the ratio Boys: Girls: Total People is 11:2:13, or 11 to 2 to 13. Similarly, in an orchard with 80 trees, if 43 of the trees were apple trees and the rest were pear trees, a resulting three-term ratio comparing Apple Trees to Pear Trees to Total Trees is 43 to 37 to 80 or 43:37:80.

Once students have an understanding that a part-to-part ratio compares one part of a set to another part of the same set, while a part-to-whole ratio compares one part of a set to the whole set, they should be able to write a part-to-part ratio as a part-to-whole ratio. For example, 1 can frozen juice concentrate to 4 cans of water can be represented as $\frac{1}{5}$, which is the ratio of concentrate to solution, or $\frac{4}{5}$, which is the ratio of water to solution.

Ratios are encountered frequently when describing real-world situations. Have students write ratios in words first. This will help students write the terms of the ratio in the correct order of comparison when expressing them symbolically.

Students have worked with ratios, which compare quantities with the same unit. Now the focus shifts to rates, which involve quantities with different units. However, the mathematics used to talk about rates is the same as the mathematics used to talk about ratios. Both represent comparisons. A graphic organizer, such as the following, may be helpful in making sense of the different types of ratios.



Remind students to include the units when writing rates.

Because a rate compares quantities measured in different units, a rate has no meaning without the units. Students should be familiar with numerous and various examples of rates already, even if they couldn't previously identify them as rates. Have the class brainstorm to identify as many real-life examples of rates as possible. Some examples that students should relate to include speed (km/h), text messaging rates ($/month), and school schedules (periods/day or days/cycle).

It is important to continue emphasizing that a rate compares two different quantities. In any context students may provide in which $\frac{a}{b}$ represents a rate, two quantities in different units are being compared.

The distinction between ratios and rates is subtle. As students work with rates, they should be encouraged to continue to examine the similarities and differences and make connections between ratios and rates. A fundamental difference between the two is the ability to represent a ratio, but not a rate, as a percent. Students should recall that part-to-whole ratios can be expressed as a percent, whereas part-to-part ratios cannot. For example, if Daniel gets a hit 2 out of every 9 times at bat, this is a ratio that can be written as a percent. If the hits are thought of as successful at bats, parts of the whole are being compared to the whole. A percent compares a number to 100. Because the units in a rate are different, there isn't a whole to make a comparison to. Therefore, a rate cannot be represented as a percent.

Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Ask students to use pattern blocks to represent the following ratios and to write the ratio symbolically (e.g., trapezoids to triangles, triangles to all).
  - 3 trapezoids to 5 equilateral triangles
  - 4 rhombi to 6 equilateral triangles
- Ask students to express the following ratio comparisons from the diagram below:
  ☺☺☺☺♥
  - The ratio of faces to hearts.
  - The ratio of hearts to faces.
  - The ratio of faces to all.
  - The ratio of all to hearts.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Write a part-to-part and part-to-whole ratio for each situation below.
  - An aquarium has 3 guppies and 5 goldfish.
  - In the parking lot there are two types of vehicles: cars and trucks. There are 30 vehicles in total and seven of them are trucks.
  - You buy a mixed variety bag of apples containing 5 Honeycrisp apples and 7 Macintosh apples.
- Determine who would get the bigger portion of pizza if 9 girls share 4 pepperoni pizzas while 7 boys share 3 vegetarian pizzas. Explain your reasoning. What assumptions are you making?
- Ask the student to select three different colours of tiles, and model the following ratios:
  - 4 to 3
  - 2:1
  - $\frac{1}{3}$
  - 2:3:5
- Write a response to the following situation. If a tap is dripping at a rate of 50 mL an hour, can you describe that as a percentage? Explain why or why not.

Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Present a collection of items that all have something in common (e.g., art supplies or types of sports balls). Ask students to describe the comparisons as ratios using part-to-part and part-to-whole ratio notation. (Note: part-to-part ratios cannot be written as fractions).
- Discuss with students that all fractions are ratios but not all ratios are fractions.
- Ensure students understand why rate cannot be represented as a percent. For example, if 200 out of 500 students in a school are going to the dance, we could also say that 40 percent will be attending. In this case the ratio is comparing the part (students attending) against the whole (the entire school population). This is different from a rate, which compares two quantities with different units such as speed in kilometres per hour. Since rates compare different quantities, they cannot be represented as a percent which compares part to whole of only one quantity (e.g. a school population).
- Discuss the best way to describe each of the following:
  - the speed you travel on the highway
  - the number of eggs a family uses in a day, a week, and a month
  - hockey player achievements per minutes played

Suggested Learning Tasks

- In your school state these ratios:
  - students to teachers
  - teachers to students
  - classrooms to total number of rooms
  - soccer balls to basketballs to total number of balls
  - window to doors
  - desks to chairs
- Sort the following as ratios or rates:
  - a store clerk is paid $11.40 per hour
  - the average test score for the previous unit was 82%
  - a recipe for cleaner calls for 1 part baking soda, 2 parts vinegar and 4 parts water
  - for every hour I spend on homework, I spend 2 hours playing basketball
  - a car drives 240 km in 3 hours
- Shade in part of a 10 × 10 grid. Exchange your grid with a partner and then represent the portion shaded on your partner's grid in more than one way.
- Find examples of rates (e.g., in shops, in flyers, on the Internet) or develop a poster displaying rates. Describe in your math journals the meaning of the rates you chose using words and symbols.
- Write each ratio as a fraction in simplest form.

- 14 to 6
- 4:22
- 18:12
- 25 to 20
- 18:21
- 18:3
- 7:21
- 20 to 9
- 4:10
- 84 to 16

- Colour the following grid so that the ratio of shaded tiles to non-shaded tiles is 5:3.

SUGGESTED MODELS AND MANIPULATIVES

- colour tiles*
- hundredths grid
- pattern blocks*
- ratio tables

*also available in *Interactive Math Tools* (Pearson n.d.)

MATHEMATICAL LANGUAGE

Teacher	Student
▪ part-to-part ratio ▪ part-to-whole ratio ▪ rate ▪ three-term ratio ▪ two-term ratio ▪ ratio	▪ part-to-part ratio ▪ part-to-whole ratio ▪ rate ▪ three-term ratio ▪ two-term ratio ▪ ratio

Resources

Print

Math Makes Sense 8 (Baron et al. 2008; NSSBB #: 2001642)

- Unit 5: Percent, Ratio, and Rate
  - Section 5.5: Exploring Ratios
  - Section 5.6: Equivalent Ratios
  - Game: *Triple Play*
  - Section 5.7: Comparing Ratios
  - Section 5.8 A: Proportional and Non-proportional Situations (Nova Scotia companion document only; found on Mathematics Learning Commons, Grades 7–9)
  - Section 5.8 B: Solving Problems That Involve Proportional Reasoning (Nova Scotia companion document only; found on Mathematics Learning Commons, Grades 7–9)
  - Section 5.9: Exploring Rates
  - Section 5.10: Comparing Rates
  - Unit Problem: What Is the Smartest, Fastest, Oldest?
- *ProGuide* (CD; Word Files) (NSSBB #: 2001643)
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Teaching Student-Centered Mathematics, Grades 5–8, Volume Three (Van de Walle and Lovin 2006), 154–175.

Math Matters: Understanding the Math You Teach, Second Edition (Chapin and Johnson 2006), 165–189.

N05 Students will be expected to solve problems that involve rates, ratios, and proportional reasoning. [C, CN, ME, PS, R]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

N05.01 Explain the meaning of $\frac{a}{b}$ within a given context.

N05.02 Provide a context in which $\frac{a}{b}$ represents a fraction, a rate, a ratio, a quotient, and a probability.

N05.03 Use pictures, models, or manipulatives to make sense of a proportional situation.

N05.04 Differentiate between proportional and non-proportional contexts.

N05.05 Use multiplicative relationships to compare quantities and to predict the value of one quantity based on the values of another.

N05.06 Use multiple methods to solve proportional tasks and understand that these methods are related to each other.

N05.07 Use estimation to determine the reasonableness of an answer.

N05.08 Solve a proportion using mental mathematics, pencil and paper, or technology, as appropriate.

N05.09 Solve a given problem involving rate, ratio, or percent using mental mathematics, pencil and paper, or technology, as appropriate.

N05.10 Create problems that are examples of proportional reasoning.

Scope and Sequence

Mathematics 7	**Mathematics 8**	**Mathematics 9**
N03 Students will be expected to solve problems involving percents from 1% to 100% (limited to whole numbers). **SP04** Students will be expected to express probabilities as ratios, fractions, and percents.	**N05** Students will be expected to solve problems that involve rates, ratios, and proportional reasoning.	**N03** Students will be expected to demonstrate an understanding of rational numbers by comparing and ordering rational numbers and solving problems that involve arithmetic operations on rational numbers.

Background

Rate, ratio, and proportion are three related concepts. They are used in many problem-solving opportunities. For example, ratios, rates, and proportions are commonly used for scale models, altering a recipe, and comparison shopping.

A ratio is a comparison of two quantities or measures. In the previous outcome, students learned about the multiplicative relationships that are the basis of a ratio. This multiplicative thinking will be necessary

as students now study proportions. A proportion is a statement of equality between two ratios. To be proportional thinkers, students

- need to develop an understanding of the multiplicative relationship within a ratio and between the equal ratios that make a proportion
- know the mathematical characteristics of proportional situations
- be able to differentiate mathematical characteristics of proportional thinking from nonproportional contexts
- understand realistic and mathematical examples of proportional situations
- realize that multiple methods can be used to solve proportional tasks and that these methods are related to each other
- know how to solve quantitative and qualitative proportional-reasoning tasks
- be unaffected by the context of the numbers in the task

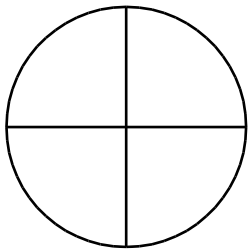
While two ratios can represent the same relationship, different notations for proportions can be used. For example,

2:5 = 4:10 or 2 to 5 = 4 to 10 or $\frac{2}{5} = \frac{4}{10}$

These can all be read as “two is to five as four is to ten.”

To effectively understand and solve ratio problems, comparing equivalent ratios or creating a proportion is necessary. Begin by using models or pictures to make sense of proportions. An example would be to use diagrams such as the following and ask students to complete a second diagram to show the same relationship as given in the first diagram.





To solve problems that can be represented as a proportion, students need to understand the multiplicative relationship found within a ratio or between two ratios. Understanding these relationships will help students solve proportions for missing information. Provide opportunities for students to gain a conceptual understanding of proportions using numbers that can be manipulated mentally with ease. Only after this understanding is gained can numbers that do not lend themselves to more intuitive methods be used.

Present students with a problem like the following.

- The ratio of indoor basketballs to outdoor basketballs at the recreation centre is 6:2. New basketballs have to be purchased. If the recreation centre is going to purchase 24 indoor basketballs, how many outdoor basketballs will they buy?

Ratios can be reviewed by asking students to describe in words the multiplicative relationship found in the ratio 6:2. We would want students to be able to say that the number of indoor basketballs is three times the number of outdoor basketballs or the number of outdoor basketballs is one-third the number of indoor basketballs. If a student says that the number of indoor basketballs is four more than the number of outdoor basketballs, they are thinking additively and not multiplicatively. Since additive comparisons are a common misconception and not part of proportional thinking, they will need more practice describing the multiplicative relationship in a ratio before they will be successful solving proportions.

A proportion that can be set up for this problem is

$$\frac{\text{6 indoor balls}}{\text{2 outdoor balls}} = \frac{\text{24 indoor balls}}{x \text{ outdoor balls}}$$

Students could get the answer by looking at the relationship within each ratio and thinking "6 is related to 2 in the same way that 24 is related to what number"?

$$\left(\frac{\text{6 indoor balls}}{\text{2 outdoor balls}} = \frac{\text{24 indoor balls}}{x \text{ outdoor balls}} \right)$$

They could also look at the relationship between each ratio and think "6 is related to 24 in the same way that 2 is related to what number." Either way, they will see the answer is 8.

$$\frac{\text{6 indoor balls}}{\text{2 outdoor balls}} = \frac{\text{24 indoor balls}}{x \text{ outdoor balls}}$$

Proportional reasoning is complex and involves the understanding that if one quantity in a ratio is multiplied or divided by a particular factor, then the other quantity must be multiplied or divided by the same factor to maintain the proportional relationship.

A ratio is usually a comparison of two quantities of like units. When the comparison is between two different units, such as litres of gas compared to kilometres travelled, that is defined as a **rate**. A **unit rate** is an equivalent rate where the second quantity is one. It allows us to compare varying sizes of quantities by examining the number of units of one quantity per 1 unit of the second quantity. To solve problems involving distance, time, and average speed, or to determine the better buy in consumer situations, it is often beneficial to use unit rates. An example would be, which is the better buy: 1.2 L of orange juice for $2.50, or 0.75 L of orange juice for $1.40? Explain why it is the better buy.

It is important for students to be aware that when they are comparing unit rates, the numbers must be in the same units. For example, if comparing one quantity measured in grams with another measured in kilograms, the options are to change both measurements to grams or kilograms. The unit of measurement used for such a unit rate is often the student's choice. A review of conversion from one unit of measurement to another may be necessary here.

Proportional reasoning can be developed through activities that compare and determine the equivalence of ratios and rates, and solving proportions in a wide variety of problem-based contexts. It is important that students see the usefulness of **proportions** and are able to efficiently set up and solve proportional reasoning problems.

Encourage estimation and thinking about a problem prior to calculating. Encourage students to think quantitatively (more than or less than a certain amount) when estimating.

Students should look for the most efficient method to set up and solve a problem, whether it be using the relationship found within a ratio, the relationship between the two ratios or the unit rate. Also if possible, determine if the problem can be solved mentally before using pencil and paper. Cross multiplication does not promote understanding and is therefore not recommend as an approach to solving problems.

"Students may need as much as three years' worth of opportunities to reason in multiplicative situations in order to adequately develop proportional reasoning skills. Premature use of rules encourages students to apply rules without thinking and, thus, the ability to reason proportionally does not develop." (Van de Walle and Lovin 2006c, 157)

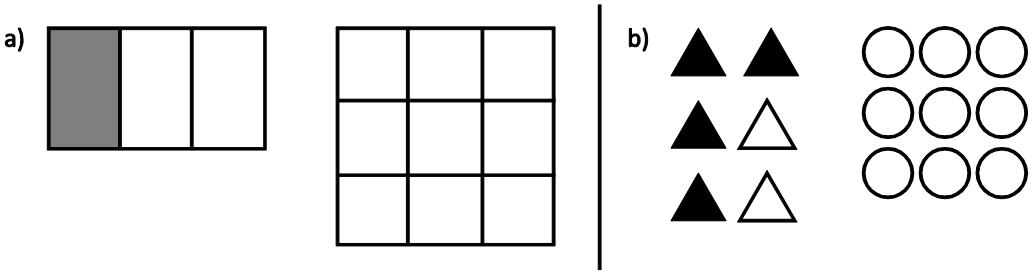
Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Tell students that the length of one rectangle is double the length of another. Their widths are the same. What happens to the area and perimeter of the rectangle? Explain.
- Have students draw line segments of 3 cm and 4 cm respectively. Then have them draw two more line segments that are longer than, and equivalent ratios to, the segments they drew.
- For each of the following, have students complete the second diagram so that it shows the same relationship as the first and to explain their reasoning.



- Have students use concrete materials to make proportional pairs.

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following sample tasks (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- During a heavy rain storm, 40 mm of rain fell in 30 minutes. How much rain would you expect to fall in one hour? In three hours? What assumptions are you making?

- Greg and Rose hammered a line of nails into different boards from one end to the other. Rose hammered more nails than Greg. Rose's board was shorter than Greg's. On which board are the nails closer together? Why?
- Jenson drives 100 km in 2 h and has 60 km to go. Will Jensen drive the other 60 km in more or less than 2 h? Justify your answer.
- Sue and Julie were running equally fast around a track. Sue started first. When she had run nine laps, Julie had run three laps. When Julie completed 15 laps, how many laps had Sue completed?
- A recipe uses 500 mL of flour for every 125 mL of sugar. How much flour would be needed when 500 mL of sugar is used?
- Provide a real-world context in which $\frac{a}{b}$ would represent: a fraction, a rate, a ratio, a quotient, and probability.
- If a package of 6 bottles of juice costs $4.50 and an individual bottle costs $1.50, how much would you save per bottle by purchasing a package?
- Dan can run 4 km in 15.2 minutes. If he keeps running at the same speed, how far can he run in 20 minutes?
- Use examples to explain how ratios and rates are the same. Then use examples to explain how ratios and rates are different.
- Jane found a good deal on soft drinks. She could buy 12 cans for $2.99. She needs 72 cans for her party. Explain how she can calculate the cost.
- Explain why 1:20 000 000 is another way to describe the ratio of 1 cm representing 200 km on a map.
- Sam measures two flowers to find they are 8 cm and 12 cm tall. Two weeks later they are 12 cm and 16 cm tall respectively. Which flower grew more?
- Jane's father drove 417 km in 4.9 hours. Leah's father drove 318 km in 3.8 hours. Who was driving faster and by how much?

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

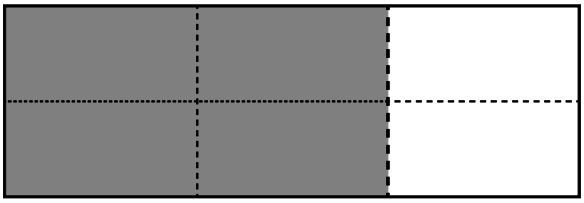
Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Use problems that encourage the use of mental mathematics, such as quantities within a ratio or between ratios that are integral multiples of each other.

- Use diagrams, such as the following to have students write two different ratios that tell what part of the rectangle is shaded and also have them write a correct proportion.



- Make comparisons of two quantities that are measured by the same unit. For example, compare the ratio of students who wear wrist watches to the ratio of those who don't wear wrist watches in other classes and then to the entire grade.

	Wrist Watch Wearers	Non-Wrist Watch Wearers	Ratio of Wrist Watch Wearers to Non-Wrist Watch Wearers
Class 1	16	8	2:1
Class 2	24	12	2:1
Entire Grade 8	varies	varies	varies

- Encourage students to use their knowledge of equivalent fractions or unit rates to solve proportion problems. Students should select the method that is most efficient to them, and appropriate to, solve problems.
- Use tables with data that contain equivalent ratios to model proportions by looking for multiplicative relationships in the rows or column. Have students examine the tables to determine if relationships are proportional. For example:

Paper Mache

Number of cups of water	Number of cups of flour
2	3
4	6
6	9
8	12

Suggested Learning Tasks

- Use the Cuisenaire rods to find proportional pairs.
- Bring in a picture of yourself standing beside a person or object. Measure the height of the images in the picture (cm) and use that ratio to find the actual height of the other person or object in the photo. One of the actual heights must be determined.
- 3 British pounds can be exchanged for 2 Canadian dollars. How many British pounds would you get for 1 Canadian dollar?
- Fred received a gift card for his birthday. He used it to download some new music for his MP3 player. Fred downloaded 12 songs in 15 minutes. At this rate, how many songs could he download in 1 hour?
- Two carafes of juice are on the table. Carafe B contains weaker juice than carafe A. Add one teaspoon of instant juice crystals to carafe A and one cup of water to carafe B. Which carafe will contain the stronger juice? Why?
- Two carafes of juice are sitting on the table. Carafe B and carafe A contain juice that tastes the same. Add one teaspoon of instant juice crystals to both carafe A and carafe B. Which carafe will contain the stronger juice? Why?
- When making lemonade Sue uses 5 scoops of powder for 6 cups of water, and Sarah uses 4 scoops of powder for 5 cups of water.
  - Are the situations proportional to each other? Explain why or why not.
  - In which situation is it likely the lemonade will be more flavourful? What assumptions did you make?
- Discuss whether or not the following could be solved using a proportion:
  - David is 6-years-old and Ellen is 2-years-old. How old will Ellen be when David is 12-years-old?
- Determine the fuel economy for a vehicle. You can prepare and use a log such as the one that follows to track fuel purchases, kilometres driven, and fuel economy over several weeks.

Amount of Gas Purchased (L)	Beginning Odometer Reading (km)	Ending Odometer Reading (km)	Total Distance Travelled	Fuel Efficiency

- Read each of the following scenarios and determine whether the fraction $\frac{a}{b}$ is representing a fraction, rate, ratio, quotient, or probability. Explain your reasoning.
  - Jeremy has a $\frac{3}{6}$ chance of rolling an even number on a six-sided regular number cube.
  - Ling travelled $\frac{27\ km}{3\ h}$ on her bike.
  - Having three girls and five boys on the intramural volleyball team is shown as $\frac{3}{5}$.
  - Pedro scored $\frac{7}{10}$ on his math quiz.
  - Allan ate $\frac{3}{4}$ of a bag of chips.
- Using the rectangles in the Fraction Factory set, place an orange rectangle and a brown rectangle side by side. Beside this pair of rectangles, place two different rectangles that are proportional; to the first pair. Explain your reasoning.

SUGGESTED MODELS AND MANIPULATIVES

- Calculator*
- coins
- 10 × 10 grid
- number line*
- open number lines
- various objects for counting (e.g. beans, counters)

*also available in *Interactive Math Tools* (Pearson n.d.)

MATHEMATICAL LANGUAGE

Teacher	Student
▪ equivalent fraction	▪ equivalent fraction
▪ part-to-part ratio	▪ part-to-part ratio
▪ part-to-whole ratio	▪ part-to-whole ratio
▪ proportion	▪ proportion
▪ rate	▪ rate
▪ three-term ratio	▪ three-term ratio
▪ two-term ratio	▪ two-term ratio
▪ unit price	▪ unit price
▪ unit rate	▪ unit rate

Resources

Print

Math Makes Sense 8 (Baron et al. 2008; NSSBB #: 2001642)

- Unit 5: Percent, Ratio, and Rate
  - Section 5.5: Exploring Ratios
  - Section 5.6: Equivalent Ratios
  - Section 5.7: Comparing Ratios
  - Section 5.8 A: Proportional and Non proportional Situations (Nova Scotia companion document only; found on Mathematics Learning Commons, Grades 7–9)
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Teaching Student-Centered Mathematics, Grades 5–8, Volume Three (Van de Walle and Lovin 2006), 154–175.

Math Matters: Understanding the Math You Teach, Second Edition (Chapin and Johnson 2006), 165–189.

Implementing the Common Core State Standards through Mathematical Problem Solving: Grades 6–8 (Gurl et al. 2013), 1–13.

Digital

- "Thinking Blocks: Model and Solve Word Problems, Ratio and Proportion Practice," *Math Playground* (MathPlayground.com 2014): www.mathplayground.com/tb_ratios/thinking_blocks_ratios.html
- "Thinking Blocks: Model Your Math Problems," Math Playground [iPad apps] (Math Playground LLC 2014): http://thinkingblocks.com/index.html

"Math Interactives: Rate/Ratio/Proportion," *LearnAlberta.ca* (Alberta Education 2003): www.learnalberta.ca/content/mejhm/index.html?ID1=AB.MATH.JR.NUMB&ID2=AB.MATH.JR.NUMB.RATE&lesson=html/video_interactives/rateRatioProportions/rateRatioProportionsInteractive.html.