Mathematics 8

Unit 6: Linear Equations and Graphing

PR01, PR02

SCO PR01 Students will be expected to graph and analyze two-variable linear relations. [C, ME, PS, R, T, V]			
[C] Communication **[T]** Technology	**[PS]** Problem Solving **[V]** Visualization	**[CN]** Connections **[R]** Reasoning	**[ME]** Mental Mathematics and Estimation

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR01.01 Determine the missing value in an ordered pair for a given equation.
PR01.02 Create a table of values by substituting values for a variable in the equation of a given linear relation.
PR01.03 Construct a graph from the equation of a given linear relation (limited to discrete data).
PR01.04 Describe the relationship between the variables of a given graph.

Scope and Sequence

Mathematics 7	Mathematics 8	Mathematics 9
PR01 Students will be expected to demonstrate an understanding of oral and written patterns and their equivalent linear relations. **PR02** Students will be expected to create a table of values from a linear relation, graph the table of values, and analyze the graph to draw conclusions and solve problems.	**PR01** Students will be expected to graph and analyze two-variable linear relations.	**PR01** Students will be expected to generalize a pattern arising from a problem-solving context using a linear equation and verify by substitution. **PR02** Students will be expected to graph a linear relation, analyze the graph, and interpolate or extrapolate to solve problems.

Background

In Mathematics 7 students used algebraic expressions to describe patterns and constructed graphs from the corresponding **table of values**. They substituted values for unknowns and evaluated algebraic expressions. The distinction between expressions and equations was made and students worked with both.

Students in Mathematics 7 used input and output tables. It will be necessary to explain that the related pair of values in a table of values is called an ordered pair of the form (x, y) and that the input values correspond to x and the output values correspond to y. Students should also recognize that if the change in the x values is constant and the change in the corresponding y values is constant, then the relation is linear. Students will not formally encounter the slope and the y-intercept in Mathematics 8. It is important for students to be able to analyze linear relationships expressed graphically. If students make these generalizations, provide them with the correct terminology. In Mathematics 8 students need many experiences focusing on contextual problems and the relationship between the variables. Although there are different types of patterns, the focus is on increasing/decreasing linear patterns. The elements that make up these patterns are called **terms**. Consequently, these patterns are often referred to as growing patterns. For example, 2, 4, 6, 8, 10 . . . and 1, 2, 4, 8, 16 . . . are two common increasing patterns. Using a table to model an increasing/decreasing pattern can help students organize their thinking. It can also help them generalize the patterns symbolically.

Students will be expected to find both missing independent variables and dependent variables for **linear relations**. Finding a dependent variable, or y-coordinate in an ordered pair, is part of the process of creating a table of values. When students have to determine the value of an independent variable, or x-coordinate in an ordered pair, they will have to apply previous work with solving linear equations.

Constructing graphs from equations allows students to visualize linear relationships. When the ordered pairs resulting from a linear relation are graphed on a coordinate plane they fall along a straight line. Although many graphs are tied to everyday situations and are mainly located in Quadrant I, students need experiences graphing an equation on a four-quadrant grid. All work with graphing in this unit is limited to **discrete** data. Discrete data can only have a finite or limited number of possible values. Generally discrete data are counts: number of students in class, number of tickets sold, number of items purchased. **Continuous** data can have an infinite number of possible values within a selected range, such as quantities of temperature and time. A graph of discrete data has plotted points, but they are not joined together.

The following example demonstrates how one problem can be used to explore several performance indicators simultaneously.

- Zachary is planning a swimming party. Pool rental will cost him $30.00 for one hour. After the swim, everyone will have a snack. The snack costs $3.00 per person.

  Write an algebraic equation to represent this problem.

Students should be able to think about this relation as the cost being equal to three times the number of guests plus $30.00 for the pool rental.

Students are accustomed to choosing variables that represent a situation. For example, c represents the cost and g represents the number of guests (e.g., $c = 3g + 30$). Now, they must recognize that if the linear equation is written as $y = 3x + 30$, it means the same thing. The development of typical algebraic language and terminology is important for the introduction of the term ordered pairs. When variables such as x and y are chosen to be used in the equation, it is important for students to state what each variable represents. In this example, students would say that x represents the number of guests and y represents the total cost.

The equation above was used to complete the following table of values.

Number of guests ***x***	**Total cost** ***y***
0	30
1	33
2	36
3	39
4	42
5	45

Consider the situation where 8 guests attend Zachary's party. In this case, have students determine the cost of the party by substituting $x = 8$ into the equation $y = 3x + 30$.

$$y = 3x + 30$$
$$y = 3(8) + 30$$
$$y = 24 + 30$$
$$y = 54$$

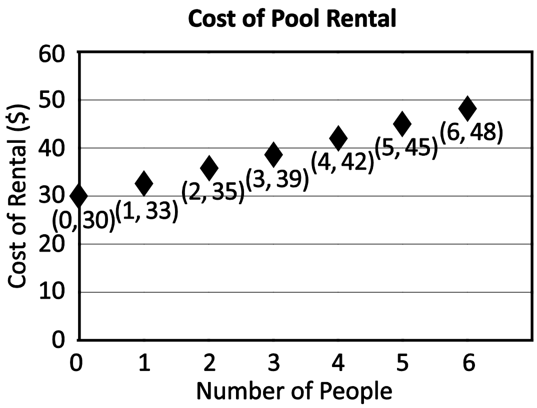
The cost of the party with 8 people is $54. Written as an ordered pair this would be (8, 54). Emphasize that all related pairs in a table of values may be written as ordered pairs. Using substitution, students may use the given equation to find missing values in any ordered pair.

Zachary has a budget of $60. Determine the maximum number of people he can invite to his party. Write this information as an ordered pair. In this case, have students determine the maximum number of guests by substituting $y = 60$ into the equation $y = 3x + 30$.

$$y = 3x + 30$$
$$60 = 3x + 30$$
$$60 - 30 = 3x + 30 - 30$$
$$30 = 3x$$
$$\frac{30}{3} = \frac{3x}{3}$$
$$10 = x$$

For $60, Zachary can invite 10 guests. Written as an ordered pair, this would be (10, 60). Encourage students to verify the value of *x* in this case by using substitution.

Construct a graph from the equation of this linear relation.



Describe the relationship between the variables in the graph.

Students should be able to make statements such as the following:

- The variables represent the number of guests and the cost of the party.
- As the number of guests increases by one, the cost of the party increases by $3.
- The points lie on a line that goes up toward the right. The points are not connected as the data is discrete.
- The graph starts at the point (0, 30) and not at the origin (0, 0) because the pool rental costs $30 even if no guests are invited.

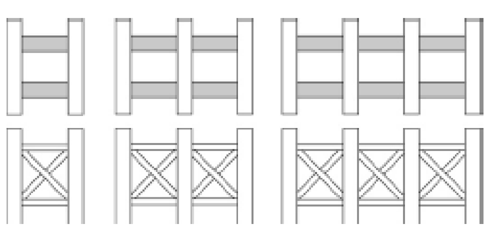
Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

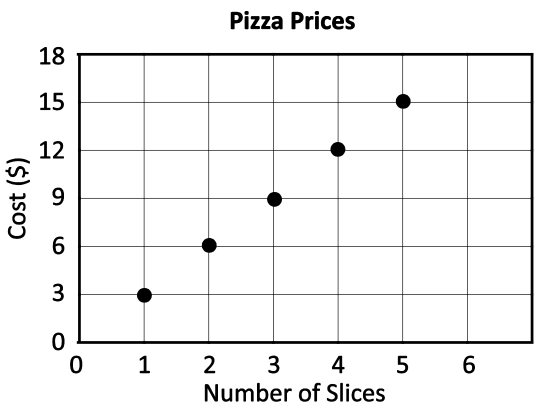
- For the following diagrams, ask students to
  - draw a representation of the pattern and extend the pattern
  - describe the pattern in their own words
  - develop a table
  - generate a graph



Whole-Class/Group/Individual Assessment Tasks

Consider the following sample tasks (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Show students the Pizza Prices graph which shows a linear relation.
  - Describe the patterns on the graph.
  - What is the cost of one slice of pizza?
  - What is the relationship between the number of slices and the cost?
  - Make a table of values from the graph.
  - If 7 slices of pizza are purchased, what is the cost?



- Provide the following linear relation: $y = -2x + 3$.
  - Create a table for values of x beginning with 0.
  - Draw the corresponding graph.
  - Determine the value of y for the ordered pair (7, y).
  - Determine the value of x for the ordered pair (x, 11).

- Tell students that Eric is organizing a skating party. He has to pay $50 to rent the rink and $4 for lunch for each person. He made a table of values, but he made an error in one of the costs. Identify the error and provide the correct value. Provide an explanation for the correction.

Number of people p	1	2	3	4	5	6	7	8
Cost in dollars c	54	58	62	68	70	74	78	82

- Mary has started a new exercise program. The first day she does 9 sit-ups, the second day she does 13, the third day 17, and the fourth day 21. This can be represented by $s = 4d + 5$.
  - Construct a graph of this linear relationship.
  - If she continues this way, how many sit-ups does she do, on the 5th day? 10th day? 20th day? 50th day?

- Provide students with a table of values that represents a linear relation.

x	–2	–1	0	1	2	3	4
y	–5	–3	–1	1	3	5	7

  - Have students graph the ordered pairs in the table of values.
  - Describe, in words, the relationship between the *x*–values and the *y*-values.
  - Write the linear relation using *x* and *y*.
- Explain the meaning of a linear relation using an example. What is the relationship between the variables?
- Use the equation $y = -3x + 4$ to complete the following:
  - Determine the missing values in the table.

x	–1	0	1	2	3	4
y						

  - Determine the value of *y* for the ordered pair (11, *y*).
  - Determine the value of *x* for the ordered pair (*x,* 13).
- James determined the mass of five pieces of a type of metal, representing a linear relationship. The table shows his results. James made one error in finding the masses.

Volume (cm^3)	8	9	10	11	12
Mass (g)	88	99	110	121	144

  - Using the relationship between the variables, identify the incorrect mass. Explain how you know this is the error. What is the correct mass?
  - Graph the ordered pairs from James' table of values.
  - How could you use the graph to show which value is incorrect?

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Encourage the use of proper algebraic vocabulary and terminology.
- Emphasize that all related pairs in a table of values may be written as ordered pairs.

- Have students arrange ordered pairs in ascending order based on the *x*-coordinate to establish a pattern to determine a missing value in a set.

SUGGESTED LEARNING TASKS

- Create a table of values for the equation, $k = 6(n + 2)$, by substituting values for 1 to 5 for *n*.

n	1	2	3	4	5
k					

- Using the table of values below, which represents a linear relation, complete the following:

x	1	2	3	4	5	6	7
y	4	8	12	16	20	24	28

  - Graph the ordered pairs in the table.
  - Determine the difference in value in consecutive *x*-values? *y*-values?
  - Describe the relationship between the *x*- and *y*-values.
  - Write an expression for *y* in terms of *x*.

x	–3	–2	–1	0	1	2
y				7	9	11

- Using the table of values below, which represents a linear relation with missing coordinates, complete the following:
  - How could you use a pattern to find the missing *y*-coordinates?
  - What are the missing coordinates?
- A community centre has a new banquet facility. It costs $8 per person to rent the centre.
  - Make a table of values showing the rental cost for 30, 60, 90, 120, and 150 people.
  - Graph the ordered pairs.
  - What is an expression for the rental cost in terms of the number of people?
- Determine the missing values in the following set of ordered pairs.

  (0, 0), (1, 12), (2, 24), (3, __)
  (–4, __),(–2, –6), (0, 2), (2, 10), (__, 18)

- An Internet company charges a basic monthly rate of $40 plus a per hour rate of $2. This can be described by the equation $c = 2h + 40$.
  - Determine the cost of using the Internet by completing the table below.

Hour (h)	Cost (c)
0	
1	
2	
3	
4	

5	
6	

- Create a graph using the data from the table of values. How did the pattern in the table of values show up in the graph?
- Matthew's Internet bill for the first month was $100. How could you use the graph to find the number of hours he used the Internet? Use the equation to determine how many hours Matthew used the Internet for the first month.

- The table of value represents a linear relation.

x	1	2	3	4	5	6
y	5	10	15	20	25	30

- Graph the ordered pairs in the table of values.
- What is the difference in consecutive *y*–values? What is the difference in consecutive *x*–values?
- Describe, in words, the relationship between the *x*–values and the *y*–values.
- What is an expression for *y* in terms of *x*?

SUGGESTED MODELS AND MANIPULATIVES

- algebra tiles*
- grid paper
- pattern blocks*

* also available in *Interactive Math Tools* (Pearson n.d.)

MATHEMATICAL LANGUAGE

Teacher	Student
▪ continuous	▪ continuous
▪ discrete	▪ discrete
▪ equation	▪ equation
▪ expression	▪ expression
▪ formula	▪ formula
▪ linear relation	▪ linear relation
▪ pattern	▪ pattern
▪ relations	▪ relations
▪ table of values	▪ table of values
▪ term	▪ term
▪ variable	▪ variable
▪ *x*-value	▪ *x*-value
▪ *y*-value	▪ *y*-value

▪ independent variable ▪ dependent variable	▪ independent variable ▪ dependent variable

Resources

Print

Math Makes Sense 8 (Baron et al. 2008; NSSBB #: 2001642)

- Student Book Unit 6: Linear Equations and Graphing
  - Section 6.6: Creating a Table of Values
  - Section 6.7: Graphing Linear Relations
  - Technology: Using Spreadsheets to Graph Linear Relations
  - Unit Problem: Planning a Ski Trip
- *ProGuide* (CD; Word Files) (NSSBB #: 2001643)
  - Assessment Masters
  - Extra Practice Masters
  - Unit Tests
- *ProGuide* (DVD) (NSSBB #: 2001643)
  - Projectable Student Book Pages
  - Modifiable Line Masters

Teaching Student-Centered Mathematics, Grades 5–8, Volume Three (Van de Walle and Lovin 2006), pp. 287–294

Digital

- *National Library of Virtual Manipulatives* (Utah State University 2015): http://nlvm.usu.edu (Algebra manipulatives for Grades 6–8.)

SCO PR02 Students will be expected to model and solve problems, concretely, pictorially, and symbolically, where *a*, *b,* and *c* are integers, using linear equations of the form

- $ax = b$
- $\frac{x}{a} = b,\ a \neq 0$
- $ax + b = c$
- $\frac{x}{a} + b = c,\ a \neq 0$
- $a(x + b) = c$

[C, CN, PS, V]

[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

PR02.01 Model a given problem with a linear equation, and solve the equation using concrete models.
PR02.02 Verify the solution to a given linear equation, using a variety of methods, including concrete materials, diagrams, and substitution.
PR02.03 Draw a visual representation of the steps used to solve a given linear equation, and record each step symbolically.
PR02.04 Solve a given linear equation symbolically.
PR02.05 Identify and correct an error in a given incorrect solution of a linear equation.
PR02.06 Apply the distributive property to solve a given linear equation.
PR02.07 Solve a given problem, using a linear equation, and record the process.

Scope and Sequence

Mathematics 7	Mathematics 8	Mathematics 9
PR06 Students will be expected to model and solve, concretely, pictorially, and symbolically, problems that can be represented by one-step linear equations of the form $x + a = b$, where *a* and *b* are integers.	**PR02** Students will be expected to model and solve problems, concretely, pictorially, and symbolically, where *a*, *b,* and *c* are integers, using linear equations of the form ▪ $ax = b$ ▪ $\frac{x}{a} = b,\ a \neq 0$ ▪ $ax + b = c$ ▪ $\frac{x}{a} + b = c,\ a \neq 0$ ▪ $a(x + b) = c$	**PR03** Students will be expected to model and solve problems, where *a*, *b*, *c*, *d*, *e*, and *f* are rational numbers, using linear equations of the form ▪ $ax = b$ ▪ $\frac{x}{a} = c,\ a \neq 0$ ▪ $ax + b = c$ ▪ $\frac{x}{a} + b = c,\ a \neq 0$ ▪ $ax = b + cx$ ▪ $a(x + b) = c$ ▪ $ax + b = cx + d$ ▪ $a(bx + c) = d(ex + f)$ ▪ $\frac{a}{x} = b,\ x \neq 0$
PR07 Students will be expected to model and solve, concretely, pictorially, and symbolically, where *a*, *b*, and *c* are whole numbers, problems that can be represented by linear equations of the form ▪ $ax + b = c$ ▪ $ax = b$ ▪ $\frac{x}{a} = b,\ a \neq 0$		

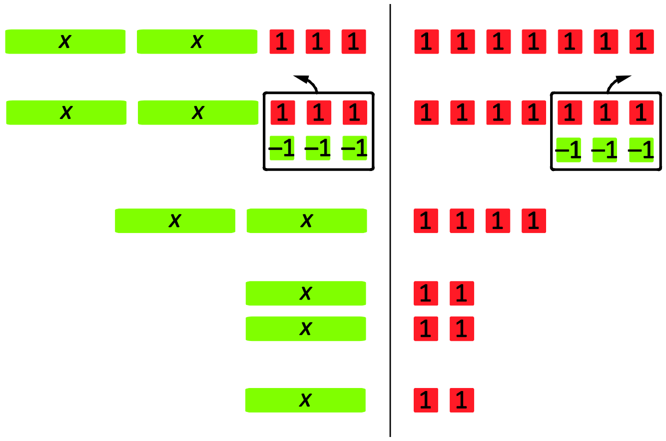
Background

Students have previously solved one-step equations of the form $x + a = b$, where *a* and *b* are integers. They have also solved two-step equations with two whole numbers. This unit builds on this previous experience and extends the numbers used to include integers. Students will also solve equations of the form $a\,(x + b) = c$, using the distributive property.

Significant work was done with concrete materials and diagrams in solving linear equations in Mathematics 7. Here, instruction should start with concrete materials and pictorial models, and then move to the symbolic, with the ultimate goal that students can solve one- and two-step equations with or without concrete or pictorial support.

When using concrete models to solve equations, the steps should also be recorded symbolically. In Mathematics 7, students applied the zero principle to add and subtract integers. Here they will use the same principle to isolate a variable in order to solve an equation. Equations must be kept balanced to maintain preservation of equality.

Algebra tiles can be used to solve $2x + 3 = 7$.



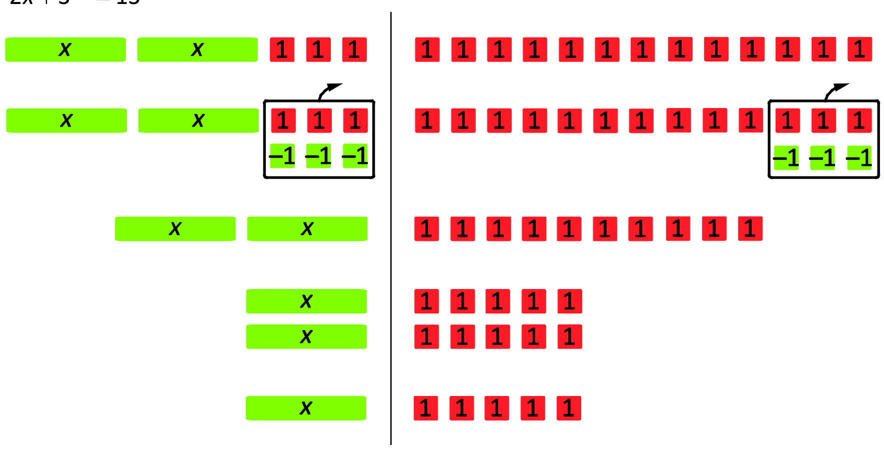
To maintain balance, record the operation that is being done to both sides of the equation on the same line as the equation. Recording the operation as a superscript (or subscript), which could be interpreted as exponents, is not an acceptable format. For example,

$$\begin{array}{l} 2x+3=13 \\ 2x+3-3=13-3 \end{array}$$

is an acceptable format,

$$\begin{array}{l} 2x+3=13 \\ 2x+3^{-3}=13^{-3} \end{array}$$

is not acceptable.



$2x + 3 = 13$

$2x + 3 - 3 = 13 - 3$

$2x = 10$

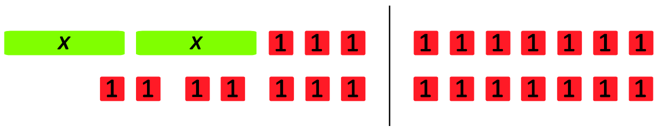
$\frac{2x}{2} = \frac{10}{5}$

$x = 5$

It is important to work with students to develop a procedure to solve equations so that they understand the mathematics involved. The important principles include the following:

- **Making zeros:** The reason you subtract 3 from both sides in the example above is to help isolate the variable. 3 – 3 creates an addition of zero, leaving 2*x* on the left-hand side. This is formally referred to as the *zero property of addition* or the *identity property of addition*.
- **Making ones:** The reason you would then divide by 2 is to isolate the variable, *x*. Dividing by 2 creates a multiplication by one, leaving just the variable on the left-hand side. This is formally referred to as the *inverse relationship* or the *identity property of multiplication*.
- **Preservation of equality:** To maintain balance and equality, you must do the same thing to both sides of the equation at the same time (on the same line of the equation).
- **Distributive property:** a property of real numbers that states that the product of the sum or the difference of two numbers is the same as the sum or difference of their products. Students should be familiar with the concept of distributive property as a quick multiplication strategy from Mathematics 5.

Have students verify all solutions to the linear equations. Verification helps develop increased understanding of the process involved. At this point, concrete materials and diagrams should be the focus when verifying solutions. To verify the previous solution symbolically, $x = 2$ will be substituted into the original equation. To model this with concrete materials, replace each variable tile with 2 positive unit tiles and then determine that both sides of the model have the same number of unit tiles, so balance has been maintained and the solution is correct.



Since both sides have the same value, the solution is demonstrated to be correct.

Students should now progress from balancing equations concretely to balancing equations symbolically. Students who have difficulty moving from models to the symbolic representation will benefit from more practice with models. Substitution can be used without models. Previously, with the aid of algebra tiles, it was determined that $x = 2$ for the equation $2x + 3 = 7$.

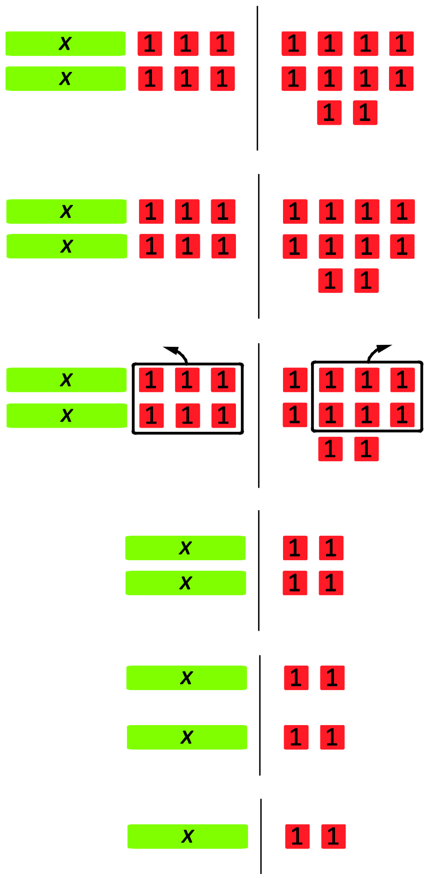
To verify this symbolically, substitute 2 into the original equation for x and evaluate.
$2x + 3 = 7$
$2(2) + 3 = 7$
$4 + 3 = 7$
$7 = 7$
Since both sides are equal, the equation is balanced and the solution is correct.

To solve a linear equation of the form $a(x + b) = c$ students will apply the distributive property. Students have used the distributive property with integers. This property is now extended to include algebraic expressions. Prior to solving equations, students should be able to explain the distributive property using diagrams or models, and use the distributive property to expand algebraic expressions. To reinforce the validity of the distributive property, it is useful to compare solutions obtained using the distributive property to solutions obtained using the order of operations.

Distributive Property	Order of Operations
4(5 + 2) (4)(5) + (4)(2) 20 + 8 28	4(5 + 2) 4(7) 28

A common error by students when using the distributive property is to distribute the negative sign over the first term only. Students may have to be reminded that an expression such as $-(6 + 3)$ can also be written as $-1(6 + 3)$ and each term in the brackets must be multiplied by -1. Having been exposed to simplifying expressions using the distributive property, students will then apply the property to solve equations. As with other types of linear equations, modelling equations of the form $a(x + b) = c$ should precede solving symbolically. An example of a model using algebra tiles to solve $2(x + 3) = 10$ follows:

Two groups of $(x + 3)$ equals 10



$$2(x + 3) = 10$$

$$2x + 6 = 10$$

$$2x + 6 - 6 = 10 - 6$$

$$2x = 4$$

$$\frac{2x}{2} = \frac{4}{2}$$

$$x = 2$$

Students should be encouraged to verify their own solutions to linear equations. It is also beneficial to provide them with worked solutions of linear equations to verify. Along with providing the correct answers, they should communicate about errors they find in solutions, including why errors might have occurred and how they can be corrected. This reinforces the importance of verifying solutions and recording solution steps, rather than only giving a final answer.

To solve problems, it is necessary for students to make connections to previous work with linear equations. Problem solving also requires communication, and can be done so through the application of a four-step process:

- understand the problem by identifying given information
- make a plan to solve the problem
- carry out the plan and record the solution
- verify that the solution is correct for the information given in the problem

Students should be able to solve problems such as the following:

- A grade 8 class held a charity car wash and earned \$368. They donated \$200 to an animal shelter, and shared the rest of the money equally among four other charities. How much money did each of the other four charities receive?
  - **Understand the Problem:** I can let *m* represent the amount of money each charity receives, since this is what I am trying to find. There are four charities. I know how much the animal shelter gets, and I also know the total amount raised.

- **Make a Plan:** I can multiply *m* by 4 and add it to the animal shelter amount to get the total. An equation to represent this is $4m + 200 = 368$, where *m* represents the amount each charity receives.
- **Carry out the Plan:** I can solve $4m + 200 = 368$ using strategies I learned previously.

$$4m + 200 = 368$$
$$4m + 200 - 200 = 368 - 200$$
$$4m = 168$$
$$\frac{4m}{4} = \frac{168}{4}$$
$$m = 42$$

Each charity will receive $42.

- **Verify the Solution:** If each charity receives $42, four of them together would receive $168. Add this to the animal shelter donation of $200 and the total is $368.

  Using substitution, the solution can be verified algebraically.

$$4m + 200 = 368$$
$$4(42) + 200 = 368$$
$$168 + 200 = 368$$
$$368 = 368$$

Assessment, Teaching, and Learning

Assessment Strategies

Assessing Prior Knowledge

Tasks such as the following could be used to determine students' prior knowledge.

- Solve each equation using algebra tiles, and by inspection. Verify each solution by substitution.
  - $c + 4 = 7$
  - $n - 3 = 6$

- Ask students to complete the following table.

Problem	Concrete Representation and Solution (When you have completed this column, show it to the teacher before moving on to the next two columns.)	Pictorial Representation and Solution	Symbolic Representation and Solution
$15 = n + 7$			
$t - 4 = 3$			
$12 = 4x$			
$\frac{p}{3} = 2$			

Whole-Class/Group/Individual Assessment Tasks

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Solve the following problems. Write the equation, solve it, and verify your answer.
  - The grade 8 students had a dance. The disc jockey charged $150 for setting up the music plus $3.00 per student who attended the dance. The disc jockey was paid $375. How many students attended the dance?
  - The high temperature today is 6°C higher than twice the high temperature yesterday. The high temperature today is 12°C. What was the high temperature yesterday?
- Tell students that Kim used the distributive property to solve the following equation: $12(x - 3) = 72$. Check her work to see if her solution is correct. If there is an error, correct it.

  $12(x - 3) = 72$
  $12x - 36 = 72$
  $12x - 36 - 36 = 72 - 36$
  $12x = 36$
  $x = \frac{36}{12}$
  $x = 3$

- Ask students which of the following produces the smallest value for *d*?

  $7d = 42$
  $\frac{d}{5} = -2 = -2$
  $3d + 4 = -5$
  $\frac{d}{4} + 12 = 36$
  $5(d + 4) = -15$

- Some cows and some chickens live on a farm. If the total number of legs is 38, and the total number of heads is 16, use algebra to find how many cows and how many chickens live on the farm.

- Explain each step in this solution to the equation $16 + 5m = 6$. Verify that the solution is correct.

  Step 1: $16 - 16 + 5m = 6 - 16$
  Step 2: $5m = -10$
  Step 3: $m = -2$

- Sandy started with the equation $4p - 14 = -46$. She covered up the $4p$ and asked herself the question: What added to −14 gives −46?
  - What should her answer have been?
  - Using her answer above, Sandy wrote a new equation which was $4p = \square$. She then asked herself: What multiplied by 4 equals ☐? What is the value of p?
- Solve $2(x - 4) = -20$ in two different ways. Record the processes you used.
- A taxicab company charges a basic rate of $3.75 plus $2.00 for every kilometre driven. If the total bill was $33.75, use algebra to find how far the cab ride was.

FOLLOW-UP ON ASSESSMENT

Guiding Questions

- What conclusions can be made from assessment information?
- How effective have instructional approaches been?
- What are the next steps in instruction for the class and for individual students?

Planning for Instruction

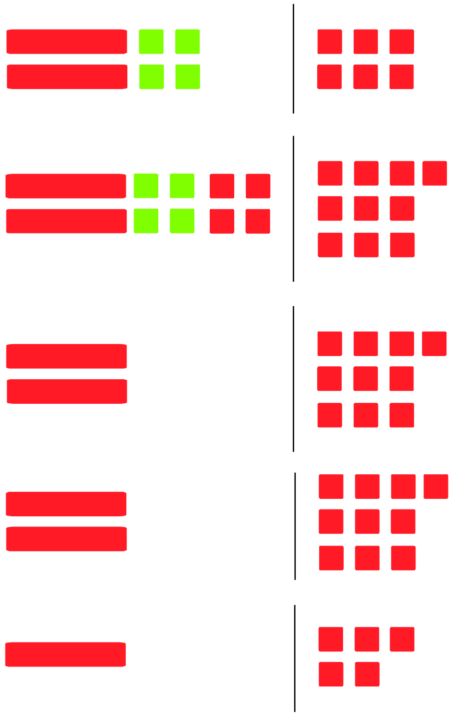
CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Three friends decided to check their answers for a homework question. They each had a different solution to the linear equation 4(*s* − 3) = 288 and wanted to determine whose answer was correct.

Sam's Solution	Leah's Solution	Paul's Solution
$4(s-3)=288$ $4s-12=288$ $4s-12-12=288-12$ $\frac{4s}{4}=\frac{276}{4}$ $s=69$	$4(s-3)=288$ $4s-12=288$ $4s-12+12=288+12$ $\frac{4s}{4}=\frac{300}{4}$ $s=75$	$4(s-3)=288$ $4s-3=288$ $4s-3+3=288+3$ $\frac{4s}{4}=\frac{291}{4}$ $s=72.75$
Sam's Verification $4(s-3)=288$ $4(69-3)=288$ $4(66)=288$ $264\neq 288$ The solution is incorrect. By examining Sam's solution students should recognize that Sam should not have subtracted 12 in the third step, but added 12.	Leah's Verification $4(s-3)=288$ $4(75-3)=288$ $4(72)=288$ $288=288$	Paul's Verification $4(s-3)=288$ $4(72.75-3)=288$ $4(69.75)=288$ $279\neq 288$ The solution is incorrect. By examining Paul's solution students should recognize that he did not correctly apply the distributive property in step two.

- Use concrete materials and diagrams to demonstrate the idea of solving for "x" as a natural progression and lead the students to an understanding of the steps needed to isolate the variable. After exploring this progression, students will be able to solve for "x" in a linear equation and record the process.



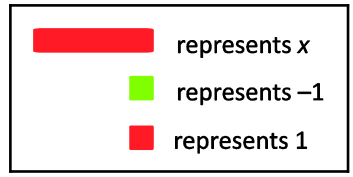
$2x - 4 = 6$

$2x - 4 + 4 = 6 + 4$

$2x = 10$

$\frac{2x}{2} = \frac{10}{2}$

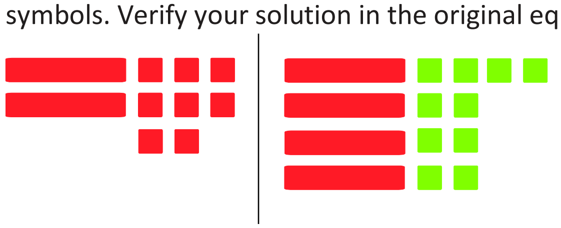
$x = 5$



- In the example above, it will not be intuitive to a student that if $-x = 5$, they can write the equivalent expression $x = -5$. This progression can be explained using the idea of opposites. Since x is the opposite of $-x$ and -5 is the opposite of 5. It will be a true statement that if $-x = 5$, then $x = -5$. The last line of the solution shows this symbolically and through modelling.
- Use an area model to expand expressions to explain the distributive property.
- Use interactive websites that allow students to explore solving linear equations, such as the algebra manipulatives for grades 6–8 from National Library of Virtual Manipulatives (http://nlvm.usu.edu/).

Suggested Learning Tasks

- Use the distributive property to write $7(c + 2)$ as a sum of terms. Sketch a diagram.
- Write and solve the equation modelled below, and write each step involved in the solution using symbols. Verify your solution in the original equation.



- Work with a partner to solve the following equations with algebra tiles. Take turns doing the following: Decide who will be the scribe and who will model the algebra tiles. The partner modelling with the tiles will tell the other person the steps to solve the equation. The scribe writes down the procedure algebraically.
  - $3x = -6$
  - $\frac{x}{3} = 4 + 2$
  - $6x = 4x - 4$

- Verify each of the following solutions. Identify and correct any errors.

  - $$\begin{aligned} x+4&=3 \\ x+4-4&=3+4 \\ x&=7 \end{aligned}$$

  - $$\begin{aligned} 5+4x-4x&=13-4x \\ 5&=9x \\ \frac{5}{9}&=\frac{9x}{9} \\ \frac{5}{9}&=x \end{aligned}$$

  - $$\begin{aligned} 5+4x&=13 \\ 56&=8(x+3) \\ 56&=8x+24 \\ \frac{32}{32}&=\frac{8x}{32} \\ x&=\frac{8}{32} \\ x&=\frac{1}{4} \end{aligned}$$

  - $$\begin{aligned} -2(x-1)&=-22 \\ -2x-2&=-22 \\ -2x-2+2&=-22+2 \\ -2x&=-20 \\ \frac{-2x}{2}&=\frac{-20}{2} \\ x&=10 \end{aligned}$$

  - $$\begin{aligned} 7x-2&=-16 \\ 7x-2+2&=-16+2 \\ 7x&=-14 \\ x&=-2 \end{aligned}$$

  - $$\begin{aligned} \frac{m}{6}+3&=11 \\ 69(\frac{m}{6})+3&=6(11) \\ m+3&=66 \\ m+3-3&=66-3 \\ m&=63 \end{aligned}$$

- Determine methods of solving the following problems using a linear equation. Be prepared to present your solution methods to the class.
  - Joe is 16 years older than Bill. Sam is the same age as Bill. Their combined age is 79. How old are Joe, Bill, and Sam?
  - Your local high school sold advance tickets to their musical for $3.00 per ticket. Tickets purchased at the door were sold for $5.00 per ticket. How many tickets were sold in advance if 20 tickets were sold at the door and $340.00 were collected in total?
  - A baker is packaging cookies in identical boxes. She has filled seven boxes with all but 5 cookies in another box. She has packaged 51 cookies. How many cookies are in a full box?

SUGGESTED MODELS AND MANIPULATIVES

- algebra tiles*
- integer tiles*

* also available in *Interactive Math Tools* (Pearson n.d.)

MATHEMATICAL LANGUAGE

Teacher	Student
▪ balance ▪ constant ▪ equation ▪ equivalent ▪ evaluate ▪ expression ▪ formula ▪ identity property of addition ▪ identity property of multiplication ▪ inverse relationship ▪ one-step linear equation ▪ opposite operation ▪ substitution ▪ two-step linear equation ▪ variable ▪ zero property of addition	▪ balance ▪ constant ▪ equation ▪ equivalent ▪ evaluate ▪ expression ▪ formula ▪ one-step linear equation ▪ opposite operation ▪ substitution ▪ two-step linear equation ▪ variable

Resources

Print

Math Makes Sense 8 (Baron et al. 2008; NSSBB #: 2001642)

- Unit 6: Linear Equations and Graphing
  - Section 6.1: Solving Equations Using Models
  - Section 6.2: Solving Equations using Algebra
  - Section 6.3: Solving Equations Involving Fractions
  - Section 6.4: The Distributive Property
  - Section 6.5: Solving Equations Involving the Distributive Property
  - Game: Make the Number
  - Unit Problem: Planning a Ski Trip
- *ProGuide* (CD; Word Files) (NSSBB #: 2001643)
  - Assessment Masters
  - Extra Practice Masters
  - Unit Tests
- *ProGuide* (DVD) (NSSBB #: 2001643)
  - Projectable Student Book Pages
  - Modifiable Line Masters

Teaching Student-Centered Mathematics, Grades 5–8, Volume Three (Van de Walle and Lovin 2006c), 287–294.

Digital

National Library of Virtual Manipulatives (Utah State University 2015): http://nlvm.usu.edu (Algebra manipulatives for Grades 6–8.)