Mathematics 9

Unit 6: Linear Equations and Inequalities

PR03, PR04
Students will be expected to model and solve problems, where \( a, b, c, d, e, \) and \( f \) are rational numbers, using linear equations of the form

- \( ax = b \)
- \( \frac{x}{a} = c, \ a \neq 0 \)
- \( ax + b = c \)
- \( \frac{x}{a} + b = c, \ a \neq 0 \)
- \( ax = b + cx \)
- \( a(x + b) = c \)
- \( ax + b = cx + d \)
- \( a(bx + c) = d(ex + f) \)
- \( \frac{a}{x} = b, \ x \neq 0 \)

\[C, \ CN, \ PS, \ V\]

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>[T] Technology</td>
<td>[V] Visualization</td>
<td>[R] Reasoning</td>
<td></td>
</tr>
</tbody>
</table>

**Performance Indicators**

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

**PR03.01** Solve the given linear equation, using concrete and pictorial representations, and record this process symbolically.

**PR03.02** Verify by substitution whether a given rational number is a solution to a given linear equation.

**PR03.03** Solve a given linear equation symbolically.

**PR03.04** Identify and correct an error in a given incorrect solution of a linear equation.

**PR03.05** Represent a given problem, using a linear equation.

**PR03.06** Solve a given problem, using a linear equation, and record the process.
Scope and Sequence

<table>
<thead>
<tr>
<th>Mathematics 8</th>
<th>Mathematics 9</th>
<th>Mathematics 10</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PR02</strong> Students will be expected to model and solve problems, concretely, pictorially, and symbolically, where a, b, and c are integers, using linear equations of the form:</td>
<td><strong>PR03</strong> Students will be expected to model and solve problems, where a, b, c, d, e, and f are rational numbers, using linear equations of the form:</td>
<td><strong>RF10</strong> Students will be expected to solve problems that involve systems of linear equations in two variables, graphically and algebraically.</td>
</tr>
<tr>
<td>• ( ax = b )</td>
<td>• ( ax = b )</td>
<td></td>
</tr>
<tr>
<td>• ( \frac{x}{a} = b, a \neq 0 )</td>
<td>• ( \frac{x}{a} = c, a \neq 0 )</td>
<td></td>
</tr>
<tr>
<td>• ( ax + b = c )</td>
<td>• ( ax + b = c )</td>
<td></td>
</tr>
<tr>
<td>• ( \frac{x}{a} + b = c, a \neq 0 )</td>
<td>• ( \frac{x}{a} + b = c, a \neq 0 )</td>
<td></td>
</tr>
<tr>
<td>• ( a(x + b) = c )</td>
<td>• ( ax = b + cx )</td>
<td></td>
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<td><strong>Background</strong></td>
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</tbody>
</table>

There are several important ideas that are addressed in this outcome:

- **Algebra** is used to represent and explain mathematical relationships as well as to describe and analyze change.

- **Equations** are used to express relationships between two quantities; both sides of the equal sign are equivalent expressions.

- **Variables** are symbols that can stand for any one of a set of numbers or other objects and can be represented by boxes or letters.

In Mathematics 8, students had experience solving one- and two-step equations, where a, b, and c are integers, in the form of

• \( ax = b \)

• \( \frac{x}{a} = b, a \neq 0 \)

• \( ax + b = c \)

• \( \frac{x}{a} + b = c, a \neq 0 \)

• \( a(x + b) = c \)

This outcome builds on previous experience to now include rational number, equations with variables on both sides of the equal sign, and using more than two steps are required to solve the equation. Students have worked extensively with concrete and pictorial representations, and include the use of algebra tiles, inspection, and systematic trial (guess and check). Instruction should start with concrete materials and pictorial models and then move to symbolic representation. Research has shown that the use of concrete models is critical in mathematics because most mathematical ideas are abstract. Students must move from the concrete to the pictorial to the symbolic, and part of instructional
planning involves making informed decisions about where students are on the continuum of concrete to symbolic thinking.

Ultimately students should be able to solve equations without concrete or pictorial support. The progression from concrete to abstract and then from integral to rational should help scaffold the skills students learned in Mathematics 8 and help them to apply their learning to solve equations with rational coefficients and constants.

When solving equations, a balance scale is an appropriate model when the coefficients and constants are positive integers. Algebra tiles can be used to represent equations with any integer coefficient and constant. As students use a concrete model, they should also record the steps symbolically. Work with models leads to solving equations using inverse operations to gather like terms and balance the equation. Equations with rational numbers such as fractions or decimals cannot be solved easily using concrete models. Students, therefore, need to be able to solve equations symbolically.

It is expected that students will initially show all the steps as they solve equations. As they develop these skills, they may be able to reduce the number of steps.

Students may use different strategies when solving equations involving fractions. When solving an equation such as \( \frac{x}{4} + \frac{1}{2} = \frac{x}{3} \), some students may eliminate the denominators by multiplying each term by the lowest common denominator. As they proceed to solve the equation, ask them the following questions:

- What is the lowest common denominator of 4, 2, and 3?
- What would happen if the lowest common denominator was multiplied on both sides of the equation? Why is this mathematically correct?
- What is the simplified equation?
- What is the solution?

Another strategy that can be used centers around the idea of undoing the operations that are being done to the variable. When students are provided with an equation such as \( 2x = 8 \), since the inverse of multiplication is division, they divide both sides of the equation by 2, thereby undoing the multiplication operation. Similarly, if students are provided with the equation \( \frac{x}{2} = 8 \), they multiply both sides of the equation by 2 since the inverse of division is multiplication.

When solving the equation \( \frac{x}{4} + \frac{1}{2} = \frac{x}{3} \), students may decide to solve this equation using the “undo” process.

\[
\begin{align*}
\frac{x}{4} + \frac{1}{2} &= \frac{x}{3} \\
4 \left( \frac{x}{4} \right) + 4 \left( \frac{1}{2} \right) &= 4 \left( \frac{x}{3} \right) \\
x + 2 &= \frac{4x}{3} \\
3(x) + 3(2) &= 3 \left( \frac{4x}{3} \right) \\
3x + 6 &= 4x \\
3x + 6 &= 4x - 3x \\
x &= 6
\end{align*}
\]
After solving several examples, discuss with students the connection between this method and the process of multiplying each term by the lowest common denominator.

When solving an equation, it does not matter if the unknown variable is isolated on the left or right hand side of the equal sign. It is preferable to add or subtract the value of $x$ that would result in a positive coefficient value. Students may show a preference to always isolate $x$ on the left hand side, resulting in the step to multiply or divide by a negative coefficient. Understanding the role of the equal sign helps in knowing that the variable need only be isolated on one side of the equation, not necessarily the left hand side. As in the above equation, some students may have done the final steps in the following manner:

$$3x + 4x + 6 = 4x + 4x + 6$$
$$x = 6$$

Have students consider in advance what might be a reasonable solution. They should be reminded that once they acquire a solution, it can be checked for accuracy by substitution into the original equation. Always encourage students to verify solutions, as this will lead to a better understanding of the process involved. Students should be provided with worked solutions of linear equations to verify. Along with providing the correct answers, they should identify any errors they find in solutions, and correct those errors. Common student errors include mistakes in using the distributive property, using sign rules incorrectly for multiplication and division, and errors in preserving equality. Error analysis reinforces the importance of verifying solutions and recording steps, rather than only producing the final answer.

To solve problems, it is necessary for students to make connections to previous work with linear equations. Students should be given the opportunity to solve problems with variables on both sides. Many problems can be solved using methods other than algebra, such as guess-and-check and systematic trial. It may, therefore, be necessary to specify the strategy to ensure that algebraic problem solving is being used. Students should be able to solve equations using rational numbers. Algebra can be used to solve problems that might otherwise be tedious to solve using methods such as guess-and-check. It would be manageable for students to solve a problem, such as the following, using guess-and check:

- John and Judy work part time. John earns $10 per day plus $6 per hour. Judy earns $8 per hour. Determine how many hours they have to work to earn the same daily pay.

It is more tedious to use guess-and-check to solve a problem such as:

- A cell phone company offers two different plans:
  - Plan A: monthly fee of $45 and $0.20 per minute
  - Plan B: monthly fee of $28 and $0.45 per minute

Determine how many minutes result in the same monthly cost for both plans.

In this case, students should realize that it is more efficient to solve the equation $0.2x + 45 = 0.45x + 28$ to determine the number of minutes.

Problem solving also requires communication through the application of a four-stage process, which was referenced in Mathematics 8:

- understand the problem by identifying given information
- make a plan to solve the problem
• carry out the plan and record the solution
• verify that the solution is correct for the information given in the problem

Assessment, Teaching, and Learning

Assessment Strategies

Assessing Prior Knowledge

Tasks such as the following could be used to determine students’ prior knowledge.

- Explain what is the same and what is different about the way that the letter m has been used in the following cases:
  - Martin ran 100 m (the m represents an abbreviation for metre)
  - $2m + 15 = 23$ (m represents a single value)
  - $3m + 2$ (m represents an infinite number of values)

- Put each equation on an individual index card and ask students to sort the cards into the following categories:
  a. Easy to solve in my head
  b. Harder, but can be solved in my head
  c. Easier to use a model or pen and paper to solve
     - $n + 2 = 7$
     - $3n = 21$
     - $2n + 7 = 19$
     - $4n - 8 = 6n + 2$
     - $5 = 2n - 7$
     - $n - 2 = 7$
     - $3n = 21$
     - $3n - 5 = n + 1$
     - $6(n - 4) = 2n + 4$
     - $5n - 2 = 9$

Ask students to discuss the following questions with a partner or in a small group:
- How did you decide which questions would be easy to solve mentally?
- What process did you use to solve these equations?
- What are the similarities and differences in the methods that each person used?

- A single shape has the same value or mass.
  - Which shape has the greatest mass?
  - Which shape has the least mass?
  - How do you know?
▪ Use algebra tiles to model and solve $3x + 4 = 10$. Record each step required to solve for the unknown variable.

**Whole-Class/Group/Individual Assessment Tasks**

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

▪ Give the equation that shows that the perimeter of a rectangle is 36 m, if the length of the rectangle is 2 m less than its width. Solve for the length and width.

▪ Solve the following, using algebra tiles, and record each step algebraically.

\[
2(-3x + 1) = 4(2x - 3) \\
-6x + 2 = 8x - 12
\]

▪ Estimate a solution to the following equation. Justify your estimate; solve and verify.

\[
\frac{x}{2} - 3 = 1 \frac{1}{6}
\]

▪ Brenda and Thomas want to buy a $199 portable mp3 player. Brenda has $45 and saves $15 per week. Thomas has $70 and saves $12.50 per week. Who will be able to buy the mp3 player first? Solve using a linear equation.

▪ *The Chronicle Herald* can be delivered to your house for $0.70 per copy plus a $25.00 yearly subscription fee. *The Globe and Mail* can be delivered to your house for $0.75 per copy plus a $20.00 yearly subscription fee. Determine how many copies are delivered before the cost is the same.

▪ Leah solved the following equation. Check for any errors. If any were made, indicate where and make the necessary changes to correct them.

\[
\frac{1}{3}(x - 2) = 5(x + 6) \\
3(x - 2) = 15(x + 6) \\
3x - 6 = 15x + 90 \\
3x - 15x = 15x - 15x + 96 \\
-12x = 96 \\
-12x = 96 \\
12x = 96 \\
x = -8
\]

▪ Solve the following:
  - $2(3x - 6) = \frac{1}{2}(4x + 2)$
  - $\frac{k}{3} - \frac{1}{2} = -1 \frac{3}{4}$
  - $2.3(h - 1.7) = 4.2(h + 1.3)$
− $-2(1 - c) = -3(2 - c)$
− $\frac{x}{4} = \frac{7}{10}$
− $8(3d - 2) = -12.32$
− $\frac{x}{3} + \frac{1}{2} = \frac{5}{6}$
− $\frac{r}{3} - \frac{3t}{4} = 10$
− $\frac{1}{3}x + 8 = -14$
− $\frac{5}{m} + 7 = -3$
− $\frac{1}{2}n - 5 = 4 - n$
− $\frac{-5}{x} = -2$

Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

▪ Use diagrams and concrete materials to help students understand the steps needed to isolate the variable.
▪ Model the solving of equations with variables on both sides, using balancing strategies of balance scales (when terms are positive) and algebra tiles (when terms are negative). Examples are shown in the Whole-Class/Group/Individual Assessment section.
▪ Solve equations with rational numbers in fraction or decimal form (not easily modelled with balance scales or algebra tiles) by doing the same action on both sides of the equation.
▪ Use equations to model and then solve problems.

SUGGESTED LEARNING TASKS

▪ The combined mass for each pair of shapes is shown on the scale below the shapes. Ask students to determine the mass of each shape if the shapes that are the same colour and size have the same mass. Ask them to represent their answer in at least two ways or show at least two strategies. Set A has integral answers, while Set B has rational answers. Ask students to solve Set A first and share their solution strategies. Then provide them with Set B and ask them to solve for the unknowns. Was their strategy the same for both sets? Many students will use guess-and-check to solve the missing values. They may choose to represent the unknowns with letters like $x$, $y$ and $z$ or they may show their solution through guess-and-check. Students should share their process in solving these questions with the rest of the class. Differentiate these activities by using different sets of numbers, including natural and rational numbers.
### Set A

![Diagram of algebra tiles](image1.png)

### Set B

![Diagram of algebra tiles](image2.png)

- Have students model the solution to each of these equations using algebra tiles; then record the steps pictorially and numerically, show all the steps.
  
  a) \(3x = -9\)
  
  b) \(2x + 3 = -1\)
  
  c) \(2(x - 3) = 4\)
  
  d) \(3x + 2 = x - 4\)
  
  e) \(3(2x + 2) = 2(2x - 4)\)
  
  f) \(\frac{x}{3} = 2\)
  
  g) \(\frac{x}{3} + 5 = -2\)

- Have students solve each of these equations, showing all steps. Have them verify the solution.
  
  a) \(\frac{1}{2}n = 12\)
  
  b) \(-0.4n + 3.6 = 12.8\)
  
  c) \(\frac{3}{4}(n - 4) = 5\)
  
  d) \(\frac{3}{4}n - \frac{1}{2} = \frac{2}{3}n - \frac{2}{3}\)
  
  e) \(0.8(6n + 6) = 0.6(4n - 12)\)
  
  f) \(\frac{x}{2} = 0.8\)
  
  g) \(\frac{2.5}{n} = 0.5\)
  
  h) \(\frac{x}{0.4} + 8.2 = 12.8\)
  
  i) \(\frac{12}{x} = 3\)

- Using two dice (one for positive numbers and one for negative), have students throw the dice twice. Substitute the value from each dice into the two boxes on the left-hand side of the equation on the first toss, and into the two boxes on the right-hand side of the equation on the second toss:

  \[ \square x + \square = \square x + \square \]

  - Students should decide which dice represents positive values and which dice represents negative values. For example, if, on the first toss, the student throws a 3 and 4 this could represent -3 and 4; and if, on the second toss, the student throws a 2 and a 3, use -2 and 3.
  
  - Students decide which box contains which dice value. The first equation to solve could be: \(-3x + 4 = -2x + 3\)
Other equation formats that help students reinforce the concept of subtraction of negative values are:

- \(-x - \square = x + \square\)
- \(-x + \square = x - \square\)
- \(-x - \square = x - \square\)

Consider using open-ended problems (with small-group discussion and then class sharing) to help students expand their repertoire of problem-solving approaches. You may also want to set up scenarios where students need to compare situations to arrive at a solution.

For example: Yolanda’s school is holding an evening fundraiser where students set up various games and competitions. There are two ways that students can participate: They can pay a $5 entry fee and play games for 75 cents each, or they can pay no entry fee and play games for $1.25 each.

- If Yolanda has $15 to spend on games that evening, what choice will allow her to play the maximum number of games?
- If Yolanda has an unlimited amount of money, would one choice be better than the other?

Develop an equation for the following situations, and use it to answer the question(s):

- To raise money the student council organized a dance. They hired a band and rented electronic equipment at a cost of $800. Participation and school spirit is important to the council so they charged only $5 per ticket. How many tickets would they need to sell to break even? To make a thousand dollars? To make two thousand dollars?
- Paul starts work 3 hours earlier than his sister Katie. Both work at the local grocery store. Katie earns $12 per hour and Paul earns $8 per hour. Paul wants to know how many hours he will have to work in order to earn the same amount of money as his sister.

Three students are having a debate about the equation \(y = \frac{1}{2}x + 5\). Student A thinks that every time \(y\) changes by 1, \(x\) changes by 2. Student B thinks that every time \(y\) changes by 1, \(x\) will change by \(\frac{5}{2}\). Student C thinks that every time \(y\) changes by \(\frac{1}{2}\), \(x\) changes by 1. Who do you think is correct and why? Use at least two representations to prove your case.

Ask students to work in pairs to solve the following equations using algebra tiles. Students should take turns doing the following: Decide who will be the scribe and who will model the algebra tiles. The partner modelling with the tiles will tell the other person the steps to solve the equation. The scribe writes down the procedure algebraically.

- \(2a + 7 = 12\)
- \(11 = 3 + 4x\)
- \(9 - 3c = 15\)

Pass the Pen: Write a multi-step problem on the board and call on one student to come up and complete the first step and explain their reasoning for this step to the class. The student then calls on another student to complete the next step and “passes the pen.” This continues until the problem is finished. When a question arises, the student holding the “pen” must answer the question, call on another student to help, or “pass the pen” to a different student. This activity can also be done on an interactive whiteboard, with hand held technology that can project, using a document camera, or in groups that pass the paper rather than the pen.
### Equation

<table>
<thead>
<tr>
<th>Equation</th>
<th>Student Explanation</th>
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</thead>
<tbody>
<tr>
<td>(2(x+3) = x-3(x-4))</td>
<td>I used the distributive property to expand</td>
</tr>
<tr>
<td>(2x+6 = x-3(x-4))</td>
<td>I used the distributive property too. A question arises about the step. There is debate among students and the step is identified as a common error.</td>
</tr>
<tr>
<td>(2x+6 = x-3x-12)</td>
<td>continue....</td>
</tr>
</tbody>
</table>

- Ask students to answer the following: Your class had to solve the equation \(4(x - 2) = -3(2x + 6)\) on a recent math test. Below are two student solutions. Did either student make any errors?

<table>
<thead>
<tr>
<th>Janicka’s Solution</th>
<th>Alison’s Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>(4(x - 2) = -3(2x + 6))</td>
<td>(4(x - 2) = -3(2x + 6))</td>
</tr>
<tr>
<td>(4x - 2 = -6x + 6)</td>
<td>(4x - 8 = -6x - 18)</td>
</tr>
<tr>
<td>(4x + 6x = 6 + 2)</td>
<td>(4x - 6x = -18 - 8)</td>
</tr>
<tr>
<td>(10x = 8)</td>
<td>(-2x = -26)</td>
</tr>
<tr>
<td>(\frac{10}{8} = \frac{4}{5})</td>
<td>(-2 = -2)</td>
</tr>
<tr>
<td>(x = \frac{8}{10}) or (\frac{4}{5})</td>
<td>(x = 13)</td>
</tr>
</tbody>
</table>

- Create a scavenger hunt around the school using QR codes. The questions should involve solving linear equations. Place a variety of QR codes around the classroom. Each group will select a question, scan it, and then write the steps of the solution. Provide each group with a scavenger hunt key where their solution informs them where to find the next code. The group is required to return all their solutions and workings to the teacher when they are finished with the activity.

**QR Code Scavenger Hunt Key #1**

If your answer is: Go to the entrance of the following rooms:

<table>
<thead>
<tr>
<th>7</th>
<th>Art Room</th>
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</thead>
<tbody>
<tr>
<td>-3</td>
<td>Main Office</td>
</tr>
<tr>
<td>-1</td>
<td>Music Room</td>
</tr>
<tr>
<td>7</td>
<td>Library</td>
</tr>
<tr>
<td>-3</td>
<td>Bulletin Board by Student Council Office</td>
</tr>
<tr>
<td>4</td>
<td>Main door of Cafeteria</td>
</tr>
<tr>
<td>5</td>
<td>Gym</td>
</tr>
</tbody>
</table>

- Ask students to complete the following: Two computer technicians both charge a fee for a home visit, plus an hourly rate for their work. Dawn charges a $64.95 fee plus $45 per hour. Alexi charges a $79.95 fee plus $40 per hour. For what length of service call do Dawn and Alexi charge the same amount?
**SUGGESTED MODELS AND MANIPULATIVES**

- 10 x 10 GRID
- algebra tiles
- balance scales (pan or beam)
- dice
- number line
- Pan Balance Tool*
  
  *also available in *Interactive Math Tools* (Pearson n.d.)

**MATHEMATICAL LANGUAGE**

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Student</th>
</tr>
</thead>
<tbody>
<tr>
<td>algebraic equation</td>
<td>algebraic equation</td>
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<tr>
<td>balance/balancing</td>
<td>balance/balancing</td>
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<tr>
<td>coefficient</td>
<td>coefficient</td>
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<td>combine</td>
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<td>constant</td>
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<td>evaluate</td>
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<tr>
<td>isolate the variable</td>
<td>isolate the variable</td>
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<tr>
<td>like terms</td>
<td>like terms</td>
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<tr>
<td>simplify</td>
<td>simplify</td>
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<tr>
<td>variable</td>
<td>variable</td>
</tr>
<tr>
<td>zero principle</td>
<td>zero principle</td>
</tr>
</tbody>
</table>

**Resources**

**Digital**

- “Algebra Balance Scale [Unnamed],” *National Library of Virtual Manipulatives* (Utah State University 2015): [http://nlvm.usu.edu/en/nav/frames_asid_324_g_4_t_2.html?open=instructions&from=category_g_4_t_2.html](http://nlvm.usu.edu/en/nav/frames_asid_324_g_4_t_2.html?open=instructions&from=category_g_4_t_2.html)
“Using Error Analysis to Teach Equation Solving,” *Mathematics Teaching in the Middle School* (Hawes 2007) pp. 238–242


*Math Makes Sense 9* (Baron et al. 2009; NSSBB #: 2001644)

- Unit 6: Linear Equations and Inequalities
  - Section 6.1: Solve Equations by Using Inverse Operations
  - Section 6.2: Solve Equations by Using Balance Strategies
  - Game: Equation Persuasion
  - Unit Problem: Raising Money for the Pep Club

- *ProGuide* (CD; Word Files; NSSBB #: 2001645)
  - Assessment Masters
  - Extra Practice Masters
  - Unit Tests

- *ProGuide* (DVD; NSSBB #: 2001645)
  - Projectable Student Book Pages
  - Modifiable Line Masters

*Making Math Meaningful to Canadian Students, K–8* (Small 2009), pp. 207–209

SCO PR04 Students will be expected to explain and illustrate strategies to solve single variable linear inequalities with rational coefficients within a problem-solving context.

[C, CN, PS, R, V]

<table>
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**Performance Indicators**

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

**PR04.01** Translate a given problem into a single variable linear inequality, using the symbols ≥, >, <, or ≤.

**PR04.02** Determine if a given rational number is a possible solution of a given linear inequality.

**PR04.03** Generalize and apply a rule for adding or subtracting a positive or negative number to determine the solution of a given inequality.

**PR04.04** Generalize and apply a rule for multiplying or dividing by a positive or negative number to determine the solution of a given inequality.

**PR04.05** Solve a given linear inequality algebraically, and explain the process orally or in written form.

**PR04.06** Compare and explain the process for solving a given linear equation to the process for solving a given linear inequality.

**PR04.07** Graph the solution of a given linear inequality on a number line.

**PR04.08** Compare and explain the solution of a given linear equation to the solution of a given linear inequality.

**PR04.09** Verify the solution of a given linear inequality, using substitution for multiple elements in the solution.

**PR04.10** Solve a given problem involving a single variable linear inequality, and graph the solution.

**Scope and Sequence**

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<thead>
<tr>
<th>Mathematics 8</th>
<th>Mathematics 9</th>
<th>Mathematics 10</th>
</tr>
</thead>
</table>
| **PR02** Students will be expected to model and solve problems, concretely, pictorially, and symbolically, where \( a, b, \) and \( c \) are integers, using linear equations of the form  
  - \( ax = b \)
  - \( \frac{x}{a} = b, a \neq 0 \)
  - \( ax + b = c \)
  - \( \frac{x}{a} + b = c, a \neq 0 \)
  - \( a(x + b) = c \) | **PR04** Students will be expected to explain and illustrate strategies to solve single variable linear inequalities with rational coefficients within a problem-solving context. | **RF10** Students will be expected to solve problems that involve systems of linear equations in two variables, graphically and algebraically. |
Background

Solving linear inequalities is new at the Mathematics 9 level and will build on previous knowledge of linear equations. Students need to realize that unlike a single variable linear equation, which has a single solution, a single variable inequality will have many solutions. Students are familiar with the symbols < and > from their work with comparing two integers in Mathematics 6. The symbols £ and £³ will have to be introduced.

Work with single variable inequalities should help students understand what the answer represents; that is, a set of values rather than a single number. If a container can hold no more than 45 kg, for example, different masses can be put in the container as long as they are less than or equal to 45, $x \leq 45$. Students should also recognize that the same inequality can be written in two different ways. For example, $x \leq 45$ and $45 \geq x$ represent the same set of numbers.

Similarly, graphing inequalities on a number line results in graphing part of the line rather than one specific point. Since there are too many points to graph when you have to consider rational numbers, shading of the number line is necessary. Ensure that students understand the difference between < and £, and > and £³, and the effect these would have on the graph.

Although the majority of the work with inequalities includes rational numbers, some applications will involve discrete data. It is necessary to discuss how this affects the graph. The concept of continuous and discrete data was discussed in PR02.

- For example, if we were to graph the admittance on a certain theme park ride as being at least 10 years old, the inequality $x \geq 10$ results in the following graph:

  ![Graph of x ≥ 10](image)

  Another application of this same inequality can modify this graph.

- Chantal’s mom said she had to invite at least 10 people to the pool party.

  This would result in the following graph for $x \geq 10$:

  ![Graph of x ≥ 10](image)

An understanding of how various operations affect the truth of an inequality should be developed before introducing variables. Students can start with true sentences such as $-2 < 4$ and $5 > 1$. They can make a chart which shows each inequality and investigate how the truth of each is affected when the following operations are performed on both sides of the inequality:

- add a positive number, add a negative number
- subtract a positive number, subtract a negative number
- multiply by a positive number, multiply by a negative number
- divide by a positive number, divide by a negative number

Through investigation, students should recognize that adding or subtracting the same number from both sides of the inequality has no effect on the truth of the inequality.
For example, if $-6 < -3$ is a true statement, then $-6 + 2 < -3 + 2$ or $(-4 < -1)$ is a true statement, and $-6 - 1 < -3 - 1$ or $(-7 < -4)$ is a true statement.

Through investigation, students should recognize that multiplying or dividing by a positive number results in a true inequality. If $10 > -2$ then $3(10) > 3(-2)$ is a true statement ($30 > -6$) as is $\frac{10}{2} > \frac{-2}{2}$ a true statement ($5 > -1$)

However, multiplying or dividing by a negative number, results in a false inequality.

For example, if $-8 < 4$, then $\frac{-8}{-2} < \frac{4}{-2}$ or $(4 < -2)$ is a false statement. The inequality sign must be reversed to keep the truth of the inequality.

Once students have generalized these rules, they can apply them to solving inequalities. The process for solving inequalities is very similar to the process for solving equations. Both need to be balanced, through the use of inverse operations, to isolate the variable. When solving inequalities, however, emphasis should be placed on applying the generalized rule of reversing the sign when multiplying or dividing by a negative number. Provide students with ample practice to reinforce this concept.

As with equations, when solving inequalities the variable can be isolated on either side of the equation. When students solved equations, they could easily see that this did not affect the meaning of their solution (i.e., $x = 3$ and $3 = x$ are equivalent solutions). Students may be confused with the different ways of writing the solution for a linear inequality. Some work may need to be done so that students understand that the solution $x > 3$ is the same as $3 < x$.

Students should understand that the main difference in the solution of an equation as compared to an inequality is the value of the variable. A single variable linear equation has only one value of the variable that makes it true. There may be many values of the variable that satisfy an inequality.

As with equations, students should be aware that once they acquire a solution for an inequality, it can be checked for accuracy by substitution into the original inequality. To reinforce the concept that solutions to inequalities are sets of numbers, students should verify the solution by substituting multiple elements into the original inequality. Students are also required to apply the skills learned earlier in this unit to graph their solutions.

Where possible an effort should be made to have students describe a problem or situation as an inequality to be solved and then represented on a number line. This is a good opportunity for teachers to discuss with students that there may or may not be limits on these inequalities that are created, depending on the context of the problem. For example, if discussing speed of a vehicle as less than 60 km/h, or $v < 60$, students should realize that speed cannot drop below zero. Students should be reminded to distinguish between discrete and continuous data and the effect these have on graphing solutions.
Assessment, Teaching, and Learning

Assessment Strategies

Assessing Prior Knowledge

Tasks such as the following could be used to determine students’ prior knowledge.

- Have students translate these statements into mathematical symbols:
  - My grandfather’s age is greater than 45 years old.
  - Four or fewer students dropped the after-school yoga course.
  - Mario’s golf score was less than 75.
  - My brother is older than my sister who turned 21 this month.
  - The speed on the highway between Truro and Halifax is less than or equal to 110 kilometres per hour.
  - Yarmouth is more than 150 km away from most larger towns in Nova Scotia.

Whole-Class/Group/Individual Assessment Tasks

Consider the following sample tasks (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Using your own example, explain the impact of adding, subtracting, multiplying, and dividing an inequality by a positive number.
- Determine if the values satisfy the corresponding given inequality.

<table>
<thead>
<tr>
<th>Inequality</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x &gt; 3$</td>
<td>5, −7, 9, 10</td>
</tr>
<tr>
<td>$−3x + 12 &lt; 36$</td>
<td>−9, −10, −15.2</td>
</tr>
<tr>
<td>$\frac{x}{4} + 6 ≥ −2$</td>
<td>−10, 15, 2, 7</td>
</tr>
</tbody>
</table>

- Graph the solution to the following inequalities on a number line.
  - $3x − 2 ≤ −20$
  - $7 − 3x ≤ 22$
  - $2 + \frac{2}{3}x > \frac{1}{2}$
  - $2 − 5x > 2x + 16$

- Glen received grades of 75%, 82%, and 78% on his first three summative assessments. They are all equally weighted. Within what range must his next mark be on the next summative assessment to achieve an average of at least 80%? (Hint: To solve the problem, set up an inequality that would help you solve the problem, understanding that there is a maximum mark that Glen could get in the final assessment.)

- Julie bought a $50 prepaid card for her cell phone usage. She has to pay a monthly rate of $15 and then $0.15 per text message. If she only texts, how many text messages can she send this month? How would you graph the solution?
• Explain how you would know whether to use a closed circle or an open circle when representing an inequality on a number line.
• Give an example of a situation or problem that can be represented by an inequality. Write the inequality that represents the problem.
• Write an inequality for the following graphs:

![Number Line 1](image1)

![Number Line 2](image2)

![Number Line 3](image3)

**Planning for Instruction**

**CHOOSING INSTRUCTIONAL STRATEGIES**

Consider the following strategies when planning daily lessons.

• Have students start with a statement they know is true, for example, $5 > -2$. (Repeat this using a variety of statements so that students might generalize a rule.) Have them explore the operations of addition, subtraction, multiplication, and division of both positive and negative integers. Discuss the results. Use the outcomes of this activity to generalize rules for solving inequalities.

• A table such as the following could be used:

<table>
<thead>
<tr>
<th>Inequality</th>
<th>Operation</th>
<th>Value of left side</th>
<th>Value of right side</th>
<th>Resulting Inequality</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5 &gt; -2$</td>
<td>+10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$5 &gt; -2$</td>
<td>-10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$5 &gt; -2$</td>
<td>$\times$ 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$5 &gt; -2$</td>
<td>$\div$ 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$5 &gt; -2$</td>
<td>$\times (-2)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$5 &gt; -2$</td>
<td>$\div (-2)$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

• Once students have an understanding of operation rules have them expand to inequalities involving a single variable, such as $-2x - 5 < 3$.

• Explore the difference between $<$, $>$, $\leq$, $\geq$ and how to represent them on the number line. For example, $x < 1$ could look like:

![Number Line 4](image4)

Whereas $x \leq 1$, would look like:
Introduce questions with answers that are discrete values and graph the solutions. For example: $x \geq 1, x \in \mathbb{Z}$ (Integers)

**Suggested Learning Tasks**

- Have students work in pairs with one student completing the A questions and the other completing the B questions, and then have them verify each other's solutions and look for patterns in the exploration.

<table>
<thead>
<tr>
<th>Student A. Solve. Show all steps. Verify your answer.</th>
<th>Student B. Solve. Show all steps. Verify your answer.</th>
<th>How are the solutions to the two inequalities similar and how are they different? [Think of the solution, sign and graph of the solution.]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n + 5 &gt; 7$</td>
<td>$n + (-5) &gt; 7$</td>
<td></td>
</tr>
<tr>
<td>$2n \leq -6$</td>
<td>$-2n \leq -6$</td>
<td></td>
</tr>
<tr>
<td>$2n + (-3) \geq 7$</td>
<td>$2n + 3 \geq 7$</td>
<td></td>
</tr>
<tr>
<td>$-2n + 3 &gt; -5$</td>
<td>$2n + 3 &gt; -5$</td>
<td></td>
</tr>
<tr>
<td>$2(n - 3) \geq 6$</td>
<td>$-2(n - 3) \geq 6$</td>
<td></td>
</tr>
<tr>
<td>$3n + 2 &lt; x - 4$</td>
<td>$-3n + 2 &lt; x - 4$</td>
<td></td>
</tr>
<tr>
<td>$\frac{n}{4} \geq 2$</td>
<td>$\frac{n}{-4} \geq 2$</td>
<td></td>
</tr>
</tbody>
</table>

- Present students with models, such as the following, and ask the questions below:

  - What values of $n$ will make this model tilt?
  - What values of $n$ will make this model balance?
  - Write a mathematics statement to illustrate each situation.
  - Explain how equality is different from an inequality.

- Have students discuss the difference between $2x + 1 = 5$ and $2x + 1 > 5$.

- Work with a partner to complete the following activity:
  - Each partner chooses a different number.
  - Decide who has the greatest number and write an inequality that compares both numbers.
  - Choose the same mathematical operation to perform on each number.
  - Decide whose resulting number is greater and record an inequality that compares these new numbers.
  - Repeat this process with different mathematical operations.
  - Try different operations until you are able to predict which operations will reverse an inequality symbol and which ones will keep it the same.
  - Organize your observations and results.
▪ Respond to the following:
  − Jason says you can solve an inequality by replacing the inequality sign with an equal sign and putting it back in after solving the equation. Do you agree? Explain.
  − Explain why \(3n - 2 > 8\) and \(3n + 4 < 14\) do not have any solutions in common. Modify one of the inequalities so that they have exactly one solution in common.

▪ Journal writing prompt: Jamaal and Nancy are discussing the inequality \(2x > -10\). Jamaal says, “The solution to the inequality is 6. When I substitute 6 for \(x\), a true statement results”. Nancy says, “I agree that 6 is a solution, but it is not the whole solution”. Ask students to explain what Nancy means.

▪ Ask students to answer the following: Christy downloads music from two online companies. Tunes4U charges $1.50 per download plus a one-time membership fee of $15. YRTunes charges $2.25 per download with no membership fee.
  − Write an expression for the cost to download \(n\) songs from Tunes4U.
  − Write an expression for the cost to download \(n\) songs from YRTunes.
  − Write and solve an inequality to determine when it costs more to download songs from Tunes4U than from YRTunes. Verify and graph the solution.
  − Which site should Christy use to download music?

**Suggested Models and Manipulatives**
- number line*
- pan balance models
- algebra tiles
- Pan Balance Tool*
*also available in Interactive Math Tools (Pearson n.d.) Mathematical Language

**Mathematical Language**

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Student</th>
</tr>
</thead>
<tbody>
<tr>
<td>coefficient</td>
<td>coefficient</td>
</tr>
<tr>
<td>combine</td>
<td>combine</td>
</tr>
<tr>
<td>constant</td>
<td>constant</td>
</tr>
<tr>
<td>equality</td>
<td>equality</td>
</tr>
<tr>
<td>expression</td>
<td>expression</td>
</tr>
<tr>
<td>evaluate</td>
<td>evaluate</td>
</tr>
<tr>
<td>isolate the variable</td>
<td>isolate the variable</td>
</tr>
<tr>
<td>inequality</td>
<td>inequality</td>
</tr>
<tr>
<td>like terms</td>
<td>like terms</td>
</tr>
<tr>
<td>satisfy</td>
<td>satisfy</td>
</tr>
<tr>
<td>simplify</td>
<td>simplify</td>
</tr>
<tr>
<td>solution set</td>
<td>solution set</td>
</tr>
<tr>
<td>variable</td>
<td>variable</td>
</tr>
<tr>
<td>verify</td>
<td>verify</td>
</tr>
<tr>
<td>zero principle</td>
<td>zero principle</td>
</tr>
</tbody>
</table>
Resources

Digital

- “Algebra Tiles [iPad App], Apps for iPads” (Braining Camp 2013): [www.brainingcamp.com/product/mobile.html](http://www.brainingcamp.com/product/mobile.html)
- “Solving Equations [iPad App], Apps for iPads” (Braining Camp 2013): [www.brainingcamp.com/product/mobile.html](http://www.brainingcamp.com/product/mobile.html)

Print

- *Math Makes Sense 9* (Baron et al. 2009; NSSBB #: 2001644)
  - Unit 6: Linear Equations and Inequalities
    > Section 6.3: Introduction to Linear Inequalities
    > Section 6.4: Solving Linear Inequalities by Using Addition and Subtraction
    > Section 6.5: Solving Linear Inequalities by Using Multiplication and Division
    > Unit Problem: Raising Money for the Pep Club
  - *ProGuide* (CD; Word Files; NSSBB #: 2001645)
    > Assessment Masters
    > Extra Practice Masters
    > Unit Tests
  - *ProGuide* (DVD; NSSBB #: 2001645)
    > Projectable Student Book Pages
    > Modifiable Line Masters