Mathematics 9

Unit 8: Circle Geometry

M01

SCO M01 Students will be expected to solve problems and justify the solution strategy, using the following circle properties: ▪ The perpendicular from the centre of a circle to a chord bisects the chord. ▪ The measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc. ▪ The inscribed angles subtended by the same arc are congruent. ▪ A tangent to a circle is perpendicular to the radius at the point of tangency. [C, CN, PS, R, T, V]
[C] Communication **[PS]** Problem Solving **[CN]** Connections **[ME]** Mental Mathematics and Estimation **[T]** Technology **[V]** Visualization **[R]** Reasoning

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

M01.01 Demonstrate that

- the perpendicular from the centre of a circle to a chord bisects the chord
- the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc
- the inscribed angles subtended by the same arc are congruent
- a tangent to a circle is perpendicular to the radius at the point of tangency

M01.02 Solve a given problem involving application of one or more of the circle properties.

M01.03 Determine the measure of a given angle inscribed in a semicircle, using the circle properties.

M01.04 Explain the relationship among the centre of a circle, a chord, and the perpendicular bisector of the chord.

Scope and Sequence

Mathematics 8	Mathematics 9	Mathematics 10
M01 Students will be expected to develop and apply the Pythagorean theorem to solve problems.	**M01** Students will be expected to solve problems and justify the solution strategy, using the following circle properties: ▪ The perpendicular from the centre of a circle to a chord bisects the chord. ▪ The measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc. ▪ The inscribed angles subtended by the same arc are congruent. ▪ A tangent to a circle is perpendicular to the radius at the point of tangency.	–

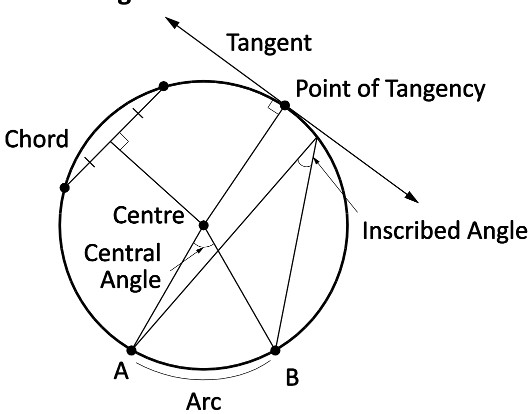
Background

Students have explored circles in Mathematics 7 in the form of radius, diameter, circumference, pi, and area.

They have developed formulas for these topics through exploration. Students are also familiar with constructing circles and central angles. While problem solving in this outcome, the Pythagorean theorem developed in Mathematics 8 will be used, and should be reviewed in context.

In Mathematics 9, students will need to develop an understanding of terms relating to circle properties. This outcome develops properties of circles and will introduce students to new terminology. Each property should be developed through a geometric exploration, which brings out the new terminology and then applies it to real life situations. Terminology includes:

- A **circle** is a set of points in a plane that are all the same distance (equidistant) from a fixed point called the centre. A circle is named for its centre.
- A **chord** is a line segment joining any two points on the circle.
- A **central angle** is an angle formed by two radii of a circle.
- An **inscribed angle** is an angle formed by two chords that share a common endpoint; that is, an angle formed by joining three points on the circle.
- An **arc** is a portion of the circumference of the circle.
- A **tangent** is a line that touches the circle at exactly one point, which is called the **point of tangency**.



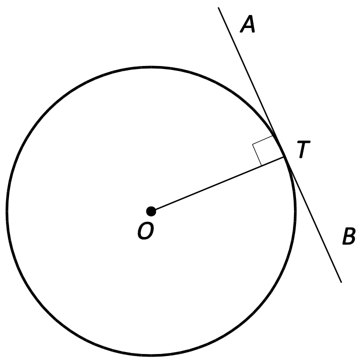
Students will be exploring circle properties around chords, inscribed and central angle relationships, and tangents to circles. The treatment of these circle topics is not intended to be exhaustive, but will be determined to a significant extent by the contexts examined.

As students use circle properties to determine angle measures, it will be necessary to apply previously learned concepts. A circle may contain an isosceles triangle, for example, whose legs are radii of the circle. Students must recognize that the angle opposite the congruent sides of the isosceles triangle have equal measures. This was introduced in Mathematics 6.

Another commonly used property is that the sum of interior angles in a triangle is 180° (Mathematics 6).

The properties of a circle can be introduced in any order. By starting with the property “A tangent to a circle is perpendicular to the radius at the point of tangency,” students are introduced to only one new term. This provides the opportunity for contextual problem solving before any other properties are developed. All properties should be developed in this manner so that students make connections with real-life situations.

- In the following diagram:
    - O is the center of the circle
    - OT is the radius
    - T is a point of tangency
    - AB is a tangent line
    - The tangent-radius property states that under the given conditions $\angle$ ATO = 90°.



Paper folding provides a good means of exploring some of the properties of circles in this outcome, such as locating the centre of a circle, determining that an inscribed angle on the diameter is a right angle, and that the perpendicular of a chord in a circle passes through the centre. (Patty paper is useful in paper folding activities.)

Locating the centre using diameters:

- Draw a large circle on a piece of paper.
- Fold the circle to form a diameter and mark endpoints A and B.
- Fold the circle again using a different mirror line mark the end points C and D.
- The point of intersection of these two diameters is the centre of the circle.

An inscribed angle on the diameter is a right angle:

- Draw a large circle on a piece of paper.
- Fold the circle to form a diameter and mark endpoints A and B.
- Mark a point C on the circumference. Fold to form chord AC.
- Fold to form chord BC.
- Measure angle C. What do you notice?

The perpendicular of a chord pass through the centre:

- Draw a large circle on a piece of paper.
- Draw two chords on the circle that are not parallel.
- Use folding to find the perpendicular bisector of each chord.
- The point of intersection of the two perpendicular bisectors is the centre of the circle.

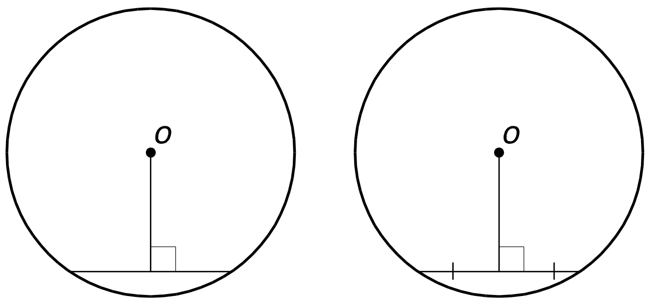
Students should come to realize that if any two of the following three conditions are in place, then the third condition is true for a given line and a given chord in a circle:

- the line bisects the chord
- the line passes through the centre of the circle
- the line is perpendicular to the chord

Illustrate the properties of a circle using the following diagrams:

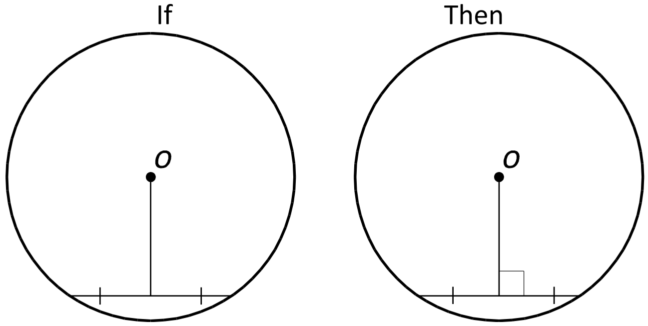
- Property 1: A line from the centre of the circle that is perpendicular to a chord will bisect the chord.

If Then

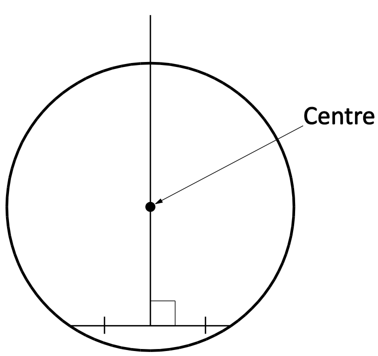


- Property 2: A line from the centre that bisects a chord is perpendicular to the chord.

If Then



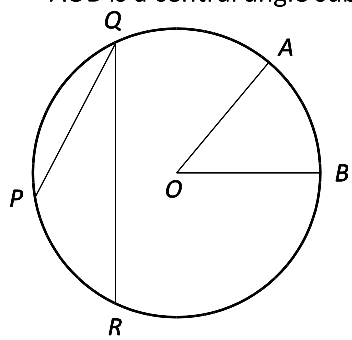
- Property 3: If a line is a perpendicular bisector of a chord, then the line passes through the centre of the circle.



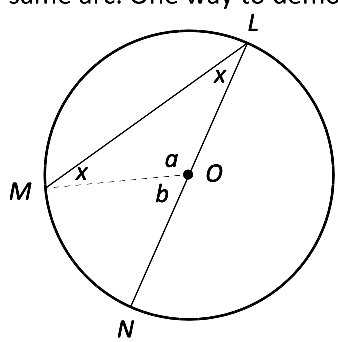
Students should also discover relationships between central and inscribed angles. Circle geometry is very visual, and students should be encouraged to draw diagrams. Some students may have difficulty identifying the arc that subtends an inscribed or central angle. They may benefit from using different colours to outline and label different lines that make angles. Reinforce the idea that an angle subtended by an arc is an angle that has common endpoints with the arc.

∠ PQR is an inscribed angle subtended by arc PR

∠ AOB is a central angle subtended by arc AB



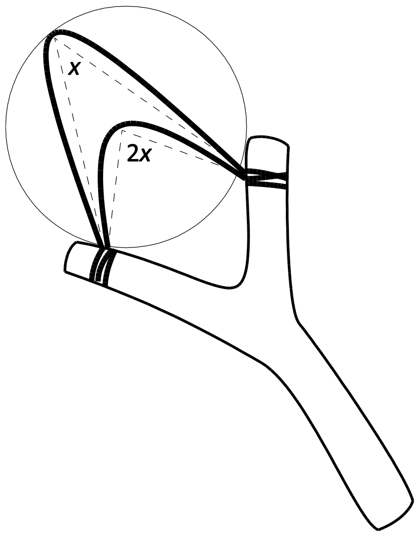
Students should discover the relationship between inscribed and central angles that are subtended on the same arc. One way to demonstrate the relationship is indicated below.



Notice $a + 2x = 180°$
Also, $a + b = 180°$
Therefore $b = 2x$

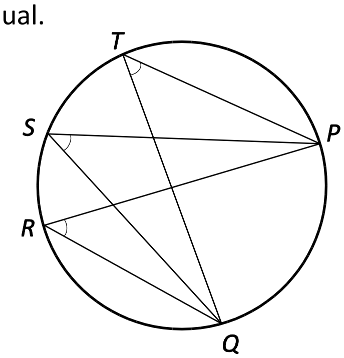
Since b represents a central angle and x represents an inscribed angle, students should conclude that inscribed angles are equal to half the measure of the central angle subtended by the same arc.

A common error occurs when students double the measure of the central angle to determine the inscribed angle. The use of diagrams is a good visual tool to show the impossibility of an inscribed angle being larger than a central angle subtending the same arc. Students could think about the act of drawing back a slingshot and measuring the angle that is formed by the elastic. The further the slingshot is pulled back the more acute (smaller) the angle becomes. This mental exercise will reinforce the notion that the inscribed angle is smaller than the central angle subtended on the same arc.



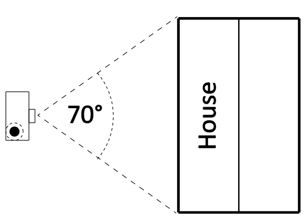
Alternatively, if students understand that the diameter is a central angle measuring 180°, they should conclude that an inscribed angle is half of the central angle subtended by the same arc and, since the central angle is 180°, the inscribed angle must be 90°.

Students should also have an opportunity to discover that inscribed angles subtended by the same arc are equal.

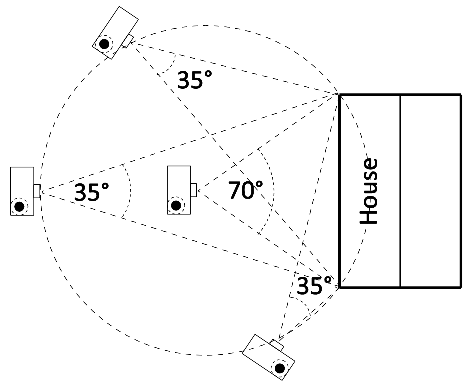


Work through the following example with students to help them develop the relationship between angles in a circle:

- Jackie works for a realtor photographing houses that are for sale. She photographed a house two months ago using a camera lens that has a 70° field of view. She has returned to the house to update the photo, but she has forgotten her lens. Today she only has a telephoto lens with a 35° field of view. From what location(s) could Jackie photograph the house with the telephoto lens, so that the entire house still fills the width of the picture? Explain your choices.



A possible solution is shown here. This also illustrates that inscribed angles subtended by the same arc are congruent.



Once all properties have been developed, students can solve problems involving a combination of properties. The use of technology is encouraged. Dynamic geometry software packages can help students explore the relationships.

Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Provide students with bull's eye compasses. Have them explore drawing circles. Remind them to label the length of the radius and the diameter of each circle.

- A student performed the following steps using the Pythagorean Theorem. **Circle** the step where the student made an error and write the corrected solution (including all steps) to the right of the student's work. For example: If ***a* = 4** and ***b* = 6**

  $4^2+6^2=c^2$

  $8+12=c^2$

  $20=c^2$

  $\sqrt{20}=\sqrt{c^2}$

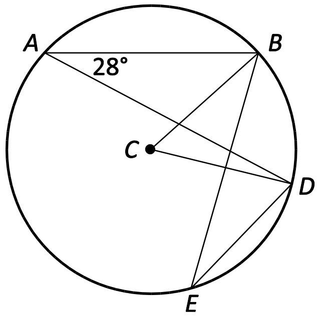
  $4.47=c$

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

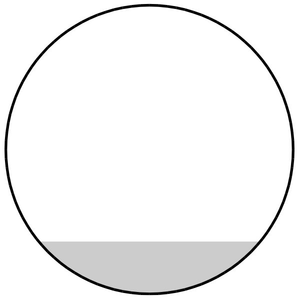
Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- You have just purchased a new umbrella to put in the centre of your wooden circular picnic table. You want to place the umbrella in the centre of the table, but the hole is not cut. Explain how you would figure out where to cut the hole for your new umbrella.

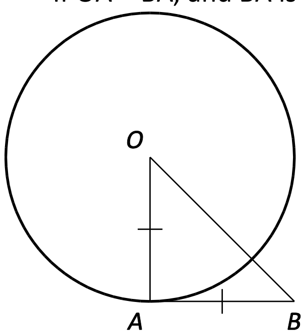
- Find $\angle$ BCD and $\angle$ BED.



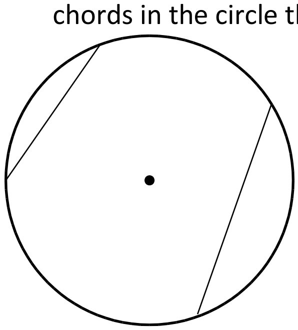
- The diagram represents the water level in a pipe. The surface of the water from one side of the pipe to the other measures 30 mm and the inner diameter of the pipe 44 mm. What is the depth of the water?



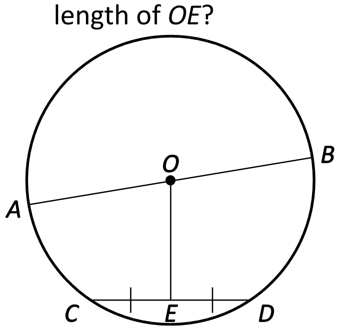
- If OA = BA, and BA is tangent to the circle at A, determine the measure of ∠ ABO.



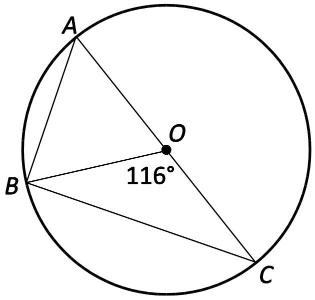
- Mike has a rock tied to the end of a 5 m rope and is swinging it over his head to form a circle with him at the center. The rock comes free of the rope and flies along a tangent from the circle until it hits the side of a building that is 14 m away from Mike. How far along the tangent did the rock travel? Determine the answer to the nearest meter.
- Ask students to explain how they could locate the center of a circle if they were given any two chords in the circle that are not parallel.



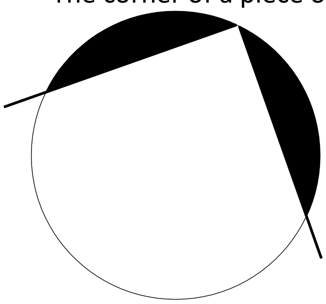
- In the following circle with center O, the diameter is 40 cm, and chord CD is 34 cm. What is the length of *OE*?



- In the following circle with centre O, ∠ BOC = 116°. What is the measure, in degrees, of ∠ ABO and ∠ BCO?

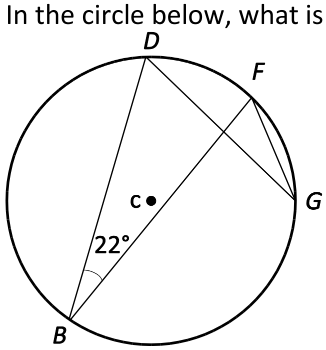


- The corner of a piece of paper is a 90° angle and is placed on a circle as shown.

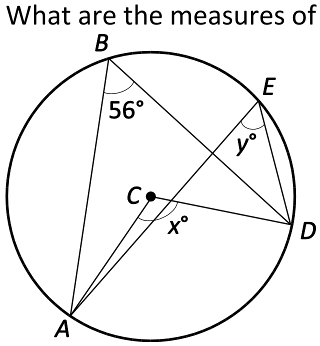


  - Why is *AB* the diameter?
  - How can the corner of the paper be used to find the centre of the circle?

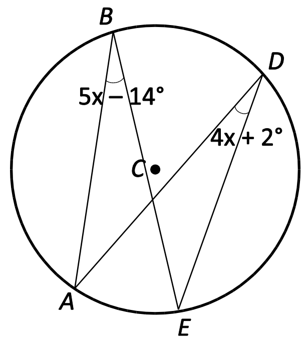
- In the circle below, what is the measure of ∠ DGF?



- What are the measures of *x* and *y*?



- What is the value of *x*? the measure of ∠ ABE?



- A city is building a pedestrian tunnel under a street using a large culvert. The culvert has a diameter of 5 meters. The city is going to fill the bottom of the culvert with concrete to create a surface for walking. Regulations state that there must be 4.2 m of space between the top of the culvert and the walking surface.
    - How deep must the city pour the concrete in the bottom of the culvert?
    - How wide will the walking surface be when it is completed?

Planning for Instruction

Choosing Instructional Strategies

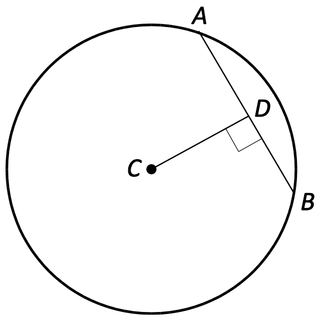
Consider the following strategies when planning daily lessons.

- Provide students with a handout of circles with labelled centres to explore circle properties.
    - Ask students to draw two non-parallel chords in the same circle. Using the triangle from a geometry set have them draw a line perpendicular to each chord passing through the centre, and then measure each part of the divided chords. This exploration should lead to establishing that the perpendicular bisector of a chord will pass through the centre of the circle and conversely that the line from the centre of the circle that meets the chord at a right angle, will also bisect the chord.
    - Provide opportunities for students to draw and measure central and inscribed angles subtended by the same arc and draw conclusions from their answers.
    - Ask students to place a point outside of one of the circles and ask them to draw the two possible tangents to the circle. From the point where each tangent touches the circle (point of tangency), ask students to draw a line to the centre of the circle. Students should then measure the angle formed by the tangent and the radius. What do the students notice about these measurements?
    - Ask students to draw a diameter on one of the circles. They should then draw and measure an inscribed angle subtended by the semi-circle.

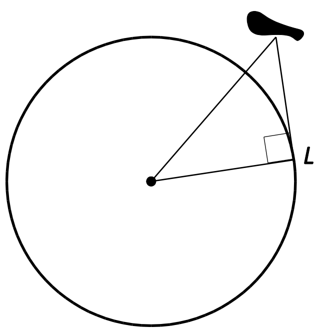
Suggested Learning Tasks

- Challenge students with the following problem: A surveillance camera is taping people coming through the entrance of the school. While reviewing the tape, school administrators realized that the camera was broken. When shopping for a new one, the cameras available have a field of view of 40° compared to the broken one that had a field of view of 80°. Where should they position the new camera to cover the same area?

- Provide students with an arc and ask them to find the radius of the circle from which the arc was taken (could be extended to a variety of arcs).
- Have students respond to the following problems:
    - The radius of the circle to the right measures 6 cm. If the distance between the centre and the chord (CD) is 4 cm, what is the length of the chord AB?



    - The radius of the earth is 6400 km. If a bird is 1500 m from the ground, how far is it from Leslie standing at point L?



- Ask students to complete the following paper folding activity to develop the relationship between the perpendicular from the centre of the circle and a chord.
    - Construct a large circle on tracing paper and draw two different chords.
    - Construct the perpendicular bisector of each chord.
    - Label the point inside the circle where the two perpendicular bisectors intersect.
    - What do you notice about the point of intersection of the two perpendicular bisectors?

SUGGESTED MODELS AND MANIPULATIVES

- circular objects to trace circles
- string
- tracing (patty) paper
- bull's-eye compass
- The Geometer's Sketchpad
- circle template (See Digital Resources)

Mathematical Language

Teacher	Student
▪ arc ▪ area ▪ bisect ▪ centre ▪ central angle ▪ chord ▪ circle ▪ circumference ▪ congruent ▪ diameter ▪ equidistant ▪ inscribed angle ▪ line segment ▪ perpendicular ▪ perpendicular bisector ▪ pi ▪ point of tangency ▪ radii ▪ radius ▪ subtend ▪ subtended ▪ tangent	▪ arc ▪ area ▪ bisect ▪ centre ▪ central angle ▪ chord ▪ circle ▪ circumference ▪ congruent ▪ diameter ▪ equidistant ▪ inscribed angle ▪ line segment ▪ perpendicular ▪ perpendicular bisector ▪ pi ▪ point of tangency ▪ radii ▪ radius ▪ subtend ▪ subtended ▪ tangent

Resources

Digital

- "Circle Template," *Illuminations: Resources for Teaching Math* (National Council of Teachers of Mathematics 2015) http://illuminations.nctm.org/uploadedFiles/Content/Lessons/Resources/9-12/FoldingCircle-CircleTemplate.pdf
- "Exploring Circle Geometry Properties: Use It," *Math Interactives* (Alberta Education, LearnAlberta.ca 2015): www.learnalberta.ca/content/mejhm/index.html?l=0&ID1=AB.MATH.JR.SHAP&ID2=AB.MATH.JR.SHAP.CIRC&lesson=html/object_interactives/circles/use_it.html
- "Folding Circles: Exploring Circle Theorems through Paper Folding," *Illuminations: Resources for Teaching Math* (National Council of Teachers of Mathematics 2015): http://illuminations.nctm.org/Lesson.aspx?id=3777
- "Point-Circle," LearnAlberta.ca (Alberta Education, LearnAlberta.ca 2015): www.learnalberta.ca/content/meda/html/pointcircle/index.html

Print

- *Math Makes Sense 9* (Baron et al. 2009; NSSBB #: 2001644)
    - Unit 8: Circle Geometry

    - > Section 8.1: Properties of Tangents to a Circle
    - > Section 8.2: Properties of Chords in a Circle
    - > Technology: Verifying the Tangent and Chord Properties
    - > Game: Seven Counters
    - > Section 8.3: Properties of Angles in a Circle
    - > Technology: Verifying the Angle Properties
    - > Unit Problem: Circle Designs
  - *ProGuide* (CD; Word Files; NSSBB #: 2001645)
    - > Assessment Masters
    - > Extra Practice Masters
    - > Unit Tests
  - *ProGuide* (DVD; NSSBB #: 2001645)
    - > Projectable Student Book Pages
    - > Modifiable Line Masters
- *Patty Paper Geometry* (Serra 2011) , pp. 103–119
- *Developing Thinking in Geometry* (Johnston-Wilder and Mason 2006), pp. 41–45
- *Geometry: Seeing, Doing, Understanding,* Third Edition (Jacobs 2003), pp. 484–485, 491–492, 497–499, 504–505