Mathematics 9

Unit 9: Statistics and Probability

SP01, SP02, SP03, SP04

SCO SP01 Students will be expected to describe the effect on the collection of data of bias, use of language, ethics, cost, time and timing, privacy, and cultural sensitivity. [C, CN, R, T]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

SP01.01   Analyze a given case study of data collection; and identify potential problems related to bias, use of language, ethics, cost, time and timing, privacy, or cultural sensitivity.
SP01.02   Provide examples to illustrate how bias, use of language, ethics, cost, time and timing, privacy, or cultural sensitivity may influence data.

Scope and Sequence

Mathematics 8	Mathematics 9	Mathematics 10
SP01 Students will be expected to critique ways in which data is presented.	**SP01** Students will be expected to describe the effect on the collection of data of bias, use of language, ethics, cost, time and timing, privacy, and cultural sensitivity.	–

Background

In Mathematics 8, students learned a variety of ways in which to present data (bar graphs, line graphs, circle graphs, pictographs), and to evaluate the strengths and limitations of different presentations of data. Terms such as discrete and continuous data, accuracy, choice of intervals, and trends were reinforced. Students learned how to justify their conclusions and identify inconsistent and misrepresented data.

In Mathematics 9, students will continue to develop data analysis and focus on factors that affect the collection of data.

There are many factors within the data collection process that have the potential to influence the results. Students should consider factors such as the method used, the reliability and usefulness of data, and the ability to make generalizations about the population from a sample. To critically analyze data collection, students must have an understanding of the factors that might lead to problems in the data collection process.

A good way to approach this would be for students to analyze survey questions showing only one problem. The following situation, for example, illustrates how timing can affect data collection:

- Free samples of sunscreen are sent to every home in the fall and winter. A mail reply card asks people if they would use the product again.

When preparing to collect data, appropriate questions and a representative sample of the whole population are important.

Students should consider the following:
- appropriate questions are clearly written, easy to answer, and effective in generating the desired data
- multiple choice questions are useful for identifying respondents' preferences
- questions should be ordered appropriately

    Students should analyze how the phrasing of questions might affect the data collected. For a given case study, they should ask questions such as:
- Does asking this question collect the required information?
- Does the question make one response sound right and another one wrong (i.e., does it have bias)?
- Is the question respectful?

Remind students that when wording survey questions, factors that may influence the responses should be considered.

Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- What graph would you use to represent the data below? Explain your choice.
    – the cost of cell phones over the past 20 years
    – prices of different brands of jeans
    – the favourite sports of boys and girls
    – the percentage of favorite pizza toppings of grade 9s

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- For the following, identify the sources of bias and suggest ways to remove it.
    – At a soccer game, a survey was given and the results showed that when asked to give their favourite sport, 85% of the youth responded it was soccer.
    – Do you think that small dogs make good pets even though they are yappy?
- Create a survey question with bias about the use of technology among teenagers.
- Your friend is unclear what the term "bias" means. Develop an example to help explain the term.

Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

▪ Use graphs from various sources (newspapers, magazines, online, etc.), computer spreadsheet applications (e.g., Microsoft Excel), and websites such as Statistics Canada (www40.statcan.gc.ca/z01/cs0002-eng.htm).

SUGGESTED LEARNING TASKS

▪ Ask students to bring in a variety of surveys and discuss the following:
  – **Bias:** How many people were surveyed? What was the age range of the population?
  – **Use of language:** Is the question clear, and not leading to a particular response?
  – **Ethics:** Will the results be used for appropriate purposes originally given?
  – **Cost:** Does the cost of the study outweigh the benefits?
  – **Time:** Is the timing of, and the time required to do the survey, appropriate?
  – **Privacy:** Are the results confidential?
  – **Cultural sensitivity:** Are the questions offensive to a particular culture?
▪ Have students, in groups, develop survey questions. For example:
  – Since students have many after school activities, do you think teachers should assign less homework?
  – Do you agree that the price of concert tickets is too high?

  Ask students to identify the potential problems associated with the questions asked.
▪ Compile some data examples (good/bad) from math texts from previous years for analysis and discussion.
▪ Investigate misleading information (e.g., the endangered tree octopus, the house hippo) and discuss the value of critical thinking skills.
▪ Students could be given a case study, such as the following, and asked to determine the factor(s) that might affect data collection. Ask students to rewrite the scenario without any bias.
  – A marketing agency wants to determine how Canadians spend their clothing dollars. Jody wrote this question to determine how much is spent on imported clothing: What does your closet contain more of? A. less expensive, foreign made clothes or B. high-quality, made-in Canada clothes.
  – What specific information is Jody trying to obtain?
  – Rewrite the question to avoid bias and sensitivity issues.
▪ Students could be asked to develop their own survey question that involves factor(s) that affect data collection. They could then identify the factor(s) involved and rewrite the question to collect accurate data.

SUGGESTED MODELS AND MANIPULATIVES

▪ computer spreadsheet applications (e.g., Microsoft Excel, Google docs)
▪ graphs from various sources (newspapers, magazines, etc.)

- computer, tablet
- websites for data sources (e.g., Statistics Canada: www.statcan.gc.ca)
- websites for interactives (e.g., Shodor: www.shodor.org\interactive)

MATHEMATICAL LANGUAGE

Teacher	Student
- bias	- bias
- bar graph	- bar graph
- circle graph	- circle graph
- continuous	- continuous
- discrete	- discrete
- distort	- distort
- double bar graph	- double bar graph
- double line graph	- double line graph
- interval	- interval
- line graph	- line graph
- misrepresentation	- misrepresentation
- pictograph	- pictograph
- trend	- trend

Resources

Digital

- "Data Analysis and Probability, [manipulatives]," *National Library of Virtual Manipulatives* (Utah State University 2015): http://nlvm.usu.edu/en/nav/topic_t_5.html
- "Tables by Subject," *Statistics Canada* (Government of Canada 2015): www40.statcan.gc.ca/z01/cs0002-eng.htm
- *Interactivate,* (Shodor 2015): www.shodor.org\interactive
- *Statistics Canada* (Government of Canada 2015): www.statcan.gc.ca

Print

- *Math Makes Sense 9* (Baron et al. 2009; NSSBB #: 2001644)
  - Unit 9: Probability and Statistics
    - > Game: Cube Master
    - > Section 9.2: Potential Problems with Collecting Data
    - > Technology: Using *Census at School* (Currently, link is not working.)
  - *ProGuide* (CD; Word Files; NSSBB #: 2001645)
    - > Assessment Masters
    - > Extra Practice Masters
    - > Unit Tests
  - *ProGuide* (DVD; NSSBB #: 2001645)
    - > Projectable Student Book Pages

> Modifiable Line Masters
- *Elementary and Middle School Mathematics: Teaching Developmentally*, 8th Edition (Van de Walle, Karp, and Bay-Williams 2013), pp. 434–448
- *Making Math Meaningful to Canadian Students, K–8* (Small 2008), pp. 525–532

SCO SP02 Students will be expected to select and defend the choice of using either a population or a sample of a population to answer a question. [C, CN, PS, R]			
[C] Communication	**[PS]** Problem Solving	**[CN]** Connections	**[ME]** Mental Mathematics and Estimation
[T] Technology	**[V]** Visualization	**[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

SP02.01 Identify whether a given situation represents the use of a sample or a population.

SP02.02 Provide an example of a situation in which a population may be used to answer a question, and justify the choice.

SP02.03 Provide an example of a question where a limitation precludes the use of a population, and describe the limitation.

SP02.04 Identify and critique a given example in which a generalization from a sample of a population may or may not be valid for the population.

SP02.05 Provide an example to demonstrate the significance of sample size in interpreting data.

Scope and Sequence

Mathematics 8	Mathematics 9	Mathematics 10
SP01 Students will be expected to critique ways in which data is presented.	**SP02** Students will be expected to select and defend the choice of using either a population or a sample of a population to answer a question.	–

Background

To analyze whether a given situation represents a **sample** or a **population**, students must clearly understand these terms in the context of data collection and analysis. Some students may not understand the difference between a population and a sample. Reinforce the meaning of each term using a familiar example. Stress the importance of the context to help determine the population or the sample. Students may think that the term population only refers to a group of people. In fact, the term population refers to a complete group of anything, such as people, objects, animals, businesses, etc.

Since it is often impractical to gather information about entire populations, sampling is a common statistical technique. Any group of individuals (or units) selected from the population would be referred to as a sample. For example, in a federal election a sample could be taken of 100 individuals chosen from each province or territory.

When a sample is representative of the population, the data collected from the sample leads to valid conclusions. Students will need to understand issues with respect to sampling strategies and **sample size** in order to properly draw inferences from sample data.

Examining different types of sampling would be beneficial to students when they are developing a

project plan, and have to consider the choice of a data collection method. Students should also be able to determine when it is best to use a population versus a sample when limitations are present. Suppose students want to conduct a survey to find out where to go on a class trip. It would be appropriate to ask everyone in the class. With a large population, however, it is impractical to survey everyone, so students need to use a representative sample group. Suppose they want to conduct a survey to determine if people in their community support year-round schooling. They would have to carefully consider whom to ask and how many people to ask. There are many factors that affect the feasibility of using the entire population.

Students should be encouraged to carefully consider any generalizations made from a sample to a population, as sometimes they may not be **valid**. For example, students could consider the following scenario:

- All grade 9 students in the province were surveyed to determine the start time for the school day. Ninety percent of the students in urban HRM wanted school to start at 7:50 a.m., as they wanted to finish up early.

  This sample might not represent the majority of students outside urban HRM because they would be considering different factors, such as length of time spent on bus travel to school, when completing their survey. On the other hand, students should realize that if proper sampling techniques were used, the survey results would be valid.

Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Collect graphs from newspapers or other sources and show them to students.
  - What are some advantages for using the graph that was chosen?
  - What are some disadvantages for using the graph that was chosen?
  - What other graphs could have been used?

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Identify the population for each situation given below, and indicate if you would survey the population or just a sample.
  - Vito's restaurant would like to know which lunch menu item its customers would prefer.
  - A phone service provider would like to know how many of its customers would like to have caller id as a feature.

- – Health Canada would like to find out reasons why some Canadians chose not to get the flu vaccine.
- For the following scenarios, what problems can you see with the generalizations that were made?
  - – At the school cafeteria, employees conducted a survey about what snacks would be offered at break time during the school day. The cafeteria worker handed out a survey to every fourth person who came through the line on a particular day and gathered the data from these. It was concluded from this that students would like to see more granola bars offered during the breaks.
  - – Student council surveyed students about how best to spend the activities budget for the coming year. It randomly surveyed students at a soccer game. Student council concluded more money should be spent on athletic teams.
- Students could be asked to explain the factors that would determine using a sample rather than a population in the following scenarios:
  - – Is there a need in Nova Scotia for a mass vaccination for the flu virus?
  - – Is there a need to check each light bulb coming off an assembly line for defects?
  - – Is there a need to survey all people in an electoral district before an election to predict the winner?

Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Consider the following two situations. In the first situation, a television survey is done to find the fraction of the public that watches a particular television program on a particular night and to determine if more women than men watch it. In the second situation, a quality engineer must estimate what percentage of bottles rolling off an assembly line is defective. In both these situations, information is gathered about a large group of people or things. The expense of contacting every person or inspecting every bottle, and the time it would take, is formidable so information is gathered about only part of the group (a sample) in order to draw conclusions about the whole group (the population).
- Ensure students recognize that choosing a representative sample from a large and varied population can be a complex task. It is important to be clear about what population is to be described and exactly what is to be measured. Areas of concern:
  - – How can a sample be chosen so that it is truly representative of the population?
  - – If a sample from a population differs from another sample from the same population, how confident can you be about predicting the true population percentage?
  - – Does the size of the sample make a difference?

SUGGESTED LEARNING TASKS

- Identify a population for a specific situation (e.g., a cell phone carrier wants to know which brand and model of phone the students are currently using) and state whether the whole population or a sample of the population should be sampled. Explain your reasoning.
- Ask students to identify whether a sample or a population is used in each of the following situations:

- All residents of a town are asked where a new school for the town should be located.
  - One out of every 100 tablets is tested for defects.
  - A student from each class in a school is questioned about removing chocolate milk from the lunch menu.
- Students could be asked to explain why data collection should include the entire population for situations such as the following:
  - Jet engines, produced by a factory, should be tested before use.
  - A government official is elected.
  - Determine whether or not a school should have a uniform for students.
- Each student is given a cut-out of a fish and is asked to label the fish as male or female. Ask students to determine the percentage of male and female in the populations. Place all fish in a fishbowl and depending on class size draw a:
  - small sample
  - medium sample
  - large sample
  - Ask students to calculate the percentage of male/female in the sample and compare this value to the experimental results of the population.

SUGGESTED MODELS AND MANIPULATIVES

- computer and Internet access

MATHEMATICAL LANGUAGE

Teacher	Student
- census - inference - population - random - sample - sample size - validity	- census - inference - population - random - sample - sample size - validity

Resources

Digital

- "Data Analysis and Probability, [manipulatives]," *National Library of Virtual Manipulatives* (Utah State University 2015): http://nlvm.usu.edu/en/nav/topic_t_5.html

Print

- *Math Makes Sense 9* (Baron et al. 2009; NSSBB #: 2001644)
  - Unit 9: Probability and Statistics

> Section 9.3: Using Samples and Population to Collect Data
(The Nova Scotia Mathematics 9 curriculum requires students to provide an example to demonstrate the significance of sample size in interpreting data [PI 9SP02.05].)

> Section 9.4: Selecting a Sample

- *ProGuide* (CD; Word Files; NSSBB #: 2001645)

  > Assessment Masters

  > Extra Practice Masters

  > Unit Tests

- *ProGuide* (DVD; NSSBB #: 2001645)

  > Projectable Student Book Pages

  > Modifiable Line Masters

- *Elementary and Middle School Mathematics: Teaching Developmentally*, 8th Edition (Van de Walle, Karp, and Bay-Williams 2013), pp. 434–448

- *Making Math Meaningful to Canadian Students, K–8* (Small 2008), pp. 525–532

SCO SP03 Students will be expected to develop and implement a project plan for the collection, display, and analysis of data by - formulating a question for investigation - choosing a data collection method that includes social considerations - selecting a population or a sample - collecting the data - displaying the collected data in an appropriate manner - drawing conclusions to answer the question [C, PS, R, T, V]	
[C] Communication **[PS]** Problem Solving **[CN]** Connections **[ME]** Mental Mathematics and Estimation **[T]** Technology **[V]** Visualization **[R]** Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

SP03.01 Create a rubric to assess a project that includes the assessment of
- a question for investigation
- the choice of a data collection method that includes social considerations
- the selection of a population or a sample and the justification for the selection
- the display of collected data
- the conclusions to answer the question

SP03.02 Develop a project plan that describes
- a question for investigation
- the method of data collection that includes social considerations
- the method for selecting a population or a sample
- the methods for display and analysis of data

SP03.03 Complete the project according to the plan, draw conclusions, and communicate findings to an audience.

SP03.04 Self-assess the completed project by applying the rubric.

Scope and Sequence

Mathematics 8	Mathematics 9	Mathematics 10
SP01 Students will be expected to critique ways in which data is presented.	**SP03** Students will be expected to develop and implement a project plan for the collection, display, and analysis of data by - formulating a question for investigation - choosing a data collection method that includes social considerations - selecting a population or a sample - collecting the data - displaying the collected data in an appropriate manner - drawing conclusions to answer the question	–

Background

In previous grades, students have collected, displayed, and interpreted data represented in various tables and graphs. In Mathematics 9, students will plan and carry out a data project to answer a question, and individually or as a group, design a rubric to assess the project. The project will include formulating an appropriate question, collecting data from a sample or population, displaying the data, and drawing conclusions..

Proper planning should identify potential problems with questions or data collection methods. Problem solving should permeate the whole process, as students decide on interesting topics, formulate questions, plan the collection of data, implement plans, and analyze results. This outcome should incorporate many of the other Statistics and Probability outcomes. This outcome is to be assessed based on the development and implementation of an individualized or group project.

The following are useful guidelines for project-based learning:
- Students may in groups or work independently.
- Allow students a choice on the topic and methods of presentation.
- Plan the project with drafts and timeline benchmarks.
- Familiarize students with the assessment plan.
- Begin stages of the project at various times of the year.

In the mathematics curriculum, this is the first exposure students have had developing rubrics. A rubric should be developed prior to the beginning of the project to clarify exactly what is expected and how the project will be assessed. A rubric should include the criteria that will be assessed and a description of each level of performance. Rubric development could be completed as a class activity. A sample rubric is shown below.

Criteria	Level 1	Level 2	Level 3	Level 4
The question for investigation				
The choice of data collection method that includes social considerations				
The selection of a population or a sample and justification of the choice				
The presentation of collected data				
Conclusions to answer the question				

Students can do a written or oral report that includes information about the project plan and conclusions. The report should also include
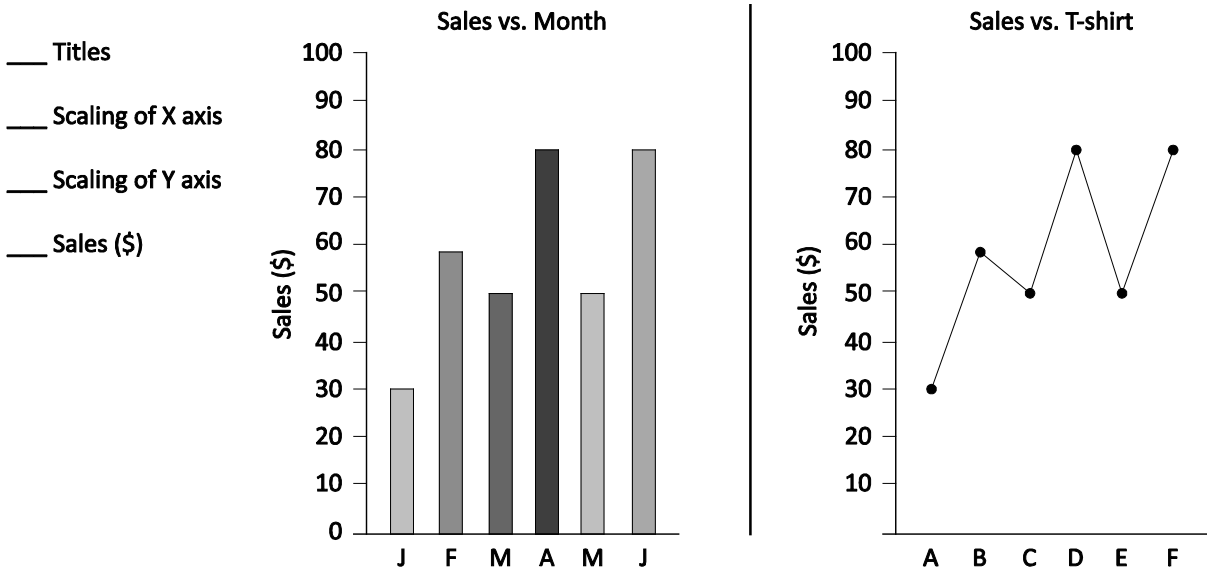- the question(s) asked in the survey
- appropriate display of the data
- valid conclusions based on the data
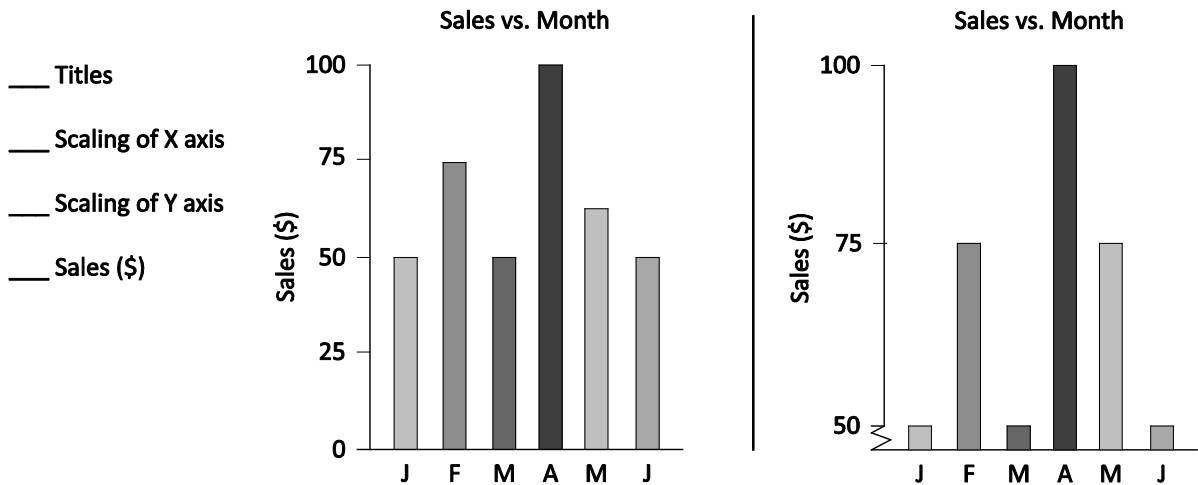
Assessment, Teaching, and Learning

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Answer **True** or **False** for each statement below:
  - The purpose of a graph is to display data in a way that is easier to interpret.
  - All graphs do not require a legend.
  - All graphs must have a vertical and a horizontal scale.
  - All graphs must have a title.
  - A circle graph is always the best way to represent data.
- For the following sets of graphs, indicate which of the necessary components match ✓ or do not match "✗". Then determine if the graphs represent the same data.

____ Titles

____ Scaling of X axis

____ Scaling of Y axis

____ Sales ($)

Sales vs. Month

Sales vs. T-shirt

Do these graphs represent the same data?   YES   NO

____ Titles

____ Scaling of X axis

____ Scaling of Y axis

____ Sales ($)

Do these graphs represent the same data?   YES   NO

WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

The following represents a list of ideas for use in the development of a statistics project. Each can be shaped by students to better reflect their interests.

- Find out how much time is spent per week on each subject area when doing homework. Does this change from grade 7 to grade 9?
- Do students in your school text their friends more or less often than they speak to them in person?
- Find out what type of transportation students in your school use to get to school. Does it differ by grade level? Does it differ with the time of year?
- Find out the most popular types of after-school activities of students in your school. Does it differ by grade level?
- Find out what the favourite cereals of students are in your class or school. Students could also compare adult versus student cereal consumption. Compare this with sales volume at the local supermarket to determine how the class compares to the rest of the community.
- Find out the favourite kind of jeans for people in your age group. Use the result of the survey to write a recommendation to a local store regarding their ordering of the types of jeans. A comparison could also be made for various age groups.
- Ask the student council or community council to suggest issues they would like investigated. Use this as a source for project work.
- Collect data to look for a relationship between average grade on students' last report cards and time spent (1) watching television; (2) time spent on homework, and (3) shoe size.
- Conduct a survey to find out information related to: students' favourite NHL team; students' favourite musical instrument; students' favourite snack food; effects of social media or technology on sleep patterns; bullying; interests; online vs. in-store shopping; etc.
- Survey or interview grade 9 students to determine preferred part-time jobs and the amount of money typically earned. They may wish to include jobs such as babysitting, lawn mowing, and paper routes.
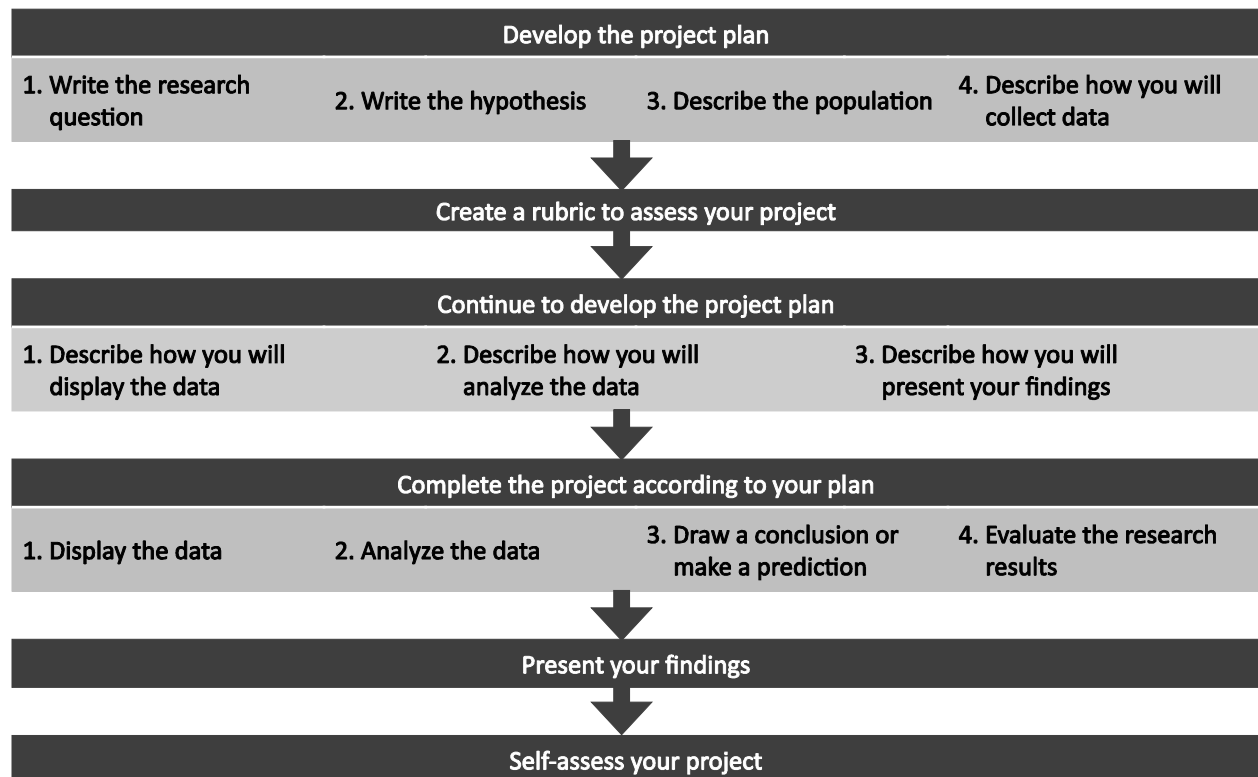
Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Create a plan with the various components of a data management project: the question that will be investigated, methods of data collection, sampling procedures, data collection, data displays, and conclusions.
- Complete statistics projects in pairs or small groups. Students could assess their projects in pairs using the rubric created for this project.
- Consider using technology to create data displays.

SUGGESTED LEARNING TASKS

- Explore and critique rubrics that have been developed so students are exposed to models.
- Ask students to brainstorm possible questions, ideas, and/or issues that could be investigated. They could organize their thoughts with mind maps.
- Ask students to create an organizer, such as the flow chart below, to help organize the research project and carry out the plan.

Develop the project plan			
1. Write the research question	2. Write the hypothesis	3. Describe the population	4. Describe how you will collect data

▼

Create a rubric to assess your project

▼

Continue to develop the project plan		
1. Describe how you will display the data	2. Describe how you will analyze the data	3. Describe how you will present your findings

▼

Complete the project according to your plan			
1. Display the data	2. Analyze the data	3. Draw a conclusion or make a prediction	4. Evaluate the research results

▼

Present your findings

▼

Self-assess your project

SUGGESTED MODELS AND MANIPULATIVES

- spreadsheets

MATHEMATICAL LANGUAGE

Teacher	Student
- population	- population
- rubric	- rubric
- sample	- sample
- sample size	- sample size
- sampling technique	- sampling technique

Resources

Print

- *Math Makes Sense 9* (Baron et al. 2009; NSSBB #: 2001644)
  - Unit 8: Circle Geometry
  - Start Where You Are: How Do I Best Learn Math
  - Unit 9: Probability and Statistics
    - > Technology: Using Census at Schools (Currently, link is not working.)
    - > Technology: Using Spreadsheets and Graphs to Display Data
    - > Section 9.5: Designing a Project Plan
  - *ProGuide* (CD; Word Files; NSSBB #: 2001645)
    - > Assessment Masters
    - > Extra Practice Masters
    - > Unit Tests
  - *ProGuide* (DVD; NSSBB #: 2001645)
    - > Projectable Student Book Pages
    - > Modifiable Line Masters
- *Elementary and Middle School Mathematics: Teaching Developmentally*, 8th Edition (Van de Walle, Karp, and Bay-Williams 2013), pp. 434–448
- *Making Math Meaningful to Canadian Students, K–8* (Small 2008), pp. 525–532

Digital

- "Data Analysis and Probability, [manipulatives]," *National Library of Virtual Manipulatives* (Utah State University 2015): http://nlvm.usu.edu/en/nav/topic_t_5.html

SCO SP04 Students will be expected to demonstrate an understanding of the role of probability in society.				
[C, CN, R, T]				
[C] Communication	[PS] Problem Solving	[CN] Connections	[ME] Mental Mathematics and Estimation
[T] Technology	[V] Visualization	[R] Reasoning	

Performance Indicators

Use the following set of indicators to determine whether students have achieved the corresponding specific curriculum outcome.

SP04.01   Provide an example from print and electronic media where probability is used.
SP04.02   Identify the assumptions associated with a given probability, and explain the limitations of each assumption.
SP04.03   Explain how a single probability can be used to support opposing positions.
SP04.04   Explain, using examples, how decisions may be based on a combination of theoretical probability, experimental probability, and subjective judgment.

Scope and Sequence

Mathematics 8	Mathematics 9	Mathematics 10
SP02 Students will be expected to solve problems involving the probability of independent events.	SP04 Students will be expected to demonstrate an understanding of the role of probability in society.	–

Background

In previous grades students have explored the difference between **theoretical** and **experimental probability**, and how to be able to express probabilities for single and independent events in fractions, percents, and decimals.

In Mathematics 9, the **focus** of study is for students to understand the role that probability plays in society by looking at the probability of events occurring, and by examining decisions that are based on those predictions. Students should be exposed to a variety of examples from daily life in which probability is used. For examples,
- insurance premiums that are established based on the historical data of a certain gender, age group, or region making claims
- warranty periods based on probable lifespan of a product
- number of units manufactured based on probable units that will sell
- projecting of winners of an election from past data
- determining probability of experiencing the side effects of a drug
- airline companies developing a schedule of flights and crew, and setting fares based on the probability of demand at different times of year
- weather forecasts and the probability of precipitation and other weather systems

Students should be focused on familiar situations. Discussions about how predictions are made (a mix of theoretical and experimental probability and subjective judgment) should be a focus. Students will quickly realize that experimental probability is the one most widely used when making predictions. The types of **assumptions** made when making these predictions should also be addressed.

Calculations of probability are based on assumptions. Students should be encouraged to identify and examine assumptions to help them determine whether the calculated probability is meaningful when making a decision.

Students should engage in evaluating situations that lend themselves to reasonably accurate predictions, those that are questionable, and those for which the unknowns are not quantifiable. Injury as a result of road accidents with/without seatbelts is a good example for safe prediction.

Health professionals predicting that people of lower socioeconomic status will have more health problems, is a more questionable situation.

There are many situations where the unknowns are too great to make probabilistic arguments. Attempting to find the probability of someone having the same name, age, and birthdate as yourself, for example, involves too many unknowns to make an accurate prediction. Teachers could discuss, with students:
- What are the reasons for the uncertainty?
- What are the important questions to ask about a situation in order to reduce it to probabilistic form?

Assessment, Teaching, and Learning

Assessment Strategies

ASSESSING PRIOR KNOWLEDGE

Tasks such as the following could be used to determine students' prior knowledge.

- Jamila and Brooke are playing a game in which Jamila rolls a pair of dice and finds the sum of the numbers rolled. If the sum is six or eight, then Brooke gets three points. If the sum is not six or eight, then Jamila receives one point. Is this game fair? Give reasons for your answer.
- Have students compare the theoretical and experimental probability for spinning pink on the spinner below and rolling a composite number on a dice.
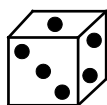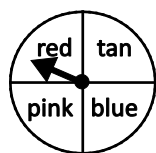
WHOLE-CLASS/GROUP/INDIVIDUAL ASSESSMENT TASKS

Consider the following **sample tasks** (that can be adapted) for either assessment for learning (formative) or assessment of learning (summative).

- Access "Climate" (Government of Canada 2015) http://climate.weather.gc.ca/data_index_e.html and search for data about your hometown to make predictions for the current month (precipitation amounts, mean temperature, etc.). Discuss any assumptions you may have had when making these predictions and explain the limitations of these assumptions.

- For your school determine the probable number of students who will go on to post-secondary education next year. Think about various ways to determine this probability. For example, use school data such as a request for transcripts from previous years, or approach the Awards Committee to see how many scholarships were obtained.

- Wayne thinks that a good way to model the performance of a baseball player who gets a hit 1 time in 4 at bats is to use a spinner with 4 sections. What assumptions is he making? Are his assumptions valid?

Planning for Instruction

CHOOSING INSTRUCTIONAL STRATEGIES

Consider the following strategies when planning daily lessons.

- Give students the opportunity to explore the media to find examples of predictions based on probability in everyday life.

- Give students the opportunity to explore decision-making based on probability. They should use a sample to determine the probability of an event, use the results and subjective judgment to make predictions and explain the reasonableness of the predictions, based on any assumptions that they made. If at all possible, the reasonableness of the predictions should be tested.

- As a class, look for examples in the media where probability is used to support or reject a position.

SUGGESTED LEARNING TASKS

- Use the sampling from outcome SP03 to make predictions about the general population by
  - identifying the assumptions made and the limitations of these assumptions
  - discussing how much reliance there was on theoretical probability, experimental probability, and subjective judgment to make that prediction
- Ask students to provide a report on examples of where probability is used in print and electronic media.
- Ask students to think of a TV game show where players consider probabilities when deciding how to proceed, and then explain the extent to which probability is involved.
- Ask students to look through print media and the Internet to find examples of cases such as the following:
  - a situation where decisions affecting your community were made that might have been based on probabilities
  - a situation where a medical organization might make a decision based on probabilities
  Ask them to describe how probabilities were involved.
- Ask students to find an article that includes probabilities and discuss the possible opposing viewpoints.
- Provide pairs of students with 2 cards containing a misleading probability statement. Ask their partner to explain the limitations of the statement.

1. I've tossed a fair quarter 3 times and got heads. It is more likely to be tails than heads if I toss it again.	**2.** The Rovers Team play against the Shooters Team. The Rovers can win, lose, or draw, so the probability that the Rovers will win is $\frac{1}{3}$.
3. There are 3 red beads and 5 blue beads in a bag. I pick a bead at random. The probability that it is red is $\frac{3}{5}$	**4.** I roll 2 dice and add the results. The probability of getting a total of 6 is $\frac{1}{12}$ because there are 12 different possibilities and 6 is one of them.
5. It is less likely to throw a 6 than a 3 with a dice.	**6.** Tomorrow it will either rain or it will not rain, so the probability that it will rain is 0.5.
7. Mr. Brown has to have a major operation. 90% of the people who have this operation make a complete recovery. There is a 90% chance that Mr. Brown will make a complete recovery if he has this operation.	**8.** If 6 fair dice are thrown at the same time, I am less likely to get 1,1,1,1,1,1 than 1,2,3,4,5,6.

- Students could be asked to give a written or oral report on scenarios such as the following:
  - Jolene's mother has an important presentation to make in the morning at a conference 200 km away. She has an evening meeting at work tonight. The weather network has reported a 50% probability of snow in the morning. The company she works for would pay for her hotel. What are the probabilities Jolene's mother has to consider when deciding whether to make the drive tonight or in the morning? Which probability do you think would have the most impact on her decision? Explain.
  - What probabilities might a government consider when deciding whether to turn a two-lane highway into a four-lane highway?
- Many insurance companies charge drivers under the age of 25 higher insurance premiums based on the probability of accidents. Ask students to find an article about car insurance costs based on the probability of collision and answer the following questions.
  - In the article, what are the assumptions associated with each probability? Explain.
  - In your opinion, is there a bias against young drivers?
  - "Discussions about car insurance costs are based on a combination of experimental probability, theoretical probability, and subjective judgement." Do you agree or disagree with this statement? Explain.
- Odette states that she has a 1 in 2 chance of getting heads when she flips a coin. Claude states that a particular coin is unfair based on the fact that, when flipped 50 times, it came up heads 40 of the 50 times. Ingrid feels that even if there is an equal chance of getting heads, heads will appear more often because she feels it is her lucky choice. Ask students to categorize whether or not the three individual's decisions are based on subjective, experimental, or theoretical probabilities, and describe how each can play a part in decision making.

SUGGESTED MODELS AND MANIPULATIVES

- dice
- spinners

MATHEMATICAL LANGUAGE

Teacher	Student
▪ assumptions	▪ assumptions
▪ experimental probability	▪ experimental probability
▪ fair	▪ fair
▪ judgment	▪ judgment
▪ subjective	▪ subjective
▪ theoretical probability	▪ theoretical probability

Resources

Digital

- ▪ "Climate," *Government of Canada* (Government of Canada 2015):
  http://climate.weather.gc.ca/data_index_e.html

Print

- ▪ *Math Makes Sense 9* (Baron et al. 2009; NSSBB #: 2001644)
  - – Unit 9: Probability and Statistics
    - > Section 9.1: Probability in Society
  - – *ProGuide* (CD; Word Files; NSSBB #: 2001645)
    - > Assessment Masters
    - > Extra Practice Masters
    - > Unit Tests
  - – *ProGuide* (DVD; NSSBB #: 2001645)
    - > Projectable Student Book Pages
    - > Modifiable Line Masters
- ▪ *Elementary and Middle School Mathematics: Teaching Developmentally*, 8th Edition (Van de Walle, Karp, and Bay-Williams 2013), pp. 454–469
- ▪ *Making Math Meaningful to Canadian Students, K–8* (Small 2008), pp. 548–560